# Robust MUI Suppression for MIMO Visible Light Communication System With Location-Aided Chaotically Rotating Orthogonal Scheme

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***Abstract*— In practical multiple-input multiple-output (MIMO) visible light communication (VLC) systems, the imperfect channel state information brings the issue that the precoding scheme may not well suppress multi-user interferences (MUIs). In this letter, with the objective of suppressing MUIs more effectively, we utilize the location information of users obtained from positioning systems, and present a real domain location-based chaotically rotating orthogonal scheme. In our design, we first develop a range-variable chaotic mapping method and embed users' location information into the chaotic sequences. Then, we exploit the chaotic rotation to construct orthogonal matrices with the aim to orthogonalize transmitted signals. At the receiver, with the aid of the known location information and the chaotically rotating orthogonalization structure, the information can be retrieved from received signals, hence achieving robust MUI suppressions. Simulations are performed and the results demonstrate that our proposed scheme can effectively combat the MUI and improve the reliability performances for VLC systems.**

***Index Terms*— Range-variable chaotic mapping method, location information, multiple-input multiple-output (MIMO) based visible light communication (VLC) system, multi-user interference (MUI), reliability.**

## I. Introduction

By EXPLOITING spatial multiplexing, multiple-input multiple-output (MIMO)-based visible light communication (VLC) systems can provide high data rate services for multiple users [1]. Although the line of sight (LOS) transmissions over visible light bands may mitigate multiple user interferences (MUIs) from users blocked by walls or obstacles, the MUIs induced by imperfect channel conditions and broadcasting transmissions still degrade performances.

In order to suppress the MUIs of MIMO-based VLC systems, many linear precoding schemes, which are originally proposed for radio frequency (RF) systems, have been applied [2]–[7]. However, different from RF systems, in VLC systems, the light emitting diodes (LEDs) require signals to be real, non-negative and constrained by both peak and average power. Then in [2], the authors take the power constraint into account, and propose a linear zero-forcing precoding technique for multi-user (MU) VLC systems. In addition, [3] sets different field of view (FOV) for PDs and proposed a block diagonalization (BD) precoding scheme. Then [4] considers the non-negativity constraint on VLC input signals and improves the BD precoding in MU-MIMO-based VLC systems, and Zeng and Du [5] further combine BD precoding with the water-filling algorithm to entirely eliminate the MUI. Moreover, the researchers study the MUI suppression for VLC systems utilizing the optical orthogonal frequency division multiplexing (OOFDM) in [6], and propose to exploit the optical spatial modulation and the spatial pulse position modulation combined with BD precoding to mitigate the MUI. However, these precoding schemes have two major drawbacks. One is that perfect channel state information (CSI) is required to be known at the transmitter. Another is that it is required that the number of transmitting antennas is no smaller than the total number of all users' receiving antennas.

In this letter, we propose to exploit the location information obtained from positioning systems [7], the outstanding initial value sensitivity and ergodicity of chaotic sequences, and the orthogonalization of VLC signals to suppress the MUIs in MIMO-VLC systems. Specifically, we firstly present a range-variable chaotic mapping method based on the Logistic map, and then the users' location information is used to generate chaotically-rotating (CR) sequences. Thanks to the chaotic sequences being ergodic and sensitive to the initial value, we subsequently rotate identity matrices using CR sequences to obtain orthogonal row vectors. By multiplying with orthogonal vectors, the signals for different users are orthogonalized. At the receiver, with the known location information and the chaotically-rotating orthogonalization structure, the information can be reliably retrieved with suppressed MUI.

Briefly, the main contributions include: 1) we exploit users' location information obtained from positioning systems to suppress MUI in an uncoordinated way, and no exact CSI or additional dedicated control channel is required at the transmitter; 2) we present a range-variable chaotic mapping method to construct rotation matrices to orthogonalize the signals; 3) the number of total receiving antennas is not required to be no larger than that of the transmitting antennas as required by conventional MUI suppressing schemes such as linear precoding methods.

The letter is organized as follows. Section II briefly introduces the MU-MIMO-based VLC system model. Then in Section III, we present the chaotically-rotating orthogonalization with the location information (CRO-LI) design in detail. Simulation results are provided and analyzed in Section IV. Finally, Section V concludes our findings.

## II. The MU-MIMO-Based VLC System Model

We consider a general MU-MIMO-VLC system, where $N$ LED arrays are used to illuminate and simultaneously transmit $N$ independent data streams to $K$ users and $N$ photodiodes (PDs) are employed at the $k$-th ($k = 1, 2, \cdots, K$) user's receiver for optical-to-electrical signal conversion. As an

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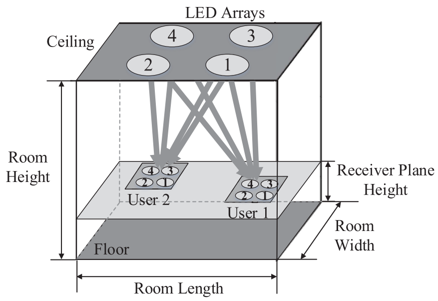


Fig. 1. A 4×(4,4) 2-user MIMO-based VLC system model.

example, Fig. 1 shows a MU-MIMO-VLC configuration of $N = 4$ and $K = 2$. At the transmitter, the input data is firstly transformed to 4 parallel data streams. After the pulse amplitude modulation (PAM), the intensity modulation and the direct detection (IM/DD) are carried out, then the resultant modulated symbols are transmitted over the VLC channels.

At the $k$-th user's receiver, after the PAM demodulation, the bandpass filter (BPF) bank can be used to separate and collect the symbols from different lamps, thus the different time delay induced by the time difference between the lamps can be removed for received signals. The received signal is obtained as

$$\mathbf{Y}_k = \mathbf{H}_k\mathbf{X} + \mathbf{Z}_k \tag{1}$$

where $\mathbf{Y}_k = [y_{k_1}, y_{k_2}, \cdots, y_{k_N}]^T$ is the $N\times1$ received signal vector across the $N$ PDs of the $k$-th user, $\mathbf{X}$ is the transmitted signal vector emitted from $N$ LEDs, $\mathbf{Z}_k$ is the $N\times1$ zero-mean additive white Gaussian noise (AWGN) vector, and $\mathbf{H}_k$ is the $N\times N$ channel gain matrix of the $k$-th user, where the element in the $i$-th row and the $j$-th column ($i, j = 1, 2, \cdots, N$) is the summation of LOS channel gain coefficients and non-LOS (NLOS) channel gain coefficients between the $j$-th transmitter and the $i$-th receiver. For example, for LOS channel, the direct current (DC) channel gain is calculated by [8]

$$h(0)_{k_{ij}} = \frac{A_{PD}(m+1)}{2\pi d_{k_{ij}}^2}\cos^m(\phi_{k_{ij}})\cos(\psi_{k_{ij}})g(\psi_{k_{ij}}) \tag{2}$$

where $A_{PD}$ is the physical area of one PD, $d_{k_{ij}}$ is the distance between the $j$-th transmitter and the $k$-th user's $i$-th receiver, $\phi_{k_{ij}}$ is the angle of irradiance with respect to the normal axis to the transmitting plane, $\psi_{k_{ij}}$ is the incidence angle with respect to the normal axis to the receiver plane, $g(\psi_{k_{ij}})$ is the gain of the optical concentrator and $m$ is the order of Lambertian emission with the half irradiance. For NLOS channel, the DC channel gain of the first reflection path is given by [9]

$$h(0)_{Ref_{k_{ij}}} = \frac{(m+1)\rho\cos^m(\theta_{k_{ij}})d_{k_{ij}}^2}{2\pi {d_{in}}_{k_{ij}}^2 {d_{ref}}_{k_{ij}}^2}h(0)_{k_{ij}} \tag{3}$$

where ${d_{in}}_{k_{ij}}$ is the incidence distance, ${d_{ref}}_{k_{ij}}$ is the reflection distance, $\rho$ is the reflection factor and $\theta$ is reflection angle.

## III. Uncoordinated Real-Domain Location Aided MUI Suppressing Scheme

In the CRO-LI MUI suppressing system, an interference elimination module denoted by $\mathbf{F}_k$ is added at the transmitter.

At the CRO-LI transmitter, the parameter representing the $k$-th user's location information is used as the initial value to generate a chaotic sequence. Subsequently, we round down the values of the chaotic sequence to constitute a CR sequence to rotate the rows of an $N \times N$ identity matrix, and the resultant CRO-LI matrix contains with only one '1' element in each row and '0' elements in the other positions. Then the transmitter constructs an orthogonal matrix denoted by $\mathbf{F}$, which satisfies

$$\mathbf{F}\mathbf{F}^T = \mathbf{I}_N, \mathbf{F}_k^T\mathbf{F}_l = 0 \tag{4}$$

where $\mathbf{I}_N$ represents an $N \times N$ identity matrix, $l = 1, 2, \cdots, K$, $l \neq k$ and $(\cdot)^T$ represents the transpose operator.

Then the transmitted signal from $K$ users is expressed as

$$\mathbf{X} = \mathbf{F}_1\mathbf{X}_1 + \cdots + \mathbf{F}_k\mathbf{X}_k + \cdots + \mathbf{F}_K\mathbf{X}_K \tag{5}$$

where $\mathbf{X}_k = [x_{k1}, x_{k2}, \cdots, x_{kN}]^T$ consists of the components to be transmitted via $N$ antennas.

At the $k$-th user's receiver, the users can easily self-recover the transmitted data with their own location information. The received signal can be obtained by substituting (5) into (1) as $\mathbf{Y}_k = \mathbf{H}_k(\mathbf{F}_1\mathbf{X}_1 + \cdots + \mathbf{F}_k\mathbf{X}_k + \cdots + \mathbf{F}_K\mathbf{X}_K) + \mathbf{Z}_k$. Then with the known $\mathbf{F}_k$ and $\mathbf{H}_k$, the estimated data at the $k$-th user's receiver can be expressed as

$$\begin{aligned}\hat{\mathbf{X}}_k &= \mathbf{F}_k^T\mathbf{H}_k^{-1}\mathbf{Y}_k \\ &= \mathbf{F}_k^T(\mathbf{F}_1\mathbf{X}_1 + \cdots + \mathbf{F}_k\mathbf{X}_k + \cdots + \mathbf{F}_K\mathbf{X}_K) + \mathbf{F}_k^T\mathbf{H}_k^{-1}\mathbf{Z}_k \\ &= \mathbf{X}_k + \mathbf{F}_k^T\mathbf{H}_k^{-1}\mathbf{Z}_k.\end{aligned} \tag{6}$$

According to (4), we can see from (6) that when $l \neq k$, we will have $\hat{\mathbf{X}}_l = \mathbf{F}_l^T\mathbf{H}_l^{-1}\mathbf{Y}_k = 0$.

It is worth pointing out that thanks to the orthogonality of $\mathbf{F}_k$, namely only one '1' element in each column and '0' elements in other positions, the multiplication of $\mathbf{F}_k^T$ in (6) will not amplify the mean and variance of the noise. Thus, the influence on the noise is negligible.

### A. The Range-Variable Chaotic Mapping Method

As mentioned above, we use the chaotic sequence to determine the circular shifting steps to rotate the $N \times N$ identity matrix, the values of elements should vary from 0 to $N$. Thus the traditional Logistic map [10], which generates chaotic sequences in the range of $(0, 1)$ can not be directly applied. We here propose a range-variable chaotic mapping method to generate the chaotic sequence as

$$c(p+1) = N\mu\frac{c(p)}{N}\big[1 - \frac{c(p)}{N}\big] = \mu c(p)\big[1 - \frac{c(p)}{N}\big] \tag{7}$$

where $c(p)$ represents the chaotic chip satisfying $0 < c(p) < N$, $p = 0, 1, 2\cdots$ and $\mu$ is the chaotic sequence control parameter satisfying $3.5699 < \mu \leq 4$.

The proposed chaotic sequence has outstanding initial value sensitivity and ergodicity. For example, when $N = 4$, $\mu = 3.99$ and there is a slight difference of $10^{-3}$ between two initial values, the obtained two chaotic sequences will diverge after 8 iterations. In order to evaluate the ergodicity, we calculate the statistical average and the time average of the proposed chaotic sequence. Using $E[\mathbf{C}(t)] = \sum_{m=1}^{M} c_m(t)P\{\mathbf{C}(t) = c_m(t)\}$ and setting $M = 500$, $t = 5000$, we calculate the statistical average for 500 range-variable chaotic sequences with randomly chosen initial values. Then we apply $\overline{\mathbf{C}} = \frac{\sum_{i=1}^{R} c(i)}{R}$ and set $R = 10000$

to evaluate the time average for a range-variable chaotic sequence with 10000 elements. The results are obtained as $E[\mathbf{C}(5000)] = 2.1246$ and $\overline{\mathbf{C}} = 2.1207$, which means that the statistical average is approximately equal to the time average. Thus good ergodicity can be achieved by the rang-variable chaotic sequences.

### *B. The Initial Value Matrix With Location Information Embedded*

To embed the users' location information within the chaotic sequences, we firstly utilize the $k$-th user's distance information to constitute the $k$-th location information matrix as

$$\mathbf{D}_k = \begin{bmatrix} d_{k_{11}} & d_{k_{12}} & \cdots & d_{k_{1N}} \\ d_{k_{21}} & d_{k_{22}} & \cdots & d_{k_{2N}} \\ \vdots & \vdots & \ddots & \vdots \\ d_{k_{N1}} & d_{k_{N2}} & \cdots & d_{k_{NN}} \end{bmatrix}. \tag{8}$$

Then we transform the elements in $\mathbf{D}_k$ to the values in the interval (0, $N$) and construct an initial value matrix $\mathbf{L}_k$ as

$$\begin{aligned} \mathbf{L}_k &= \mathbf{D}_k \oslash \left(\frac{d_{k_{\max}} + d_{k_{\min}}}{N}\right) \\ &= \begin{bmatrix} \frac{Nd_{k_{11}}}{d_{\max}+d_{\min}} & \frac{Nd_{k_{12}}}{d_{\max}+d_{\min}} & \cdots & \frac{Nd_{k_{1N}}}{d_{\max}+d_{\min}} \\ \frac{Nd_{k_{21}}}{d_{\max}+d_{\min}} & \frac{Nd_{k_{22}}}{d_{\max}+d_{\min}} & \cdots & \frac{Nd_{k_{2N}}}{d_{\max}+d_{\min}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{Nd_{k_{N1}}}{d_{\max}+d_{\min}} & \frac{Nd_{k_{N2}}}{d_{\max}+d_{\min}} & \cdots & \frac{Nd_{k_{NN}}}{d_{\max}+d_{\min}} \end{bmatrix} \end{aligned} \tag{9}$$

where $\oslash$ is the element-by-element operator, $d_{k_{\max}}$ and $d_{k_{\min}}$ represent the maximum and minimum elements in $\mathbf{D}_k$, i.e. $d_{k_{\max}} = \max\{d_{k_{11}}, d_{k_{12}}, \cdots, d_{k_{NN}}\}$ and $d_{k_{\min}} = \min\{d_{k_{11}}, d_{k_{12}}, \cdots, d_{k_{NN}}\}$, where $\max\{\cdots\}$ denotes the maximum operator and $\min\{\cdots\}$ represents the minimum operator. Then the initial value to generate the chaotic sequence is randomly chosen from the elements in $\mathbf{L}_k$.

Notably, thanks to the high precision positioning provided by [7] and [11], the location information error is neglected in our design. More explicitly, in [11], the user can be exacted localized in an area of 0.0036m$^2$, while in our system, the position resolution is 0.01m$^2$, hence exact $\mathbf{D}_k$ and $\mathbf{L}_k$ without location errors can be constructed. Moreover, we here assume that PDs are placed horizontally. In practical scenarios where PDs randomly placed, accelerometers equipped in positioning systems can identify their orientations and then estimate user's location information [11].

### *C. The Chaotically-Rotating Orthogonal Matrix*

There are three reasons why we propose to rotate identity matrices by the chaotic sequences. Firstly, rotated identity matrices will not change the non-negative property of the transmitted signals to meet the requirements of non-negative inputs for LEDs. Secondly, by exploiting the non-periodic property of chaotic sequences, we could construct different CRO-LI matrices for different users. Thirdly, implementing chaotic rotation on an identity matrix is equivalent to the elementary transformation of the matrix.

Specifically, using Eq. (7) wherein a randomly chosen element in $\mathbf{L}_k$ as the initial value, a chaotic sequence with $N$ values in the range (0, $N$) can be generated, which is denoted by $\mathbf{C}_k$. Then, using the floor function, we round down the values in $\mathbf{C}_k$ as $\mathbf{S}_k = [s_k(1), s_k(2), \cdots, s_k(N)]^T = \left[\lfloor c_k(1)\rfloor, \lfloor c_k(2)\rfloor, \cdots, \lfloor c_k(N)\rfloor\right]^T$, where $\lfloor\cdot\rfloor$ represents the floor operator.

Notably, each element of $\mathbf{S}_k$ determines the circular shifting step length. For example, when $\mu = 3.99$, $c_k(0) = 2.13$ and $N = 4$, using (7), we obtain $\mathbf{C}_k = [3.97, 0.12, 0.46, 1.62]^T$. Then a CR sequence can be calculated as $\mathbf{S}_k = [3, 0, 0, 1]^T$. For a $4 \times 4$ identity matrix, since $s_k(1) = 3$, all the four elements in the first row need to shift 3 step lengths, i.e., $[1, 0, 0, 0]$ becomes $[0, 0, 0, 1]$. The second row and third row remain unshifted since $s_k(2) = 0$ and $s_k(3) = 0$. For $s_k(4) = 1$, we get $[1, 0, 0, 0]$. Thus after rotating all the 4 rows in the $4 \times 4$ identity matrix, the CRO-LI matrix can be obtained as

$$\mathbf{R}_k = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}. \tag{10}$$

Then we extract the last $t = \frac{N}{K}$ rows in $\mathbf{R}_k$, transpose the rows to columns and construct matrix $\mathbf{F}_k$. Subsequently, we unite $\mathbf{F}_k$ with the transposed vectors from the other users' CRO-LI matrices to construct an $N \times N$ matrix $\mathbf{F}$ given by $\mathbf{F}^T = [\mathbf{F}_1 \mathbf{F}_2 \cdots \mathbf{F}_K]$. The transmitter will examine if $\mathbf{F}$ is an orthogonal matrix, namely if (4) holds. If the vectors in matrix $\mathbf{F}$ are not mutually orthogonal, the transmitter will choose another element in the matrix $\mathbf{L}_k$ to generate another chaotic sequence. Then similar process is carried out to construct $\mathbf{F}$ till it becomes orthogonal. Besides, the initial value selected is transmitted to the users in an uncoordinated way [12], thus the same $\mathbf{F}$ can be used at transceivers.

Therefore, with the orthogonal matrix $\mathbf{F}$, the $k$-th user will recover the received signal by exploiting (6), while the data for the other users will turn zeros thank to the orthogonality. In this way, the MUIs are mitigated and the reliability and practicability are improved. Furthermore, it is worth mentioning that the computation complexity of our proposed CRO-LI MUI suppressing scheme is at an order of $O(n)$, which is not time-consuming.

## IV. Simulation Results

In the simulations, we assume that the 4×(4,4) MIMO-VLC system serves 2 users and the 6×(6,6,6) MIMO-VLC system serves 3 users. In the 4×(4,4) system, 4 LED arrays are used as transmit antennas, and each user is equipped with 4 PDs. We adopt a $5m \times 5m \times 3m$ indoor room model. One LED array is equipped with 3600 LEDs and the transmit power of each LED is 20mW. The coordinates of the LED arrays of 4×(4,4) MIMO system are (1.3, 1.3, 2.5), (1.3, 3.7, 2.5), (3.7, 1.3, 2.5) and (3.7, 3.7, 2.5) and the coordinates of the LED arrays of 6×(6,6,6) MIMO system are (1.3, 2, 2.5), (1.3, 2.5, 2.5), (1.3, 3, 2.5), (3.7, 2, 2.5), (3.7, 2.5, 2.5) and (3.7, 3, 2.5), where all numbers have the unit of meter. At the PDs, the semi-angle is 70° and the field-of-view (FOV) is 90°. The PD's responsivity is 0.4 with an active area of 1cm$^2$. The data

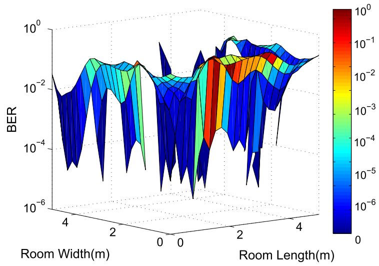


Fig. 2. BER performance of 4×(4,4) 2-user MIMO-based VLC system with CRO-LI scheme where user 1 is located at (1.8, 2.6, 0.8).

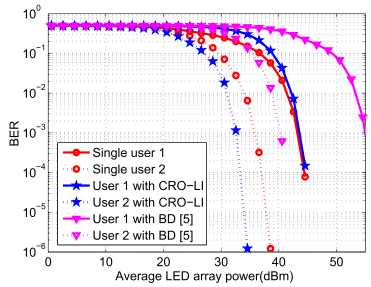


Fig. 3. BER performance comparison among single-user system, CRO-LI aided 2-user system and BD precoding [5] aided 2-user system.

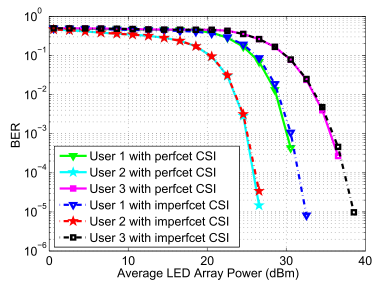


Fig. 4. BER performance comparison between CRO-LI aided 3-user systems with perfect and imperfect CSI.

rate is 10 Mb/s. The background current is 5.84 mA. The shot noise bandwidth factor is 0.562 and the thermal noise bandwidth factor is 0.086. The bandwidth is 30 MHz. The modulation scheme is PAM. In addition, we assume light is only provided by the LED arrays.

Firstly, Fig. 2 illustrates the bit error rate (BER) performances of CRO-LI aided VLC systems. It can be seen that when user 1's location is fixed at (1.8, 2.6, 0.8) and user 2 is mobile at the receiver plane, user 2's BER performances are closely dependent on the signal-to-noise ratio (SNR), i.e., when user 2 is located at the room margin, worse BER performances are provided than that located at the room center.

Then we compare BER performances among single-user 4×4 MIMO-VLC system, CRO-LI aided 4×(4,4) 2-user system and BD precoding [5] aided 8×(4,4) 2-user system, where user 1 is located at (0.8, 3.1, 0.8) and user 2 is located at (1.0, 2.0, 0.8) in Fig. 3. It can be observed that the BER performances of CRO-LI aided system are better than those of BD precoding aided 8×(4,4) system, although the latter one is equipped with more antennas at the transmitter. The reason is the orthogonalization of users' data can effectively eliminate MUI and does not have the problem of noise amplification.

Subsequently, Fig. 4 compares BER performances between CRO-LI aided 6×(6,6,6) 3-user MIMO-VLC systems with perfect and imperfect CSI, where user 1 is located at (1.6, 3.8, 0.8), user 2 is located at (1.6, 2.8, 0.8) and user 3 is located at (1.6, 0.7, 0.8). Let $\Delta\mathbf{H}_k$ denote the channel estimation error, which follows the Gaussian distribution with zero mean and variance of $10^{-15}$ [13]. It can be observed that only slight performance degradations occurs with imperfect CSI, which demonstrates the robustness of the CRO-LI design.

## V. Conclusion

In this letter, we propose to use the location information as the initial value to generate range-variable chaotic sequences, which are then used for rotating orthogonalization in the real domain to construct interference elimination modules to suppress MUIs for MIMO-based VLC systems. With our design, the users can recover the data with their own location information. Simulation results demonstrate that CRO-LI aided MU-MIMO-VLC systems can achieve better BER performances than those in the BD precoding [5] aided system. Additional benefits of our design include no channel estimates are required, and the transmit antenna number is not required to be smaller than the receiving antenna number. Hence the presented scheme can be generally applied in practical MIMO-based VLC systems for robust MUI suppression.

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