Coastal Dynamics and Adaptation to Uncertain Sea Level Rise: Optimal Portfolios for Salt Marsh Migration

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Abstract

The sustainability of dynamic natural systems often depends on their capacity to adapt to uncertain climate-related changes, where different management options may be combined to facilitate this adaptation. Salt marshes exemplify such a system. Marsh sustainability under rapid sea level rise requires the preservation of transgression zones - undeveloped uplands onto which marshes migrate. Whether these uplands eventually become marsh depends on uncertain sea level rise and natural dynamics that determine migration onto different land types. Under conditions such as these, systematically diversified management actions generally outperform ad hoc or non-diversified alternatives. This paper develops the first adaptation portfolio model designed to optimize the benefits of a migrating coastal system. Results are illustrated using a case study of marsh conservation in Virginia, USA. Results suggest that models of this type can enhance adaptation benefits beyond those available through current approaches.

KEYWORDS: benefit; diversification; risk; spatial; climate change; wetland; optimal conservation; geomorphology.

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1 Introduction

The sustainability of dynamic natural systems often depends on their capacity to adapt to climate-related changes. The uncertainties associated with such changes and the effect of management responses pose challenges for economic analysis and decision-making (Ando and Mallory 2012; Heal and Millner 2014; LaRiviere et al. 2018; Newbold and Marten 2014; Pindyck 2007). Migrating coastal systems such as salt marshes exemplify the type of dynamics for which climate-related uncertainty is relevant for management. Salt marshes are regularly flooded intertidal habitats that provide multiple ecological functions (Vernberg 1993). The value of these systems is well established and has been recognized as an important motivation for coastal management (Barbier et al. 2011, 2013; Gopalakrishnan et al. 2018; Interis and Petrolia 2016; Johnston et al. 2002a,b, 2005; Milon and Scrogin 2006; Petrolia et al. 2014). Until recently salt marshes have been largely resilient to changes in sea level due to natural adjustments in elevation via vegetation growth and sediment accretion, and by migrating landward as sea levels rise (Kirwan et al. 2010, 2016a). However, there is now widespread concern about the loss of marsh benefits given the accelerated and uncertain rise in sea level, with regional and global analyses forecasting a $20 - 45\%$ marsh loss by 2100 (Craft et al. 2009; McFadden et al. 2007).

Efforts to sustain salt marshes typically emphasize the preservation of transgression zones - undeveloped uplands onto which marshes can migrate landward as sea levels rise. Given limits in the extent to which marshes can build elevation naturally, the preservation of these zones—often via fee-simple purchase by conservation organizations—is necessary to ensure marsh persistence in many areas (Enwright et al. 2016; Field et al. 2017; Kirwan and Megonigal 2013; Kirwan et al. 2016b; Torio and Chmura 2013). The extent to which preserved transgression zones eventually become marsh, however, depends on uncertain future sea level rise (SLR) and natural dynamics that determine when and where marshes migrate (Enwright et al. 2016; Feagin et al.

2010; Kirwan et al. 2016b; Smith 2013). Different types of transgression zones, e.g., different land types at different elevations and/or locations, will hence differ in their expected productivity of future marsh "supply," where this dynamic productivity is subject to uncertainty. For example, preserving low-elevation land at marsh edges may allow marsh migration at low SLR, but will be ineffective at high SLR, which causes these low-lying areas to submerge or drown. Preserving higher-elevation land will be ineffective at low SLR (because these areas remain above the marsh edge), but can enable migration at high SLR.

Because the tendency of marshes to migrate onto different types of preserved upland depends on uncertain SLR, systematic diversification of this preservation offers a potential means to enhance management benefits. This reflects the capacity of diversified approaches to hedge against risk (Ando and Mallory 2012). Insufficiently diversified preservation increases the risk that marshes and their benefits will decline or even vanish, despite efforts to ensure marsh migration. Optimal diversification can minimize such risks, much as it does in financial portfolios (Markowitz 1952; Merton 1969, 1971). The potential benefits of diversification are further evident if one recognizes that decision-makers are often risk averse (Berrens 2001; Holt and Laury 2002).

Despite recognition of concepts related to asset diversification within economics and the relevance of uncertainty for decision-making, optimal portfolio models are relatively sparse within environmental economics. Nonetheless, this literature demonstrates that often substantial gains are available through systematic treatment of diversification. For example, Ando and Mallory (2012) demonstrate the benefits of optimal portfolio design for habitat conservation subject to climate-related uncertainty (cf. Mallory and Ando 2014; Shah and Ando 2015). Sanchirico et al. (2008) find similar benefits within ecosystem-based fishery management. Leroux and Martin (2016) and Leroux et al. (2018) demonstrate the advantages of optimal water supply portfolios. Related work by Van 't Veld and Plantinga (2005) finds that optimal portfolios of greenhouse-gas mitigation depend on carbon-price paths.[1] Yet while the capacity to reduce risk via hedging can have first-order implications for management (LaRiviere et al. 2018), economic analyses of conservation decisions rarely capitalize on the insights available from portfolio theory. Moreover, we are aware of no economic model able to inform management diversification of this type for dynamic, migrating coastal systems such as salt marshes.[2]

This paper develops the first portfolio model designed to optimize the benefits of a migrating coastal system. The model focuses on the diversification of transgression zone investments to maximize marsh conservation benefits, while hedging risk across transgression zone types. The model also provides insight into how diversification can be adapted to address factors such as differing SLR expectations and preservation additionality, among other features. Unlike most diversification models in the environmental literature that derive empirical solutions using modern portfolio theory (Markowitz 1952; e.g., Ando and Mallory 2012; Mallory and Ando 2014; Sanchirico et al. 2008), here we develop a dynamic model following Merton (1969, 1971) that provides closed-form analytical solutions (Bretschger and Vinogradova 2017; Leroux and Martin 2016, Leroux et al. 2018).[3] An empirical illustration is provided using an application to the Virginia Coast Reserve Long Term Ecological Research (LTER) site on the Eastern Shore of Virginia, USA, focusing on portfolios that preserve agri-

[1] Portfolio analyses in ecology and engineering address such topics as species biodiversity and flood management (Aerts et al. 2008; Crowe and Parker 2008; Figge 2004; Koellner and Schmitz 2006; Moore et al. 2010; Schindler et al. 2010; Yemshanov et al. 2014; Zhou et al. 2012). Additional work in economics considers diversification in environmental and resource management, but does not develop optimal portfolios (e.g., Gourguet et al. 2014; Jardine and Sanchirico 2015; Kasperski and Holland 2013; Sethi et al. 2014).

[2] Even analyses proposed as a means to inform salt marsh conservation under uncertainty fail to consider the role of diversification (e.g., Propato et al. 2018).

[3] Bretschger and Vinogradova (2017) derive analytical solutions for the optimal consumption profile and aggregate income share allocated to national emission abatement with uncertain benefits. In the context of water, closed-form solutions for the optimal water consumption path and composition of an urban water supply are derived by Leroux and Martin (2016) and augmented in Leroux et al. (2018) to allow for stochastic habit formation in water consumption. Our theoretical specification is adapted to represent dynamic coastal resource, and unlike previous dynamic portfolio models allows for the portfolio to be comprised of only risky assets.

cultural land, forest land and intertidal land with established marsh. Our empirical results point to the benefits of emphasizing a particular land type for marsh transgression (higher-elevation agricultural land) that currently represents only a small portion of coastal conservation portfolios. The underlying biophysical model accounts for the dynamic nature of salt marsh geomorphology via a spatial, dynamic, process-based approach. Although we develop the model for marshes, it may be adapted to other systems whose sustainability depends on migration, including beaches, dunes and mangroves (Barbier et al. 2011; Gopalakrishnan et al. 2018; Millar et al. 2007; Parsons et al. 2013).

2 A Portfolio Model for Salt Marsh Migration

The goal of portfolio design is to identify the combination of land types for preservation that exploits risk hedging opportunities to maximize expected marsh benefits, subject to decision-makers' risk aversion and budget. A number of financial portfolio models, including Markowitz (1952) and Merton (1971), could provide a basic foundation for a model of this type. Common to all of these is the premise that in an uncertain world the optimal portfolio efficiently trades off expected portfolio returns against the variance of those returns. Markowitz' framework enables the identification of an efficient portfolio frontier within a static variance-return space, while Merton's model yields the efficient portfolio that maximizes the flow of benefits over time for a given set of risk preferences. The former is predominantly empirical; results are typically obtained via numerical methods given an assumed model structure. The present model is based on the latter approach, which generates closed-form analytical solutions from which general economic intuition can be derived, as well as empirical results from a case study.

Like all models of this type in finance or elsewhere, some aspects of the problem are simplified to promote tractability and to enable a focus on issues that are most relevant to the decision context. The model is also designed around parameters for which empirical estimates are commonly available. The goal is a readily applicable model that provides practical insights for marsh conservation. At the same time, we acknowledge key assumptions and identify mechanisms through which they might be relaxed as part of model extensions.

2.1 Salt Marsh Dynamics Under Sea-Level Rise

Development of the model requires an understanding of biophysical marsh dynamics. The dynamics of salt marsh evolution depend on interactions among hydrology, plant growth, and sediment transport (Fagherazzi et al. 2012; Kirwan and Megonigal 2013; Reed 1995). The change in marsh area over time depends on the ability of marshes to build elevation vertically (or accrete) at rates greater than relative SLR, and/or to migrate onto upland areas at rates faster than erosion at their seaward edge (Kirwan et al. 2016b). The ability of marshes to build elevation naturally is limited by factors such as sediment supply and vegetation growth, and hence vertical accretion can only sustain marshes up to a certain threshold rate of SLR (Kirwan et al. 2010).[4] When the vertical elevation of marsh at any given point cannot keep up with SLR, it eventually "drowns" and becomes open water.

When this occurs, the only way that marsh can be sustained is if it migrates onto adjacent uplands (Feagin et al. 2010; Kirwan et al. 2016b; Kirwan and Megonigal 2013; Torio and Chmura 2013). That is, as seas rise, upland areas adjacent to marshes obtain the ecological conditions (e.g., degree and frequency of inundation, soil salinity) that enable them to become marsh (Anisfeld et al. 2017; Brinson et al. 1995; Raabe and Stumpf 2015; Schieder et al. 2018). The ability of marshes to "migrate" in this way depends on an array of biophysical factors, and on the absence of coastal development

[4]These thresholds differ across marshes as a function of spatially varying factors that determine marsh accretion. Kirwan et al. (2016a, p. 256) estimate these thresholds for marshes on the Gulf and Atlantic coasts of North America and Europe. Their results indicate that "marshes will generally survive relative SLR rates of 10–50 $mm\ yr^{-1}$ during the twenty-first century, depending on tidal range and suspended sediment availability."

or armoring. If adjacent upland areas are armored (e.g., using sea walls), developed, or topographically altered (via artificial sediment deposition), marshes can no longer migrate and will be progressively drowned as they are "squeezed" between hardened uplands and rising seas (Enwright et al. 2016; Torio and Chmura 2013). Hence, preservation of undeveloped and unarmored uplands for marsh migration, typically called preserved marsh transgression zones, is necessary for marsh sustainability, and particularly in areas with rapid coastal development (Kirwan et al. 2016b; Kirwan and Megonigal 2013).

The speed of SLR and coastal development has motivated calls for "urgent attention" and "pre-emptive planning to set aside key coastal areas for wetland migration" (Runting et al. 2017, p. 49). This urgency, also seen in marsh conservation strategies (e.g., The Nature Conservancy in Virginia 2011), informs the economic model that is developed. If marsh conservation actions were seen as less urgent, other model types might be more salient. For example, in some cases the question of optimal preservation might be viewed as a real options problem (Arrow and Fisher 1974) wherein the optimal timing and allocation of land purchases for marsh preservation depend on the speed with which new SLR information becomes available. Models of this general type have been developed to inform conservation under climate change (Leroux and Whitten 2014). However, two properties of the marsh conservation context imply that an optimal portfolio approach is better suited to provide relevant information. First, coastal development pressures in many areas are such that conservation agencies do not have the luxury to "wait and see" for SLR uncertainty to resolve before making decisions. Second, conservation decisions are not irreversible as portfolios can be rebalanced at any time to account for updated information as it becomes available. Reflecting this situation, marsh conservation decisions are typically viewed in terms that are consistent with an optimal portfolio perspective (i.e., how to allocate conservation investments now, based on available information). Hence, we proceed with a portfolio model, while acknowledging that other adaptation contexts may be conducive to real options or other approaches that account for decision time paths.

2.2 Defining Portfolio Assets

To examine natural resource conservation from a portfolio perspective requires that all relevant management options are identified and grouped into classes pertinent for the decision context. These options define the assets over which the portfolio is optimized. Among the choices to be made are (1) how many assets to include, and (2) whether assets are defined in explicit spatial terms. Closed-form models of this type generally include a small number of assets to maintain tractability. Assets are further chosen to match the primary conservation choices under consideration (e.g., Ando and Mallory 2012; Leroux and Martin 2016). In the case of salt marsh conservation these choices include different types of land preservation that provide areas on which marsh may exist now or in the future. These land types are defined as mutually exclusive combinations of land cover or use, elevation, location, and other land characteristics that determine marsh dynamics. Preserving a particular land type yields specific marsh benefits subject to uncertainty. In this sense, land types may be viewed as equivalent to financial assets in an investment portfolio, where each asset is characterized by a mean return and standard deviation.

Here, asset classes are defined based on marsh dynamics and the primary land categorizations used within conservation decision-making. Biophysical models of salt marsh evolution—including the one used for our illustration below—are spatial and designed to reproduce the natural migration of marshes (Fargherazzi et al. 2012). Models of this type spatially implement equations that characterize marsh migration processes over a gridded topography representing conditions at each study site (Appendix B). For example, marsh accretion rates depend on local elevation relative to mean sea level and on the distance to the closest channel, both of which are spatially variable and change with time. The spatial nature of these forecasts is implied by the patchy nature of marsh changes forecast over time (e.g., see case study below, Figure 2).

General patterns emerge from these spatial models that can provide insights into the characteristics of the portfolio assets. Low-elevation uplands (largely forest in our case study area)—if preserved—can enable effective marsh migration at low rates of SLR. At higher SLR these low-elevation areas transition from marsh to open water (i.e., drown). Hence, when higher SLR is expected, higher-elevation transgression zones (largely agricultural land in our case study) are better able to support marsh migration and retain marsh properties. The varying slopes of higher versus lower elevation uplands also determine how much land is inundated by any given SLR. For example, lower and flatter areas tend to yield relatively more marsh on average as seas rise, *ceteris paribus*, but the variance of marsh migration due to changes in sea level is also greater—leading to a mean versus variance trade-off. In addition to preserving uplands for marsh migration, one can preserve intertidal land on which marsh is already established. This preserves current marsh independent of upland migration, and is an effective way to retain marsh at very low SLR, where there is minimal risk of marsh drowning.

These patterns are used to define assets within the model. Most marsh conservation sites include intertidal land and adjacent uplands. In our case study, upland areas are characterized by high correlation in elevation and land use: agricultural land is found at higher elevations and at a greater distance from the Atlantic Ocean than lower-elevation forested land. Conservation decision-making emphasizes these three land types (agricultural land, forest and intertidal salt marsh) in areas suitable for marsh migration (The Nature Conservancy 2011; The Nature Conservancy in Virginia 2011; Bruce and Crichton 2014). Reflecting this situation, the model is designed around these three preservation land types.

Although we illustrate the model using this land classification, the model structure is general and can accommodate any classification for which marsh migration can be modeled. It is conceptually straightforward to extend the model to more than three land types or to other classifications.[5] For example, if explicitly spatial categorizations are desired, the model can be adapted accordingly. In general, however, practical limits on asset numbers and data availability constrain the use of portfolio models for high-resolution spatial planning. As noted by Mallory and Ando (2014, p. 4), these models are "most useful for coarse-scale conservation targeting exercises and less well-suited to function as a parcel-level targeting tool." However, portfolios emerging from these models can be combined with supplementary analyses that support spatial targeting within each portfolio class; an example is provided in Section 4.4 below.

2.3 Model Structure and Objective Function

To develop the portfolio model based on these asset classes, let N_i be the total area of conserved land of type i in the portfolio, of which proportion S_i is currently marsh. Subscript $i = a, f, m$, references the three land types (or asset classes) in the portfolio, where a denotes agricultural land, f is forest and m is intertidal marsh. Commensurate with financial portfolio theory (Merton 1969, 1971), we define the total size of the marsh portfolio in monetary terms as M, where

$$M = \sum_{i=1}^{n} c_i N_i S_i. \tag{1}$$

The costs of preserving units of land type i for the purpose of marsh preservation are explicit in the form of marginal land costs $c_i(S_i)$. These costs are sensitive to marsh intrusion, or the proportion of each land type that is marsh at any given time. Marsh intrusion is expected to reduce the productivity of transgression zone land for its current land use (e.g., for agriculture), resulting in reduced land costs $\lambda_i \equiv dc_i/dS_i < 0$,

[5] Although these land classes are not defined based on explicit spatial attributes (e.g., proximity to channels), the spatial attributes of these land types are implicit in the underlying biophysical model. Hence, spatial influences are captured within the portfolio model, even though our current selection of land types does not include explicit spatial designations.

$i = a, f$.[6] Marsh drowning on intertidal land on the other hand, implies that land mass is lost and with it all marsh and alternative land use benefits, hence $\lambda_m \equiv dc_m/dS_m > 0$. Following standard nomenclature for models of this type (Leroux and Martin 2016), we refer to M as the "marsh budget."[7]

Asset or portfolio shares θ_i, are defined as the proportion of the marsh budget allocated to marsh conservation on land type i at time t, such that

$$\theta_i \equiv \frac{c_i N_i S_i}{M}, \qquad \{i = a, f, m\}. \tag{2}$$

We refer to these as marsh shares, which satisfy the normalization $\theta_a + \theta_f + \theta_m = 1$. Marsh shares may be further translated into shares of physical land type i in the portfolio, as shown in Section 2.4.

From this foundation, the purpose of portfolio optimization is to maximize the expected discounted utility from salt marsh ecosystem benefits, $x(t)$, over time t. The inter-temporal utility function is maximized with respect to $x(t)$ and asset shares θ_i according to

$$\max_{x,\theta_i} E_0 \int_0^\infty e^{-\delta t} U(x(t))\, dt, \qquad \{i = a, f, m\}. \tag{3}$$

We assume a standard expected utility specification, where utility, $U(x)$, is strictly concave in $x(t)$ and is given by $U(x) = x(t)^{(1-\gamma)} / (1-\gamma)$. The parameter γ is a constant relative risk aversion parameter that characterizes the conservation planner's risk attitude associated with the flow of marsh benefits. Given this specification, the conservation planner has logarithmic preferences for $\gamma = 1$ and is risk averse when $\gamma > 0$. The parameter δ denotes the discount rate.

[6] To the extent that marsh intrusion is correlated with saltwater intrusion, this may also capture effects related to the latter. Although the biophysical effects of saltwater intrusion on agricultural land have been studied (Tully et al. 2019), we are aware of no research that models the effect of saltwater intrusion (or anticipated SLR itself) on the cost of different types of land preservation. Hence, we leave related extensions of the model for future work.

[7] This terminology does *not* imply a traditional budget constraint of the type encountered in static utility maximization; the formal dynamic constraints for this model are derived below.

2.4 Dynamic Constraints

Equation (3) is maximized subject to two dynamic constraints. The first governs how marsh migrates from a biophysical perspective. The second governs how the size of the marsh porfolio (or the marsh budget) changes over time. These two dynamic constraints are formalized below.

2.4.1 Biophysical Migration Constraint

The dynamic constraint on biophysical marsh migration is given by

$$dS_i = \mu_i dt + \sigma_i dz_i, \qquad \{i = a, f, m\}. \tag{4}$$

This describes the change in marsh area on each preserved land type, with S_a, S_f, S_m representing respectively the proportion of a unit (km^2) of agricultural, forest and intertidal land that is marsh at a given point in time, where the subscript t has been dropped for expositional convenience. At time zero we expect a large proportion of intertidal land to be marsh, $S_m \precsim 1$, owing to very limited localized marsh drowning. In contrast, we expect most transgression zone land to be actively used as agricultural or forest land, with only marginal marsh intrusion at the current time, $S_a, S_f \simeq 0$. Marsh migration, dS_i, is a purely biophysical process, depending on elevation, location, vegetation cover and SLR, all of which are modelled separately (Section 4.1). Here we use the biophysical model's aggregated dynamics by land class, describing the change in marsh proportion on land class i as a Brownian motion. The mean proportional change of marsh on land type i is denoted μ_i and the uncertainty associated with marsh dynamics on this land type is described by the variance σ_i^2. The Brownian motion z_i is normally distributed with mean zero and variance dt, $z_i \sim N(0, dt)$. We allow for marsh risk to be correlated across the different land types, $dz_a dz_f = \rho_{af} dt$, $dz_a dz_m = \rho_{am} dt$ and $dz_f dz_m = \rho_{fm} dt$. We expect migration onto agricultural and forest land to be positively correlated, $0 < \rho_{af} < 1$, as both are driven by SLR. In contrast, SLR is likely to cause existing marsh on intertidal land to drown, so that the correlation between marsh dynamics on transgression zone and intertidal land are expected to be negative, $-1 < \rho_{am}, \rho_{fm} < 0$.

We initially assume that marshes can only migrate onto land that has been preserved as part of the portfolio. This assumption implies that private landowners will take actions to prevent unpreserved land from becoming marsh (e.g., armoring). Similarly, it is assumed that existing marsh on unpreserved intertidal land will be developed and converted into some other land use. Hence, the only way that managers can ensure the future persistence of marsh is to preserve land.[8]

2.4.2 Dynamic Marsh Budget Constraint

To derive the dynamic constraint on the size of the marsh portfolio, denoted as the dynamic (marsh) budget constraint, we begin with the marsh budget as described above and defined by (1). Applying Ito's lemma to equation (1) yields the change in M over time

$$dM = \sum_{i=1}^{n} dc_i N_i S_i + \sum_{i=1}^{n} c_i dN_i S_i + \sum_{i=1}^{n} c_i N_i dS_i + \sum_{i=1}^{n} dc_i N_i dS_i + o\left(dt\right), \qquad (5)$$

where the last term arises as c_i is stochastic, being a function of S_i, and the term $o\left(dt\right)$ groups higher order terms in dt.

Models in financial portfolio theory (Merton 1969, 1971) acknowledge a consumption-investment trade-off (cf. Kamien and Schwartz 1981, p. 248), whereby expansion of marsh in the portfolio via new land purchases, represented by dN_i, requires a consumption sacrifice (Leroux and Martin 2016). Here it is the consumption of marsh ecosystem benefits, xdt, valued at v at the margin, that is sacrificed (i.e., sold) to finance these purchases, where v reflects the marginal willingness to pay for these benefits (or services). We initially assume a simple case in which each unit area of marsh provides identical benefits regardless of its location and the land type from which it originated.

[8] A more general approach is adopted in Section 2.5 by allowing for a non-zero probability that marsh can persist and migrate onto different types of unpreserved land.

This assumption is relaxed in Section 2.5.[9] Unlike the returns from a financial portfolio, salt marsh benefits have quasi-public good characteristics, meaning that only a proportion ϕ of these benefits can be assigned tradeable property rights in markets for ecosystem services, such as recreational, hunting or fishing access, or salt marsh hay harvesting (Bromberg Gedan et al. 2009).[10] The associated consumption-investment trade-off between consuming all marsh benefits and privatizing some in order to finance additional marsh preservation is hence given by $\sum_{i=1}^{n} c_i dN_i S_i = -\phi vxdt$.

Using the above expression in (5) yields

$$dM = \sum_{i=1}^{n} dc_i N_i S_i + \sum_{i=1}^{n} c_i N_i dS_i + \sum_{i=1}^{n} dc_i N_i dS_i - \phi vxdt + o\left(dt\right). \tag{6}$$

Substituting (4) for dS_i in (6) and using the definition of marsh shares θ_i, in (2) gives

$$\begin{aligned} dM &= \sum_{i=1}^{n} \theta_i \frac{\lambda_i}{c_i} \mu_i M dt + \sum_{i=1}^{n} \theta_i \frac{\lambda_i}{c_i} \sigma_i M dz_i + \sum_{i=1}^{n} \theta_i \mu_i \frac{M}{S_i} dt + \sum_{i=1}^{n} \theta_i \sigma_i \frac{M}{S_i} dz_i \\ &\quad + \sum_{i=1}^{n} \theta_i \frac{\lambda_i}{c_i} \sigma_i^2 \frac{M}{S_i} dt - \phi vxdt + o\left(dt\right). \end{aligned} \tag{7}$$

Imposing the normalization restriction, the resulting dynamic budget constraint may be rewritten as

$$\begin{aligned} dM &= \theta_a \widetilde{\mu}_a M dt + \theta_f \widetilde{\mu}_f M dt + \widetilde{\mu}_m M dt + \theta_a \left(\widetilde{\sigma}_a dz_a - \widetilde{\sigma}_m dz_m\right) M \\ &\quad + \theta_f \left(\widetilde{\sigma}_f dz_f - \widetilde{\sigma}_m dz_m\right) M + \widetilde{\sigma}_m M dz_m - \phi vxdt + o\left(dt\right), \end{aligned} \tag{8}$$

where

$$\widetilde{\mu}_m = \frac{\mu_m}{S_m} + \frac{\lambda_m}{c_m}\mu_m + \frac{\lambda_m}{c_m}\frac{\sigma_m^2}{S_m}, \tag{9}$$

$$\widetilde{\mu}_a = \frac{\mu_a}{S_a} + \frac{\lambda_a}{c_a}\mu_a + \frac{\lambda_a}{c_a}\frac{\sigma_a^2}{S_a} - \widetilde{\mu}_m, \tag{10}$$

$$\widetilde{\mu}_f = \frac{\mu_f}{S_f} + \frac{\lambda_f}{c_f}\mu_f + \frac{\lambda_f}{c_f}\frac{\sigma_f^2}{S_f} - \widetilde{\mu}_m. \tag{11}$$

[9]There is no conclusive evidence suggesting that the long run economic value of marsh area is affected by land use prior to marsh transition. This is consistent with the wetland valuation literature, none of which identifies a systematic effect of marsh provenance on value. Hence, average marsh value is initially assumed to be independent of its provenance (i.e., the original land type), and hence optimizing over marsh area is formally equivalent to optimizing over marsh value, if one considers a typical area and type of marsh. However, this need not always be the case.

[10]As the parameters v and ϕ are model scalars and independent of land class i, they have no effect on the optimal portfolio composition.

The term $\widetilde{\mu}_m$ represents the risk-adjusted return from existing marsh on intertidal land, and $\widetilde{\mu}_a$ and $\widetilde{\mu}_f$ are respectively the risk-adjusted excess returns of agricultural and forest land relative to $\widetilde{\mu}_m$. Marsh migration per proportion of salt marsh on the relevant land type is captured by the first term in equations (9) to (11), while the second term represents mean migration adjusted for the change in land costs due to marsh drowning on intertidal land, λ_m, and salt marsh intrusion on transgression zone land, λ_a, λ_f. The third term adjusts land type's i mean return by type i's migration risk and land cost. Finally, $\widetilde{\sigma}_i$ in equation (8) is the cost-adjusted standard deviation given by

$$\widetilde{\sigma}_i = \sigma_i \left(\frac{1}{S_i} + \frac{\lambda_i}{c_i} \right), \qquad \{i = a, f, m\}. \tag{12}$$

2.5 Model Solution

The model solution is obtained by maximizing expected utility (3) with respect to $x\,(t)$ and θ_i, subject to the two dynamic constraints (4) and (8), and initial condition for the marsh budget $M\,(0) = M_0$. As shown in Appendix A, the optimal solution is obtained by solving the following Hamilton-Jacobi-Bellman equation

$$\begin{aligned} \delta V \;=\; & \max_{x,\theta_a,\theta_f} \Big(x^{(1-\gamma)} / \left(1-\gamma\right) \Big) - \phi v x V_M + \left[\theta_a \widetilde{\mu}_a + \theta_f \widetilde{\mu}_f + \widetilde{\mu}_m \right] M V_M \\ & + \frac{1}{2} \left[\theta_a^2 \left(\widetilde{\sigma}_a^2 - 2\widetilde{\sigma}_{am} + \widetilde{\sigma}_m^2 \right) + \widetilde{\sigma}_m^2 + \theta_f^2 \left(\widetilde{\sigma}_f^2 - 2\widetilde{\sigma}_{fm} + \widetilde{\sigma}_m^2 \right) \right. \\ & + 2\theta_a \theta_f \left(\widetilde{\sigma}_{af} - \widetilde{\sigma}_{am} - \widetilde{\sigma}_{fm} + \widetilde{\sigma}_m^2 \right) \\ & \left. + \; 2\theta_a \left(\widetilde{\sigma}_{am} - \widetilde{\sigma}_m^2 \right) + 2\theta_f \left(\widetilde{\sigma}_{fm} - \widetilde{\sigma}_m^2 \right) \right] M^2 V_{MM} \Big) + o\,(dt)\,, \end{aligned} \tag{13}$$

where $\widetilde{\sigma}_{ij} = \rho_{ij} \widetilde{\sigma}_i \widetilde{\sigma}_j$, represents the cost-adjusted covariance between land types i and j, and V is the time-invariant value function. The derivatives V_M and V_{MM} denote the first and second derivatives of the value function with regard to the marsh budget, M. Maximizing (13) with respect to θ_a and θ_f and using the closed-form solution $V = AM^{1-\gamma}$, to solve for the optimal marsh portfolio yields optimal marsh shares from agricultural, forest and intertidal land (see Appendix A)

$$
\begin{aligned}
\theta_a \;=\;& \left[\left(\frac{\widetilde{\mu}_a}{\gamma} - \widetilde{\sigma}_{am} + \widetilde{\sigma}_m^2\right)\left(\widetilde{\sigma}_f^2 - 2\widetilde{\sigma}_{fm} + \widetilde{\sigma}_m^2\right)\right. \\
& \left. - \left(\frac{\widetilde{\mu}_f}{\gamma} - \widetilde{\sigma}_{fm} + \widetilde{\sigma}_m^2\right)\left(\widetilde{\sigma}_{af} - \widetilde{\sigma}_{am} - \widetilde{\sigma}_{fm} + \widetilde{\sigma}_m^2\right)\right] \qquad (14) \\
& \times \left[\left(\widetilde{\sigma}_a^2 - 2\widetilde{\sigma}_{am} + \widetilde{\sigma}_m^2\right)\left(\widetilde{\sigma}_f^2 - 2\widetilde{\sigma}_{fm} + \widetilde{\sigma}_m^2\right) - \left(\widetilde{\sigma}_{af} - \widetilde{\sigma}_{am} - \widetilde{\sigma}_{fm} + \widetilde{\sigma}_m^2\right)^2\right]^{-1},
\end{aligned}
$$

$$
\begin{aligned}
\theta_f \;=\;& \left[\left(\frac{\widetilde{\mu}_f}{\gamma} - \widetilde{\sigma}_{fm} + \widetilde{\sigma}_m^2\right)\left(\widetilde{\sigma}_a^2 - 2\widetilde{\sigma}_{am} + \widetilde{\sigma}_m^2\right)\right. \\
& \left. - \left(\frac{\widetilde{\mu}_a}{\gamma} - \widetilde{\sigma}_{am} + \widetilde{\sigma}_m^2\right)\left(\widetilde{\sigma}_{af} - \widetilde{\sigma}_{am} - \widetilde{\sigma}_{fm} + \widetilde{\sigma}_m^2\right)\right] \qquad (15) \\
& \times \left[\left(\widetilde{\sigma}_a^2 - 2\widetilde{\sigma}_{am} + \widetilde{\sigma}_m^2\right)\left(\widetilde{\sigma}_f^2 - 2\widetilde{\sigma}_{fm} + \widetilde{\sigma}_m^2\right) - \left(\widetilde{\sigma}_{af} - \widetilde{\sigma}_{am} - \widetilde{\sigma}_{fm} + \widetilde{\sigma}_m^2\right)^2\right]^{-1},
\end{aligned}
$$

with the share allocated to intertidal marsh obtained from the normalization constraint

$$\theta_m = 1 - \theta_a - \theta_f. \qquad (16)$$

These model solutions provide insight into the characteristics of optimal portfolios. For example, close inspection of marsh-share equations (14) to (16) reveals that under most conditions, land type i's share is higher the higher is its risk-adjusted excess mean return $d\theta_i/d\widetilde{\mu}_i > 0$ and the lower is the risk-adjusted excess mean migration onto the alternative transgression zone land type, $d\theta_i/d\widetilde{\mu}_j < 0$. Optimal marsh share on land type i decreases with uncertainty surrounding marsh migration onto land type i, $d\theta_i/d\sigma_i^2 < 0$. Expressions (14) and (15) further show that the relationship between risk aversion, γ, and marsh share is ambiguous, and depends on the relative size of excess returns, variances and covariances.

Equations (14) to (16) represent the optimal share of *marsh* from land type i in the marsh portfolio and are a function of all parameters of the model, except the marsh budget M. However, as the conservation planner is ultimately interested in the shares of each land type that comprise the optimal *land* conservation portfolio, we use (2) to convert marsh shares θ_i, into land shares θ_i^l, according to

$$\theta_i^l = \frac{\theta_i M}{c_i S_i \sum N_i}, \qquad \{i = a, f, m\}, \qquad (17)$$

where $\sum N_i$ represents the total conservation area that is preserved for a given M and θ_i. Equations (14) to (16) and (17) constitute a key result of the theoretical model. These equations characterize the portfolio shares that maximize the expected value of marsh conservation subject to uncertain marsh dynamics and SLR. It is straightforward to show that corresponding land shares θ_i^l, decrease with respective land costs $d\theta_i^l/dc_i < 0$, as one would expect. Assuming that the key conditions of the model continue to hold, these shares are applicable to a land portfolio of any given size.

The optimal marsh benefit x, from solving (13) is given by (see Appendix A)

$$x = (\phi v)^{-\frac{1}{\gamma}} (1-\gamma)^{-\frac{1}{\gamma}} A^{-\frac{1}{\gamma}} M, \tag{18}$$

where A is a constant defined implicitly in terms of the parameters of the model as

$$\begin{aligned} A^{-\frac{1}{\gamma}} \;=\;& \delta\frac{1-\gamma^{\frac{1}{\gamma}}}{\gamma}(\phi v)^{\frac{1}{\gamma}-1} - \frac{(1-\gamma)^{1+\frac{1}{\gamma}}}{\gamma}(\phi v)^{\frac{1}{\gamma}-1}\left(\theta_a\widetilde{\mu}_a + \theta_f\widetilde{\mu}_f + \widetilde{\mu}_m\right) \\ &+\frac{1}{2}(1-\gamma)^{1+\frac{1}{\gamma}}(\phi v)^{\frac{1}{\gamma}-1}\left[\theta_a^2\left(\widetilde{\sigma}_a^2 - 2\widetilde{\sigma}_{am} + \widetilde{\sigma}_m^2\right)\right. \\ &+\theta_f^2\left(\widetilde{\sigma}_f^2 - 2\widetilde{\sigma}_{fm} + \widetilde{\sigma}_m^2\right) + \widetilde{\sigma}_m^2 + 2\theta_a\theta_f\left(\widetilde{\sigma}_{af} - \widetilde{\sigma}_{am} - \widetilde{\sigma}_{fm} + \widetilde{\sigma}_m^2\right) \\ &\left. + 2\theta_a\left(\widetilde{\sigma}_{am} - \widetilde{\sigma}_m^2\right) + 2\theta_f\left(\widetilde{\sigma}_{fm} - \widetilde{\sigma}_m^2\right)\right], \end{aligned} \tag{19}$$

and where θ_a and θ_f are respectively the solutions for the agricultural land and forest marsh shares from (14) and (15). As $(\phi v)^{-\frac{1}{\gamma}} (1-\gamma)^{-\frac{1}{\gamma}} A^{-\frac{1}{\gamma}}$ is a constant, equation (18) shows that marsh benefits x, are proportional to the marsh budget M. Moreover, given the definition of the $\widetilde{\mu}_i$ in equations (9) to (11) as well as the definition of $\widetilde{\sigma}_i$ in (12), equation (18) implies a non-linear relationship between the optimal marsh benefit x and the proportion of marsh on land type i, S_i.

2.6 Portfolios Under Additionality and Provenance Effects

The baseline model may be extended in various ways. For example, thus far we have assumed that privately owned land will be armored with probability $p_i = 1$ to prevent marsh from migrating onto that land. Similarly, we have assumed that existing marsh will be developed or otherwise lost in the absence of conservation. Under these assumptions, the additional benefit from purchasing a unit of land for marsh transgression is equal to the expected marsh benefit on that land. In practice, however, not all private land will likely be armored against SLR, nor will all unpreserved salt marsh be developed. Where private land remains unarmored, marsh can migrate onto that land in the same way as it migrates onto preserved land. Consequently, the economic benefit of spending conservation funds to preserve private land that will *never* be armored or developed in the future is zero, because preserving this type of land provides no additional benefit. These issues relate to the concept of additionality, or whether the environmental services provided by a given policy intervention would have been provided in the absence of that intervention (Pattanayak et al. 2010).

To allow for additionality, we extend the model to consider the probabilities p_a, p_f and p_m of armoring private agricultural land and forest or developing intertidal land as being equivalent to the additionality provided by conserving that land. Suppose that conservation of a piece of forest land has a probability of being armored of $p_f = 0.25$. This would imply that marsh transgression *in the absence* of conservation would occur with probability of $1 - p_f = 0.75$, thereby reducing the value of marsh benefit from conserving this land by 3/4. The effect of additionality matters for the analysis if the probability p_i varies across land types. Formally, this implies defining the unit value of marsh benefit from land type i as $v_i \equiv p_i v$, and rewriting dM in (6) as

$$dM = \sum_{i=1}^{n} d\frac{c_i}{v_i} N_i S_i + \sum_{i=1}^{n} \frac{c_i}{v_i} N_i dS_i + \sum_{i=1}^{n} d\frac{c_i}{v_i} N_i dS_i - \phi x dt, \tag{20}$$

with marsh shares now re-defined as $\theta_i \equiv c_i S_i N_i / (v_i M)$ and solving as before. This solution parallels that shown previously in equations (14) - (16) and (17), but with c_i replaced by c_i/v_i and $\lambda_i \equiv d(c_i/v_i)/dS_i$, while M is interpreted as the physical marsh budget and expressed in km^2.

An identical model structure may be used to account for cases in which the value of marsh depends on provenance, or the type of land from which the marsh originated.

For example, one might consider a case in which marsh that was originally agricultural land has higher or lower average value than marsh that was originally forest. In this case, the equation introduced above, $v_i \equiv p_i v$, may be reinterpreted as a measure of different underlying unit values of marsh benefit from land type i, whereby $p_i = 1$ is assigned to the land type that generates the highest-value marsh, and $p_i < 1$ to other land types, reflecting the proportionally lower value of marsh from those land types.

Depending on the interpretation of this model structure (i.e., additionality or provenance effects on marsh benefits), the formal solution leads to an intuitive result that higher shares of land type i in the optimal portfolio are associated with (a) higher probability of armoring on land type i (and hence greater additionality), and/or (b) higher value of marsh originating from land type i. Illustrative empirical results of this effect for the additionality case are shown below.

Additional insight may be gained through empirical applications of the model. These applications can be particularly informative when analytical results alone provide insufficient or inconclusive insight on particular effects of interest, or when quantitative insights on portfolio shares are desired. Because the presented approach provides closed-form, analytical solutions, it may be applied to any site for which sufficient data are available to calibrate the model.

3 Empirical Illustration - Salt Marsh Conservation at the Virginia Coast Reserve

Illustrative empirical results of the model are demonstrated for a case study application to a 100 km^2 coastal area within the US Virginia Coast Reserve LTER on the Eastern Shore of Virginia, USA, as depicted in Figure 1 (Hayden et al. 1991). Current land cover within the case study site includes 49 km^2 of existing marsh and 51 km^2 of upland, of which 2.2 km^2 has been developed and armored. Remaining upland areas are comprised of 27.3 km^2 of agricultural land and 21.5 km^2 of forest onto which marshes can potentially migrate, provided that the land is preserved in a way that enables migration (Brinson et al. 1995). This migration is necessary to offset losses due to erosion and drowning at the seaward edge (Deaton et al. 2017). Given development pressures, a major focus of conservation in the area is the preservation of marsh transgression zones (Bruce and Crichton 2014; The Nature Conservancy 2011).

This case study site is similar to many other coastal areas in the US and elsewhere. Hence, while the specific empirical results of the analysis are limited to our case study site, the general patterns and economic intuition are likely applicable to other areas. The biophysical model is run over a 90-year time horizon until 2100.

Figure 1 about here

3.1 Biophysical Dynamics of Marsh Migration

Net marsh migration (marsh migration minus marsh drowning) subject to uncertain SLR is simulated using a dynamic process-based model with 30m grid spacing, based on standard approaches (Fagherazzi et al. 2012; Kirwan et al. 2010, 2016b; Schile et al. 2014). Details are given in Appendix B. The spatial model predicts when and where SLR leads to marsh migration, and the conditions under which portions of marsh drown due to insufficient accretion. This approach incorporates dynamic processes affecting the vertical evolution of a salt marsh (e.g., feedbacks between flooding frequency, vegetation growth, organic accretion, and sediment deposition) and inundation-driven marsh migration, as influenced by spatial features such as channel and marsh location.

We begin with a set of four possible mean SLR trajectories, based on regionally appropriate SLR scenarios from low to intermediate-high developed by the Sea Level Rise and Coastal Flood Hazard Scenarios and Tools Interagency Task Force (Sweet et al. 2017).[11] These scenarios involve local mean SLR projections that correspond to

[11] Developed from updated global mean SLR scenarios, the corresponding regional scenarios of SLR are incorporated into the U.S. coastal risk management tools and capabilities for deployment by individual U.S. agencies (Sweet et al. 2017).

a total local rise of 0.32, 0.60, 0.90 and 1.7 meters by 2100. These scenarios reflect uncertainty (or more formally, risk) related to mean SLR over long periods. We also incorporate random annual fluctuation in mean sea level (with a standard deviation of 0.04m) based on annual variation of mean sea level observed at Wachapreague, VA. This provides four stochastic SLR projections over which marsh changes are forecast. To generate distributions of marsh extent for each year over the 90-year period, the dynamic model is repeated 50 times for each SLR projection and land type, with each run corresponding to a unique set of stochastic annual fluctuations in sea level.[12]

Migration means and variances are estimated from these stochastic time series. These reflect the net gain or loss of marsh for each grid across the entire modeled area, at each point in time, distinguished by land type. These are central inputs to portfolio model calibration. For illustration, Figure 2 shows projected results for agricultural, forest and marsh land, by the year 2100, compared to current land cover. The leftmost map shows current land cover, followed by projected mean model results for projected SLRs of 0.32, 0.60, 0.90 and 1.7m, moving rightward. These illustrative forecasts assume that all undeveloped land is preserved for migration, and illustrate a situation in which marsh migration exceeds drowning at low SLR, but drowning exceeds migration at higher SLR. At the highest level of 1.7m, virtually all existing marsh is projected to be lost by 2100.[13]

Figure 2 about here

[12] Independent calibrations are provided for each SLR projection, based on standard modeling approaches (Kirwan et al. 2016b). Each projection is grounded in a unique, path-dependent set of "underlying ... socioeconomic conditions and technological considerations" (Sweet et al. 2017, p. 13). Because future conditions cannot instantaneously jump between the socio-technical scenarios underlying different SLR projections (given their path-dependence), separate migration models are estimated and portfolios identified for each one. We then consider the sensitivity of results to discrete probability distributions over the four SLR projections, allowing for cases in which policy makers have different perceptions regarding the relative likelihood that each will occur (Section 4.2). This approach parallels the treatment of climate scenarios within prior portfolio models (e.g., Ando and Mallory 2012; Mallory and Ando 2014).

[13] To calculate patterns of *potential* marsh migration the *biophysical* model considers the case in which all private land has been purchased as transgression zone land. This does not affect the optimal marsh or land shares calculated from the *economic* model, which can be applied to a land portfolio of any size.

3.2 Calibration of the Portfolio Model

The resulting parameter values used for the baseline model calibration are summarized in Table 1. The initial proportions of marsh on agricultural $S_a = 0.03$, and on forest land $S_f = 0.16$, were approximated from historic aerial records.[14] The same records suggest that approximately 5% of historic salt marsh has drowned such that $S_m = 0.95$ of what was once marsh remains. Allowing for these initial marsh proportions yields transgression areas of $N_a = 28.1\ km^2$ and $N_f = 25.6\ km^2$ for agricultural and forest land respectively and an intertidal marsh area of $N_m = 44.5km^2$.

For the baseline model land costs c_i per km^2, and the marginal changes in these costs for a change in marsh proportion λ_i, are drawn from Gardner and Johnston (2018). This prior study reports average purchase costs of different types of undeveloped land parcels using data on all raw land transactions from 2014 to 2016 in the two counties that encompass the study site (Accomack and Northampton, Virginia).[15] The average costs for each land type are $c_a = 1,425,302\ \$/km^2$, $c_f = 904,651\ \$/km^2$ and $c_m = 567,320\ \$/km^2$. These are the expected costs of purchasing raw land for preservation via fee-simple market purchase, which is the predominant method of land preservation in the area. For the given values of N_i, S_i and c_i, the initial marsh budget is calculated as $M_0 = 28.82\ \$m$.

The marginal change in land costs with respect to marsh intrusion is calculated based on a linear interpolation of these average costs, assuming a 1 percent change of each km^2 of land from one type to another (e.g., farm to marsh). This yields $\lambda_a \equiv dc_a/dS_a = -8,580\ \$/km^2$ and $\lambda_f \equiv dc_f/dS_f = -3,373\ \$/km^2$. The change

[14] Under initial conditions each land type includes some marsh, as explained above. Most of the intertidal land is covered by marsh. Recently formed marsh on adjacent land, where the prior land use is still clearly identifiable, retains its original classification as agricultural or forest land.

[15] The total amount of agricultural land in the modeled area is 27.3 km^2, which represents less than 6 percent of the active farmland in these two counties, based on GIS land use data retrieved from https://www.acrevalue.com/ on March 5, 2019. Given this small percentage we do not expect that conservation in the area (that would realistically target only a portion of this small area) would have a significant effect on agricultural land prices.

in intertidal land cost as marsh drowns and becomes open water is measured as the marginal value of marsh land, $\lambda_m \equiv dc_m/dS_m = 5,673$ $\$/km^2$. For comparison, and to evaluate the robustness of portfolio results to other land cost assumptions, we also calibrate the model using alternative cost estimates drawn from hedonic models of undeveloped land sales in Gardner and Johnston (2018) and Allen et al. (2006). These alternative calculations lead to similar land cost estimates and nearly identical optimal portfolio results (Appendix C).

The value of marsh, v in equation (8), is not pivotal to the baseline model results as this estimate does not influence portfolio shares because benefits do not vary by marsh provenance.[16] Hence, we approximate these benefits using the published meta-analytic results of Ghermandi et al. (2010). This meta-analysis allows estimation of wetland value per km^2 per year, for a wetland of specified attributes, generating an estimated value of $3,104,403$ $\$/km^2$ of marsh.[17] The marsh value reported in Table 1 includes public as well as private benefits, where only the private benefits can be sold to increase the marsh budget as per equation (8). We assume that these privatizeable benefits account for about $\phi = 0.05$ of total annual marsh benefits. We illustrate the model for an annual discount rate of $\delta = 0.03$, although the model can be readily adapted to any desired rate of discount.[18]

While anecdotal evidence points to a high degree of risk aversion among conservationists (e.g., Berrens 2001), there are no empirical studies of the risk attitudes of marsh conservation planners that could be used to determine γ. Our baseline calibration assumes $\gamma = 1.5$. This value implies a relatively high level of risk aversion in many experimental settings (Holt and Laury 2002), but falls within the lower range of

[16] Section 2.6 illustrates how the model can be adapted to allow v to vary across land classes.

[17] We use this model to estimate annual value in 2018 USD per km^2 for a salt marsh in the study area, subject to medium/low human pressure and providing water quality improvements, habitat and nonuse, amenity and aesthetic values. Other variable values are representative of the study site, including income (from US Census data for Virginia) and total wetland area within 50 km of the study site (https://www.fws.gov/wetlands/Data/Mapper.html).

[18] This rate is consistent with the central discount rate recommended by the U.S. EPA for climate-related projects (U.S. EPA 2014, pp. 6-19).

estimates that are implied by natural resource planners' decisions in related settings (Leroux and Martin 2016; Leroux et al. 2018). Sensitivity analyses of the results under alternative degrees of risk aversion suggest that the basic results of the model are robust to a wide range of assumptions regarding this parameter. Hence, these results are omitted for conciseness.

Table 1 about here

The means and standard deviations of the change in marsh proportions by land type and SLR scenario are estimated using biophysical model outputs for years 0 to 90, and are reported in Table 2. Higher SLR leads to greater marsh migration onto transgression zone land, with migration onto forest being faster, $\mu_f > \mu_a$, and more volatile, $\sigma_f > \sigma_a$, than migration onto agricultural land across all scenarios. Uncertainty with respect to migration increases with higher SLR for migration onto agricultural land, but decreases for migration onto forest. In contrast, the aerial extent of existing marsh remains mostly steady for low and medium SLR and decreases as a result of marsh drowning for higher SLR. The uncertainty surrounding existing marsh also increases with higher SLR.

Correlation coefficients in Table 2 show that marsh migration onto agricultural and forest land is positively correlated $\rho_{af} > 0$. However, across the four scenarios ρ_{af} decreases steadily, which implies that the risks associated with marsh migration are increasingly driven by transgression zone characteristics rather than by uncertainties surrounding marsh migration more generally. Transgression zone migration is negatively correlated with existing salt marsh dynamics, $\rho_{am}, \rho_{fm} < 0$, signalling that investment in transgression zone land may be an effective way to hedge against the risk of marsh drowning from rising sea levels. The relative effect of SLR on ρ_{am} and ρ_{fm}, suggests that agricultural land could be an increasingly important hedge against the risk of existing marsh drowning.

Table 2 about here

4 Empirical Results

This section presents the results of the illustrative empirical calibration. We first illustrate results for the baseline model. This is followed by sensitivity analyses of SLR perceptions among conservation planners and additionality effects. We also illustrate an extension of the model that enables spatial targeting of land preservation within each asset class.

4.1 Optimal Portfolio as a Function of SLR

Table 3 presents the optimal portfolio, based on the parameter values summarized in Tables 1 and 2. Two types of portfolios are presented for each scenario. The first is the optimal *marsh* portfolio based on equations (14) to (16), where the share θ_i represents the optimal share of the total marsh budget allocated to land type i. The second is the corresponding physical *land* portfolio, where θ_i^l represents the share of a particular transgression zone in the total preserved land portfolio according to equation (17). In some instances the difference between marsh and land shares is noticeable. For example, while agricultural marsh shares range from $0.07 \leq \theta_a \leq 0.51$, for rises in sea level of $0.32m$ to $0.90m$, the corresponding agricultural land shares are considerably higher, between $0.47 \leq \theta_a^l \leq 0.93$. This is because $S_a < S_f < S_m$. Hence, the land share required to achieve the optimal marsh share from agricultural land is relatively larger than for the other land types. For practical reasons marsh conservation planners are primarily interested in the optimal mix of land in their conservation portfolios, and so the discussion that follows focuses on land shares, θ_i^l.[19]

As shown in Table 3, optimal land portfolios across all scenarios are dominated by agricultural land. Agricultural land share increases from just below 0.5 to the maximum of 1.0 between the lowest and highest scenarios, as marsh on agricultural land is least likely to drown. The preservation of some current salt marsh area is optimal provided

[19]These results are similarly applicable to the optimal marsh portfolio, defined by marsh shares θ_i.

SLR remains low to moderate. Similarly, the benefits from investing in forest land for marsh migration are limited to low and medium SLR, as marsh on forest land also becomes highly susceptible to drowning at higher SLR. This illustrates the potentially important role of SLR uncertainty for marsh conservation decisions.

Table 3 about here

4.2 Optimal Portfolios under Combinations of SLR Scenarios

The optimal conservation portfolio depends on SLR expectations, which could involve some probability distribution over a number of possible scenarios. While decision-makers' expectations regarding SLR are generally unknown, an optimal portfolio can be generated for any given decision-maker with a specified set of assumptions.[20] Table 4 shows three illustrative cases. The optimistic case assumes that the likelihood of the lowest SLR scenario is 70% with the remaining 30% equally distributed across the other three scenarios. The middle case assumes equal weighting across all four scenarios and the pessimistic case assumes that the highest scenario occurs with a probability of 70% while the three lower scenarios have equal weight of 10%. As shown in columns 2 to 4 in Table 4, greater optimism by the conservation planner with respect to future SLR implies more diversified portfolios with investments in all three land classes being optimal. As one becomes more pessimistic about SLR, portfolios are re-weighted more heavily towards agricultural land.

Table 4 about here

The portfolio of land that is currently preserved in the study area is reported in the last line of Table 4, and consists of 14% agricultural land, 7% forest land, and

[20] Because alternative SLR projections are grounded in different socio-technical assumptions regarding the future world, objective probabilities for these distributions have not been established (Sweet et al. 2017). Hence, we follow standard practice (e.g., Ando and Mallory 2012) and conduct sensitivity analysis according to various possible assumptions regarding these probabilities.

79% intertidal land.[21] In comparison, all optimal land portfolios for SLRs of $0.32m$ and above place greater emphasis on preserving agricultural land. This result suggests that current conservation strategies may have led to an over-investment in existing salt marsh and under-investment in transgression zone land.

When interpreting the discrepancy between optimal and observed portfolios, it must be recognized that conservation decisions are subject to a number of considerations that are not modeled here, such as supply- and demand-side constraints. For example, not all land types may be available for purchase in required quantities at all times, a conservation agency may not always be in a position to purchase land when it becomes available, and agencies may consider other conservation benefits beyond those associated with salt marsh. Planners may have different expectations regarding SLR than those considered above (although the observed portfolio is non-optimal even under very optimistic SLR expectations). Other considerations that may also influence decisions inlcude the relative likelihood of agricultural versus forest land being armored against SLR or existing marsh being developed in the absence of preservation (see Section 4.4). Moreover, despite a sense of urgency concerning the need to preserve transgression zones (Runting et al. 2017), some conservation planners might nonetheless espouse a "wait and see" approach, delaying the preservation of higher-elevation land until sea levels approach these elevations. These and other factors may represent some of the difference between the seemingly non-optimal conservation strategies that are currently observed and the optimal solutions derived here.

Such caveats aside, the large discrepancy between the observed and optimal portfolios suggests that current conservation decisions may not be optimal. This is not surprising as there is no means to characterize optimal diversification in the absence of model results such as these.

[21] These estimates are based on land conservation data from the NOAA Coastal Change Analysis Program, (https://www.coast.noaa.gov/digitalcoast/).

4.3 Benefits of Diversification

To assess the importance of optimal portfolio investment, column 5 of Table 4 reports the ratio of the benefits from the optimal x^{opt}, and observed portfolios x^{obs}, based on (18).[22] Given that the model objective is to identify the portfolio that maximizes marsh benefits x in (3), it is not surprising that the optimal portfolio outperforms the observed portfolio in this criterion in all combinations of SLR scenarios considered. The relative benefits of portfolio optimization increase the more pessimistic the conservation agency is with respect to SLR. In the optimistic case the benefits from the optimal portfolio exceed those obtainable from the observed portfolio by 8%, while the benefits from the optimal portfolio are 46% higher than those obtainable from the observed portfolio in the pessimistic case.

The observed portfolio includes 79% of intertidal land which is the least expensive land type. Portfolio optimization requires significant investment in comparatively more expensive agricultural land, resulting in 58% to 92% higher land costs than the observed portfolio (column 6, Table 4). Implications for net benefits (i.e., benefits minus costs), are calculated following Hallegatte et al. (2012). We report the difference in net benefits per km^2 of conserved land from the optimal and observed portfolios in the last column of Table 4.[23] For the optimistic case we observe marginally higher net benefits from the optimal portfolio than the observed portfolio. As greater weight is put on to the possibility of high SLR, the optimal portfolio yields net benefits that are USD $0.54m$ higher in the balanced case, and USD $1.55m$ higher in the pessimistic case than the net benefits that result from the observed portfolio.[24]

[22] The optimal and observed marsh benefits are calculated based on the initial marsh wealth per km^2 of preserved land.

[23] The net benefit for a 90-year time horizon is calculated as $NB = xv\left(1 - e^{-90\delta}\right) / \left(10^6 \times \delta\right) - \sum_i^3 \theta_i^l c_i$ and reported in USD m per representative km^2 of preserved land.

[24] A sensitivity analysis with respect to a realistic range of the privatizeable proportion of marsh benefits, $0.01 \leq \phi \geq 0.15$, reveals for the pessimistic case that the difference in net benefits between the optimal and observed portfolio range from USD $10.34m$ to USD $0.08m$. As discussed above, this parameter has no impact on the optimal portfolio, only on the benefits that are realized.

4.4 Allowance for Spatial Targeting

A natural extension of the model is to a case in which spatial targeting of conservation is desired. Although optimal portfolio models alone are not suitable for high-resolution spatial targeting (Ando and Mallory 2014)[25], the model may be integrated with supplemental analyses that inform such targeting, conditional on land classes and optimal shares identified by the portfolio model. An approach of this type allows one to capitalize on the rich spatial information provided by the underlying biophysical model.

This section illustrates such a model extension for the agricultural land class, although a parallel exercise can be conducted for any land type. The approach applies standard performance ratio methods developed within financial modeling to select individual assets optimally within asset classes (Farinelli et al. 2008; Sharpe 1994; Stoyanov et al. 2007). We illustrate this targeting for marsh conservation using a reward-risk performance ratio μ_{ak}/σ_{ak}, where μ_{ak} is the mean marsh return on individual land area k within land type a (agricultural land), and σ_{ak} is the corresponding standard deviation. This is equivalent to the Sharpe Ratio compared to a default riskless asset (zero investment) that provides zero marsh return (Sharpe 1994). Individual land areas within an asset class may be ranked in terms of this performance ratio, enabling these areas to be prioritized in terms of the reward-risk ratio.

To illustrate the approach empirically we disaggregate agricultural land in the study site into 30 areas of roughly equal size, each containing a minimum of 600 modelled grid cells, each spaced $30m$ apart. The performance ratio is calculated for each of these areas following parallel methods to those discussed above. For the sake of conciseness, we illustrate results only for the $0.6m$ SLR scenario, although analogous results may be generated for any scenario. Under this low-to-moderate SLR scenario, 14 of the

[25]Closed-form solutions are infeasible for large number of asset classes, precluding fine-scale spatial targeting within the portfolio model itself. Similar constraints apply to financial models, which provide guidance for diversification over broad asset classes (e.g., stocks, bonds) rather than specific investments (e.g., one company's stock).

30 areas are characterized by $\mu_{ak}/\sigma_{ak} > 0$, and would hence be suitable for marsh conservation. Figure 3 shows the reward-risk prioritization for these areas mapped across the case study region, along with information on area elevation in meters.

As shown by the portfolio shares in Table 3, 76% of the optimal portfolio is comprised of agricultural land under the $0.60m$ SLR scenario. Figure 3 demonstrates how specific areas can be prioritized within this agricultural land class, subject to exogenous factors such as the availability of parcels for conservation at any given time. Results are consistent with expectations. Under the $0.60m$ SLR scenario, conservation within the agricultural land class should target relatively low elevation areas close to current marsh edges - as higher elevation agricultural areas have little marsh migration under low-to-moderate SLR. Although most high priority agricultural parcels occur in low-lying easterly regions where agriculture is interspersed with current marsh, some also occur in upland areas, particularly towards the northern end of the case study area. Results such as these provide a readily accessible means to help decision-makers optimally target individual areas within general land classes (or portfolio assets), based on standard reward-risk ratios that are consistent with the underlying portfolio model.[26]

Figure 3 about here

4.5 Portfolio Optimization under Additionality

This section illustrates the effects of additionality assumptions for our case study, grounded in the model adaptation introduced by Section 2.6. For illustrative purposes, we first assume that armoring on private agricultural and forest land occurs with probabilities $p_a = p_f = 1$, while existing salt marsh is converted to some other land use with varying probability $0 \leq p_m \leq 1$. The result of this sensitivity analysis is shown

[26]Because performance ratios of this type do not consider the covariances across asset types that are central to portfolio optimization, they are not well suited to identifying underlying portfolio shares. Moreover, any within-asset targeting exercise that causes mean returns, variances or covariances of the resulting asset class to differ significantly from those used to optimize the original portfolio shares, may influence the optimality of those shares. These and related issues are discussed by Sharpe (1994), inter alia.

in the left panel of Figure 4 for the case of a $0.6m$ SLR. As the probability of salt marsh conversion increases, so does the optimal share of existing salt marsh in the land conservation portfolio. This rebalancing of the optimal portfolio occurs primarily at the expense of agricultural land shares. The right-hand panel of Figure 4 demonstrates an alternative case where existing marsh is developed with $p_m = 0.2$, agricultural land is always armored $(p_a = 1)$, while forest is armored with varying probability $0 \leq p_f \leq 1$. It shows that lower additionality from forest preservation leads to larger shares of agricultural land being held in the optimal land portfolio, as expected.[27] If marsh is allowed to migrate freely on all privately held forest land $(p_f = 0)$ and all private farm land is armored $(p_a = 1)$, then the optimal portfolio contains no forest land.[28]

Figure 4 about here

4.6 Summary of Key Results

The results presented above demonstrate the type of insight that can be provided through systematic consideration of diversification within coastal climate adaptation. Theoretical results demonstrate how optimal portfolios respond to biophysical and socio-economic dimensions, including factors such as land costs and characteristics of both the landscape and conservation planners. Empirical results point to the benefits of emphasizing a particular land type for marsh transgression (higher-elevation agricultural land) that currently represents only a small portion of coastal conservation portfolios. Other key empirical results include the following.

Result 1 Drivers: Optimal portfolios for marsh conservation depend nonlinearly on expected SLR and corresponding rates of marsh migration onto different land

[27] This analysis is based on $\lambda_i \equiv d\frac{c_i}{v_i}/dS_i = 0$, as we are unable to estimate the effect of marsh migration on the probability of armoring or marsh conversion.

[28] The observed portfolio is sub-optimal over the wide range of additionality probabilities. To obtain optimal portfolios that resemble the observed portfolio, one would have to assume very low additionality in agricultural land conservation and very high additionality in forest and marsh conservation, which is not consistent with observed patterns at the study site.

types. They also depend on economic factors such as preservation additionality across land types.

Result 2 Portfolio Composition: Greater SLR is associated with larger proportions of higher-elevation agricultural land in the marsh conservation portfolio, ranging from 0.47 at low SLR to 1.00 at high SLR in the baseline scenario. Forest land has the lowest share, at a maximum of 0.12 under low SLR.

Result 3 Policy Relevance: Current conservation strategies may have under-invested in transgression zone land, especially in agricultural land, for which the observed share is 14% compared to $60\% - 92\%$ for the optimal share.

Result 4 Benefits and Costs: Portfolio optimization yields higher conservation costs per km^2, but also results in higher marsh benefits. Considering both benefits and costs, the estimated net benefit of optimal diversification relative to observed management increase with SLR, ranging from USD $0.5m$ to $1.55m$ per km^2 of preserved land.

Result 5 Spatial Targeting: Allowing for spatial targeting, conservation within the agricultural land class for the low-medium SLR should target relatively low elevation areas close to current marsh edges.

Result 6 Additionality: The greater is the additionality in marsh conservation from a given land type the greater is its share in the optimal land portfolio.

5 Discussion and Conclusions

Although illustrated for a particular case study, the structure of the theoretical model facilitates applications to diverse marsh conservation contexts. For example, it can be adapted to other transgression zone classes or numbers, including cases in which assets are defined using explicit spatial attributes. One might also consider applications in which a lower-risk but higher-cost management option exists—such as alternatives in which marshes are artificially constructed and then maintained in perpetuity via management interventions to increase sediment delivery and accretion. Although the model is demonstrated using four illustrative SLR scenarios, it can accommodate alternative stochastic paths. The model can also be adapted to marsh migration projections provided via other biophysical modeling approaches. In addition, the model can be updated to consider patterns of marsh benefits not considered here, but that might be relevant in different coastal contexts. Parallel models could be developed for other types of migrating assets vulnerable to climate change.

Naturally, the presented results must be viewed with respect to the implicit and explicit assumptions of the model, together with the characteristics and limitations of the case study. Results of any model of this type may be affected by model specification, including the treatment of marsh migration dynamics. The model is also estimated contingent upon constant means and variances for marsh migration, whereas the inclusion of exogenous factors and structural breaks can lead to instances in which the means and variances of environmental economic phenomena are time-varying. Generalizations of the model for such cases is left for future work. The insights of the model are also contingent upon the set of management alternatives considered—here represented by the preservation of alternative land classes for marsh migration. Finally, as emphasized above, the model is designed for a case in which conservation planners wish to identify an optimal portfolio of conservation actions based on information available today. Portfolios can then be rebalanced at any time to accommodate new information as it becomes available. We do not, however, formally consider the optimal *timing* of marsh conservation decisions, which would require a different modeling approach and is beyond the scope of the current analysis.

In closing, we emphasize that despite the complexity of the underlying mathematics, portfolio models of dynamic, natural assets provide concrete empirical guidance that is readily understood by decision-makers, and for which sensitivity analysis can be conducted. This facilitates the direct use and exploration of results to inform adaptation decisions.

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A Figures and Tables

Figure 1: Aerial photograph of the VCR Study Site, Virginia, USA (Google Earth).

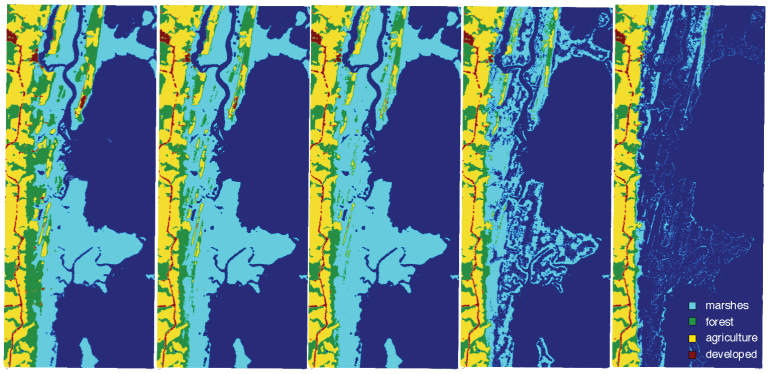

Figure 2: Forecast Land Use Cover by the year 2100 for the VCR Study Site (maps left to right are current conditions and projections at 0.32, 0.60, 0.90 and 1.7m sea level rise, respectively).

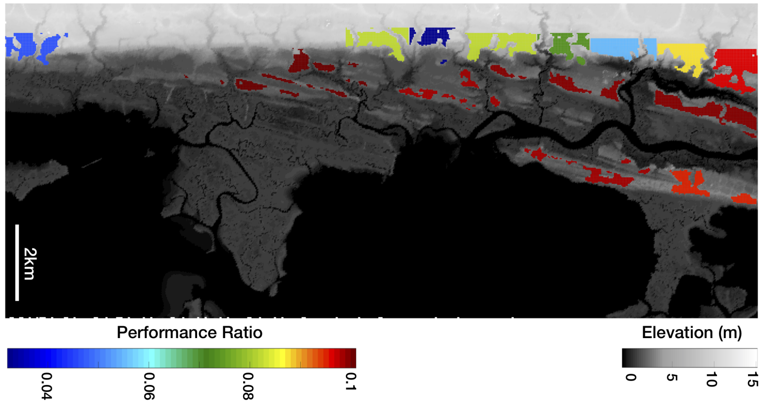

Figure 3: Conservation ranking of individual agricultural land areas based on the marsh migration reward-risk performance ratio, μ_{ak}/σ_{ak}. Only areas for which $\mu_{ak}/\sigma_{ak} > 0$ are shown.

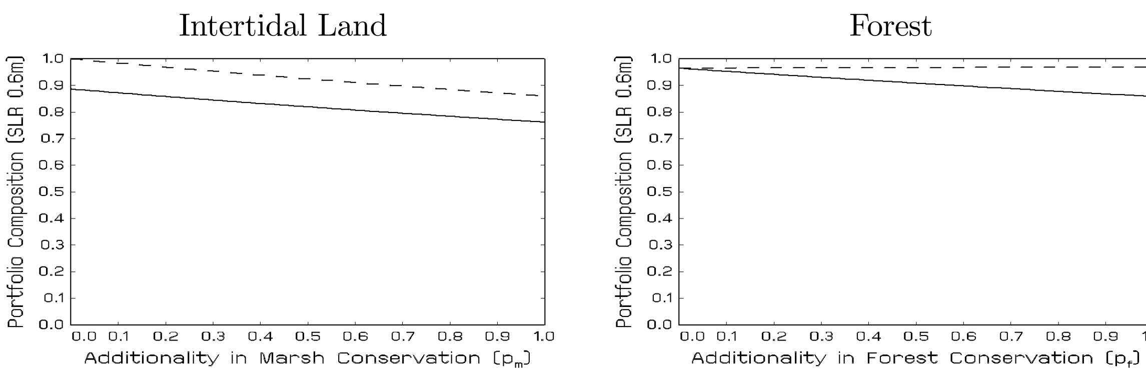

Figure 4: Optimal land portfolio composition under varying probabilities of marsh benefits being lost in the absence of land class conservation. The continuous line is θ_a^l and the dashed line is $\theta_a^l + \theta_f^l$.

Table 1:

Base case parameter values.

Description	Parameter	Value	Unit	Source[(a)]
Agricultural transgression area	N_a	28.08	km^2	emp. biophys. model
Forest transgression area	N_f	25.61	km^2	emp. biophys. model
Intertidal marsh area	N_m	44.50	km^2	emp. biophys. model
Prop. of marsh on agric. land	S_a	0.028		emp. biophys. model
Prop. of marsh on forest land	S_f	0.161		emp. biophys. model
Prop. of marsh on intertidal land	S_m	0.950		emp. biophys. model
Agricultural land cost	c_a	$1,425,302$	$\$/km^2$	Gardner & Johnston (2018)
Forest land cost	c_f	$904,651$	$\$/km^2$	Gardner & Johnston (2018)
Intertidal land cost	c_m	$567,320$	$\$/km^2$	Gardner & Johnston (2018)
Change in agricultural land cost	dc_a/dS_a	$-8,580$	$\$/km^2$	comp.
Change in forest land cost	dc_f/dS_f	$-3,373$	$\$/km^2$	comp.
Change in intertidal land cost	dc_m/dS_m	$5,673$	$\$/km^2$	comp.
Willingness to pay for marsh (p.a.)	v	$3,104,403$	$\$/km^2$	Ghermandi et al. (2010)
Privatizeable marsh benefits (prop.)	ϕ	0.05		Ghermandi et al. (2010)
Discount rate (p.a.)	δ	0.03		U.S. EPA. (2014)
Risk aversion parameter	γ	1.50		Leroux et al. (2016, 2018)

(a) 'emp. biophys. model': the parameter value is empirically derived from the biophysical marsh model, 'comp.': the parameter value is computed from empirical estimates.

Table 2:

Annual mean change (μ_i), standard deviation (σ_i) and correlation $\left(\rho_{ij}\right)$ of the proportion of marsh on agricultural land ($i = a$), forest ($i = f$) and intertidal land ($i = m$) by SLR scenario.

SLR	Marsh Migration						Correlation Coefficients		
	Agriculture		Forest		Intertidal				
	μ_a	σ_a	μ_f	σ_f	μ_m	σ_m	ρ_{af}	ρ_{am}	ρ_{fm}
$0.32m$	0.000	0.005	0.001	0.038	0.001	0.011	0.930	-0.545	-0.652
$0.60m$	0.001	0.006	0.002	0.027	0.000	0.012	0.832	-0.778	-0.896
$0.90m$	0.001	0.007	0.003	0.025	-0.002	0.029	0.771	-0.867	-0.672
$1.70m$	0.002	0.007	0.003	0.025	-0.010	0.061	0.624	-0.895	-0.399

Table 3:

Optimal marsh and land portfolio shares for agricultural land $\left(\theta_a, \theta_a^l\right)$, forest $\left(\theta_f, \theta_f^l\right)$ and intertidal land $\left(\theta_m, \theta_m^l\right)$ by SLR scenario.

SLR	Marsh Shares$^{(a)}$			Land Shares$^{(b)}$		
	θ_a	θ_f	θ_m	θ_a^l	θ_f^l	θ_m^l
$0.32m$	0.07	0.07	0.86	0.47	0.12	0.41
$0.60m$	0.25	0.12	0.63	0.76	0.10	0.14
$0.90m$	0.51	0.00	0.49	0.93	0.00	0.07
$1.70m$	1.00	0.00	0.00	1.00	0.00	0.00

(a) Based on equations (14) to (16). (b) Based on equation (17).

Table 4:

Optimal land portfolio shares for agricultural land (θ_a^l), forest (θ_f^l) and intertidal land (θ_m^l) under alternative weights on a combined SLR scenario.

Combined SLR	Land Shares$^{(a)}$			Benefit Ratio$^{(b)}$	Cost Ratio$^{(c)}$	Net Benefit Difference$^{(d)}$
	θ_a^l	θ_f^l	θ_m^l	$\frac{x^{opt}}{x^{obs}}$	$\frac{cost^{opt}}{cost^{obs}}$	$NB^{opt} - NB^{obs}$
Optimistic$^{(e)}$	0.60	0.09	0.31	1.08	1.58	0.05
Balanced$^{(f)}$	0.79	0.05	0.16	1.21	1.79	0.54
Pessimistic$^{(g)}$	0.92	0.02	0.06	1.46	1.92	1.55
Observed	0.14	0.07	0.79	1	1	0

(a) Based on equation (17). (b) Based on equation (18). (c) Ratio of total optimal and observed portfolio land cost based per km^2. (d) Difference in net benefit between the optimal and observed portfolio in USD m per km^2. Combined SLR based on weights of (e) 0.7, 0.1, 0.1 and 0.1; (f) 0.25, 0.25, 0.25 and 0.25 and (g) 0.1, 0.1, 0.1 and 0.7 for SLR scenarios of 0.32m, 1.60m, 1.90m and 1.70m.

A Appendix: Derivations of Marsh Portfolio Model

This appendix contains derivations of key equations shown in the main text. The optimal solution (see Kamien and Schwartz 1981, p. 248) is obtained by solving the Hamilton-Jacobi-Bellman equation given in (13). Maximizing this equation with respect to x and rearranging gives the following expression for marsh benefit

$$x = (\phi v)^{-\frac{1}{\gamma}} V_M^{-\frac{1}{\gamma}}, \tag{A6}$$

where V_M is the derivative of the value function V with respect to the marsh budget M. Maximizing the right-hand side of (13) with respect to θ_a and θ_f and solving the system of linear equations yields the portfolio shares for agricultural land and forest

$$\begin{pmatrix} \theta_a \\ \theta_f \end{pmatrix} = \begin{pmatrix} a & b \\ b & c \end{pmatrix}^{-1} \begin{pmatrix} z \\ y \end{pmatrix}, \tag{A7}$$

where

$$\begin{aligned}
a &= \widetilde{\sigma}_a^2 - 2\widetilde{\sigma}_{am} + \widetilde{\sigma}_m^2, \\
b &= \widetilde{\sigma}_m^2 + \widetilde{\sigma}_{af} - \widetilde{\sigma}_{am} - \widetilde{\sigma}_{fm}, \\
c &= \widetilde{\sigma}_f^2 - 2\widetilde{\sigma}_{fm} + \widetilde{\sigma}_m^2, \\
z &= -\widetilde{\mu}_a \frac{V_M}{MV_{MM}} - \widetilde{\sigma}_{am} + \widetilde{\sigma}_m^2, \\
y &= -\widetilde{\mu}_f \frac{V_M}{MV_{MM}} - \widetilde{\sigma}_{fm} + \widetilde{\sigma}_m^2,
\end{aligned}$$

and V_{MM} is the second derivative of V with respect to M. Substituting optimal consumption from (A6) and the optimal share expressions from (A7) into the right-hand side of (13), yields the second-order differential equation

$$\begin{aligned}
\delta V &= \frac{\gamma}{1-\gamma} (\phi v)^{1-\frac{1}{\gamma}} V_M^{1-\frac{1}{\gamma}} + \left[\theta_a \widetilde{\mu}_a + \theta_f \widetilde{\mu}_f + \widetilde{\mu}_m\right] MV_M \\
&+ \frac{1}{2} \left[\theta_a^2 \left(\widetilde{\sigma}_a^2 - 2\widetilde{\sigma}_{am} + \widetilde{\sigma}_m^2\right) + \widetilde{\sigma}_m^2 + \theta_f^2 \left(\widetilde{\sigma}_f^2 - 2\widetilde{\sigma}_{fm} + \widetilde{\sigma}_m^2\right)\right. \\
&+ 2\theta_a\theta_f \left(\widetilde{\sigma}_{af} - \widetilde{\sigma}_{am} - \widetilde{\sigma}_{fm} + \widetilde{\sigma}_m^2\right) \\
&\left. + 2\theta_a \left(\widetilde{\sigma}_{am} - \widetilde{\sigma}_m^2\right) + 2\theta_f \left(\widetilde{\sigma}_{fm} - \widetilde{\sigma}_m^2\right)\right] M^2 V_{MM}.
\end{aligned} \tag{A8}$$

A closed-form solution to (A8) is given by

$$V = AM^{1-\gamma}, \tag{A9}$$

where A is an unknown constant as implicitly defined in (19). Substitution of the derivatives V_M and V_{MM} into (A7) yields the optimal portfolio shares (14) to (16). Substituting the derivative of (A9) into (A6) yields the expression for optimal marsh benefits given in (18), where the analytical solution for A given in (19) is obtained by substituting (A9) and its corresponding derivatives into (A8) and rearranging. Note that the marsh benefits arising from any non-optimal marsh portfolio are also determined by (18), except that θ_a, θ_f and θ_m now represent the non-optimal portfolio shares.

B Appendix: Biophysical Dynamics of Marsh Migration

The biophysical model of marsh migration is adapted from prior work (Kirwan and Murray 2008; Kirwan et al. 2010, 2016b) and a spatial model by Langston et al. (2019). The model describes the response to relative sea level rise (SLR) of a spatial 2-dimensional topography $Z(x, y, t)$ subject to different land uses. Land uses are grouped into five general classifications: open (salt) water, marsh, forest, agricultural and developed land. Given a change in mean sea level (MSL) the model computes the change in elevation, dominated by marsh accretion, which is adjusted for land use. The parameter values of the model are calibrated to measurements taken from the Virginia Coast Reserve LTER study area.

Change in Mean Sea Level

We assume the mean sea level Z_{MSL} at a given time t is given by

$$Z_{\mathrm{MSL}} = \int_0^t \dot{Z}_{\mathrm{MSL}} dt + \delta Z_{\mathrm{MSL}}, \tag{B1}$$

where $t = 0$ denotes the initial condition, $\dot{Z}_{\mathrm{MSL}} = R(t)$ is the average rate of relative sea level rise, and δZ_{MSL} is the fluctuation in mean sea level. The fluctuation in MSL was found to be normally distributed with standard deviation σ, based on the de-trended, de-seasonalised interannual variation of MSL from Wachapreague, VA, a site in close proximity to the study area. Assuming a typical response time of marshes and forests to changes in MSL of 6 months, the distribution of MSL fluctuations averaged over that time has a standard deviation $\sigma \approx 0.04m$. In the simulations we use σ to generate an annual time series of random fluctuations (δZ_{MSL}) for different scenarios j of sea level rise parameterized by a set of curves $R_j(t)$.

Marsh Accretion

We assume marsh habitat is defined by flooding frequency and is confined to a fixed range in elevations: $Z^m_{\min} < Z_r < Z^m_{\max}$, where $Z_r = Z - Z_{\mathrm{MSL}}(t)$ is the elevation relative to mean sea level. The limits $Z^m_{\min}$ and $Z^m_{\max}$, defined relative to mean sea level, are calculated empirically.

Marshes naturally promote organic and inorganic accretion in response to SLR. The surface accretion rate $\dot{Z} \equiv \mathrm{d}Z/\mathrm{d}t$ can be written as the sum of the organic A_o and the inorganic A_i accretion rates

$$\dot{Z} = A_o(Z_r) + A_i(Z_r, \ell), \tag{B2}$$

where we assume: (1) the organic accretion rate A_o is a function of the local elevation relative to MSL ($Z_r = Z - Z_{MSL}$), and (2) the inorganic accretion rate A_i is function of both, the local relative elevation Z_r, and the distance $\ell(x, y)$ to the nearest sediment source, i.e. channels or flats, defined in the current context as 'open water'.

Organic accretion rate: The organic accretion rate $A_o(Z_r)$ is defined empirically by calibrating to observed surface accretion rates (after removing the mineral contribution) and assuming its continuity ($A_o = 0$) along marsh elevation limits, $Z^m_{\min}$ and $Z^m_{\max}$. We find accretion rates change drastically from a typical value A_o^{low} to A_o^{high} at a 'critical' elevation $Z_r = Z_c$ and can be approximated by

$$A_o(Z_0 < Z_r < Z_1) = A_o^{\mathrm{low}}\Theta(Z_c - Z_r) + A_o^{\mathrm{high}}\Theta(Z_r - Z_c) \tag{B3}$$

for Z_r within the range (Z_0, Z_1) defined by the quadratic equation $(Z^m_{\max} - Z_{0,1})(Z_{0,1} - Z^m_{\min}) = (Z^m_{\max} - Z^m_{\min})^2/8$. Outside that range, we assume accretion rates decay to zero according to

$$A_o(Z_1 < Z_r < Z^m_{\max}) = A_o^{\mathrm{high}}\frac{8(Z^m_{\max} - Z_r)(Z_r - Z^m_{\min})}{(Z^m_{\max} - Z^m_{\min})^2}, \tag{B4}$$

and

$$A_o(Z^m_{\min} < Z_r < Z_0) = A_o^{\text{low}} \frac{8(Z^m_{\max} - Z_r)(Z_r - Z^m_{\min})}{(Z^m_{\max} - Z^m_{\min})^2}. \tag{B5}$$

Inorganic accretion rate: Following reported field measurements and the solution of simplified conservation equations (Langston et al. 2019) we assume the inorganic accretion rate A_i decays exponentially with the distance $\ell(x, y, t)$ to the nearest sediment source:

$$A_i(Z_r, \ell) = A_i^0(Z_r) \exp\left(-\ell/L_c^{\text{basin}}\right), \tag{B6}$$

where L_c^{basin} is the decay length and $A_i^0(Z_r)$ is the accretion rate at the marsh edge ($\ell = 0$).

The decay length L_c^{basin} in (B6) for a given basin is

$$L_c^{\text{basin}} = 1.5 L^{\text{basin}} \tau/(T w_f^e), \tag{B7}$$

where L^{basin} is the size of the local basin, τ is the tidal range, T is the tidal period and w_f^e is the particle effective settling velocity. The size of the local basin L^{basin} is defined as the local maximum of $\ell(x, y, t)$.

The accretion rate $A_i^0(Z_r)$ at the marsh edge is

$$A_i^0(Z_r) = A_i^{\max} F\left(Z_r\right) \left(1 + \left(1 + T w_f^e/\tau\right)^{-1}\right)/2, \tag{B8}$$

with flooding frequency $F\left(Z_r\right) \approx (1/2 - Z_r/\tau)$ and theoretical maximum accretion rate $A_i^{\max} = C_0 w_f^e/\rho_m$, where C_0 is the average suspended sediment concentration at the marsh edge and ρ_m is the characteristic density of mineral sediments inside the marsh root layer.

Parameters for the marsh accretion model: Field data from Phillips Creek, VA (a location within the study area) is consistent with the following set of parameters: $Z^m_{\min} = 0.1$m, $Z^m_{\max} = 1.2$m, $Z_c = 0.8$m, $A_o^{\text{low}} = 2.4\,10^{-3}$m/s, $A_o^{\text{high}} = 6\,10^{-3}$m/s, $\tau = 1.4$m, $w_f^e = 10^{-4}$m/s, $T = 12.5$h and $C_0 = 5\,10^{-2}$kg/m^3. For the other parameters we use $w_f^e = 10^{-4} m/s$ and $\rho_m = 2\,10^3$kg/m^3.

Spatial character of the model: The marsh accretion rate, defined by equations (B2) and (B6), decreases with the distance to the marsh edge (sediment source) and thus changes within the marsh platform. Under SLR, this could lead to local marsh loss, such as the formation of ponds and channels, as shown in Fig. 2. Furthermore, as explained below, the dependence of the marsh accretion rate on the elevation Z_r relative to the mean sea level, ensures the upland marsh migration under SLR as the inter-tidal region characterizing marsh habitat propagates upland with an increase in sea level.

Change in Land Cover

We model only changes in land cover driven by long-term SLR and interannual fluctuations in mean sea level. Since the location of the mean sea level essentially controls marsh dynamics, those changes are of three types: (1) conversion to open waters due to marsh drowning at lower elevations; (2) conversion to marsh due to increasing flooding; and (3) recovery of forest and agricultural land or developed areas due to temporal marsh loss at higher elevation.

Conversion to open waters: Following our definition of the marsh habitat, we assume for elevations in the range $Z < Z^m_{\min} + Z_{\mathrm{MSL}}(t)$, marshes drown within the time interval $\Delta t = 1$yr used to integrate the model. In that case, land is converted to 'open water', a classification that includes coastal lagoons, tidal flats and tidal channels.

Conversion to marshes: We assume elevations in the range $Z^m_{\min} + Z_{\mathrm{MSL}}(t) < Z(t) < Z^m_{\max} + Z_{\mathrm{MSL}}(t)$, convert into marsh within the time interval $\Delta t = 1$yr, which leads to marsh upland migration into forest and agricultural land.

Recovery of uplands due to marsh loss: A temporal decrease in mean sea level due to a random fluctuation can lead to the temporary loss of marsh for elevations in the range $Z > Z^m_{\max} + Z_{\mathrm{MSL}}(t)$, and the partial recovery of the previous land use in that location.

C Appendix: Sensitivity Analysis to Land Costs

This appendix evaluates the robustness of the empirical results in the main text to alternative land cost estimates, c_a, c_f and c_m, and their sensitivity to marsh encroachment, dc_a/dS_a, dc_f/dS_f and dc_m/dS_m. To conduct the analysis, we re-calibrate the portfolio model using alternative cost estimates drawn from two prior hedonic models of undeveloped land prices in the Northeastern US. The first, Gardner and Johnston (2018), predicts costs of different types of undeveloped land that could be purchased to ensure marsh migration, including farm, forest and intertidal marsh. This model was estimated using data on all raw land transactions from 2014 to 2016 in the two coastal counties that encompass our case study site (Accomack and Northampton, Virginia), and incorporates spatial variables such as elevation and coastal distance. The second, Allen et al. (2006), predicts easement prices for similar types of conservation land (farm, forest and wetland) in three Delaware (USA) coastal counties. This model has been used previously to inform published models of optimal land conservation (e.g., Duke et al. 2014). The applied hedonic models are taken from Table 2 in Gardner and Johnston (2018) and Table 9 in Allen et al. (2006). We estimate costs assuming a 1 km^2 parcel of agricultural and forest land at 1.96m of elevation, with a parcel centroid within 100m of the Atlantic coast. The latter two assumptions approximate anticipated locations and elevations of marsh by 2100, under a high SLR scenario. For the Allen et al. (2006) model, we further assume a parcel that is 20 miles from the nearest urban area, approximating the mean distance from the study site to the town of Cape Charles, Virginia. Other variables are held at mean values for each dataset. All cost estimates are updated to 2018 USD.

Table 5 compares the resulting land cost estimates to those used in the main text. Cost estimates are similar regardless of source, with the only notable difference being somewhat lower costs of forest land estimated using Allen et al. (2006). As shown in Table 6, re-calibrating the model according to these alternative land cost estimates yields very similar land shares to those shown in the main text, with no portfolio share showing more than a 5 percentage point difference across the three calibrations, and shares virtually identical for higher rates of SLR. None of the fundamental results discussed in the main text change when alternative land cost estimates are used for calibration, suggesting a high degree of robustness to these alternative sources of cost information.

Table 5:

Alternative land costs.

Description	Parameter	Base Costs$^{(a)}$	Gardner & Johnston	Allen et al.	Unit
Agricultural land cost	c_a	$1,425,302$	$1,376,070$	$1,310,174$	$\$/km^2$
Forest land cost	c_f	$904,651$	$945,749$	$608,635$	$\$/km^2$
Salt marsh land cost	c_m	$567,320$	$511,648$	$433,130$	$\$/km^2$
Change in ag. land cost	dc_a/dS_a	$-8,580$	$-8,651$	$-8,770$	$\$/km^2$
Change in forest land cost	dc_f/dS_f	$-3,373$	$-4,342$	$-1,755$	$\$/km^2$
Change in marsh land cost	dc_m/dS_m	$5,673$	$5,116$	$4,331$	$\$/km^2$

(a) Average land costs drawn from Gardner & Johnston (2018). (b) Based on the hedonic land cost model of Gardner & Johnston (2018). (c) Based on the hedonic land cost model of Allen et al. (2006).

Table 6:

Optimal land portfolio shares for agricultural land (θ_a^l), forest (θ_f^l) and intertidal land (θ_m^l) for alternative sea level rise (SLR) scenarios and alternative land costs, based on equation (17).

SLR	Base Costs$^{(a)}$			Gardner & Johnston$^{(b)}$			Allen et al.$^{(c)}$		
	θ_a^l	θ_f^l	θ_m^l	θ_a^l	θ_f^l	θ_m^l	θ_a^l	θ_f^l	θ_m^l
$0.32m$	0.47	0.12	0.41	0.47	0.11	0.42	0.42	0.15	0.44
$0.60m$	0.76	0.10	0.14	0.76	0.09	0.15	0.72	0.13	0.16
$0.90m$	0.93	0.00	0.07	0.93	0.00	0.07	0.92	0.00	0.08
$1.70m$	1.00	0.00	0.00	1.00	0.00	0.00	1.00	0.00	0.00

(a) Based on the average land costs drawn from Gardner & Johnston (2018). (b) Based on the hedonic land cost models of Gardner & Johnston (2018). (c) Based on the hedonic land cost model of Allen et al. (2006).