Engaging Preservice Secondary Mathematics Teachers in Authentic Mathematical Modeling: Deriving Ampere's Law

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Incorporating modeling activities into classroom instruction requires flexibility with pedagogical content knowledge and the ability to understand and interpret students' thinking, skills that teachers often develop through experience. One way to support preservice mathematics teachers' (PSMTs) proficiency with mathematical modeling is by incorporating modeling tasks into mathematics pedagogy courses, allowing PSMTs to engage with mathematical modeling as *students* and as future *teachers*. Eight PSMTs participated in a model-eliciting activity (MEA) in which they were asked to develop a model that describes the strength of the magnetic field generated by a solenoid. By engaging in mathematical modeling as *students*, these PSMTs became aware of their own proficiency with and understanding of mathematical modeling. By engaging in mathematical modeling as future *teachers*, these PSMTs were able to articulate the importance of incorporating MEAs into their own instruction.

Keywords: Modeling, Model-eliciting activities, STEM-related modeling, Preservice education

Using mathematics to solve real-world problems is consistently identified as one of the most important applications of mathematics, and students should be regularly provided the opportunity to experience mathematics in context (National Council of Teachers of Mathematics [NCTM], 2000; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Mathematical modeling requires one to be able to move fluidly between the real world and the mathematized world and is rooted in the "assumption that humans interpret their experiences using internal conceptual systems (or constructs) whose functions are to select, filter, organize, and transform information, or to infer patterns and regularities beneath the surface of things" (Lesh & Lehrer, 2003). To create a meaningful representation of the given situation when modeling a real-world situation, students must interpret and make sense of complex and imperfect information (Daher & Shahbari, 2015).

Mathematical modeling requires facility with mathematics beyond computational proficiency. Previous success with routine tasks does not imply proficiency with mathematical modeling. Garofalo and Trinter (2013) found that preservice mathematics teachers (PSMTs) who were able to successfully complete textbook trigonometry exercises were not as successful when generating mathematical models to represent the projectile motion of a softball and the periodic motion of a pendulum. Similarly, Delice and Kertil (2015) found that preservice teachers struggled with applying their prior knowledge related to linear and angular speed when asked to develop a mathematical model to describe the change in radii in a cassette tape.

Mathematical modeling also requires the ability to critically analyze the applicability of proposed models. Zbiek (1998) found that PSMTs who relied too heavily on technological tools to generate mathematical models for data had difficulty explaining the appropriateness of models in mathematical terms. These PSMTs tended to choose their models based on "goodness of fit" (i.e., r^2 values) regardless of whether or not the model reflected the relationship visible in the data's scatterplot. When one is modeling with mathematics, the situation for which the model is being generated must remain at the forefront. Yet, for some preservice teachers, connecting mathematics with real-world situations is difficult even when solving tasks that do not require the use of mathematical models (Verschaffel, De Corte, & Borghart, 1997).

Lesh and Lehrer (2003) argue that the use of modeling perspectives in classroom instruction emphasizes the idea that "expertise in teaching is reflected not only in what teachers can 'do,' but also what they 'see' in teaching, learning, and problem-solving situations" (p. 111). Successfully integrating modeling activities into classroom instruction requires teachers to have facility with their pedagogical content knowledge, the ability to understand and interpret students' thinking, and knowledge of the types of contexts and situations that can be modeled. Because of this, teachers have a tendency to avoid using modeling tasks in their own classrooms (Delice & Kertil, 2015). Because these skills are often developed through experience, implementing modeling activities is particularly difficult for preservice and early career teachers.

One of the indicators of a well-prepared beginning teacher is having "solid and flexible knowledge of mathematical processes and practices" (Association of Mathematics Teacher Educators [AMTE], 2017, p. 9). Effective mathematics teacher preparation programs provide teacher candidates with "opportunities . . . to learn mathematics that enable them to engage in mathematical practices and processes that are appropriate to the content being studied" (p. 31). In particular, mathematics teacher preparation programs should incorporate the practice of mathematical modeling throughout a teacher candidate's course of study (AMTE, 2017).

Theoretical Framework

The documented challenges of PSMTs applying their prior mathematical knowledge to modeling situations clearly indicate that this is a skill that does not develop naturally and must be developed and supported in teacher preparation courses. Furthermore, modeling with mathematics is something that can be taught through the use of appropriate tasks that encourage multiple solutions (Blum & Ferri, 2009). Incorporating modeling tasks into mathematics teacher preparation courses provides preservice teachers with the opportunity to engage with mathematical modeling, both as student and as teacher, and to become proficient. Anhalt and Cortez (2016) found that preservice teachers who participated in a mathematical modeling module that was integrated into their content pedagogy course developed a deeper conceptual understanding of mathematical modeling, better preparing them to integrate mathematical modeling into their future classroom instruction. Our approach to mathematical modeling is related but different as we incorporated a STEM-related modeling activity that we developed using the design principles of the Model-Eliciting Activity (MEA) theory into a pedagogy course for PSMTs.

Model-Eliciting Activities

The underlying assumption of the *models and modeling perspective* is that people use conceptual systems as a way to interpret mathematical experiences (Lesh, Doerr, Carmona, & Hjalmarson, 2003). From this perspective, problem-solving tasks are those that emphasize interpreting meaningful situations. The process of generating these complex models is the primary product of such tasks.

Delice and Kertil (2015) propose a five-phase cyclical process to describe the steps involved in developing a mathematical model (Figure 1). According to this process, one needs to first decide how a real-world problem should be mathematized and then interpret what information given in the real-world problem is relevant and which mathematical techniques are appropriate in developing the model (Crouch & Haines, 2004). As mathematical models are developed, they are tested and revised, and the initial real-world problem itself is revisited and reinterpreted as the model is amended (Delice & Kertil, 2015).

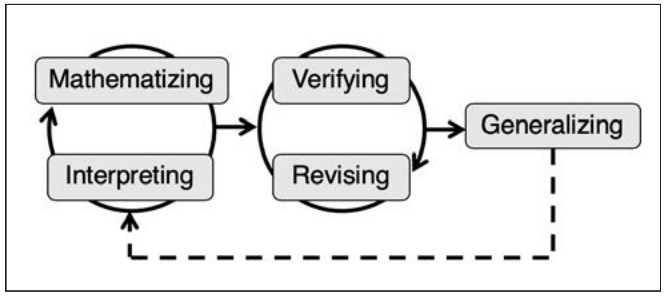

Figure 1. **Mathematical modeling cycle.**

A MEA is "a problem-solving activity constructed using specific principles of instructional design in which students make sense of meaningful situations, and invent, extend, and refine their own mathematical constructs" (Kaiser & Sriraman, 2006, p. 306). By emphasizing the modeling process rather than merely applying known procedures, MEAs encourage students to think mathematically and provide them with the opportunity to showcase their mathematical understanding and capabilities (Daher & Shahbari, 2015). MEAs also provide students with multiple entry points to a problem because they encourage authentic problem solving, defined as "engaging in a task for which the solution method is not known in advance" (NCTM, 2000, p. 52). Because there is no prescribed procedure for MEAs, mathematical modeling tasks are open-ended, and the final models themselves can vary (Bliss & Libertini, 2016). When meaningfully incorporated into classroom instruction, MEAs can support students' ability to transition between abstract representations in mathematics and applications of mathematical reasoning to real-world problems.

The *models and modeling perspective* also informs the rationale for incorporating MEAs into mathematics teacher preparation coursework. According to this perspective, professional development of teachers should emphasize "designing effective and sophisticated ways of helping teachers see and interpret children's thinking and support the development of that thinking" (Lesh et al., 2003, p. 228). Teachers need to be able to develop tasks that encourage children to develop models. Thus, providing PSMTs with authentic modeling experiences during their teacher preparation program can help uncover their own proficiency with and attitudes toward incorporating modeling activities into their future classroom instruction. By incorporating MEAs into pedagogy courses, mathematics teacher educators can support PSMTs' development of their own conceptual understandings of mathematical modeling and the knowledge required to incorporate such tasks into their future teaching.

The MEA on which this study is based was designed to provide students with the opportunity to develop a mathematical model to describe a scientific phenomenon (Ampere's law). Preservice mathematics teachers were asked to relate the magnetic field strength generated by a solenoid to its different attributes (explained below). The research questions we explored in this study were (a) What strategies do preservice secondary mathematics teachers use when experimentally deriving Ampere's law? and (b) What are preservice secondary mathematics teachers' thoughts about incorporating mathematical modeling tasks in classroom instruction after participating in this model-eliciting activity? The intent of the first question was to explore preservice teachers' engagement with mathematical modeling as students, whereas the intent of the second question was to explore their engagement with modeling as future teachers.

Methodology

A solenoid is a coil of conductive wire; when electric current flows through the wire, the coil generates a magnetic field (see Figure 2). Ampere's law ($B = \mu \frac{N}{L} I$) relates the strength of the magnetic field produced by a solenoid (B) to the number of coils of wire (N), the length of the solenoid (L), and the current passing through the wire (I). Solenoids can be found in a number of modern-day technologies, including telephones, speakers, and electric guitar pickups.

Ampere's law can be derived experimentally by systematically varying the different attributes of a solenoid. The number of wraps of wire and the current are directly related to magnetic field strength, whereas the length of the solenoid is inversely related to magnetic field strength.

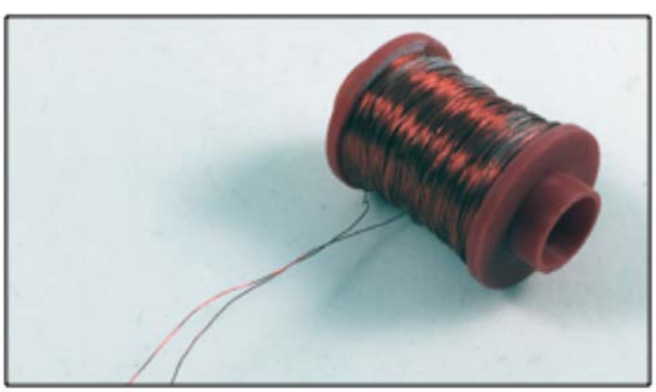

Figure 2. **Example of a solenoid.**

Setting

The setting for this study was a major public university located in central Virginia. Within this university is a self-contained college of education that offers both graduate and undergraduate programs of study, including a dual-degree teacher preparation program and a postgraduate teacher preparation program. The Deriving Ampere's Law activity took place at this university during the 2016–2017 academic year.

Participants

Eight preservice secondary mathematics teacher education students (PSMTs) completed the Deriving Ampere's Law activity in two phases. These students were selected because they were enrolled in the yearlong secondary mathematics pedagogy course at the time of the study. Of the eight PSMTs who completed the activity, two PSMTs volunteered to complete the activity in the fall 2016 semester outside of the mathematics pedagogy course (Phase 1), and the remaining six PSMTs completed the activity in the spring 2017 semester during one session of the mathematics pedagogy course (Phase 2).

Activity Development

With support from the National Science Foundation, the University of Virginia's Make to Learn Lab partnered with the Smithsonian Institution to develop a series of invention kits: Middle school students reconstruct historical inventions (e.g., solenoid, motor, speaker) using modern technology (e.g., computer-aided design software, three-dimensional [3D] printing, laser cutting). These invention kits were created as a way to meaningfully integrate science, technology, engineering, and mathematics. The Deriving Ampere's Law activity was developed to further extend the mathematics connections within the Solenoid Invention Kit.

When working with solenoids, many students shared qualitative observations that indicated an intuitive understanding of the relationship between the different attributes of a solenoid and the strength of the resulting magnetic field. For example, one might correctly conclude that a solenoid with fewer wraps of wire would produce a weaker magnetic field than a solenoid that is more densely wrapped. These qualitative observations inspired us to explore whether a mathematical model existed that was accessible to middle school students and that would describe the magnetic field strength generated by a solenoid. When developing the Deriving Ampere's Law activity, we engaged in the MEA design process of expressing, testing, and revising our own understanding of what it would mean for students to understand the underlying mathematics of Ampere's law (Lesh, Middleton, Caylor, & Gupta, 2008). Through this process, we realized that because the relationships between the independent variables and the dependent variable were either proportional or inversely proportional, this model could be derived experimentally.

Unlike other MEAs, the Deriving Ampere's Law activity challenges students to develop a model of an existing law. Because of this, it has been suggested that this activity might be better described as a "function-eliciting activity" (R. Zbiek, personal communication, February 7, 2019) rather than a model-eliciting activity. However, the participants who completed the Deriving Ampere's Law activity did not know that they were deriving an existing model; not until after the participants derived their own model was it revealed that the model they generated was, in fact, Ampere's law. Although the nature of the Deriving Ampere's Law activity may not appear to be completely consistent with published literature related to MEAs, this activity nevertheless preserves the underlying goals of MEAs. The Deriving Ampere's Law activity provides students with the opportunity to "develop powerful, sharable, and reusable models . . . for accomplishing specific goals in mathematically interesting situations" (Lesh et al., 2008, p. 118).

Activity Description

To complete the Deriving Ampere's Law activity, the PSMTs were provided with a set of solenoids that varied in both the number of wraps of wire and the solenoid length. They were also provided with a variable DC power supply and a magnetic field sensor. To generate magnetic field measurements that would allow a wide range of participants to be successful with this activity, the solenoids were calibrated during activity development (Table 1). The power supply used for this activity consistently and reliably held 3.16 A, which is why the solenoids were calibrated using this current value.

In previous iterations of the Deriving Ampere's Law activity, the activity was divided into four separate investigations for use with several groups of middle school students, who successfully derived the final model (see Corum & Garofalo, 2018). Given the nature of the research questions for this project, a less scaffolded version of the Deriving Ampere's Law activity was presented to the PSMTs. They received all of the solenoids up front and were asked to develop a model that related number of wraps of wire, solenoid length, and electric current to magnetic field strength.

Data Collection

Because of the nature of the research questions for this study, data collection was separated into two phases. The goal of Phase 1 was to understand the PSMTs' engagement with mathematical modeling as *students*; the goal of Phase 2 was to understand the PSMTs' engagement with mathematical modeling as future *teachers*. During Phase 1, only two PSMTs participated in the Deriving Ampere's Law activity to allow for fine-grained data collection. Collected data included video recordings, audio transcriptions, the PSMTs' written work, and observational field notes. On completing the activity, these two PSMTs participated in a debriefing interview to further explore

Table 1
Solenoid Data Collected During Activity Development

Number of Wraps (N)	Solenoid Length (L)	Electric Current (I)	Field Strength (B)	Constant (μ)
50	2 in	3.16 A	35.97 G	0.455
100	2 in	3.16 A	71.87 G	0.455
150	2 in	3.16 A	107.8 G	0.455
50	1 in	3.16 A	71.87 G	0.455
50	2 in	3.16 A	35.97 G	0.455
50	3 in	3.16 A	24.20 G	0.459
50	4 in	3.16 A	18.10 G	0.458
50	2 in	0.79 A	9.0 G	0.456
50	2 in	1.58 A	17.97 G	0.455
50	2 in	3.16 A	35.97 G	0.455

their opinions about the activity and how this activity compared to their previous classroom experiences. During Phase 2, the six remaining PSTMs completed the Deriving Ampere's Law activity in a classroom setting and were given a postactivity questionnaire to report their experiences with the activity.

Data Analysis

The primary goals of this project were to understand how PSTMs' prior experiences influenced their approach to completing a MEA, which strategies they used when developing their models, and their thoughts about incorporating mathematical modeling tasks after participating in a MEA. Data analysis for Phase 1 began after the first two PSMTs completed the Deriving Ampere's Law activity.

During preliminary data analysis, the first author attempted to code the transcripts on the basis of the PSMTs' application of prior knowledge (e.g., slope, variables, direct variation, linear equations). However, differentiating among applications of prior knowledge is difficult when mathematical concepts are interconnected (e.g., slope and linear equations), and coding efforts resulted in the data becoming disjointed. The first author then analyzed the data more holistically by reading through the transcript multiple times and aligning the PSMTs' progression through the task with the different phases of the mathematical modeling framework: mathematizing, interpreting, verifying, revising, and generalizing (see Figure 1). In particular, the first author focused on the PSMTs' interpretation of the activity, strategies for data collection and model development, applications of prior knowledge, use of technology, and beliefs about the nature of mathematics and mathematical tasks. The first author prepared narrative descriptions of the PSMTs' solution strategies. After completing the initial round of data analysis, the second author reviewed the narrative descriptions separately from the first author.

The two authors then met to confirm the first author's interpretation. They reread parts of the transcript, reanalyzed the PSMTs' written work, and reviewed their separately collected field notes. During this meeting, both authors regularly revisited their multiple data sources to ensure that their analysis and interpretations were warranted. The two authors triangulated the narrative descriptions with the observational field notes, audio transcripts, and the PSMTs' written work, and they agreed that the narrative accurately captured what the PSMTs had done to complete the Deriving Ampere's Law activity.

Data analysis for Phase 2 began once the remaining PSMTs had completed the postactivity debriefing survey, which was administered electronically several weeks after the in-class activity implementation. The second author reviewed the survey responses and identified several common themes. He then shared the original survey responses with the first author, who analyzed the survey responses separately from the second author. The two authors then met to confirm the second author's interpretations.

Findings

Overall, in both phases of the activity, the PSMTs had little difficulty recognizing the *structure* of their final model. By identifying the relationships between the independent variables and dependent variables separately (i.e., direct or inverse variation), the PSMTs hypothesized how the variables should be organized in the final model and then confirmed this using their collected data. Modeling strategies identified during Phase 1 included establishing a strategy prior to beginning data collection, being systematic in their data collection and analysis, and regularly testing and revising their model using their collected data. These are illustrated in Figure 3 and described through the work of Emily and Anna below. We then report on the Phase 2 findings focused on PSMTs' beliefs related to incorporating mathematical modeling tasks in their future mathematics teaching.

Phase 1. Emily and Anna

Anna and Emily's progression through the Deriving Ampere's Law activity can be summarized in four parts: collecting and analyzing solenoid data, structuring the model, recognizing the need for a constant, and testing and revising the model. Each of these parts aligned with the different phases of the mathematical modeling cycle (see Figure 3). Given the nature of this task, the processes of mathematizing and interpreting were intertwined and are reported as such.

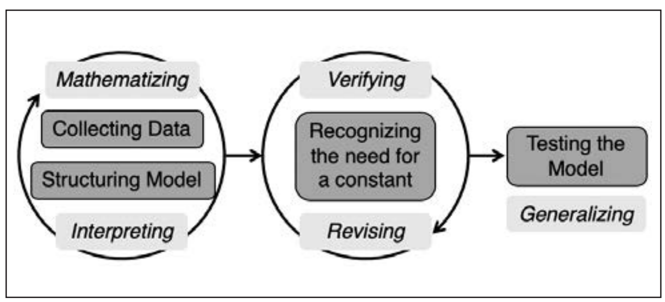

Figure 3. **Anna and Emily's task progression.**

Collecting and analyzing data (*Mathematizing* and *Interpreting*). To begin the activity, Emily suggested that they isolate the independent variables as they collected their data to better understand the relationships between the independent and dependent variables. They first measured the 2-inch solenoids at 3.16 A and 3.0 A. Emily noticed that at 3.16 A, the rate of change in the number of wraps equaled the rate of change in the magnetic field strength (Figure 4) and that this relationship also held true at 3.0 A.

Emily summarized this relationship: "As the number of wraps increases by a factor of *x*, so does the magnet strength," to which Anna responded, "We have a direct variation." Emily noted that because the relationship between number of wraps of wire and magnetic field strength was a direct variation, the formula should be in the form "wraps over magnet is some factor," and she hypothesized that the relationship might be *wraps/gauss* = 1/2 · amps. Anna suggested that they measure the 2-inch solenoids at 2.0 A to see whether the relationship still held true. Although the data collected at 2.0 A did not fit Emily's hypothesized relationship, both Emily and Anna noticed that the direct variation they observed previously also held true at 2.0 A. At this point, Emily abandoned her initial model but confirmed the direct variation between number of wraps of wire and magnetic field strength. She then suggested, "If we find out all the different ways they [the independent variables] vary, we can just kind of put it together."

Anna and Emily then collected data for solenoids of varying lengths (see Figure 5). They noticed that at 1.0 A, the 1-inch, 50-wrap solenoid generated a magnetic field strength of 23 G, which was a field strength they had previously observed with other solenoids. Anna commented that the magnetic field strength was increasing by approximately 10 G as the current increased by 1.0 A.

Emily analyzed the data they had collected so far and summarized the relationship between current and magnetic field strength. She explained, "There's direct variation again. . . . Current over Gauss equals *x*, so both the number of wraps and the current vary directly with the strength." Emily also observed, "The length increases, the strength decreases. That makes sense because it's [wraps of wire] not as close together." Once Emily had articulated the relationship between electric current and magnetic field strength in this way, she immediately recognized the relationship as inverse variation. She then confirmed this relationship by looking at the relationship across different levels of current to see whether the relationship still held true. Emily recalled that the inverse relationship is represented by the equation $xy = k$, and Anna recalled that in an inverse relationship, "as one goes up, the other goes down." They decided to check the relationship using their collected data.

Looking at their data, Emily and Anna multiplied the length of the solenoid by the magnetic field strength to determine whether this resulted in a constant. They did

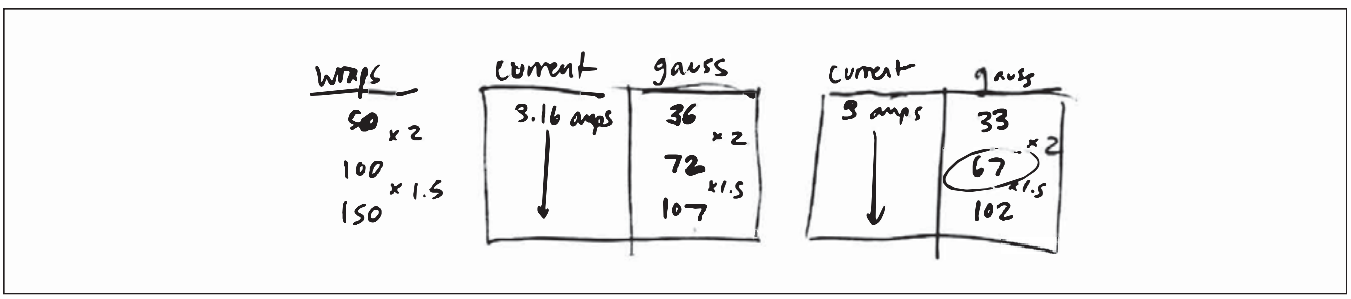

Figure 4. **Emily's data table for solenoids of varying wraps of wire.**

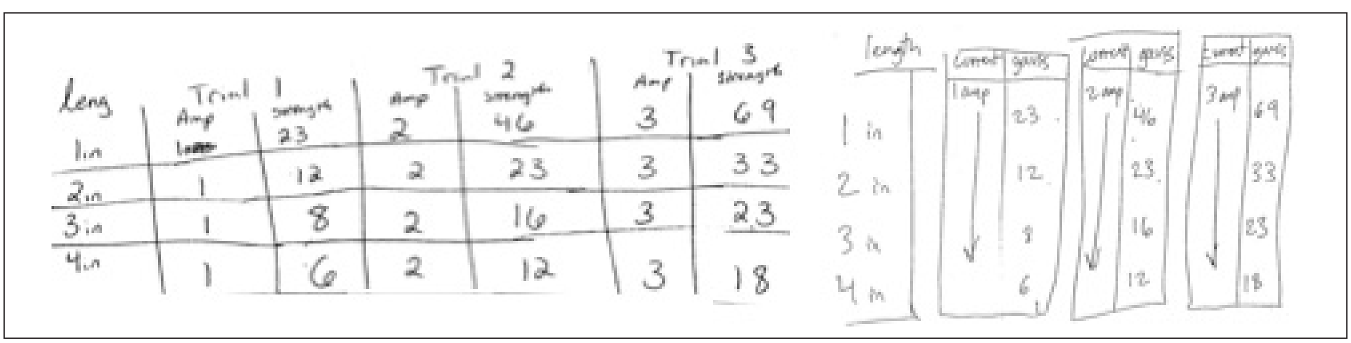

Figure 5. **Anna's (L) and Emily's (R) data for solenoids of varying length.**

this for the data they collected at each of the different current values (see Figure 6). Emily questioned how they were verifying whether the relationship between electric current and magnetic field strength was inverse variation. She suggested that they review all the relationships they had identified with the three independent variables.

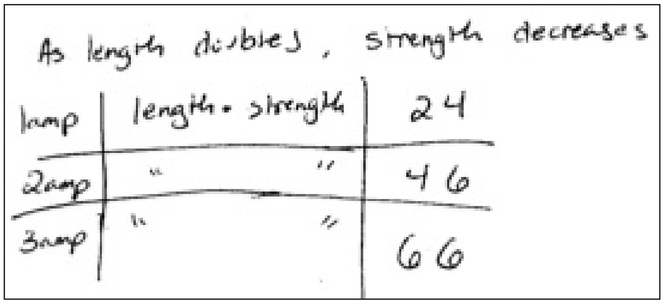

Figure 6. **Anna's attempt to verify *xy* = *k* for data collected at different current values.**

Structuring their model (*Mathematizing* and *Interpreting*). The relationships Emily and Anna observed were as follows: (a) as length increases, strength decreases; (b) as current increases, strength increases; and (c) as wraps increase, strength increases. Emily assigned letters to represent the different variables and recommended that they think about the relationships in terms of strength increasing; she drew the following diagram on her paper (Figure 7).

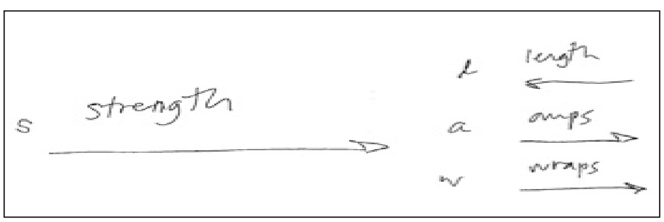

Figure 7. **Emily's summary of the relationships between the variables.**

Anna suggested that they could use the types of variation they identified to determine the equation's structure: "Alright, so *a* and *w*, in some way or form, have to be on top." Because every relationship they had seen thus far involved a ratio, they decided that they would need to divide by the length of the solenoid. This resulted in the following structure for their final model: $s = \frac{aw}{l}$.

Recognizing the need for a constant (*Verifying* and *Revising*). From this model, Anna noticed that the magnetic field strength divided by the number of wraps of wire should equal the current divided by the length and that this could be used to help them determine the constant. Returning to the observation that all the relationships involved ratios, Emily predicted that the constant would be multiplied. Anna agreed that the constant should be multiplied but for a different reason. She explained, "[In] science, every single time you add something [to an equation], it's a variable of some kind. All of our variables are already taken up, so it can't be that."

To determine the constant, Anna suggested using the data they collected for the 2-inch, 50-wrap solenoid at 2.0 A. She used $s/w = a/l$ and set up the equation below (Figure 8).

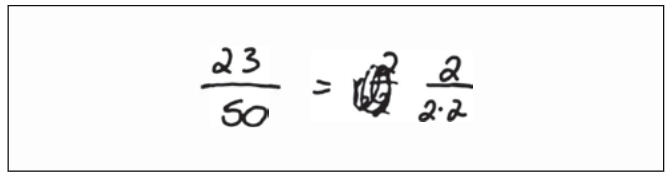

Figure 8. **Anna's initial attempt at determining the constant for their equation.**

Emily also calculated s/w and a/l and, using their collected data, she found that those two were not equal. However, she noticed that the two expressions were approximately equal if they multiplied the denominator by two. Emily explained, "We need to find a relationship to make this true. Our strength needs to be multiplied by two, which means it's this over 2*l*." She then proposed the equation $s = \frac{aw}{2l}$ and used their collected data to see whether that equation held true (see Figure 9).

Emily saw that $s = \frac{aw}{2l}$ held true for some of their collected data but not all of it, which led her to question whether their constant was correct. After testing the equation using all their collected data, both Anna and Emily acknowledged that the model was not entirely correct, but they were unsure whether the discrepancy

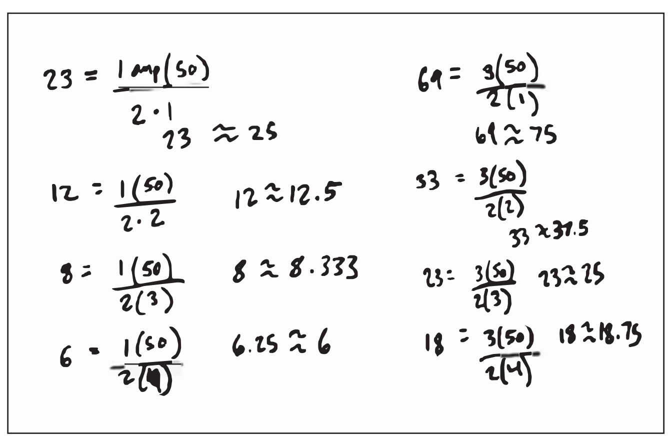

Figure 9. **Emily's verification of their initial equation.**

between the predicted values and their collected data could be attributed to measurement error.

Testing and revising their model (*Generalizing*). Anna and Emily were confident that their model's structure was correct, but they questioned the accuracy of their constant, as demonstrated in the following exchange:

Emily: We found the direct and inverse variations . . . so we know that our variables are in the right places, so the only thing we're not 100% sure about is this constant.

Anna: I'm wondering if it's a little over two.

Emily: Or what if it's a decimal? Oh god, what are we gonna do? How are we gonna tell?

Thinking that the constant might be a value greater than two, Anna recalled that the metric system is most commonly used in science and suggested that the constant in their equation might be the conversion factor between inches and centimeters (2.54). Emily tested their new model ($s = \frac{aw}{2l}$) with their collected data and saw that this did not work.

Anna no longer believed that the constant was related to a conversion factor, but she still wanted to find a value for their constant that was more accurate; Emily disagreed. Emily liked the fact that the constant in the denominator was a "nice number" (an integer) because "that's how a lot of science things look." Anna, however, suggested that they rearrange their equation so that the constant was isolated.

With the equation $c = \frac{aw}{sl}$, Emily suggested that they calculate values for their constant using their collected data. Anna and Emily worked together and found that the constant ranged from 2.08 to 2.27. Emily suggested that if the constant was an irrational number, they would never be able to calculate the constant exactly. Anna suggested that the constant could be either pi ($\pi \approx 3.14$) or e ($e \approx 2.72$), but it did not take much time for them to recognize that these values did not work in their equation.

Anna and Emily now had $s = \frac{aw}{2.2l}$ as their model. Emily suggested that the only way they could get a more accurate constant would be if they collected more data. She recalled that they had determined the value of their constant using the data collected at 1.0 A, 2.0 A, and 3.0 A, but they still had the data they collected at 3.16 A. Using ($s = \frac{aw}{2.2l}$), Emily confirmed that the magnetic field strength predicted by this equation matched the data they collected at 3.16 A. At this point, both Anna and Emily agreed that their final model was $s = aw/2l$.

Phase 2: Reflections From All Eight PSMTs

A few weeks after completing the Deriving Ampere's Law activity in the mathematics pedagogy course, the PSMTs reflected on the experience. A number of themes emerged from their responses to the postactivity questionnaire (see **Appendix A**) related to (a) the value of the activity for PSMTs themselves, (b) perceived benefits for students working through the activity, and (c) issues connected with implementing similar model-eliciting activities in their own future classrooms. The PSMTs said that working through the Deriving Ampere's Law activity helped them learn more about mathematical modeling, the importance of planning and collaborating with colleagues, and the benefits of group work.

The value of the activity for PSMTs. By participating in an MEA themselves, the PSMTs developed a better understanding of different aspects of doing mathematics, including the nature of mathematical formulas and models, the value of collecting sufficient data and taking repeated measurements when developing mathematical models, and the role of collaborative group work when engaging in authentic mathematical problem solving:

Alice: Math can be used to model things in the natural world. It was great to see how formulas are derived.

Erin: I learned that a lot of data is required to make sure your models are accurate and show the correct relationships.

Simon: I think for students experiencing this for the first time, there's enough in this activity to supply them [with] the idea that one or two measurements won't be enough for accurate mathematical models of physical phenomena.

Julie: It was good to get insights from other students instead of possibly being unsure of what to do if working independently.

Erin: Working with the group was helpful because a few people were needed to work the technology and collect the data. But my group members were also helpful when it came to making the hypotheses. It is always good to discuss and talk about hypotheses before collecting the data, and being able to talk with my group members was beneficial to understanding what was happening.

Perceived benefits for secondary students working through the activity. Besides being useful for their own development, these PSMTs expressed thoughts related to the benefits of modeling tasks for students. They mentioned that such activities would engage students and help them see connections between mathematics and science. In particular, the PSMTs thought that MEAs would be more engaging for students and would help students see mathematics as more authentic and purposeful:

Alice: Tasks like this could help students make real connections with material.

Sean: It keeps students engaged with the content. It counteracts the notion that only geniuses can derive mathematical postulates.

Simon: When students have measured and recorded data values for themselves, they might see them as more than arbitrarily chosen numbers picked from a textbook. This gives them an ownership of what they're examining and could allow them to help feel a personal connection to the problem.

Emily: This could help students realize that the subjects they learn in school are often divided a little arbitrarily and are actually way more similar than they might realize.

Naija: One advantage is that it connects mathematical concepts with the real world, which makes the knowledge more meaningful to students. Also it can connect with other classes, in this case physics, so students would be exposed to the same information more than once.

Alice: It allows students the chance to feel like career mathematicians, deriving formulas the way a mathematician would have, and I believe that's a really valuable experience.

Constraints to classroom implementation. By participating in an MEA themselves, these PSMTs realized that implementing such activities is not easy. Because they were concurrently enrolled in a field experience course, almost all of them commented on the difficulties and constraints due to their own inexperience, the materials needed, the necessary class time, and potential classroom management issues:

Anna: These activities require a lot more thought and planning than a simple lecture. There are many facets to think about and account for in order to make the investigation worth it.

Emily: This task also showed me that I really lack the skills to create my own authentic tasks, at least in many science areas.

Sean: I am still worried, personally, about implementing this activity in the classroom, but I think that'll change over time as I become more acclimated to the classroom environment.

Alice: I absolutely loved the hands on aspect of this task, but it may be challenging to implement in a large scale in a classroom.

All the PSMTs saw great value in this activity, not only for themselves but also for secondary mathematics students. They reported that completing the activity helped them better understand the nature of mathematical modeling with real data, experience firsthand the benefits of working as a group on complex tasks, and appreciate the importance of preparation and collaboration in teaching. These PSMTs said that activities like this one would be exciting and engaging for students and, furthermore, could help students see mathematics as authentic and useful. But they acknowledged that implementing such activities in a classroom would not be easy because of the time, materials, and expertise required.

Discussion

Participating in the Deriving Ampere's Law activity was a valuable experience for these PSMTs because it not only revealed their own understanding of the nature of mathematical modeling and their own strategies for solving MEAs (engaging with modeling as *students)*, but it also brought to light their own concerns about integrating MEAs into their future classrooms (engaging with modeling as *teachers*).

Modeling as Students: Strategies for Experimentally Deriving Ampere's Law

Emily and Anna understood that the goal of the activity was to develop a mathematical model that could describe their collected data. Their productive modeling strategies included (a) planning a systematic approach to data collection, (b) identifying direct and inverse variations, and (c) routinely verifying and revising their developed models. However, the two struggled with determining the constant for their final model. And although they were ultimately successful, they experienced several instances when their initial conjectures regarding the constant were completely unrelated to the context at hand.

Emily and Anna analyzed their collected data by looking for relationships between the independent variables and the dependent variable, which was an approach that came about organically for these PSMTs. Emily and Anna also considered informal models for each of the independent variables before arriving at their model. Both Emily and Anna referred to direct and inverse variation by name when describing the relationships between the independent variables and the dependent variable. With an initial model structure in mind, Emily and Anna used their collected data to confirm whether their model was accurate. Their conclusion then informed their next steps for model revision. They continued this cycle of testing and revising until they settled on a generalizable model that they felt predicted their collected data accurately.

Grappling with the constant. Although Emily and Anna had no trouble with recognizing whether the independent variables were directly or inversely related to their dependent variable, they did struggle with identifying the constant in their final model. Recall that before using their collected data to solve for the constant, they tried the conversion factor from inches and centimeters (2.54) and the mathematical constants pi and e. Although recognizing that these values did not work in their equation did not take much time, the fact that these values had no relation whatsoever to the problem at hand did not deter them from testing the values in their final model.

Emily and Anna seemed to focus on "obvious candidates" for the constant. They initially chose numbers they were familiar with and that were reasonably close to the actual constant, even though these numbers had nothing to do with the situation they were asked to model. Emily and Anna were able to quickly rule out these proposed values for the constant after testing the model using their collected data, yet they seemed to focus on numbers because of their magnitude rather than the real-world context of the problem.

Modeling as Teachers: Incorporating MEAs Into the Classroom

To these PSMTs, the Deriving Ampere's Law activity was unique and nontraditional. Working through the task was a truly integrated STEM experience for them because this activity not only involved developing a mathematical model of a scientific law but also helped them better understand the science behind solenoids and how they can be designed to meet identified specifications. These experiences helped the PSMTs see how engaging mathematical modeling activities can be to students and what can be learned through model-eliciting activities. The PSMTs reported that participating in the activity encouraged them to reconsider how these types of activities could be used in their future classrooms.

Because the Deriving Ampere's Law activity was so different from what these PSMTs had previously experienced, participating in an MEA encouraged them to consider developing such activities themselves. Many PSMTs might not be in a position to come up with these sorts of tasks without support, so incorporating tasks like this into teacher education courses can be a worthwhile endeavor. Other MEAs that we have implemented in our mathematics methods course include modeling the relationship between voltage and sound pressure level, the relationship between the ratio of resistors in an amplifier circuit and the increase in loudness, and equal temperament fret spacing on stringed instruments. Incorporating MEAs into a mathematics methods course gives preservice teachers the chance to begin visualizing how they might create authentic contexts for their future students to explore mathematical modeling.

Limitations

This research study has several potential limitations. The first limitation is that researcher bias may have influenced data analysis. The first author developed the materials for the Deriving Ampere's Law activity, giving her an intimate understanding of the task as well as her own modeling strategies. The second author was the co-PI on the supporting National Science Foundation grant. To mitigate how their roles may have influenced data analysis, both authors recorded field notes separately and also triangulated their findings across multiple data sources.

The second limitation is related to the participants' recent experiences with the mathematics and science content related to the activity. The PSMTs who participated in the activity were also enrolled in a concurrent "maker" course where they designed artifacts that incorporated solenoids (e.g., motor, speaker), giving them prior experience with solenoids that other PSMTs may not possess. However, on the basis of our experiences implementing the Deriving Ampere's Law activity with other groups of participants, we concluded that limited knowledge of either direct and inverse variation or solenoids does not make the activity inaccessible (see Corum & Garofalo, 2018). Similarly, having prior knowledge of Ampere's law itself does not prohibit someone from participating in the activity, as the Deriving Ampere's Law activity challenges students to develop a mathematical model to describe data collected experimentally.

The third limitation is related to the amount of time required to develop and calibrate the materials used during the Deriving Ampere's Law activity. All the solenoid bobbins were printed using a 3D printer, and the solenoids themselves were then hand-wound and calibrated by the first author. In addition to the solenoids, supplemental materials (e.g., variable DC power supply, magnetic field sensor) are required to complete the activity. To attend to this limitation, the authors have made available the related 3D printing file and materials list (see **Appendix B** and download the 3D printer file for Deriving Ampere's Law activity here: Ampere's Law activity: **https://www.nctm.org/Publications/Mathematics-Teacher-Educator/2019/Vol8/Issue1/Engaging-Preservice-Secondary-Mathematics-Teachers-in-Authentic-Mathematical-Modeling_-Deriving-Ampere_s-Law/**).

Conclusion

The eight preservice mathematics teachers who participated in the Deriving Ampere's Law activity were able to experimentally derive Ampere's Law and develop a mathematical model that related three independent variables (number of wraps of wire, electric current, and solenoid length) to a single dependent variable (strength of the magnetic field produced by a solenoid). Despite the limitations described above, the findings from this study indicate the value of incorporating mathematical modeling activities into teacher preparation courses. Incorporating MEAs into our secondary mathematics pedagogy course allowed our preservice teachers to experience mathematical modeling as both *students* and future *teachers*. By engaging in the modeling process themselves, the PSMTs who participated in this study not only became more aware of their own understanding of mathematical modeling but also considered how they might incorporate model-eliciting activities into their future classroom instruction.

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Appendix A: Ampere's Law Task Reflection Questionnaire

Self-Reflection:

1. What kinds of mathematical thinking did you do while completing the task? Please briefly explain.

2. What mathematics concepts or procedures were used or needed in doing the task? Please briefly explain.

3. What did you learn about the nature of mathematical modeling and the processes involved in creating mathematical models?

4. What did you learn about formulas, science, or the nature of science?

5. Please describe the experience of working with your group. What was the value of group work in doing this task?

6. What technology features did you learn, or relearn, when doing this task?

7. Did you struggle with any aspects of this task? Please describe or explain. What, if anything, did you learn from that struggle?

Task Reflection:

1. How did doing this task affect your thinking about developing and providing authentic tasks for students?

2. What learning-related advantages or benefits do you see in giving assignments like this to middle school or high school students?

3. What possible drawbacks or problems do you see in giving assignments like this?

4. Do you have any further thoughts or comments on using such tasks in class or for homework?

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Appendix B: List of Materials Used When Implementing the Deriving Ampere's Law Activity

Solenoids of various lengths/windings

(3) 2-inch solenoids varying in coil windings (50, 100, 150)

(3) 50-wrap solenoids varying in length (1-inch, 3-inch, 4-inch)

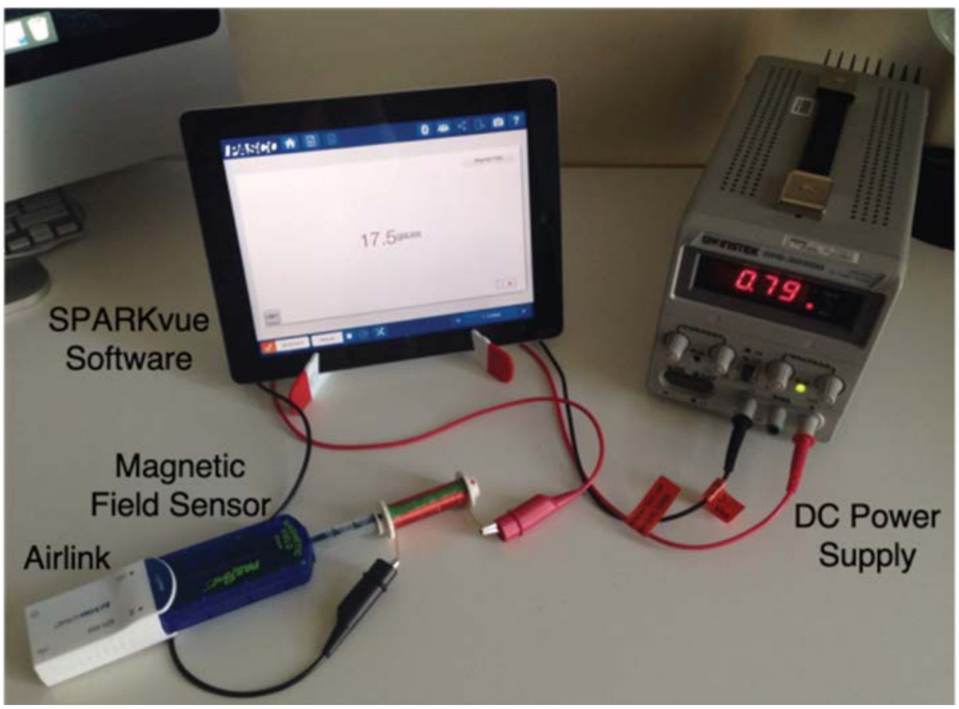
PASCO PASPORT Magnetic Field Sensor (PS-2112)

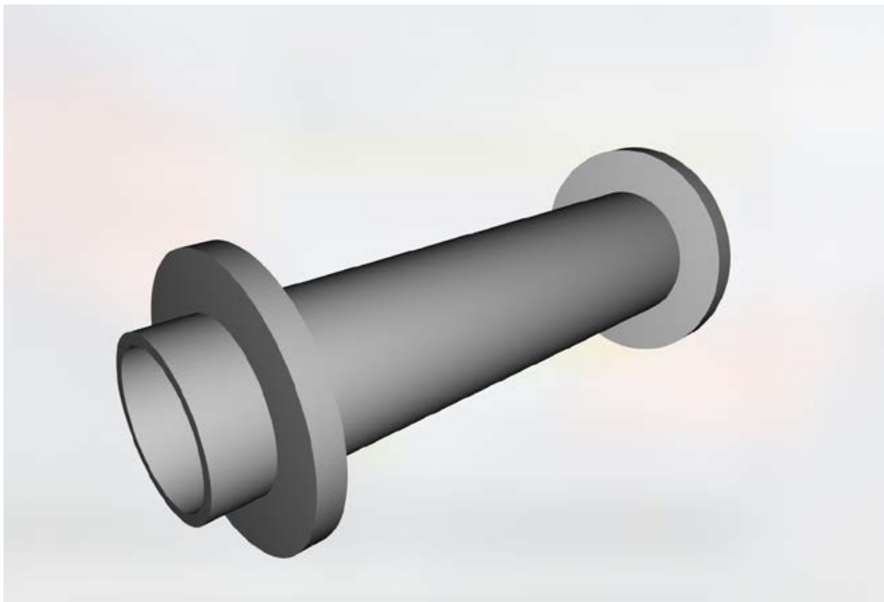
PASCO Airlink (PS-3200) or PASCO Universal Interface

SPARKvue software (can be installed on a computer, tablet, or mobile device)

DC power supply

Set of banana plug to alligator clip test leads





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Appendix C: Adapted Version of the Deriving Ampere's Law Activity

Create a single mathematical model that relates the strength of the magnetic field generated by a solenoid to: (1) number of coils of wire, (2) length of the solenoid, and (3) current output.

Data Table 1. Magnetic field strengths generated by 2-inch solenoids with *varying coils of wire*

Wraps	Current	Field Strength	Current	Field Strength
50	3.16 Amps	36 Gauss	3 Amps	33 Gauss
100	3.16 Amps	72 Gauss	3 Amps	67 Gauss
150	3.16 Amps	107 Gauss	3 Amps	102 Gauss

Data Table 2. Magnetic field strengths generated by 50-wrap solenoids of *varying length*

Length	Current	Field Strength	Current	Field Strength	Current	Field Strength
1 inch	1 Amp	23 Gauss	2 Amps	46 Gauss	3 Amps	69 Gauss
2 inches	1 Amp	12 Gauss	2 Amps	23 Gauss	3 Amps	33 Gauss
3 inches	1 Amp	8 Gauss	2 Amps	16 Gauss	3 Amps	23 Gauss
4 inches	1 Amp	6 Gauss	2 Amps	12 Gauss	3 Amps	18 Gauss

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Using STEM-Related Modeling Tasks to Support Preservice Teachers' Understanding of Mathematical Modeling

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