

Impact of Non-Orthogonal Multiple Access on the Offloading of Mobile Edge Computing

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Abstract—This paper considers the co-existence of two important communication techniques, non-orthogonal multiple access (NOMA) and mobile edge computing (MEC). Both NOMA uplink and downlink transmissions are applied to MEC, and analytical results are developed to demonstrate that the use of NOMA can efficiently reduce the latency and energy consumption of MEC offloading. In addition, various asymptotic studies are carried out to reveal the impact of the users' channel conditions and transmit powers on the application of NOMA to MEC are quite different from those in conventional NOMA scenarios. Computer simulation results are also provided to facilitate the performance evaluation of NOMA-MEC and also verify the accuracy of the developed analytical results.

Index Terms—Non-orthogonal multiple access (NOMA), mobile edge computing (MEC), offloading delay, energy consumption.

I. INTRODUCTION

NON-ORTHOGONAL multiple access (NOMA) is viewed as a key enabling technology in next-generation wireless networks due to its superior spectral efficiency [1]. On the one hand, the principles of NOMA bring fundamental changes to the design of future multiple access techniques [2], [3]. In particular, compared to conventional orthogonal multiple access (OMA) which allocates orthogonal bandwidth resource blocks to users, NOMA encourages the users to share the same spectrum, where multiple access interference is handled by applying advanced transceiver designs, such as superposition coding and successive interference cancellation (SIC). Hence compared to OMA, NOMA offers better flexibility for efficiently utilizing the scarce bandwidth resources.

On the other hand, NOMA has also been shown to be important to the evolution of many other types of

communication techniques. For example, the spectral efficiency of multiple-input multiple-output (MIMO) systems can be significantly improved by designing sophisticated MIMO-NOMA transmission schemes and harvesting the spatial degrees of freedom in a more efficient way compared to MIMO-OMA [4]–[6]. Another example is the application of NOMA to millimeter-wave (mmWave) communication systems, and the existing studies show that the directional transmission feature of mmWave propagation is ideal for the application of NOMA, where users with strongly correlated channels are grouped together for the implementation of NOMA [7]. Wireless caching is one of the latest examples for the applications of NOMA to other communication techniques, where NOMA assisted content pushing and delivery schemes have been developed to improve the cache hit probability and ensure that the files stored in the local caches are frequently updated during on-peak hours [8]. It is worth pointing out that NOMA and OMA are compatible, and many practical forms of NOMA, including sparse code multiple access (SCMA) and pattern division multiple access (PDMA), are able to take advantage of both OMA and NOMA [2], [9]–[12].

This paper focuses on the coexistence of NOMA and mobile-edge computing (MEC) which is another important communication technique for future wireless networks [13], [14]. The use of MEC is motivated by the fact that emerging mobile applications, such as virtual reality, augmented reality, and interactive gaming, make mobile networks computationally constrained. Take virtual reality as an example. Mobile nodes need to carry out object recognition, pose estimation, vision-based tracking, etc. Furthermore, for virtual reality assisted gaming, the mobile devices are also expected to facilitate mixed reality and human computer interaction [15]. However, most mobile devices are computation and power limited, which means that relying on the mobile devices to locally complete the computationally intensive tasks will result in two disadvantages. One is that the batteries of the devices can be drained quickly, and the other is that the devices might not be able to complete the tasks before their deadlines. The key idea of MEC is to employ more resourceful computing facilities at the edge of mobile networks, such as access points and small-cell base stations integrated with MEC servers, and ask the mobile users to offload their computationally intensive tasks to the MEC facilities. In order to improve the energy-latency tradeoff of MEC, a dynamic computation offloading scheme has been developed under the assumption that mobile devices can carry out energy harvesting by using renewable energy sources [16]. In [17] and [18] a similar MEC scenario

Manuscript received April 17, 2018; revised July 11, 2018 and August 28, 2018; accepted September 6, 2018. Date of publication September 18, 2018; date of current version January 15, 2019. The work of Z. Ding was supported by the UK EPSRC under grant number EP/P009719/2 and H2020-MSCA-RISE-2015 under grant number 690750. The work of P. Fan was supported by the National Natural Science Foundation of China (NSFC) under grant No.61731017, and the 111 Project under grant No.111-2-14. The work of H. V. Poor was supported by the U.S. National Science Foundation under Grant CNS-1702808. The associate editor coordinating the review of this paper and approving it for publication was W. Chen. (Corresponding author: Zhiguo Ding.)

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Digital Object Identifier 10.1109/TCOMM.2018.2870894

was considered, in which simultaneous wireless information and power transfer was applied to MEC to facilitate user cooperation. In [19], a user scheduling scheme was proposed to MEC in order to achieve a balanced tradeoff between latency and reliability of task offloading. In [20], a more challenging multi-user MEC scenario was considered, where the users offload their tasks to the MEC server in an asynchronous manner.

Initial studies in [21] and [22] have already demonstrated the benefits of applying NOMA to MEC, by developing various optimization frameworks. However, there is still a lack of theoretical performance analysis for a better understanding of the impact of NOMA on MEC, which is the motivation of this paper. The contributions of this paper can be summarized as follows:

- The application of NOMA uplink transmission to MEC is considered, where the impact of NOMA on the latency of MEC is considered first. When there are multiple users and a single MEC server, the use of NOMA can ensure that multiple users complete their offloading at the same time, which effectively reduces the offloading latency. The probability that a strong user completes its offloading by using the time that would be solely occupied by a weak user in the OMA mode is characterized first and then used to identify the impact of the users' channel conditions and transmit powers on the offloading latency. Asymptotic studies reveal that, in the low signal-to-noise ratio (SNR) regime, it is almost sure that the use of NOMA will guarantee a superior latency performance, i.e., the strong user does not need extra time, but just uses the time allocated to the weak user for offloading. However, this conclusion is not valid in the high SNR regime, as shown by the asymptotic studies. These observations are quite different from conventional NOMA scenarios, where the benefit of using NOMA is more pronounced in the high SNR regime.
- From the energy perspective, NOMA-MEC is not energy efficient if a strong user is forced to complete its offloading by using only the time that would be solely occupied by a weak user in the OMA mode. A more energy efficient offloading approach is to ask the strong user to first offload parts of its task while the weak user is offloading, and then offload its remaining data to the server by using a dedicated time slot. Our developed analytical results show that this modified NOMA assisted MEC protocol can offload more data than OMA, while using less energy. This conclusion is surprising since it is commonly believed that more energy is needed for many NOMA transmission schemes compared to their OMA counterparts, in order to combat strong multiple access interference.
- The application of NOMA downlink transmission to MEC is also considered, where a user uses NOMA to offload its multiple tasks to multiple MEC servers simultaneously. For NOMA uplink transmission, admitting an additional node into the system will not bring any performance degradation to the existing nodes, as long as the newcomer's signal is decoded correctly at the first step

of SIC. However, this is not valid to NOMA downlink transmission, which motivates the use of a cognitive radio inspired power allocation policy. The analytical results are developed to demonstrate that NOMA-MEC with cognitive radio power allocation can simultaneously reduce the energy consumption for offloading and also increase the amount of data offloaded to the servers, particularly in the high SNR regime. In addition, asymptotic studies show that, for the application of NOMA downlink transmission to MEC, it is important to group servers with strong channel conditions, in order to realize the performance gain of NOMA-MEC over OMA-MEC, whereas, for the application of NOMA uplink transmission to MEC, it is preferable to schedule users with diverse channel conditions, i.e., a user with poor channel conditions is paired with a user with strong channel conditions.

II. SYSTEM MODEL

Consider a general MEC communication scenario with M users and K access points having integrated MEC servers. Each node is assumed to have a single antenna, and to operate in half duplex mode. Each user needs to complete computationally intensive latency-critical tasks. Because of the users' limited computation capabilities, carrying out those tasks locally can consume a significant amount of time and energy, which is the motivation for the use of MEC. In order to clearly illustrate the impact of NOMA on MEC, the following assumption is made in this paper:

Assumption 1: The users always prefer to offload their tasks to the MEC servers.

With this assumption, the cost of using OMA-MEC for offloading will be compared to that of using NOMA-MEC in this paper, so the performance gain of NOMA over OMA can be clearly demonstrated. We assume that each user has L tasks, where each task is inseparable and task l belonging to user m contains $N_{m,l}$ bits.

Typically, MEC consists of the following two phases. The first phase is the offloading phase, during which a user transmits its tasks to one or more than one MEC server. The second phase is the feedback phase, during which the MEC servers carry out the offloaded tasks and feed the outcomes of these computations back to the users. In this paper, the focus is on the impact of NOMA on the first phase of MEC, and the following assumption is used:

Assumption 2: The costs of the second phase of MEC are negligible for the purposes of our analysis.

Note that in the literature on MEC, this assumption has been commonly used due to the following two reasons [16]–[18]. Firstly, the delay caused by the second phase of MEC, i.e., the time for a server to compute an offloaded task and the time for a user to download the computation results from a server, is negligible, because of the superior computation capabilities of the servers as well as the small sizes of the computation results. Secondly, the energy required for a MEC server to compute the offloaded tasks as well as the transmission energy consumption during the second phase of MEC can also be omitted, since the MEC servers are not energy constrained.

The performance of MEC can be evaluated from both latency and energy perspectives:

- *Latency of MEC*: Denote the data rate for user i to offload task l by $R_{i,l}$ which is a function of the transmit power. In particular, denote the transmit power used by user i to offload task l by $P_{i,l}^{ow}$. The time required for offloading task l of user i is given by

$$T_{i,l} = \frac{N_{i,l}}{R_{i,l}}. \quad (1)$$

Due to Assumption 1, all the tasks will be offloaded, and hence there is no delay cost for local computing.

- *Energy Consumption of MEC*: Recall that the offloading transmit power is $P_{i,l}^{ow}$, which is determined by $R_{i,l}$. Therefore, the total energy consumed by offloading all the L tasks of user i is given by

$$E_i = \sum_{l=1}^L P_{i,l}^{ow} \frac{N_{i,l}}{R_{i,l}}, \quad (2)$$

where the use of Assumption 1 means that there is no energy cost for local computing, and the energy consumption during the second phase of MEC is omitted due to Assumption 2.

III. APPLICATION OF NOMA UPLINK TRANSMISSION TO MEC

This section focuses on one particular type of MEC scenario, in which M users offload their tasks to a single MEC server ($K = 1$) and each user has a single task for offloading ($L = 1$). Offloading in this MEC scenario can be viewed as a special case of uplink transmission, to which both OMA and NOMA can be applied. Depending on the user's quality of service (QoS) requirements, different MEC offloading strategies can be applied, as described in the following two subsections.

Without loss of generality, assume that the users are ordered as follows:

$$|h_1|^2 \leq \dots \leq |h_M|^2, \quad (3)$$

where h_m denotes the channel gain between user m and the MEC server. In this paper, the users' channels are assumed to be quasi-static Rayleigh fading. In order to avoid overloading the MEC server in a single resource block, such as a time slot or a frequency channel, we assume that only two users, user m and user n , are scheduled to be served by the MEC server in the same resource block where $m < n$.

A. Impact of NOMA on Offloading Latency

If the users' tasks are delay sensitive, i.e., using less offloading time has higher priority than energy consumption, OMA-MEC and NOMA-MEC can be implemented as follows.

In OMA-MEC, the users are allocated with dedicated time slots for offloading their tasks to the MEC server individually, i.e., each user needs the following time interval for delivering its task to the server¹:

$$T_i \triangleq \frac{N}{\log \left(1 + \frac{P_{i,l}^{ow}}{P_{N0}} |h_i|^2 \right)}, \quad (4)$$

¹For notational simplicity, the subscript l is omitted since each user has a single task for offloading ($L = 1$).

for $i \in \{m, n\}$, where P_{N0} denotes the receiver noise power. To facilitate performance analysis, we assume that the users' tasks have the same size, i.e., $N = N_{i,1}$, for $i \in \{m, n\}$.

In NOMA-MEC, user n is admitted to time slot T_m which would be solely occupied by user m in the OMA mode, and hence both the users can offload their tasks simultaneously. If user n can finish its offloading within T_m , the advantage of NOMA-MEC over OMA-MEC is that user n does not need extra time for offloading, and hence the offloading latency is reduced. However, it is important to point out that admitting user n to time slot T_m cannot cause any performance degradation to user m , otherwise user m may block user n from being admitted to time slot T_m . This condition can be satisfied if user n 's signals are decoded before user m 's at the MEC server and also user n uses the following rate constraint [23]:

$$R_n \leq \log \left(1 + \frac{P_n^{ow} |h_n|^2}{P_m^{ow} |h_m|^2 + P_{N0}} \right). \quad (5)$$

The following lemma provides a closed-form expression for the probability $P_n = P(R_n T_m \geq N)$, which measures the likelihood of the event that user n can complete its offloading within T_m , for given P_n^{ow} and P_m^{ow} .

Lemma 1: For given P_n^{ow} and P_m^{ow} , the probability that user n can complete offloading by using the time slot that would be solely occupied by user m in the OMA mode is given by

$$\begin{aligned} P_n = & c_{mn} \sum_{p=0}^{n-1-m} \frac{c_p}{M-m-p} \sum_{l=0}^{m-1} c_l e^{\frac{l^2}{4a}} \\ & \times \frac{\sqrt{\pi}}{2\sqrt{a}} \left(1 - \Phi \left(\frac{\max\{0, \rho_n - \rho_m\}}{\rho_m^2} + \frac{b\sqrt{a}}{2a} \right) \right) + 1 \\ & - \frac{M!}{(m-1)!(M-m)!} \sum_{l=0}^{m-1} \frac{c_l e^{-(M-m+l+1)\frac{\max\{0, \rho_n - \rho_m\}}{\rho_m^2}}}{M-m+l+1}, \end{aligned} \quad (6)$$

where $\rho_i = \frac{P_i^{ow}}{P_{N0}}$, $i \in \{m, n\}$, $c_{mn} = \frac{M!}{(m-1)!(n-1-m)!(M-n)!}$, $c_p = \binom{n-1-m}{p} (-1)^{n-1-m-p}$, $c_l = \binom{m-1}{l} (-1)^l$, $a = \frac{\rho_m^2}{\rho_n} (M-m-p)$, $b = p+l+1 + (M-m-p)\frac{\rho_m}{\rho_n}$, and $\Phi(\cdot)$ denotes the Gaussian probability integral.

Proof: Please refer to Appendix A. \square

Remark 1: The analytical result in Lemma 1 serves the following two purposes. One is to provide a closed-form and exact expression for the offloading probability and hence the performance analysis can be facilitated by using this closed-form expression instead of carrying out computationally complex Monte Carlo simulations. The other is to ensure that greater insight into NOMA-MEC can be obtained, since the closed-form expression makes asymptotic performance studies possible, as shown in the following.

In order to carry out asymptotic studies, we first present the following proposition which will be used for the development of high and low SNR approximations to P_n .

Proposition 1: For $m < M$, the following equality holds:

$$\frac{M!}{(m-1)!(M-m)!} \sum_{l=0}^{m-1} c_l \frac{1}{M-m+l+1} = 1. \quad (7)$$

Proof: Please refer to Appendix B. \square

By using Lemma 1 and Proposition 1, the high and low SNR approximations to P_n can be obtained in the following lemmas.

Lemma 2: When both P_n^{ow} and P_m^{ow} approach infinity and $\eta \triangleq \frac{P_n^{ow}}{P_m^{ow}}$ is a constant, the probability that user n can complete offloading within T_m can be approximated as follows:

$$P_n \approx \frac{1}{\rho_m^{\frac{m}{2}}} \sum_{p=0}^{n-1-m} \frac{\eta^{\frac{m}{2}} c_{mn} c_p}{(M-m-p)^{\frac{m}{2}+1}} (\tilde{Q}_1 - \tilde{Q}_2). \quad (8)$$

The two parameters, \tilde{Q}_1 and \tilde{Q}_2 , are given by

$$\tilde{Q}_1 \approx \begin{cases} \frac{\sqrt{\pi}(-1)^{m-1}(m-1)!}{\left(\frac{m-1}{2}\right)!2^m}, & \text{if } m \text{ is an odd number} \\ \frac{\sqrt{\pi}\mu_m}{\left(\frac{m}{2}\right)!2^{m+1}a^{\frac{m}{2}}}, & \text{if } m \text{ is an even number,} \end{cases} \quad (9)$$

and

$$\tilde{Q}_2 \approx \begin{cases} \frac{\mu_m}{m!!2^{\frac{m+1}{2}}a^{\frac{1}{2}}}, & \text{if } m \text{ is an odd number} \\ \frac{(-1)^{m-1}(m-1)!}{(m-1)!!2^{\frac{m}{2}}}, & \text{if } m \text{ is an even number,} \end{cases} \quad (10)$$

where $\lambda = \left[p + 1 + \frac{(M-m-p)}{\eta} \right]$ and $\mu_m = \left(\sum_{l=0}^{m-1} c_l l^m + m! \lambda (-1)^{m-1} \right)$.

Proof: Please refer to Appendix C. \square

Lemma 3: At low SNR, i.e., when both P_n^{ow} and P_m^{ow} approach zero and η is a constant, the probability that user n can complete offloading within T_m approaches one, i.e., $P_n \rightarrow 1$.

Proof: Please refer to Appendix C. \square

Following steps similar to those in the proof for Lemma 3, we can obtain the following corollary.

Corollary 1: When P_n^{ow} approaches infinity and P_m^{ow} is fixed, the probability that user n can complete offloading within T_m approaches one, i.e., $P_n \rightarrow 1$.

Remark 2: Lemma 2 indicates that P_n approaches zero at high SNR. This phenomenon can be explained as follows. When P_m^{ow} approaches infinity, user m 's rate also approaches infinity, and hence T_m approaches zero. On the other hand, the data rate for user n to transmit during time slot T_m becomes a constant at high SNR, i.e., $\log \left(1 + \frac{\rho_n |h_n|^2}{\rho_m |h_m|^2 + 1} \right) \rightarrow \log \left(1 + \frac{\eta |h_n|^2}{|h_m|^2} \right)$ for $P_n^{ow} \rightarrow \infty$ and $P_m^{ow} \rightarrow \infty$. Therefore, with $T_m \rightarrow 0$ and a constant R_n , it will be difficult for user n to complete offloading within T_m . Note that T_m can be increased at high SNR if user m 's message is decoded first at the MEC server. However, with this change of decoding order, user m 's rate in NOMA becomes a constant at high SNR and hence is much smaller than that in OMA, which means that user m might block user n from being admitted to T_m , i.e., user m simply adopts the OMA mode and solely occupies the time slot².

²Note that the discussion provided in Remark 2 is based on the assumption that user m 's data rate approaches infinity as the transmission power approaches infinity. Provided that user m 's symbol rate is fixed and its modulation type is not adaptive to the available transmission power, the constellation constrained capacity should be used and it is possible to use a different SIC decoding order, which is beyond the scope of this paper.

Remark 3: The decay rate of P_n can be obtained as follows. At high SNR, a also approaches infinity. Therefore, \tilde{Q}_1 is dominant when m is an odd number, otherwise \tilde{Q}_2 becomes dominant. As a result, P_n in Lemma 2 can be further approximated as follows:

$$\begin{cases} \frac{\sum_{p=0}^{n-1-m} \frac{c_{mn} c_p \sqrt{\pi} (m-1)!}{(M-m-p)^{\frac{m}{2}+1} \left(\frac{m-1}{2}\right)! 2^m}}{\rho_m^{\frac{m}{2}}}, & m \in \{1, 3, \dots\} \\ \frac{\sum_{p=0}^{n-1-m} \frac{c_{mn} c_p (m-1)!}{(M-m-p)^{\frac{m}{2}+1} (m-1)!! 2^{\frac{m}{2}}}}{\rho_m^{\frac{m}{2}}}, & m \in \{2, 4, \dots\}, \end{cases} \quad (11)$$

which means that the decay rate of P_n is $\rho^{-\frac{m}{2}}$, i.e., scheduling a user with poor channel conditions to act as the NOMA weak user is beneficial to increase P_n .

Remark 4: Lemma 3 indicates that, in the low SNR regime, it is almost sure that user n can complete its data offloading by using T_m only. The reason is that, at low SNR, a user with poor channel conditions needs to use a significant amount of time for offloading, which provides an ideal opportunity for using NOMA, i.e., user n has more time to offload its task to the MEC server. For a similar reason, another ideal situation for the application of NOMA-MEC is that P_n^{ow} approaches infinity and P_m^{ow} is fixed, as indicated by Corollary 1.

Remark 5: If user n completes its offloading within T_m , the latency of NOMA-MEC offloading can be significantly reduced, but at a price that more energy is consumed compared to OMA-MEC. Particularly, in order to strictly ensure that N bits are offloaded within T_m , the power used by user n needs to satisfy the following constraint:

$$\log \left(1 + \frac{P_n^{ow} |h_n|^2}{P_m^{ow} |h_m|^2 + 1} \right) T_m > \log (1 + P_m^{ow} |h_m|^2) T_m. \quad (12)$$

Therefore, the minimal power for user n is given by

$$P_n^{ow} = \frac{|h_m|^2}{|h_n|^2} P_m^{ow} (1 + P_m^{ow} |h_m|^2). \quad (13)$$

In OMA, if user n is given the same amount of time (T_m) for offloading N bits, user n 's power needs to satisfy the following:

$$P_n^{OMA} = \frac{|h_m|^2}{|h_n|^2} P_m^{ow}. \quad (14)$$

So the price for the improved latency is for user n to consume more energy, i.e., $P_n^{ow} - P_n^{OMA} = \frac{(P_m^{ow} |h_m|^2)^2}{|h_n|^2}$.

B. Impact of NOMA on Offloading Energy Consumption

The energy inefficiency pointed out in Remark 5 is due to the imposed constraint that user n has to complete its offloading within T_m . By removing this constraint, a modified NOMA-MEC protocol with better energy efficiency can be designed as described in the following.

In order to have a fair comparison between OMA and NOMA, first consider the following modified OMA benchmark. In particular, assume that each user is allocated an equal-duration time slot with T seconds for offloading.

Furthermore, denote user n 's transmit power in OMA by P_i^{ow} , $i \in \{m, n\}$, which means that the energy consumption for user i in OMA is TP_i^{ow} and the amount of data sent within T is $T \log(1 + P_i^{ow}|h_i|^2)$.

For the modified NOMA-MEC protocol, the two users use NOMA to transmit simultaneously during the first time slot, and user n solely occupies the second time slot³. Assume that user n 's power in NOMA is only a portion of that in OMA, i.e., βP_n^{ow} . Therefore, the overall energy consumption for user n in NOMA is $2T\beta P_n^{ow}$ and the amount of data sent within $2T$ is given by

$$T \log \left(1 + \frac{\beta P_n^{ow} |h_n|^2}{P_m^{ow} |h_m|^2 + 1} \right) + T \log(1 + \beta P_n^{ow} |h_n|^2). \quad (15)$$

β is an energy reduction parameter and needs to be smaller than $\frac{1}{2}$ since the constraint that NOMA-MEC is more energy efficient than OMA-MEC is equivalent to the following:

$$2T\beta P_n^{ow} < TP_n^{ow}. \quad (16)$$

However, with β satisfying (16), it is not guaranteed that NOMA-MEC can deliver the same amount of data as OMA-MEC, and the probability for this event can be expressed as follows:

$$\tilde{P}_n \triangleq P \left(T \log \left(1 + \frac{\beta \rho_n |h_n|^2}{\rho_m |h_m|^2 + 1} \right) + T \log(1 + \beta \rho_n |h_n|^2) \right. \\ \left. \leq T \log(1 + \rho_n |h_n|^2) \right). \quad (17)$$

The following corollary provides a closed-form expression for \tilde{P}_n .

Corollary 2: If $(1 - \beta)\rho_m \geq \beta^2 \rho_n$, the probability \tilde{P}_n can be expressed as follows:

$$\tilde{P}_n = 1 - c_{mn} \sum_{p=0}^{n-1-m} c_p \sum_{l=0}^{m-1} c_l \frac{e^{-(M-m-p)\frac{1-2\beta}{\beta^2 \rho_n}}}{(M-m-p)\tilde{a}}, \quad (18)$$

otherwise

$$\tilde{P}_n = 1 - \frac{M! \sum_{l=0}^{m-1} c_l \frac{e^{-(M-m+l+1)\kappa_1}}{M-m+l+1}}{(m-1)!(M-m)!} - c_{mn} \sum_{p=0}^{n-1-m} c_p \\ \times \sum_{l=0}^{m-1} c_l \frac{e^{-(M-m-p)\frac{1-2\beta}{\beta^2 \rho_n}} (1 - e^{-\tilde{a}\kappa_1})}{\tilde{a}(M-m-p)}, \quad (19)$$

where $\kappa_1 = \frac{1-2\beta}{\beta^2 \rho_n - (1-\beta)\rho_m}$ and $\tilde{a} = \frac{\rho_m(1-\beta)(M-m-p)}{\beta^2 \rho_n} + p + l + 1$.

Proof: With some algebraic manipulations, \tilde{P}_n can be rewritten as follows:

$$\tilde{P}_n = P \left(|h_n|^2 \leq \frac{(1-\beta)(1 + \rho_m |h_m|^2) - \beta}{\beta^2 \rho_n} \right). \quad (20)$$

If $(1 - \beta)\rho_m \geq \beta^2 \rho_n$, $\frac{(1-\beta)(1 + \rho_m |h_m|^2) - \beta}{\beta^2 \rho_n} \geq |h_m|^2$ always holds. Applying the joint probability density function (pdf) of h_m and h_n and also following steps similar to those in the proof for Lemma 1, the first part of the corollary can be obtained.

³Since admitting user n into the first time slot will not cause any performance degradation to user m , only user n 's performance is considered.

If $(1 - \beta)\rho_m < \beta^2 \rho_n$, whether $\frac{(1-\beta)(1 + \rho_m |h_m|^2) - \beta}{\beta^2 \rho_n} \geq |h_m|^2$ holds depends on the value of $|h_m|^2$. Particular, if $|h_m|^2 < \kappa_1$, then $\frac{(1-\beta)(1 + \rho_m |h_m|^2) - \beta}{\beta^2 \rho_n} \geq |h_m|^2$ holds, otherwise $\frac{(1-\beta)(1 + \rho_m |h_m|^2) - \beta}{\beta^2 \rho_n} < |h_m|^2$. Hence the probability \tilde{P}_n can be rewritten as follows:

$$\tilde{P}_n = P(|h_m|^2 \leq \kappa_1) \\ - P \left(|h_m|^2 \leq \kappa_1, |h_n|^2 \leq \frac{(1-\beta)(1 + \rho_m |h_m|^2) - \beta}{\beta^2 \rho_n} \right). \quad (21)$$

Following steps similar to those in the proof for Lemma 1, the second part of the corollary can also be obtained and hence the proof for the corollary is complete. \square

Remark 6: It is desirable to have $\tilde{P}_n \rightarrow 0$ which means that NOMA-MEC can deliver more data while using less energy compared to OMA-MEC. However, the asymptotic behavior of \tilde{P}_n depends on whether $(1 - \beta)\rho_m < \beta^2 \rho_n$ holds.

- For the case ρ_m is a constant and $\rho_n \rightarrow \infty$, we have $(1 - \beta)\rho_m < \beta^2 \rho_n$. In this case, \tilde{P}_n approaches zero, since

$$\tilde{P}_n \leq 1 - \frac{M! \sum_{l=0}^{m-1} c_l \frac{e^{-(M-m+l+1)\kappa_1}}{M-m+l+1}}{(m-1)!(M-m)!} \quad (22) \\ \stackrel{(10)}{\approx} \frac{M!}{(M-m)!} \frac{(1-2\beta)^m}{\beta^{2m} \rho_n^m} \rightarrow 0,$$

where step (10) follows from steps similar to those in the proof for Lemma 2. For the case that both ρ_m and ρ_n approach infinity and $\frac{\rho_m}{\rho_n} < \frac{\beta^2}{1-\beta}$, the same conclusion can be obtained.

- For the case that both ρ_m and ρ_n approach infinity and $\frac{\rho_m}{\rho_n} \geq \frac{\beta^2}{1-\beta}$, \tilde{P}_n approaches a non-zero constant, since

$$P_n \rightarrow 1 - c_{mn} \sum_{p=0}^{n-1-m} c_p \sum_{l=0}^{m-1} c_l \frac{1}{(M-m-p)\tilde{a}}.$$

By applying Proposition 1, the following holds:

$$\sum_{l=0}^{m-1} c_l \frac{1}{(\tilde{b} + p + l + 1)} = \frac{m!}{(\tilde{b} + p + 1) \cdots (\tilde{b} + p + m)}, \quad (23)$$

where $\tilde{b} = \tilde{a} - p - l - 1$. Therefore, at high SNR, the probability can be approximated as follows:

$$P_n \rightarrow 1 - \sum_{p=0}^{n-1-m} \frac{m! c_p c_{mn}}{(M-m-p) \prod_{i=1}^m (\tilde{b} + p + i)},$$

which is a non-zero constant and not a function of the SNR. If ρ_n is a constant, we will have $(1 - \beta)\rho_m \geq \beta^2 \rho_n$ when $\rho_m \rightarrow \infty$. In this case, \tilde{P}_n also approaches a non-zero constant.

IV. THE APPLICATION OF NOMA DOWNLINK TRANSMISSION TO MEC

This section considers another type of MEC scenario with $M = 1$ and $L = K$, i.e., a single user has K tasks to be

offloaded to K MEC servers. Assume that the MEC servers are ordered as follows:

$$|g_1|^2 \leq \dots \leq |g_K|^2, \quad (24)$$

where $|g_m|^2$ denotes the channel gain between the user and MEC server m . If OMA is used, the user uses K dedicated time slots with T seconds each to offload its tasks to the servers individually. By using NOMA downlink transmission, the user can offload multiple tasks to multiple servers simultaneously. Similar to the previous section, we assume that two MEC servers, server m and server n , are scheduled to perform NOMA.

A. Impact of NOMA on Offloading Latency

By imposing the constraint that the user offloads the task intended to MEC server n within the time slot that would be solely occupied by server m in the OMA mode, the overall offloading latency can be significantly reduced. Particularly, in NOMA, the numbers of bits transmitted to the two MEC servers within one time slot are given by

$$N_m^{NOMA} = T \log \left(1 + \frac{P^{ow} \alpha_m^2 |g_m|^2}{P_{N0} + P^{ow} \alpha_n^2 |g_n|^2} \right), \quad (25)$$

and

$$N_n^{NOMA} = T \log \left(1 + \frac{P^{ow} \alpha_n^2 |g_n|^2}{P_{N0}} \right), \quad (26)$$

where α_m and α_n denote the NOMA power allocation coefficients, and P^{ow} denotes the user's transmit power⁴.

Therefore, the probability that the user can finish offloading its tasks to the MEC servers can be expressed as follows:

$$P_m^D = P \left(T \log \left(1 + \frac{P^{ow} \alpha_m^2 |g_m|^2}{1 + P^{ow} \alpha_n^2 |g_n|^2} \right) \geq N_m \right), \quad (27)$$

and

$$P_n^D = P \left(T \log (1 + P^{ow} \alpha_n^2 |g_n|^2) \geq N_n \right), \quad (28)$$

where the index for the user, i , is omitted since there is a single user, i.e., $N_{i,l}$ is simplified to N_l . When T and N_l are fixed, the above probabilities can be obtained straightforwardly from the existing literature on NOMA [24].

B. Impact of NOMA on Offloading Energy Consumption

Similar to Section III-B, a modified NOMA-MEC scheme is considered by using two time slots. During the first time slot, the user offloads one task to server m and parts of a task to server n simultaneously. The second time slot is dedicated for the user to offload the remaining parts of the task intended to server n . In OMA, the user offloads the two tasks in two time slots separately. Denote the overall energy consumption in the OMA and NOMA modes by E^{OMA} and E^{NOMA} , respectively.

It is important to point out that the use of a NOMA downlink changes the expressions for the offloading rates. On the one

hand, in OMA, the numbers of bits transmitted to the two MEC servers are given by

$$\tilde{N}_i^{OMA} = T \log(1 + \rho |g_i|^2), \quad (29)$$

for $i \in \{m, n\}$, where $\rho = \frac{P^{ow}}{P_{N0}}$, and it is assumed that the user uses the same transmit power during the two equal-length time slots. Since T seconds are used, the overall energy consumed in the two time slots in OMA is $E^{OMA} = 2TP^{ow}$.

On the other hand, in NOMA, it is assumed that during the first time slot, the user uses the same transmit power as in the OMA mode, and uses $\tilde{\beta}P^{ow}$ as the transmit power during the second time slot, where $\tilde{\beta}$ denotes a parameter determining the energy reduction. Therefore, in NOMA, the numbers of bits transmitted to the two MEC servers are given by [25]

$$\tilde{N}_m^{NOMA} = T \log \left(1 + \frac{\rho \alpha_m^2 |g_m|^2}{1 + \rho \alpha_n^2 |g_n|^2} \right), \quad (30)$$

and

$$\tilde{N}_n^{NOMA} = T \log (1 + \rho \alpha_n^2 |g_n|^2) + T \log (1 + \tilde{\beta} \rho |g_n|^2), \quad (31)$$

respectively. Since $2T$ seconds are used, the overall consumed energy is $E^{NOMA} = (1 + \tilde{\beta})TP^{ow}$ which implies

$$E^{NOMA} < E^{OMA}, \quad (32)$$

if $\tilde{\beta} < 1$.

To ensure that server m is connected in the NOMA mode with the same reliability as in OMA, the cognitive ratio power allocation policy is used as follows [24]:

$$\alpha_n^2 = \max \left\{ 0, \frac{\rho |g_m|^2 - \epsilon}{\rho |g_m|^2 (1 + \epsilon)} \right\}, \quad (33)$$

where it is assumed that the user's tasks contain the same number of bits, i.e., $N_m = N_n \triangleq N$ and $\epsilon = 2^{\frac{N}{T}} - 1$. Since MEC server m experiences the same reliability in the OMA and NOMA modes, we will focus only on the performance of server n in the following.

With a choice of $\tilde{\beta}$ satisfying (32), NOMA-MEC uses less energy than OMA-MEC, but it is not guaranteed that NOMA can deliver the same amount of data as OMA-MEC, which is measured by the following probability:

$$\tilde{P}_n^D \triangleq P \left(T \log (1 + \alpha_n^2 |g_n|^2) + T \log (1 + \tilde{\beta} \rho |g_n|^2) \leq T \log (1 + \rho |g_n|^2) \right). \quad (34)$$

The following lemma provides a closed-form expression for \tilde{P}_n^D .

Lemma 4: For a fixed choice of $\tilde{\beta}$, the probability that OMA-MEC can deliver more data to server n than NOMA-MEC can be approximated as follows:

$$\begin{aligned} \tilde{P}_n^D = 1 - & \frac{K! \sum_{l=0}^{m-1} c_l e^{-\frac{(K-m+l+1)\frac{\epsilon}{\rho}}{K-m+l+1}}}{(m-1)!(K-m)!} \\ & + c_{mn} \sum_{p=0}^{n-1-m} c_p \sum_{i=1}^{N_{CG}} \frac{\pi}{N_{CG}} \left(\frac{\epsilon}{2\tilde{\beta}\rho} - \frac{\epsilon}{2\rho} \right) \\ & \times f \left(\left(\frac{\epsilon}{2\tilde{\beta}\rho} + \frac{\epsilon}{2\rho} \right) + \left(\frac{\epsilon}{2\tilde{\beta}\rho} - \frac{\epsilon}{2\rho} \right) \theta_i \right) \sqrt{1 - \theta_i^2}, \end{aligned} \quad (35)$$

⁴For notational simplicity, the subscript i is omitted since there is a single user ($M = 1$).

where $\theta_i = \cos\left(\frac{2i-1}{2N}\pi\right)$, N_{CG} denotes the Chebyshev-Gauss approximation parameter, and

$$f(x) = e^{-(p+1)x}(1-e^{-x})^{m-1} \times \frac{e^{-(K-m-p)x} - e^{-(K-m-p)\frac{\rho x[(1-\beta)(1+\epsilon)-1]+\epsilon}{\rho\beta(\rho x-\epsilon)}}}{K-m-p}. \quad (36)$$

Proof: Please refer to Appendix E. \square

The high SNR behavior of \tilde{P}_n^D is shown in the following lemma.

Lemma 5: At high SNR, i.e., $\rho \rightarrow \infty$, \tilde{P}_n^D can be approximated as follows:

$$\tilde{P}_n^D \doteq \rho^{-m}, \quad (37)$$

where $\tilde{f}(\rho) \doteq \rho^{-d}$ denotes exponential equality, i.e., $d = -\lim_{\rho \rightarrow \infty} \frac{\log \tilde{f}(\rho)}{\log \rho}$ [26].

Proof: Please refer to Appendix F. \square

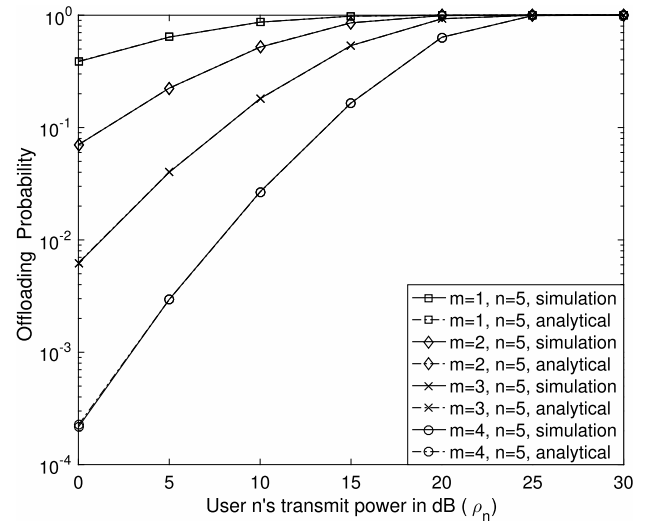
Remark 7: Lemma 5 shows that at high SNR, the probability that NOMA-MEC outperforms OMA-MEC becomes one, which can be explained as follows. Recall that the use of the cognitive radio power allocation policy is to satisfy server m 's requirements before allocating any power to server n . At high SNR, more power becomes available to server n , which means that a significant amount of data can be offloaded to server n during the first time slot, and hence the overall amount of the offloaded data over the two time slots is also improved.

Remark 8: Lemma 5 also indicates that MEC server m 's channel condition has a critical impact on the probability \tilde{P}_n^D . In particular, scheduling a server with better channel conditions to act as server m improves the probability that NOMA-MEC outperforms OMA-MEC. It is worth pointing out that, for the application of NOMA uplink transmission to MEC, a different conclusion was made in Lemma 2 which states that scheduling a user with poor channel conditions is beneficial to the implementation of NOMA.

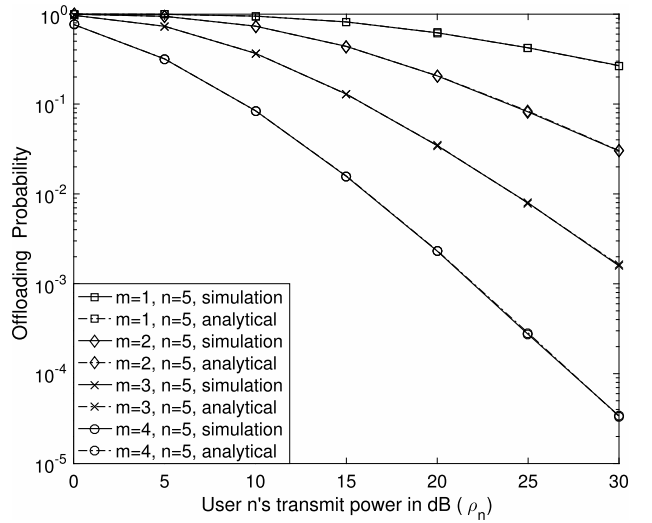
V. SIMULATION RESULTS AND DISCUSSION

In this section, the performance of NOMA-MEC is evaluated by using computer simulations, where the accuracy of the developed analytical results is also verified.

The impact of NOMA uplink transmission on MEC is examined first. Recall that the NOMA-MEC schemes described in Section III ensure that user n is served without causing any performance degradation to user m , so only user n 's performance is considered. In Fig. 1, the offloading probability P_n is shown as a function of user n 's transmit power. Note that the noise power is assumed to be normalized, which means that user n 's transmit power is the same as ρ_n . Fig. 1 shows that the behavior of P_n depends on the relationship between the two users' transmit powers. When user m 's transmit power (ρ_m) is fixed, Fig. 1(a) demonstrates that increasing user n 's transmit power can increase P_n . This phenomenon can be explained as follows. When ρ_m is fixed, the time duration required by user m to offload its task, T_m , is also fixed. On the other hand, increasing ρ_n increases user n 's offloading data rate, which makes it more likely that user n can complete its offloading within the fixed time duration T_m .



(a) $\rho_m = 10$ dB



(b) $\eta = \frac{\rho_n}{\rho_m} = 2$

Fig. 1. The probability that user n can complete its offloading by using the time slot allocated to user m , P_n . There are five users $M = 5$.

If both ρ_m and ρ_n approach infinity and the ratio of the two users' powers is held constant, Fig. 1(b) shows that P_n goes to zero. This phenomenon is due to the fact that increasing ρ_m reduces T_m , the time duration required by user m to complete its offloading. On the other hand, recall that P_n measures the likelihood that user n can complete its offloading by using only T_m , the time slot that would be solely occupied by user m in the OMA mode. Therefore, reducing T_m means that there is less opportunity for user n to use NOMA for offloading, which leads to the reduction of P_n . It is worth pointing out that the two subfigures in Fig. 1 show that the curves for the analytical results perfectly match the ones for the simulation results, which verifies the accuracy of the developed analytical results.

In Fig. 2, the impact of the parameters, such as m , n , and η , on the offloading probability P_n is shown. Recall that the users are ordered according to their channel conditions as in (3), which means that the user index, m (or n), indicates how strong user m 's (or user n 's) channel is. As pointed out

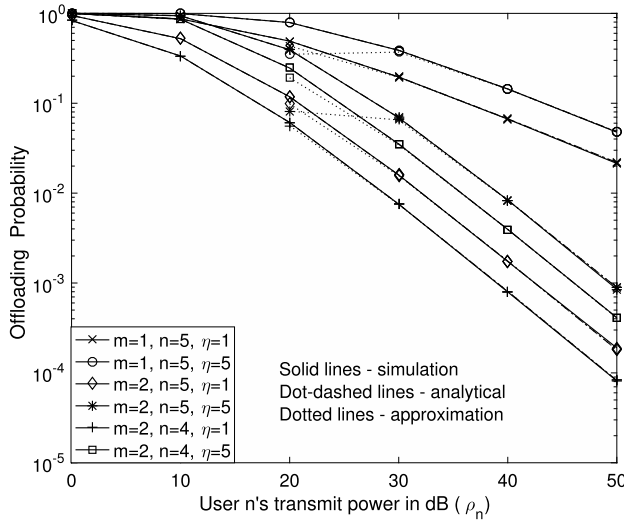
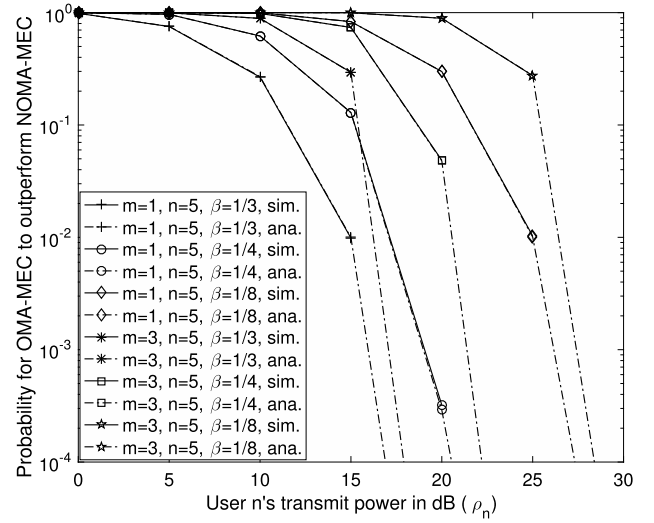


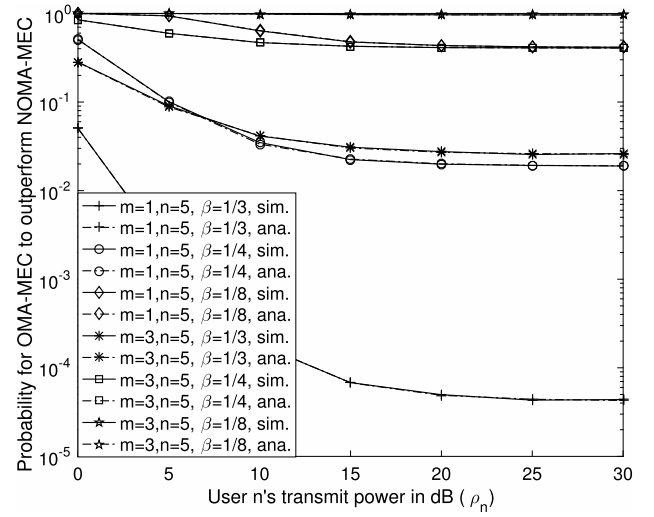
Fig. 2. The impact of the parameters, such as m , n , and η on the offloading probability, P_n . There are five users $M = 5$ and $\eta = \frac{\rho_n}{\rho_m}$.

in the remarks for Lemma 2, the probability P_n is inversely proportional to $\rho_n^{\frac{m}{m-1}}$. This conclusion is confirmed by Fig. 2 as one can observe that the choice of m has a critical impact on P_n . On the other hand, reducing n also reduces the probability, but its impact on the probability is not as significant as m . For a fixed ρ_n , increasing η reduces user m 's transmit power, which means that user m needs more time for offloading, i.e., T_m is increased. Since there is more time available for user n to offload, the offloading probability is improved, as can be observed from Fig. 2. Furthermore, the high SNR approximation obtained in Lemma 2 is also verified in the figure. While this approximation is not accurate in the low SNR regime, it matches the simulation results perfectly at high SNR.

The impact of NOMA-MEC on the energy consumption is examined in Fig. 3. As can be observed from the figure, the use of NOMA can significantly reduce user n 's energy consumption for offloading. In particular, first recall from (16) that the ratio between the energy consumption in the OMA and NOMA modes is 2β . As shown in Fig. 3(a), if the energy used by NOMA-MEC is only a quarter of the energy used by OMA-MEC, i.e., $\beta = \frac{1}{8}$, the probability that OMA-MEC outperforms NOMA-MEC, \tilde{P}_n , reduces to 10^{-2} when $\rho_n = 25$ dB and $m = 1$. If the energy of NOMA-MEC is just half of the energy used in the OMA mode, it becomes almost sure that NOMA-MEC outperforms OMA-MEC, after ρ_n is larger than 15 dB. Recall that Remark 6 points out that for the case that both ρ_m and ρ_n approach infinity and $\frac{\rho_m}{\rho_n} \geq \frac{\beta^2}{1-\beta}$, \tilde{P}_n approaches a non-zero constant, which is confirmed by Fig. 3(b). It is worth pointing out that user pairing has a significant impact on energy saving of NOMA-MEC, as can be seen from the figure. For example, in Fig. 3(a), when $\rho_n = 15$ dB, the case with $m = 1$ and $\beta = \frac{1}{4}$ can even realize a smaller \tilde{P}_n than the case with $m = 3$ and $\beta = \frac{1}{3}$, i.e., scheduling user 1 as the NOMA weak user can save more energy than the case of $m = 3$. Note that the subfigures in Fig. 3 also demonstrate the accuracy of the analytical results developed in Corollary 2.



(a) $\rho_m = 10$ dB



(b) $\eta = \frac{\rho_n}{\rho_m} = 5$

Fig. 3. The probability that OMA-MEC outperforms NOMA-MEC, \tilde{P}_n . There are five users $M = 5$.

In Fig. 4, the impact of NOMA downlink transmission on MEC is illustrated. Because the cognitive radio power allocation policy is used, user m 's performance is not affected even though user n is admitted to the time slot that would be solely occupied by user m in the OMA mode. Therefore, only user n 's performance is evaluated. As can be observed from Fig. 4, the probability that OMA-MEC outperforms NOMA-MEC approaches zero as the transmit power increases. This phenomenon is due to the fact that, at high SNR, more power becomes available to user n for its offloading. One can also observe that the slope of the probability, \tilde{P}_n^D , is determined by the choice of m . This observation is consistent with Lemma 5, which states that the decay rate of \tilde{P}_n^D depends on m .

It is interesting to note that the effects of m in different NOMA-MEC scenarios are different. In particular, for the MEC scenario considered in Fig. 1, increasing m degrades the performance of NOMA-MEC, but for the scenario considered in Fig. 4, increasing m improves the performance

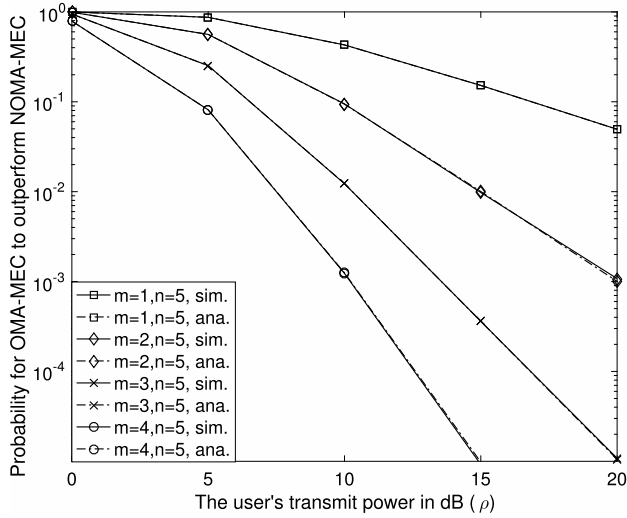
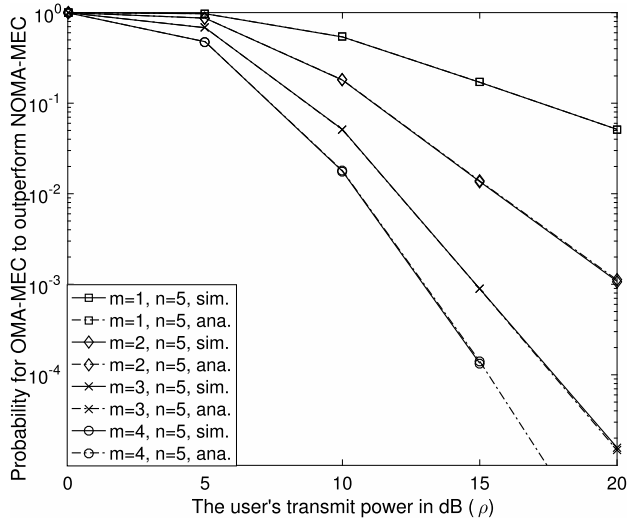
(a) $\tilde{\beta} = \frac{1}{2}$ (b) $\tilde{\beta} = \frac{1}{5}$

Fig. 4. The probability that OMA-MEC outperforms NOMA-MEC, \tilde{P}_n^D . There are five users $M = 5$.

of NOMA-MEC. The reason for the two different effects is explained in the following. For the scenario considered in Fig. 1, i.e., the application of NOMA uplink transmission to MEC, increasing m , i.e., scheduling a user with better channel conditions to act as the NOMA weak user, reduces T_m , the offloading time required by user m . Therefore, it is less likely for user n to offload its task to the server within the shortened time interval T_m . In the scenario considered in Fig. 4, i.e., the application of NOMA downlink transmission to MEC, increasing m , i.e., scheduling a server with better channel conditions to act as the NOMA weak user, reduces the power consumed by server m , and hence there is more power available to perform NOMA and offload the user's task to server n .

VI. CONCLUSIONS

In this paper, we have investigated the combination of NOMA and MEC. The application of NOMA uplink

transmission to MEC was considered first, where the use of NOMA ensures that multiple users can perform offloading at the same time. Then, the application of NOMA downlink transmission to MEC was studied, where one user uses NOMA to offload multiple tasks to multiple MEC servers simultaneously. Analytical results have been developed to demonstrate that the use of NOMA can efficiently reduce the latency and energy consumption of MEC offloading. In addition, various asymptotic studies have also been carried out to reveal the impact of the users' channel conditions and transmit powers on the performance of the combined NOMA and MEC system. Note that the implementation of the proposed NOMA-MEC strategies requires perfect knowledge of channel state information (CSI) at the transmitter. Existing studies in [27] and [28] have demonstrated that NOMA is robust to imperfect CSI compared to OMA in non-MEC scenarios, and it is an important topic for future research to study the impact of imperfect CSI on NOMA-MEC. In addition, scheduling more users, instead of only two users as in this paper, into one NOMA group can potentially improve the performance gain of NOMA over OMA, as confirmed by the existing studies in [25]. However, with more users in the same group, the implementation complexity is also increased. Therefore, a more practical method is to divide the users into multiple small-size groups, where different groups are allocated different OMA resource blocks and the users within one group are served via NOMA. Advanced signal processing methods, such as game theory and matching theory [29], [30], can be applied to user clustering, which is a promising topic for future research.

APPENDIX A PROOF OF LEMMA 1

Recall that P_n can be rewritten as follows:

$$\begin{aligned} P_n &= P(R_n T_m \geq N) \\ &= P\left(\log\left(1 + \frac{\rho_n |h_n|^2}{\rho_m |h_m|^2 + 1}\right) \geq \log(1 + \rho_m |h_m|^2)\right). \end{aligned} \quad (38)$$

With some algebraic manipulations, the probability can be further rewritten as follows:

$$P_n = P\left(|h_n|^2 \geq \frac{\rho_m}{\rho_n} |h_m|^2 + \frac{\rho_m^2}{\rho_n} |h_m|^4\right). \quad (39)$$

Recall that there is an implicit constraint, $|h_n|^2 \geq |h_m|^2$, which means that the event $\left\{|h_n|^2 \geq \frac{\rho_m}{\rho_n} |h_m|^2 + \frac{\rho_m^2}{\rho_n} |h_m|^4\right\}$ can be divided into the following two events:

$$\begin{aligned} &\left\{|h_n|^2 \geq \frac{\rho_m}{\rho_n} |h_m|^2 + \frac{\rho_m^2}{\rho_n} |h_m|^4\right\} \\ &= \left\{|h_n|^2 \geq |h_m|^2, |h_m|^2 \geq \frac{\rho_m}{\rho_n} |h_m|^2 + \frac{\rho_m^2}{\rho_n} |h_m|^4\right\} \\ &\quad + \left\{|h_n|^2 \geq \frac{\rho_m}{\rho_n} |h_m|^2 + \frac{\rho_m^2}{\rho_n} |h_m|^4, \right. \\ &\quad \left. \frac{\rho_m}{\rho_n} |h_m|^2 + \frac{\rho_m^2}{\rho_n} |h_m|^4 \geq |h_m|^2\right\}. \end{aligned} \quad (40)$$

Due to the space limitations, we focus only on the case with $\rho_n \geq \rho_m$, where the results for the case with $\rho_n < \rho_m$ can be obtained similarly. P_n can be expressed as the sum of the following two probabilities:

$$P_n = \underbrace{P\left(|h_n|^2 > \frac{\rho_m}{\rho_n}|h_m|^2 + \frac{\rho_m^2}{\rho_n}|h_m|^4, |h_m|^2 \geq \frac{\rho_n - \rho_m}{\rho_m^2}\right)}_{T_1} + \underbrace{P\left(|h_n|^2 > |h_m|^2, |h_m|^2 \leq \frac{\rho_n - \rho_m}{\rho_m^2}\right)}_{T_2}. \quad (41)$$

By using the order statistics, the joint pdf of $|h_m|^2$ and $|h_n|^2$ can be written as follows [31]:

$$f_{|h_m|^2, |h_n|^2}(x, y) = c_{mn} e^{-x} e^{-(M-n+1)y} (1 - e^{-x})^{m-1} \times (e^{-x} - e^{-y})^{n-1-m}, \quad (42)$$

where c_{mn} is defined in the lemma.

Therefore, the probability T_1 can be expressed as follows:

$$T_1 = c_{mn} \sum_{p=0}^{n-1-m} c_p \int_{\frac{\rho_n - \rho_m}{\rho_m^2}}^{\infty} e^{-(p+1)x} (1 - e^{-x})^{m-1} \times \frac{1}{M-m-p} e^{-(M-m-p)(\frac{\rho_m}{\rho_n}x + \frac{\rho_m^2}{\rho_n}x^2)} dx, \quad (43)$$

where c_p is defined in the lemma.

By applying the binomial expansion, the probability T_1 can be expressed as follows:

$$T_1 = c_{mn} \sum_{p=0}^{n-1-m} \frac{c_p}{M-m-p} \sum_{l=0}^{m-1} \int_{\frac{\rho_n - \rho_m}{\rho_m^2}}^{\infty} c_l e^{-(p+l+1+(M-m-p)\frac{\rho_m}{\rho_n})x - (M-m-p)\frac{\rho_m^2}{\rho_n}x^2} dx. \quad (44)$$

To make the Gaussian probability integral applicable, the probability can be further expressed as follows:

$$T_1 = c_{mn} \sum_{p=0}^{n-1-m} \frac{c_p}{M-m-p} \times \sum_{l=0}^{m-1} \int_{\frac{\rho_n - \rho_m}{\rho_m^2}}^{\infty} c_l e^{-a(x + \frac{b}{2a})^2 + \frac{b^2}{4a}} dx. \quad (45)$$

By applying [32, eq. (3.321.2)] to the above equation, the first part in (6) is proved.

Again applying the joint pdf in (42), the probability T_2 can be obtained as follows:

$$T_2 = 1 - \frac{M!}{(m-1)!(M-m)!} \sum_{l=0}^{m-1} c_l \frac{e^{-(M-m+l+1)\frac{\rho_n - \rho_m}{\rho_m^2}}}{M-m+l+1}. \quad (46)$$

By substituting (45) and (46) into (41), the proof of the lemma is complete.

APPENDIX B PROOF OF PROPOSITION 1

We first rewrite the sum of the binomial coefficients as follows:

$$\begin{aligned} & \frac{M!}{(m-1)!(M-m)!} \sum_{l=0}^{m-1} c_l \frac{1}{(M-m+l+1)} \\ &= \frac{M!}{(m-1)!(M-m)!(M-m+1)} \sum_{l=0}^{m-1} c_l \frac{1}{(1 + \frac{l}{M-m+1})}. \end{aligned} \quad (47)$$

From [33], we have the following property for the binomial coefficients:

$$\sum_{i=0}^t (-1)^i \binom{t}{i} \frac{1}{1+ix} = \frac{t!x^t}{\prod_{i=1}^t (1+ix)}, \quad (48)$$

for $x > 0$ and where t is a non-negative integer.

By letting $x = \frac{1}{M-m+1}$ and $t = m-1$, the above property can be rewritten as follows:

$$\begin{aligned} & \frac{M!}{(m-1)!(M-m)!} \sum_{l=0}^{m-1} c_l \frac{1}{(M-m+l+1)} \\ &= \frac{M!}{(m-1)!(M-m)!(M-m+1)} \frac{(m-1)! \frac{1}{(M-m+1)^{m-1}}}{\prod_{i=1}^{m-1} (1 + \frac{i}{M-m+1})}, \end{aligned} \quad (49)$$

which can be further simplified as follows:

$$\begin{aligned} & \frac{M!}{(m-1)!(M-m)!} \sum_{l=0}^{m-1} c_l \frac{1}{(M-m+l+1)} \\ &= \frac{M!}{(M-m)!(M-m+1)} \frac{1}{\prod_{i=1}^{m-1} (i + M-m+1)} \\ &= \frac{M!}{(M-m)!(M-m+1)} \frac{1}{\prod_{i=M-m+2}^M l} = 1. \end{aligned} \quad (50)$$

The proof of the proposition is complete.

APPENDIX C PROOF OF LEMMA 2

Recall that the probability $P_n = T_1 + T_2$. In the following, the approximation for T_1 is obtained first, and then the approximation for T_2 is developed.

Since both ρ_m and ρ_n approach infinity and η is a constant, we have the following approximation:

$$\frac{\max\{0, \rho_n - \rho_m\}}{\rho_m^2} + \frac{b\sqrt{a}}{2a} \approx \frac{b\sqrt{a}}{2a}, \quad (51)$$

which implies that whether $\eta \geq 1$ or $\eta < 1$ has no impact on the high SNR approximation for T_1 .

We recall that the probability integral function $\Phi(x)$ has the following series representation:

$$\Phi(x) = \frac{2}{\sqrt{\pi}} e^{-x^2} \sum_{k=0}^{\infty} \frac{2^k x^{2k+1}}{(2k+1)!!}. \quad (52)$$

By using the approximation in (51) and the series representation in (52), the first term of the probability P_n , T_1 , can be rewritten as follows:

$$T_1 \approx c_{mn} \sum_{p=0}^{n-1-m} \frac{c_p}{M-m-p} \sum_{l=0}^{m-1} c_l e^{\frac{b^2}{4a}} \frac{\sqrt{\pi}}{2\sqrt{a}} \times \left(1 - \frac{2}{\sqrt{\pi}} e^{-\left(\frac{b\sqrt{a}}{2a}\right)^2} \sum_{k=0}^{\infty} \frac{2^k \left(\frac{b\sqrt{a}}{2a}\right)^{2k+1}}{(2k+1)!!} \right). \quad (53)$$

To facilitate the asymptotic studies, the series representation of the exponential functions, $e^{\frac{b^2}{4a}}$, is used and the probability T_1 can be expressed as follows:

$$T_1 = c_{mn} \sum_{p=0}^{n-1-m} \frac{c_p}{M-m-p} \left(\underbrace{\sum_{l=0}^{m-1} c_l \frac{\sqrt{\pi}}{2\sqrt{a}} \sum_{s=0}^{\infty} \frac{b^{2s}}{s! 4^s a^s}}_{Q_1} - \underbrace{\sum_{l=0}^{m-1} c_l \frac{1}{\sqrt{a}} \sum_{k=0}^{\infty} \frac{2^k \left(\frac{b\sqrt{a}}{2a}\right)^{2k+1}}{(2k+1)!!}}_{Q_2} \right),$$

where the two terms, Q_1 and Q_2 , are evaluated separately in the following two subsections.

A. High SNR Approximation of Q_1

Recall that $b = l + \lambda$. To facilitate the high SNR approximation, the binomial expansion is applied to the term b^{2s} and we have the following expression:

$$Q_1 = \sum_{s=0}^{\infty} \frac{\sqrt{\pi}}{s! 2^{2s+1} a^{s+\frac{1}{2}}} \sum_{l=0}^{m-1} c_l b^{2s} = \sum_{s=0}^{\infty} \frac{\sqrt{\pi}}{s! 2^{2s+1} a^{s+\frac{1}{2}}} \sum_{l=0}^{m-1} c_l \sum_{q=0}^{2s} \binom{2s}{q} \lambda^{2s-q} l^q. \quad (54)$$

By exchanging the order of the sums, Q_1 can be rewritten as follows:

$$Q_1 = \sum_{s=0}^{\infty} \frac{\sqrt{\pi} \sum_{q=0}^{2s} \binom{2s}{q} \lambda^{2s-q}}{s! 2^{2s+1} a^{s+\frac{1}{2}}} \sum_{l=0}^{m-1} c_l l^q. \quad (55)$$

1) *If m Is an Odd Number:* Recall the following properties of the binomial coefficients:

$$\sum_{l=0}^{m-1} c_l l^t = 0, \quad (56)$$

for $0 \leq t \leq m-2$, and

$$\sum_{l=0}^{m-1} c_l l^{m-1} = (-1)^{m-1} (m-1)!. \quad (57)$$

Note that when m is an odd number, $(m-1)$ is an even number. In this case, Q_1 can be approximated at high SNR as

follows:

$$Q_1 \approx \sum_{s=0}^{\frac{m-1}{2}} \frac{\sqrt{\pi}}{s! 2^{2s+1} a^{s+\frac{1}{2}}} \sum_{q=0}^{2s} \binom{2s}{q} \lambda^{2s-q} \sum_{l=0}^{m-1} c_l l^q \approx \frac{\sqrt{\pi}}{\left(\frac{m-1}{2}\right)! 2^m a^{\frac{m}{2}}} \sum_{q=0}^{m-1} \binom{m-1}{q} \lambda^{m-1-q} \sum_{l=0}^{m-1} c_l l^q \quad (58)$$

$$\stackrel{(2)}{=} \frac{\sqrt{\pi} (-1)^{m-1} (m-1)!}{\left(\frac{m-1}{2}\right)! 2^m a^{\frac{m}{2}}}, \quad (59)$$

where step (1) follows from (56), and step (2) follows from (56) and (57).

2) *If m Is an Even Number:* In this case, $(m-1)$ becomes an odd number and $2 \lceil \frac{m-1}{2} \rceil = m$, where $\lceil \cdot \rceil$ denotes the ceiling function. Therefore, Q_1 can be approximated at high SNR as follows:

$$Q_1 \approx \sum_{s=0}^{\lceil \frac{m-1}{2} \rceil} \frac{\sqrt{\pi}}{s! 2^{2s+1} a^{s+\frac{1}{2}}} \sum_{q=0}^{2s} \binom{2s}{q} \lambda^{2s-q} \sum_{l=0}^{m-1} c_l l^q \approx \frac{\sqrt{\pi}}{\lceil \frac{m-1}{2} \rceil! 2^{m+1} a^{\lceil \frac{m-1}{2} \rceil + \frac{1}{2}}} \sum_{q=0}^m \binom{m}{q} \lambda^{m-q} \sum_{l=0}^{m-1} c_l l^q \stackrel{(4)}{=} \frac{\sqrt{\pi}}{\lceil \frac{m-1}{2} \rceil! 2^{m+1} a^{\lceil \frac{m-1}{2} \rceil + \frac{1}{2}}} \times \left(m\lambda \sum_{l=0}^{m-1} c_l l^{m-1} + \sum_{l=0}^{m-1} c_l l^m \right), \quad (60)$$

where (56) is used to obtain steps (3) and (4). After applying (57), Q_1 can be approximated as follows:

$$Q_1 \approx \frac{\sqrt{\pi}}{\left(\frac{m}{2}\right)! 2^{m+1} a^{\frac{m+1}{2}}} \left(m\lambda (-1)^{m-1} (m-1)! + \sum_{l=0}^{m-1} c_l l^m \right).$$

B. High SNR Approximation of Q_2

On the other hand, after applying the binomial expansion to b^{2s} , Q_2 can be expressed as follows:

$$Q_2 = \sum_{k=0}^{\infty} \frac{1}{(2k+1)!! 2^{k+1} a^{k+1}} \sum_{l=0}^{m-1} c_l b^{2k+1} = \sum_{k=0}^{\infty} \frac{1}{(2k+1)!! 2^{k+1} a^{k+1}} \sum_{p=0}^{2k+1} \binom{2k+1}{p} \times \lambda^{2k+1-p} \sum_{l=0}^{m-1} c_l l^p. \quad (61)$$

Depending on the value of m , Q_2 can be evaluated differently as seen in the following subsections.

1) *If m Is an Odd Number:* In this case, $(m-2)$ is still an odd number, and $2 \lceil \frac{m-2}{2} \rceil = m-1$. Therefore, Q_2 can be

written as follows:

$$\begin{aligned}
Q_2 &\approx \sum_{k=0}^{\lceil \frac{m-2}{2} \rceil} \frac{1}{(2k+1)!!2^{k+1}a^{k+1}} \sum_{p=0}^{2k+1} \binom{2k+1}{p} \\
&\quad \times \lambda^{2k+1-p} \sum_{l=0}^{m-1} c_l l^p \\
&\stackrel{(5)}{\approx} \frac{1}{m!!2^{\lceil \frac{m-2}{2} \rceil+1}a^{\lceil \frac{m-2}{2} \rceil+1}} \sum_{p=0}^m \binom{m}{p} \lambda^{m-p} \sum_{l=0}^{m-1} c_l l^p \\
&\stackrel{(6)}{=} \frac{1}{m!!2^{\frac{m+1}{2}}a^{\frac{m+1}{2}}} \left(\sum_{l=0}^{m-1} c_l l^m + m\lambda(-1)^{m-1}(m-1)! \right), \tag{62}
\end{aligned}$$

where step (5) follows from (56), and step (6) follows from (56) and (57).

2) *If m Is an Even Number:* In this case, $(m-2)$ is also an even number. Following steps similar to those in the previous subsections, Q_2 can be evaluated as follows:

$$\begin{aligned}
Q_2 &\approx \sum_{k=0}^{\frac{m-2}{2}} \frac{1}{(2k+1)!!2^{k+1}a^{k+1}} \\
&\quad \times \sum_{p=0}^{2k+1} \binom{2k+1}{p} \lambda^{2k+1-p} \sum_{l=0}^{m-1} c_l l^p \\
&\approx \frac{1}{(m-1)!!2^{\frac{m-2}{2}+1}a^{\frac{m-2}{2}+1}} \\
&\quad \times \sum_{p=0}^{m-1} \binom{m-1}{p} \lambda^{m-p} \sum_{l=0}^{m-1} c_l l^p \\
&= \frac{1}{(m-1)!!2^{\frac{m}{2}}a^{\frac{m}{2}}} (-1)^{m-1}(m-1)!. \tag{63}
\end{aligned}$$

Combining (59), (60), (62) and (63), the approximation for T_1 can be obtained.

On the other hand, the approximation for T_2 can be obtained by first rewriting T_2 as follows:

$$\begin{aligned}
T_2 &= 1 - \frac{M!}{(m-1)!(M-m)!} \sum_{l=0}^{m-1} c_l \\
&\quad \times \frac{\sum_{k=0}^{\infty} (-1)^k \frac{(M-m+l+1)^k (\eta-1)^k}{k! \rho_m^k}}{M-m+l+1} \\
&= 1 - \frac{M!}{(m-1)!(M-m)!} \sum_{l=0}^{m-1} c_l (M-m+l+1)^{-1} \\
&\quad - \frac{M!}{(m-1)!(M-m)!} \sum_{l=0}^{m-1} c_l (-1)^k \\
&\quad \times \sum_{k=1}^{\infty} \frac{(M-m+l+1)^{k-1} (\eta-1)^k}{k! \rho_m^k}. \tag{64}
\end{aligned}$$

By applying Proposition 1 and also using the fact that ρ_m approaches infinity, T_2 can be approximated

as follows:

$$\begin{aligned}
T_2 &\approx -\frac{M!}{(m-1)!(M-m)!} \sum_{k=1}^{\infty} (-1)^k \frac{(\eta-1)^k}{k! \rho_m^k} \\
&\quad \times \sum_{q=0}^{k-1} \binom{k-1}{q} (M-m+1)^{k-1-q} \sum_{l=0}^{m-1} c_l l^q. \tag{65}
\end{aligned}$$

Again applying (57), T_2 can be approximated as follows:

$$T_2 \approx \frac{M!}{(M-m)!} \frac{(\eta-1)^m}{m! \rho_m^m}. \tag{66}$$

One can observe that the decay rate of T_1 is $\rho_m^{-\frac{m}{2}}$, but the decay rate of T_2 is ρ_m^{-m} . Therefore, at high SNR, T_1 is dominant and the proof of the lemma is complete.

APPENDIX D PROOF OF LEMMA 3

Depending on whether $\rho_n > \rho_m$ holds, the low SNR approximation for P_n can be obtained differently, as shown in the following subsections.

A. For the Case of $\rho_n \leq \rho_m$

In this case, $T_2 = 0$, and the probability P_n is given by

$$\begin{aligned}
P_n &= c_{mn} \sum_{p=0}^{n-1-m} \frac{c_p}{M-m-p} \sum_{l=0}^{m-1} c_l \\
&\quad \times e^{\frac{b^2}{4a}} \frac{\sqrt{\pi}}{2\sqrt{a}} \left(1 - \Phi \left(\frac{b\sqrt{a}}{2a} \right) \right). \tag{67}
\end{aligned}$$

Recall the probability integral function can be approximated as follows:

$$\Phi(x) \approx 1 - \frac{e^{-x^2}}{\sqrt{\pi}x} \sum_{k=0}^{k_x} (-1)^k \frac{(2k-1)!!}{(2x^2)^k}, \tag{68}$$

for $x \rightarrow \infty$, where k_x determines how many terms are to be kept for the approximation. At low SNR, i.e., $\rho_m \rightarrow 0$, $\frac{b\sqrt{a}}{2a}$ also approaches infinity, and therefore, the probability P_n can be approximated as follows:

$$\begin{aligned}
P_n &\approx c_{mn} \sum_{p=0}^{n-1-m} \frac{c_p}{M-m-p} \sum_{l=0}^{m-1} c_l \frac{1}{b} \\
&\quad \times \sum_{k=0}^{k_x} (-1)^k \frac{(2k-1)!!2^k a^k}{b^{2k}} \\
&\stackrel{(7)}{\rightarrow} c_{mn} \sum_{p=0}^{n-1-m} \frac{c_p}{M-m-p} \sum_{l=0}^{m-1} \frac{c_l}{M-m+l+1} \\
&\stackrel{(8)}{=} 1, \tag{69}
\end{aligned}$$

where step (7) follows by using $k_x = 0$ and ignoring those terms of the order of $\frac{a}{b^2}$ or higher contained in $\Phi \left(\frac{b\sqrt{a}}{2a} \right)$, and step (8) follows from the following fact:

$$\begin{aligned}
1 &= \int_0^\infty \int_x^\infty f_{|h_m|^2, |h_n|^2}(x, y) dy dx \\
&= c_{mn} \sum_{p=0}^{n-1-m} \frac{c_p}{M-m-p} \sum_{l=0}^{m-1} \frac{c_l}{M-m+l+1}. \tag{70}
\end{aligned}$$

B. For the Case of $\rho_n > \rho_m$

Recall that the probability P_n is the sum of the two terms, T_1 and T_2 . For the case of $\rho_n \geq \rho_m$, T_1 is given by

$$T_1 = c_{mn} \sum_{p=0}^{n-1-m} \frac{c_p}{M-m-p} \sum_{l=0}^{m-1} c_l e^{\frac{b^2}{4a}} \frac{\sqrt{\pi}}{2\sqrt{a}} \times \left(1 - \Phi \left(\frac{\eta-1}{\rho_m} + \frac{b\sqrt{a}}{2a} \right) \right), \quad (71)$$

At low SNR, i.e., $\rho_m \rightarrow 0$, we have the following approximation:

$$\begin{aligned} \frac{\eta-1}{\rho_m} + \frac{b\sqrt{a}}{2a} &= \frac{\eta-1}{\rho_m} + \frac{p+l+1 + \frac{M-m-p}{\eta}}{2\sqrt{\frac{\rho_m}{\eta}}(M-m-p)} \\ &\approx \frac{\eta-1}{\rho_m} \rightarrow \infty. \end{aligned} \quad (72)$$

Again applying the approximation of the probability integral function, in the low SNR regime, the probability can be approximated as follows:

$$T_1 \approx c_{mn} \sum_{p=0}^{n-1-m} \frac{c_p}{M-m-p} \sum_{l=0}^{m-1} c_l \times \frac{e^{\frac{b^2}{4\frac{\rho_m}{\eta}(M-m-p)} - \left(\frac{\eta-1}{\rho_m}\right)^2}}{2\sqrt{\frac{\eta-1}{\eta\rho_m}(M-m-p)}} \rightarrow 0, \quad (73)$$

where we set $k_x = 0$. The last approximation follows from the facts that ρ_m^{-2} is more dominant than ρ_m^{-1} for $\rho_m \rightarrow 0$, and $x^{\frac{1}{2}}e^{-x} \rightarrow 0$ for $x \rightarrow \infty$. It is easy to show that $T_2 \rightarrow 1$ since

$$\frac{M!}{(m-1)!(M-m)!} \sum_{l=0}^{m-1} c_l \frac{e^{-(M-m+l+1)\frac{\eta-1}{\rho_m}}}{M-m+l+1} \rightarrow 0, \quad (74)$$

for $\rho_m \rightarrow 0$. Since $T_1 \rightarrow 0$ and $T_2 \rightarrow 1$, we have $P_n \rightarrow 1$.

Therefore, no matter whether $\rho_n > \rho_m$, P_n always approaches 1 and the proof of the lemma is complete.

APPENDIX E PROOF OF LEMMA 4

With some algebraic manipulations, the probability \tilde{P}_n^D can be written as follows:

$$\tilde{P}_n^D = P \left(|g_m|^2 \leq \frac{\epsilon}{\rho} \right) + Q_{mn}, \quad (75)$$

where

$$Q_{mn} \triangleq P \left(|g_m|^2 > \frac{\epsilon}{\rho}, T \log(1 + \alpha_n^2 \rho |g_n|^2) + T \log(1 + \tilde{\beta} \rho |g_n|^2) \leq T \log(1 + \rho |g_n|^2) \right). \quad (76)$$

Note that in (75), we use the fact that when $|g_m|^2 \leq \frac{\epsilon}{\rho}$, MEC server n cannot be admitted during the first time slot and hence its rate in NOMA is always smaller than that of OMA due to $\beta < 1$.

By using the marginal pdf of $|g_m|^2$, $P \left(|g_m|^2 \leq \frac{\epsilon}{\rho} \right)$ can be calculated as follows:

$$P \left(|g_m|^2 \leq \frac{\epsilon}{\rho} \right) = 1 - \frac{K! \sum_{l=0}^{m-1} c_l \frac{e^{-(K-m+l+1)\frac{\epsilon}{\rho}}}{K-m+l+1}}{(m-1)!(K-m)!}. \quad (77)$$

The second term in (75), denoted by Q_{mn} , can be rewritten as follows:

$$Q_{mn} = P \left(|g_m|^2 > \frac{\epsilon}{\rho}, \left(1 + \frac{\rho |g_m|^2 - \epsilon}{\rho |g_m|^2 (1 + \epsilon)} \rho |g_n|^2 \right) \times (1 + \tilde{\beta} \rho |g_n|^2) \leq (1 + \rho |g_n|^2) \right),$$

where the equation follows by using the cognitive radio power allocation coefficient in (33). With some algebraic manipulations, the term Q_{mn} can be expressed as follows:

$$Q_{mn} = P \left(|g_m|^2 > \frac{\epsilon}{\rho}, |g_n|^2 \leq \frac{\rho |g_m|^2 [(1 - \tilde{\beta})(1 + \epsilon) - 1] + \epsilon}{\rho \tilde{\beta} (\rho |g_m|^2 - \epsilon)} \right).$$

Due to the channel ordering assumption made in (24), we have the following inequality:

$$|g_m|^2 \leq |g_n|^2 \leq \frac{\rho |g_m|^2 [(1 - \tilde{\beta})(1 + \epsilon) - 1] + \epsilon}{\rho \tilde{\beta} (\rho |g_m|^2 - \epsilon)}, \quad (78)$$

which leads to the following constraint on $|g_m|^2$:

$$|g_m|^2 \leq \frac{\rho |g_m|^2 [(1 - \tilde{\beta})(1 + \epsilon) - 1] + \epsilon}{\rho \tilde{\beta} (\rho |g_m|^2 - \epsilon)}. \quad (79)$$

With some algebraic manipulations, one can find that $-\frac{1}{\rho}$ and $\frac{\epsilon}{\tilde{\beta}\rho}$ are the two roots of the following quadratic form:

$$\rho \tilde{\beta} (\rho x - \epsilon) x - \left(\rho x [(1 - \tilde{\beta})(1 + \epsilon) - 1] + \epsilon \right) = 0. \quad (80)$$

Therefore, the constraint in (79) can be surprisingly written in a very simplified form as follows:

$$|g_m|^2 \leq \frac{\epsilon}{\tilde{\beta}\rho}. \quad (81)$$

Note that $\tilde{\beta} \leq 1$, which means $\frac{\epsilon}{\tilde{\beta}\rho} \geq \frac{\epsilon}{\rho}$. Therefore Q_{mn} can be further expressed as follows:

$$Q_{mn} = P \left(\frac{\epsilon}{\rho} < |g_m|^2 \leq \frac{\epsilon}{\tilde{\beta}\rho}, |g_n|^2 \leq \frac{\rho |g_m|^2 [(1 - \tilde{\beta})(1 + \epsilon) - 1] + \epsilon}{\rho \tilde{\beta} (\rho |g_m|^2 - \epsilon)} \right),$$

where we use the fact that

$$P \left(|g_m|^2 > \frac{\epsilon}{\tilde{\beta}\rho}, |g_n|^2 \leq \frac{\rho |g_m|^2 [(1 - \tilde{\beta})(1 + \epsilon) - 1] + \epsilon}{\rho \tilde{\beta} (\rho |g_m|^2 - \epsilon)} \right) = 0.$$

After applying the joint pdf in (42), the term Q_{mn} can be written as follows:

$$\begin{aligned} Q_{mn} &= c_{mn} \sum_{p=0}^{n-1-m} c_p \int_{\frac{\epsilon}{\rho}}^{\frac{\epsilon}{\beta\rho}} e^{-(p+1)x} (1 - e^{-x})^{m-1} \\ &\quad \times \int_x^{\frac{\rho x[(1-\tilde{\beta})(1+\epsilon)-1]+\epsilon}{\rho\tilde{\beta}(\rho x-\epsilon)}} e^{-(K-m-p)y} dy dx \\ &= c_{mn} \sum_{p=0}^{n-1-m} c_p \int_{\frac{\epsilon}{\rho}}^{\frac{\epsilon}{\beta\rho}} e^{-(p+1)x} (1 - e^{-x})^{m-1} \\ &\quad \times \frac{e^{-(K-m-p)x} - e^{-(K-m-p)\frac{\rho x[(1-\tilde{\beta})(1+\epsilon)-1]+\epsilon}{\rho\tilde{\beta}(\rho x-\epsilon)}}}{K-m-p} dx. \end{aligned}$$

After applying the Chebyshev-Gauss approximation, Q_{mn} can be approximated as follows:

$$\begin{aligned} Q_{mn} &\approx c_{mn} \sum_{p=0}^{n-1-m} c_p \sum_{i=1}^{N_{CG}} \frac{\pi}{N_{CG}} \left(\frac{\epsilon}{2\tilde{\beta}\rho} - \frac{\epsilon}{2\rho} \right) \\ &\quad \times f \left(\left(\frac{\epsilon}{2\tilde{\beta}\rho} + \frac{\epsilon}{2\rho} \right) + \left(\frac{\epsilon}{2\tilde{\beta}\rho} - \frac{\epsilon}{2\rho} \right) \theta_i \right) \sqrt{1 - \theta_i^2}. \end{aligned} \quad (82)$$

By substituting (77) and (82) into (75), the proof of the lemma is complete.

APPENDIX F PROOF OF LEMMA 5

Recall that \tilde{P}_n^D is the sum of two terms, i.e., $\tilde{P}_n^D = P(|g_m|^2 \leq \frac{\epsilon}{\rho}) + Q_{mn}$. By using the proof of Lemma 2, the first part of \tilde{P}_n^D can be approximated as follows:

$$\begin{aligned} P\left(|g_m|^2 \leq \frac{\epsilon}{\rho}\right) &= 1 - \frac{K! \sum_{l=0}^{m-1} c_l \frac{e^{-(K-m+l+1)\frac{\epsilon}{\rho}}}{K-m+l+1}}{(m-1)!(K-m)!} \\ &\approx \frac{K!}{(K-m)!} \frac{2^{2m}(1-\beta)^m}{m! \beta^{2m} \rho^m} \doteq \rho^{-m}. \end{aligned} \quad (83)$$

In the following, the approximation of Q_{mn} will be considered. According to the mean value theorem for integrals, \tilde{P}_n^D can be evaluated as follows:

$$Q_{mn} = c_{mn} \sum_{p=0}^{n-1-m} c_p e^{-(p+1)\frac{\epsilon}{\xi\rho}} (1 - e^{-\frac{\epsilon}{\xi\rho}})^{m-1} \frac{e^{-\frac{\xi_1}{\rho}} - e^{-\frac{\xi_2}{\rho}}}{K-m-p},$$

for a parameter ξ satisfying

$$\frac{\epsilon}{\tilde{\beta}\rho} \leq \frac{\epsilon}{\xi\rho} \leq \frac{\epsilon}{\rho}, \quad (84)$$

where $\tilde{\beta} \leq \xi \leq 1$.

To simplify the notation, we define $\xi_1 = (K-m-p)\frac{\epsilon}{\xi}$ and $\xi_2 = (K-m-p)\frac{[(1-\tilde{\beta})(1+\epsilon)-1]+\epsilon}{\tilde{\beta}(1-\xi)}$. Note that neither of the parameters, ξ_1 and ξ_2 , is a function of the SNR. Therefore, \tilde{P}_n^D can be approximated as follows:

$$\begin{aligned} Q_{mn} &\approx c_{mn} \sum_{p=0}^{n-1-m} c_p \sum_{l=0}^{m-1} c_l e^{-(p+1)\frac{\epsilon}{\xi\rho}} e^{-\frac{l\epsilon}{\xi\rho}} \frac{e^{-\frac{\xi_1}{\rho}} - e^{-\frac{\xi_2}{\rho}}}{K-m-p} \\ &= c_{mn} \sum_{p=0}^{n-1-m} c_p e^{-(p+1)\frac{\epsilon}{\xi\rho}} \frac{e^{-\frac{\xi_1}{\rho}} - e^{-\frac{\xi_2}{\rho}}}{K-m-p} \sum_{l=0}^{m-1} c_l e^{-\frac{l\epsilon}{\xi\rho}}. \end{aligned}$$

By applying the series representation for the exponential function, \tilde{P}_n^D can be approximated as follows:

$$\begin{aligned} Q_{mn} &\approx c_{mn} \sum_{p=0}^{n-1-m} c_p e^{-(p+1)\frac{\epsilon}{\xi\rho}} \frac{e^{-\frac{\xi_1}{\rho}} - e^{-\frac{\xi_2}{\rho}}}{K-m-p} \sum_{l=0}^{m-1} c_l \sum_{k=0}^{\infty} \frac{l^k \epsilon^k}{\xi^k \rho^k k!} \\ &= c_{mn} \sum_{p=0}^{n-1-m} c_p e^{-(p+1)\frac{\epsilon}{\xi\rho}} \frac{e^{-\frac{\xi_1}{\rho}} - e^{-\frac{\xi_2}{\rho}}}{K-m-p} \sum_{k=0}^{\infty} \frac{\epsilon^k \sum_{l=0}^{m-1} c_l l^k}{\xi^k \rho^k k!}. \end{aligned}$$

Now applying the properties in (56) and (57), we have the following approximation:

$$\begin{aligned} Q_{mn} &\approx c_{mn} \sum_{p=0}^{n-1-m} c_p e^{-(p+1)\frac{\epsilon}{\xi\rho}} \frac{e^{-\frac{\xi_1}{\rho}} - e^{-\frac{\xi_2}{\rho}}}{K-m-p} \\ &\quad \times \frac{\epsilon^{m-1} (-1)^{m-1} (m-1)!}{\xi^{m-1} \rho^{m-1} (m-1)!}. \end{aligned}$$

In order to remove the sum with respect to p , we first rewrite \tilde{P}_n^D as follows:

$$\begin{aligned} Q_{mn} &\approx c_{mn} \frac{\epsilon^{m-1} (-1)^{m-1} (m-1)!}{\xi^{m-1} \rho^{m-1} (m-1)!} \\ &\quad \times \sum_{p=0}^{n-1-m} c_p e^{-(p+1)\frac{\epsilon}{\xi\rho}} \frac{e^{-\frac{\xi_1}{\rho}} - e^{-\frac{\xi_2}{\rho}}}{K-m-p} \\ &\approx \frac{1}{\rho^m} \frac{c_{mn} \epsilon^{m-1} (-1)^{m-1} (m-1)!}{\xi^{m-1} (m-1)!} \\ &\quad \times \sum_{p=0}^{n-1-m} c_p \frac{\xi_2 - \xi_1}{K-m-p}. \end{aligned} \quad (85)$$

In order to make Proposition 1 applicable, the sum in \tilde{P}_n^D can be first rewritten as follows:

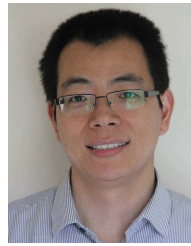
$$\begin{aligned} Q_{mn} &\approx \frac{1}{\rho^m} \frac{c_{mn} \epsilon^{m-1} (-1)^{m-1} (m-1)!}{\xi^{m-1} (m-1)!} \\ &\quad \times (\xi_2 - \xi_1) \sum_{t=0}^{n-1-m} \binom{n-1-m}{t} \frac{(-1)^t}{K-n+1+t} \\ &\stackrel{(9)}{=} \frac{1}{\rho^m} \frac{c_{mn} \epsilon^{m-1} (-1)^{m-1} (m-1)!}{\xi^{m-1} (m-1)!} \\ &\quad \times (\xi_2 - \xi_1) \frac{(n-m-1)!(K-n)!}{(K-m)!} \doteq \rho^{-m}, \end{aligned} \quad (86)$$

where step (9) follows by using Proposition 1. Since both the terms in (35) have the same order of m , the proof of the lemma is complete.

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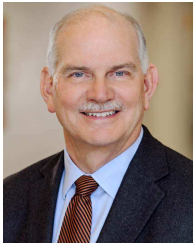
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