

# What does inflation say about dark energy given the swampland conjectures?

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We discuss the relations between swampland conjectures and observational constraints on both inflation and dark energy. Using the requirement  $|\nabla V| \geq cV$ , with  $c$  as a *universal* constant whose value can be derived from inflation, there may be no observable distinction between constant and nonconstant models of dark energy. However, the latest modification of the above conjecture, which utilizes the second derivative of the potential, opens up the opportunity for observations to determine if the dark energy equation of state deviates from that of a cosmological constant. We also comment on the observability of tensor fluctuations despite the conjecture that field excursions are smaller than the Planck scale.

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## I. INTRODUCTION

The discovery of the accelerating expansion of the Universe [1,2] was a huge surprise to the community. Because gravity only *pulls*, it should hinder the expansion of the Universe after the big bang, and hence the expansion should decelerate. Acceleration implies that there is a substance in the Universe that *pushes* the expansion. It was dubbed dark energy. The most discussed candidate for dark energy is the cosmological constant  $\Lambda$ , a finite energy density of the vacuum, due to the simple way it can be implemented into cosmological models based on general relativity. However, despite being consistent with data [3], the 120 orders of magnitude difference between the observed vacuum energy density ( $\rho \approx (\text{meV})^4$ ) and the naive theoretical expectation ( $\rho \approx M_{\text{Pl}}^4$ ) still remains the most challenging problem in modern physics [4].

Since dark energy and the cosmological constant problem inevitably involve quantum gravity, string theory, as a theory of quantum gravity, should address these topics. The attempts to construct de Sitter solutions (spacetime solutions to general relativity with a positive  $\Lambda$ ) in string theory [5–7] have led to the notion of the string landscape.

The landscape consists of an enormous number of vacua, each described by different low-energy effective field theories (EFTs) of different fields and parameters. String theory therefore supports the anthropic argument [8], namely, that the value of the observed dark energy density is what it is because otherwise human civilization could not exist. If we really live in a (meta)stable vacuum in the string landscape where a constant vacuum energy explains dark energy, then there is no point in measuring the dark energy equation-of-state parameter  $w = p/\rho$ , where  $p$  and  $\rho$  are the pressure and energy density of the dark energy, respectively.

String theory seems to lead to many possible low-energy EFTs, so conversely one can ask what criteria a given low-energy EFT should satisfy in order to be contained in the string landscape. For the last decade, several criteria of this kind, dubbed swampland conjectures, have been proposed [9–11]. These can have important cosmological implications. For instance, one of the relatively well-established conjectures is the distance swampland conjecture [10,12–24], which implies that scalar fields in a low-energy EFT of a consistent theory of quantum gravity cannot have field excursions much larger than the Planck scale since otherwise an infinite tower of states becomes exponentially light and the validity of the EFT breaks down. In other words, one has the constraint [25]

$$\Delta\phi \lesssim \alpha M_{\text{Pl}}, \quad \alpha \approx O(1). \quad (1)$$

In the context of inflation, field excursions are related to the tensor-to-scalar ratio  $r$  by the Lyth bound [26],

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$$\frac{\Delta\phi}{M_{\text{Pl}}} \simeq \sqrt{\frac{r}{8}} \mathcal{N} \quad (2)$$

where  $\mathcal{N}$  is the number of  $e$ -folds of inflationary expansion. Clearly the distance conjecture, Eq. (1), limits the possibility of measuring tensor modes and hence primordial  $B$ -modes in the cosmic microwave background (CMB). Naively, with  $\mathcal{N} \gtrsim 50$ , we find  $r \lesssim 0.003$ , which is on the edge of observability for future experiments [27,28].

The attempts to construct de Sitter solutions or inflationary models in string theory [7,29–39] have sparked discussions on various issues with such constructions, as well as no-go theorems [40–65]. Motivated by the obstructions encountered in various attempts, the de Sitter swampland conjecture was proposed [66], which states that the scalar potential of a low-energy limit of quantum gravity must satisfy

$$M_{\text{Pl}} |\nabla V| \geq cV, \quad c \approx O(1) > 0 \quad (3)$$

where  $\nabla$  denotes the gradient with respect to the field space, and the norm of the gradient is defined by the metric on field space. Whether the conjecture holds true is still an open debate [67–87]. Yet, even before the debate is settled, it is interesting and important to investigate both its consequences in cosmology and potential modifications or extensions [88–124]. The primary implication of this condition is that the observed positive energy density of our Universe should correspond to the potential of a rolling quintessence field rather than a positive  $\Lambda$  [125]. The fact that one can easily embed any quintessence model into supergravity [126,127] in a rather simple fashion, despite the fact that supersymmetry breaking generically spoils the flatness of the quintessence potential, is also encouraging. This raises the hope that  $w \neq -1$  might be detected.

The de Sitter conjecture forbids (meta)stable vacua with positive energy density, so it is not surprising that the inflationary paradigm has apparent conflicts with the conjecture and one may call for a paradigm shift. Nonetheless, one can also adopt a conservative approach and regard the conjecture as a parametric constraint where the inequality holds but the number  $c$  may not be strictly  $\mathcal{O}(1)$  [99]. From this perspective, constraints on inflation can then be used to constrain  $c$ .

However, if we follow this route, the optimism that one can observe  $w \neq -1$  is greatly diminished. To see this, recall that in single-field slow-roll inflation, the slow-roll parameters of the potential are defined as

$$\epsilon_V \equiv \frac{M_{\text{Pl}}^2}{2} \left( \frac{V'}{V} \right)^2, \quad \eta_V \equiv M_{\text{Pl}}^2 \frac{V''}{V}, \quad (4)$$

where the primes denote derivatives with respect to the inflaton. The distance conjecture limits the inflaton field excursion  $\Delta\phi \approx \sqrt{2\epsilon_V} \mathcal{N} \lesssim \mathcal{O}(1)$ , and therefore the

necessary number of  $e$ -folds  $\mathcal{N} \approx 50$  forces  $c \lesssim \sqrt{2\epsilon_V} \lesssim \mathcal{N}^{-1} \sim 0.02$ . On the other hand, the number  $c$  in Eq. (3) is meant to be *universal* in a given EFT. Therefore, the current accelerating expansion must involve a quintessence field  $Q$  whose potential  $V_Q$  must satisfy

$$1 + w = \frac{2(V'_Q)^2}{(V'_Q)^2 + 6V_Q^2} > \frac{2c^2}{6 + c^2} \equiv \Delta \gtrsim 1.33 \times 10^{-4}. \quad (5)$$

Although this does not exclude observable quintessence, given the fact that so far almost all observations are consistent with a cosmological constant, such a small lower bound on a possible deviation of  $w$  from  $-1$  makes it questionable if it is worthwhile to push the sensitivity of the observations further. We may never know whether the Universe is de Sitter or quintessence.

However, the original de Sitter conjecture, Eq. (3), was so strong that even the Higgs potential was in tension with it [98]. The conjecture was also in tension with the well-understood supersymmetric AdS solutions [81]. Recently the refined de Sitter swampland conjecture was proposed [101,128], which states that the scalar potential of a low-energy theory that can be consistently coupled to quantum gravity should satisfy *either*

$$M_{\text{Pl}} |\nabla V| \geq cV, \quad c \approx O(1) > 0, \quad (6)$$

or

$$M_{\text{Pl}}^2 \min(\nabla_i \nabla_j V) \leq -c'V, \quad c' \approx O(1) > 0, \quad (7)$$

where  $\min(\dots)$  denotes the minimum eigenvalue of the Hessian  $\nabla_i \nabla_j V$  in an orthonormal frame of the scalar field space. With this refinement, the aforementioned conflicts with the Higgs potential and the SUSY AdS solutions are resolved. The refined conjecture also raises new possibilities for inflation. In particular, one can evade the strict bound on  $c$  arising from the distance conjecture by having the scalar potential satisfy the second condition, Eq. (7), of the new conjecture during part (or all) of inflation. As such, one may regain the hope that observable time-varying dark energy with  $w \neq -1$  can be obtained. See also [129] for a recent discussion on  $w$  considering the refined dS conjecture.

## II. SINGLE-FIELD SLOW-ROLL INFLATION MODELS

Due to the above tension between the de Sitter conjecture and the requirements of inflation, we assume that the inflaton potential switches from one de Sitter condition to another as the inflaton rolls, an idea also utilized in [117]. To be specific, we take the following step-function approach to keep the discussion general and simple: we apply the first condition, Eq. (6), for the initial  $\mathcal{N}_1$   $e$ -folds and apply the second condition, Eq. (7), for the remaining

$\mathcal{N}_2 = \mathcal{N}_{\text{tot}} - \mathcal{N}_1$   $e$ -folds. In our analysis we set  $\mathcal{N}_{\text{tot}} = 50$ . We assume  $\epsilon_V$  and  $\eta_V$  are approximately constant for each interval so that we have

$$\sqrt{2\epsilon_V^{(1)}} \geq c \quad \text{and} \quad \eta_V^{(2)} \leq -c'. \quad (8)$$

Additionally, Eq. (1) requires that

$$\sqrt{2\epsilon_V^{(1)}}\mathcal{N}_1 + \sqrt{2\epsilon_V^{(2)}}\mathcal{N}_2 \leq \alpha \sim \mathcal{O}(1). \quad (9)$$

To maximize  $c$ , we assume  $\epsilon_V^{(2)} < 10^{-4}$  so that the contribution of the second era to Eq. (1) is negligible. Combining Eqs. (8) and (9), we have

$$c < \frac{\alpha - \sqrt{2\epsilon_V^{(2)}}\mathcal{N}_2}{\mathcal{N}_1}. \quad (10)$$

We can also obtain a bound for  $c'$  from the spectral tilt  $n_s = 1 - 2\epsilon - \eta$ , where the Hubble slow-roll parameters are

$$\epsilon = -\frac{\dot{H}}{H^2}, \quad \eta = \frac{\dot{\epsilon}}{H\epsilon}. \quad (11)$$

For single-field inflation models, these are related to the slow-roll parameters of the potential as  $\epsilon_V = \epsilon$  and  $\eta_V = 2\epsilon - \frac{1}{2}\eta$ . Therefore, we can constrain  $\eta_V$  and hence the second parameter of the refined de Sitter conjecture as

$$c' < \frac{1}{2}(1 - n_s(k) - 6\epsilon_V^{(2)}), \quad (12)$$

where we allow for a  $k$ -dependent spectral tilt. Since we assume  $\epsilon_V^{(2)}$  is small, our bounds simplify to

$$(c', c) < \left( \frac{1 - n_s(k)}{2}, \frac{\alpha}{\mathcal{N}_1} \right). \quad (13)$$

Equation (13) is valid until  $\mathcal{N}_1 = \mathcal{N}_{\text{tot}}$ , at which point the derivation on the bound of  $c'$  above no longer applies, and the only constraint one finds is that  $c < \alpha/\mathcal{N}_{\text{tot}}$ . To proceed, we utilize the Planck analysis based on TT, TE, EE, lowE, lensing and BAO [3], which gives

$$dn_s/d\ln k = -0.0041 \pm 0.0067, \quad (14)$$

$$n_s = 0.9659 \pm 0.0040, \quad (15)$$

at  $k_* = 0.05 \text{ Mpc}^{-1}$ . We add errors in quadrature, ignoring correlations, and use

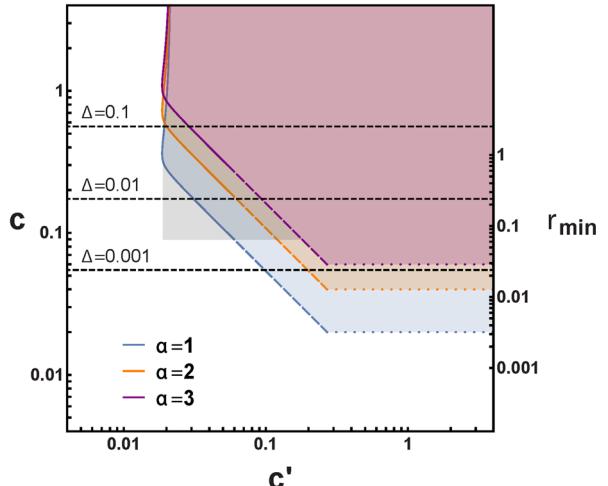


FIG. 1. Bounds on swampland parameters for generic single-field inflation models at the  $1\sigma$  level assuming the running of  $n_s$  can be extended to  $\mathcal{N}_{\text{tot}} = 50$   $e$ -folds. The unshaded region is the allowed parameter space. The solid lines are for  $\mathcal{N}_1 \leq 10$ , the dashed lines are for  $10 < \mathcal{N}_1 < 50$ , and the horizontal dotted lines correspond to  $\mathcal{N}_1 = 50$ ; i.e. the first constraint, Eq. (6), applies to the whole inflationary period. The values of  $c$  excluded by [131] are shaded in grey. We use the distance conjecture with  $\Delta\phi \leq \alpha M_{\text{Pl}}$  and display the minimum values for  $1 + w \geq \Delta$  with black dashed lines. With the original de Sitter conjecture,  $c$  has to be below the dotted horizontal lines, but there are no constraints on  $c'$ .

$$n_s(k) = 0.9659 - 0.0041 \ln \frac{k}{k_*} \pm \sqrt{(0.0040)^2 + \left( 0.0067 \ln \frac{k}{k_*} \right)^2}. \quad (16)$$

A smaller  $n_s$  allows for larger  $c'$  in Eq. (13), so we take the  $1\sigma$  allowed lower end in order to place our bounds. The weak correlation between  $n_s$  and  $dn_s/d\ln k$  we see in Fig. 26 of [3] actually works in our favor, and ignoring the correlation is therefore the more conservative approach (i.e., it gives a smaller allowed range) [130]. Using the simple relationship  $\mathcal{N}_1 = \ln(k/a_0 H_0)$ , where  $a_0$  is the present scale factor and  $H_0$  is the present Hubble scale, we can constrain the swampland parameters in single-field inflation as shown in Fig. 1. The current CMB constraints on the spectral index and its running are limited to  $\mathcal{N}_1 \lesssim 10$ . This range is denoted by the solid lines in Fig. 1. Beyond this there are no strong observational constraints, and we extend our analysis by extrapolating Eq. (16) to  $\mathcal{N}_1 \geq 10$  shown by the dashed lines in Fig. 1. The unshaded regions indicate values of  $(c', c)$  that satisfy the above inequalities. The vertical asymptotes correspond to satisfying Eq. (7) for the entirety of the inflationary epoch,  $\mathcal{N}_1 = 0$ , so that  $c$  is left completely arbitrary but  $c'$  has a strict upper bound that is much less than the  $\mathcal{O}(1)$  expectation. The horizontal dotted lines correspond to

satisfying the first constraint, Eq. (6), for all of inflation,  $\mathcal{N}_2 = 0$ , which leaves  $c'$  arbitrary but severely limits  $c$ . The horizontal black dashed lines indicate the lowest values of  $c$  that yield the given  $\Delta$  defined in Eq. (5) as the lower bound on  $1+w$  from the constraint, Eq. (6). Finally, the grey region excludes values of  $c$  that may satisfy Eq. (13), depending on the value of  $\alpha$ , but conflicts with the constraint  $r_{0.002} < 0.064$  [131], as  $r = 16\epsilon \geq 8c^2$ . The grey excluded region has a left vertical boundary since the constraint applies only to  $k > 0.002 \text{ Mpc}^{-1}$ .

We also comment on the observability of the tensor mode  $r$ . The swampland distance conjecture, Eq. (1), combined with the Lyth bound, Eq. (2), is normally believed to disfavor observably large  $r$ , assuming  $\alpha \approx 1$ . The best sensitivity anticipated in the future is  $r \sim 10^{-3}$  [27,28]. There is a parameter region in Fig. 1 where  $r \geq r_{\min} \equiv 8c^2$  is close to the current observational bound. Physically this is because, in our spirit of a step function approximation, we can allow for a brief initial period, say  $\mathcal{N}_0 \sim 4$ , where the upper bound on  $\epsilon$  from the distance conjecture,  $\epsilon \lesssim \mathcal{N}_0^{-2}/2 \sim 0.03$ , is relaxed. Thus, it is possible to have  $r$  large enough to saturate the observational bound at low  $\ell$ . This is encouraging, especially for space-borne CMB  $B$ -mode experiments such as LiteBIRD [28].

### III. MULTIFIELD SLOW-ROLL INFLATION MODELS

The constraints discussed above are due to the tight relations between  $n_s$ ,  $\epsilon_V$ ,  $\eta_V$ , and  $r$  in single-field slow-roll inflation models. It is natural to ask whether the constraints can be relaxed in multifield models. In our analysis below, we take the conservative assumption that the swampland distance conjecture applies to the proper length of the trajectory, instead of the geodesic distance between the starting and ending points in the field space.

We discuss here a class of multifield models where directions orthogonal to the slow-roll direction are massive,  $M \gtrsim H$ . The inflaton therefore rolls near the bottom of the valley, which has “bends” in the multidimensional field space. The main difference here is that the local angular velocities of the inflaton around the bends can modify the effective sound speed  $c_s$  of fluctuations. As a result, we have the modified relation [132]

$$12\eta_V = (c_s^{-2} - 1) \frac{M^2}{H^2} + 2 \frac{M^2}{H^2} + 3(4\epsilon - \eta) - 2\sqrt{\left(\frac{M^2}{H^2} - \frac{3}{2}(4\epsilon - \eta)\right)^2 + 9(c_s^{-2} - 1) \frac{M^2}{H^2}}. \quad (17)$$

Here,  $\eta_V$  is the minimum eigenvalue of the Hessian,  $M$  is the effective mass of the field orthogonal to the slow-roll direction, and  $c_s$  is given by

$$c_s^{-2} = 1 + \frac{4\Omega^2}{M^2}, \quad (18)$$

where  $\Omega$  is the local angular velocity describing the bend of the inflaton trajectory in the potential. Note that in the limit  $\Omega \rightarrow 0$ , the sound speed reduces to unity and  $\eta_V$  to the expression of the single-field models. Allowing for a significant deviation of  $c_s$  from unity relaxes the constraints on  $(c, c')$ , as shown in Fig. 2, where we set  $M = H$ . This allows for larger values of  $c$  and  $c'$  compared to the single-field case, which are preferred by the swampland conjecture. Note that lowering the sound speed further will not achieve  $\mathcal{O}(1)$  values for  $c'$  because our scenario relies on having negative  $\eta_V$ . As  $c_s$  is reduced from unity,  $\eta_V$  initially becomes more negative and widens the allowed parameter space. Beyond some critical value  $c_s \approx 0.3$ , further reduction of  $c_s$  makes  $\eta_V$  less negative, thereby narrowing the allowed parameter space. For  $c_s \lesssim 0.2$ ,  $\eta_V$  becomes positive and our analysis no longer holds. Empirically, we find that  $c_s \sim 0.24$  maximizes the allowed parameter region in the  $(c', c)$  plane. The grey shaded regions again correspond to experimental constraints on  $r = 16\epsilon c_s$ , but their area is greatly reduced as  $c_s$  decreases.

It is also interesting to note that we expect primordial equilateral and orthogonal non-Gaussianities once  $c_s \neq 1$  in this class of models [132],

$$f_{\text{NL}}^{\text{equil}} = -(c_s^{-2} - 1)(0.275 + 0.078c_s^2), \quad (19)$$

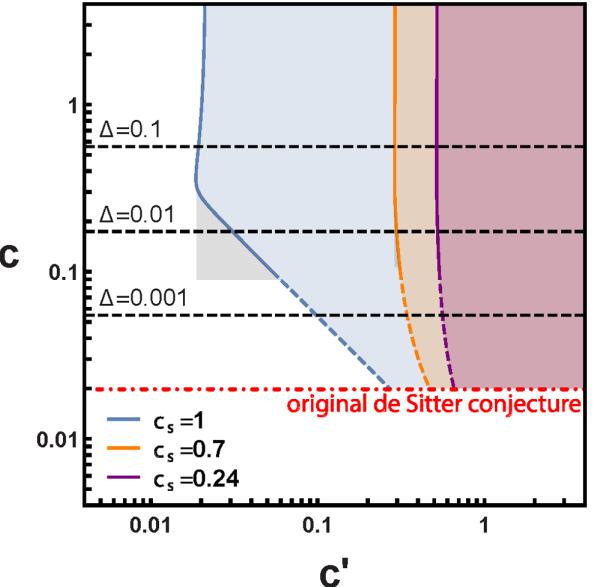


FIG. 2. Bounds on swampland parameters for generic multifield inflation models. We take  $\alpha = 1$  and  $M = H$ . Here,  $c_s$  is the sound speed for fluctuations, and the rest is the same as in Fig. 1. With the original de Sitter conjecture, Eq. (3), and single-field slow-roll models,  $c$  has to be below the red dot-dashed horizontal line.

$$f_{\text{NL}}^{\text{ortho}} = (c_s^{-2} - 1)(0.0159 - 0.0167c_s^2). \quad (20)$$

Here we have ignored the third order parameter. The current observational constraint on the sound speed is  $c_s \geq 0.024$  [see Eq. (89) of [133]], which is an order of magnitude below the limit we can reach in our setup, as shown in Fig. 2. Future observations combining CMB lensing, galaxy and 21 cm surveys, Lyman  $\alpha$  forest, etc., have the potential to improve the constraint on  $f_{\text{NL}}$  by an order of magnitude or more [134].

#### IV. IMPLICATIONS FOR DARK ENERGY

The de Sitter conjecture states that constants  $c$  and  $c'$  are *universal* and should apply to all sectors in a given EFT. Therefore, we can use inflationary physics to get a handle on the values of  $c$  and  $c'$  and apply this knowledge to the quintessence potential  $V_Q$ . When this argument is applied to single-field inflation models with conjectures Eqs. (3) and (1), one deduces that there may be little hope in finding  $w \neq -1$  due to the small lower bound seen in Eq. (5). This depressing outlook is drastically changed in light of Eqs. (6) and (7), as Fig. 1 illustrates. We see that the refined de Sitter conjecture has allowed for the possibility of having  $\Delta$  bounded from below such that it must be larger than a few percent and should be observable to experiments. Current and future experiments, such as DES [135], HSC [136], DESI [137], PFS [138], LSST [139], Euclid [140], andWFIRST [141], are aiming for an accuracy of about a percent in  $w$ . The cost for this is that  $c'$  must be much lower than the  $\mathcal{O}(1)$  expectation of [101,128] in the single-field case. This seems to indicate that single-field inflation falls more in line with the modified de Sitter conjecture discussed in [92], where the smallest Hessian eigenvalue needs to only be negative when  $|\nabla V| < cV$ .

This state of affairs is altered by considering multifield inflation models. Not only could  $\Delta$  be forced to be as large

as several percent, it is also possible to have both  $c$  and  $c'$  approximately  $\mathcal{O}(1)$  as long as the sound speed is low enough, as seen in Fig. 2. In either the single-field or multifield scenario, a better theoretical understanding of the magnitude of  $c'$  is essential to understand the consistency of the swampland conjectures and inflation.

#### V. CONCLUSIONS

In this paper, we studied the consequences of the latest swampland conjecture on inflation and dark energy. The original de Sitter conjecture raised the hope that measuring the dark energy equation of state  $w$  would be promising but simultaneously dashed that hope since the consistency with single-field inflation suggests that the deviation from  $w = -1$  would likely be unobservable. As we have shown, this situation is much more encouraging with the refined de Sitter conjecture. Not only could  $w \neq -1$  be observable even with a single-field inflationary scenario, but tensor modes could be as well. If one considers multifield inflationary scenarios, then the prospect for observing  $w \neq -1$  is better, and one gains improved agreement with the swampland conjectures.

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