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Coded Energy-Efficient Beam-Alignment for Millimeter-Wave Networks

Muddassar Hussain, Nicolò Michelusi

***Abstract*— Millimeter-wave communications rely on narrow-beam transmissions to cope with the strong signal attenuation at these frequencies, thus demanding precise alignment between transmitter and receiver. However, the beam-alignment procedure may entail a huge overhead and its performance may be degraded by detection errors. This paper proposes a coded energy-efficient beam-alignment scheme, robust against detection errors. Specifically, the beam-alignment sequence is designed such that the error-free feedback sequences are generated from a codebook with the desired error correction capabilities. Therefore, in the presence of detection errors, the error-free feedback sequences can be recovered with high probability. The assignment of beams to codewords is designed to optimize energy efficiency, and a water-filling solution is proved. The numerical results with analog beams depict up to 4dB and 8dB gains over exhaustive and uncoded beam-alignment schemes, respectively.**

I. Introduction

Millimeter-wave (mm-wave) technology is emerging as a promising solution to enable multi-Gbps communication, thanks to abundant bandwidth availability [1]. However, signal propagation at these frequencies poses several challenges to the design of future communication systems supporting high throughput and mobility, due to high isotropic path loss and sensitivity to blockages [2]. To compensate the propagation loss [1], mm-wave systems will leverage narrow-beam communications, by using large antenna arrays at both base stations (BSs) and user-ends (UEs).

However, narrow beams are susceptible to frequent loss of alignment due to mobility or blockage, which necessitate the use of beam-alignment protocols. These use significant communication resources, and it is therefore imperative to design schemes to mitigate the overhead. Beam-alignment in mm-wave has been a subject of intensive research. The simplest and yet most popular scheme is *exhaustive* search [3], which sequentially scans through all possible BS-UE beam pairs and selects the one with maximum signal power. A version of this scheme has been adopted in existing mm-wave standards including IEEE 802.15.3c [4] and IEEE 802.11ad [5]. An interactive version has been proposed in [6], wherein the beam-alignment phase is terminated once the power of the received beacon is above a certain threshold. The second popular scheme is *iterative* search [7], where scanning is first performed using wider beams followed by refinement using narrow beams. In [8], a multiuser beam-alignment scheme is proposed that performs beam-alignment by exploring the angle of departure (AoD) and angle of arrival (AoA) domain through pseudorandom multi-finger beam patterns. The authors use tools from compressed sensing to estimate the AoD/AoA pair.

In the aforementioned papers, the optimality of the corresponding search scheme is not established and the energy cost of beam-alignment is neglected, which may be significant when targeting high detection accuracy. To address it, in our previous works [9]–[11], we designed optimal energy-efficient beam-alignment protocols. In [11], we prove the optimality of a *fractional search* method. In [10], we account for the UE mobility by widening the BS beam to mitigate the uncertainty on the UE position, and optimize the trade-off between data communication and the cost of beam-sweeping. The algorithms [9]–[12] are designed based on the assumption that no detection errors occur in the beam-alignment phase. However, the performance may deteriorate due to mis-detection and false-alarm errors, causing a loss of alignment during the communication phase. Therefore, it is of great interest to design beam-alignment algorithms robust to detection errors and, at the same time, energy-efficient.

Motivated by these observations, in this paper we consider the design of an energy-efficient beam-alignment protocol robust to detection errors. To do so, we restrict the solution space for the beams such that the error-free feedback sequence can only be generated from a codebook with error correction capabilities. Thus, if detection errors occur, the error-free feedback sequence may still be recovered with high probability by leveraging the structure of the error correction code. We pose the beam-alignment problem as a convex optimization problem to minimize the average power consumption, and provide its closed-form solution that resembles a "water-filling" over the beamwidths of the beam-alignment beam patterns. The numerical results depict the superior performance of the proposed coded technique, with up to 4dB and 8dB gains over exhaustive and uncoded beam-alignment schemes, respectively. Open- and closed-loop error control sounding schemes have been studied in [13], but with no consideration on energy-efficient design. *To the best of our knowledge, this paper is the first to propose a coded beam-alignment scheme, which is both energy-efficient and robust to detection errors.*

In [14], beam-alignment is treated as a beam discovery problem in which locating beams with strong path reflectors is analogous to locating errors in a linear block code. Unlike [14], we use error correction to correct errors during the beam-alignment procedure, rather than to detect strong signal clusters. Unlike [9], [11], [12] which rely on continuous feedback from UEs to BS, we consider a scheme where the feedback is generated only at the end of the beam-alignment

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Authors are with the School of Electrical and Computer Engineering, Purdue University. email: {hussai13,michelus}@purdue.edu.

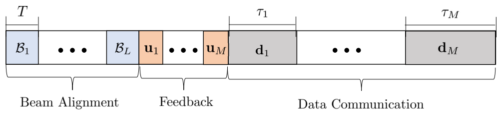

Fig. 1: Timing Diagram.

phase, which scales well to multiple users scenarios.

The rest of the paper is organized as follows. In Sec. II, we present the system model; in Sec. III, we present the optimization problem and analysis; in Sec. IV, we present numerical results, followed by concluding remarks in Sec. V.

II. SYSTEM MODEL

We consider a mm-wave cellular network with a single base-station (BS) and M users (UEs) denoted as $\text{UE}_i, i = 1, 2, \dots, M$, in a downlink scenario. UE_i is at distance $d_i \leq d_{\max}$ from BS, where $d_{\max} > 0$ is the coverage radius of the BS. We assume that there is a single strongest path between the BS and each UE_i, whose angle of departure (AoD) and angle of arrival (AoA) are denoted by $\theta_{\text{t},i} \sim \mathcal{U}[-\pi/2, \pi/2]$ and $\theta_{\text{r},i} \sim \mathcal{U}[-\pi/2, \pi/2]$, respectively. $\mathcal{U}[a, b]$ denotes the uniform distribution over the interval $[a, b]$. We use the *sectored antenna* model to approximate the beam patterns of the BS and UEs [15]. Under such model, the beamforming gain is characterized by the angular support of the BS and UE beams, denoted as $\mathcal{B}_{\text{t},k} \subseteq [-\pi/2, \pi/2]$ and $\mathcal{B}_{\text{r},k} \subseteq [-\pi/2, \pi/2]$, respectively, and is given by

$$G(\mathcal{B}_k, \boldsymbol{\theta}_i) = \frac{\pi^2}{|\mathcal{B}_k|} \chi(\boldsymbol{\theta}_i \in \mathcal{B}_k), \tag{1}$$

where $\mathcal{B}_k \equiv \mathcal{B}_{\text{t},k} \times \mathcal{B}_{\text{r},k}$ and $\boldsymbol{\theta}_i \triangleq (\theta_{\text{t},i}, \theta_{\text{r},i})$; $\chi(\boldsymbol{\theta} \in \mathcal{A})$ is the indicator function of the set $\mathcal{A}$, and $|\mathcal{A}| \triangleq \int_{\mathcal{A}} \mathrm{d}\boldsymbol{\theta}$ is its Lebesgue measure. In other words, if the AoD/AoA $\boldsymbol{\theta}$ lies in the beam support $\mathcal{B}_k$ of the BS and UE, then the signal is received with gain $\frac{\pi^2}{|\mathcal{B}_k|}$; otherwise, only noise is received. The received signal at UE_i can thus be expressed as

$$\mathbf{y}_k^{(i)} = h_k^{(i)} \sqrt{P_k G(\mathcal{B}_k, \boldsymbol{\theta}_i)} \mathbf{s}_k + \mathbf{n}_k^{(i)}, \tag{2}$$

where k is the slot index, $\mathbf{s}_k$ is the transmitted sequence, P_k is the transmission power of the BS, $h_k^{(i)}$ is the complex channel gain between the BS and UE_i, and $\mathbf{n}_{k,i} \sim \mathcal{CN}(\mathbf{0}, N_0 W_{\text{tot}} \mathbf{I})$ is complex additive white Gaussian noise (AWGN). The quantity N_0 denotes the one-sided power spectral density of the AWGN channel and W_{tot} is the system bandwidth. We assume Rayleigh fading channels $h_k^{(i)} \sim \mathcal{CN}(0, 1/\ell(d_i)), \forall i, k$, independent across UEs and i.i.d over slots, where $\ell(d_i)$ is the path loss between the BS and UE_i.

We consider a time-slotted system where the frame duration $T_{\text{fr}}[s]$ is divided into three phases: beam-alignment, feedback and data communication, of duration T_s, T_{fb} and T_d, respectively, with $T_s + T_{\text{fb}} + T_d = T_{\text{fr}}$, as depicted in Fig. 1. Data transmission is orthogonalized across users according to a TDMA strategy. We now describe these phases in more detail.

Beam-Alignment Protocol: The beam-alignment phase, of duration T_s, is divided into L slots, each of duration $T = T_s/L$, indexed by the set $\mathcal{I}_s = \{1, \dots, L\}$. In each beam-alignment slot, the BS sends a pilot sequence $\mathbf{s}_k$ using the sequence of beams $\{\mathcal{B}_{\text{t},k}, k=1, \dots, L\}$. Simultaneously, each UE receives using the sequence of beams $\{\mathcal{B}_{\text{r},k}, k=1, \dots, L\}$. In each beam-alignment slot, UE_i tests whether $\boldsymbol{\theta}_i \in \mathcal{B}_k$ (alignment) or $\boldsymbol{\theta}_i \notin \mathcal{B}_k$ (mis-alignment). This can be expressed as the following hypothesis testing problem:

$$\mathcal{H}_1: \ \mathbf{y}_k^{(i)} = h_k^{(i)} \sqrt{\frac{\pi^2 P_k}{|\mathcal{B}_k|}} \mathbf{s}_k + \mathbf{n}_k^{(i)}, \ \text{(alignment)}, \tag{3}$$
$$\mathcal{H}_0: \ \mathbf{y}_k^{(i)} = \mathbf{n}_k^{(i)}, \ \text{(mis-alignment)}.$$

Under no CSI ($h_k^{(i)}$ unknown), the optimal Neyman-Pearson detector for the above binary problem is the threshold detector

$$\frac{|\mathbf{s}_k^H \mathbf{y}_k^{(i)}|^2}{N_0 W_{\text{tot}} \|\mathbf{s}_k\|_2^2} \underset{\mathcal{H}_1}{\overset{\mathcal{H}_0}{\lessgtr}} \tau_{\text{th}}. \tag{4}$$

If UE_i infers that $\mathcal{H}_1$ is true, then it generates $u_k^{(i)}=1$, otherwise $u_k^{(i)}=0$. Each UE generates its detection sequence $\mathbf{u}_i \triangleq (u_1^{(i)}, u_2^{(i)}, \dots, u_L^{(i)}) \in \{0,1\}^L$ with the above detector. This is used to infer the AoD/AoA $\boldsymbol{\theta}_i$, and to design the beams for the data communication phase, as detailed below.

$$\text{Let } \mathbf{c}_i \triangleq (c_1^{(i)}, c_2^{(i)}, \dots, c_L^{(i)}) \text{ with } c_k^{(i)} = \chi(\boldsymbol{\theta}_i \in \mathcal{B}_k) \tag{5}$$

denote the error-free detection sequence. The detected $\mathbf{u}_i$ may incur mis-detection ($u_k^{(i)}=0$ but $c_k^{(i)}=1$) or false-alarm errors ($u_k^{(i)}=1$ but $c_k^{(i)}=0$) , with probabilities (these can be obtained from the signal model (3))

$$p_{\text{md},i} = 1 - \exp\left(-\frac{\tau_{\text{th}} |\mathcal{B}_k| N_0 W_{\text{tot}} \ell(d_i)}{|\mathcal{B}_k| N_0 W_{\text{tot}} \ell(d_i) + P_k \pi^2 \|\mathbf{s}_k\|_2^2}\right), \tag{6}$$
$$p_{\text{fa},i} = \exp\left(-\tau_{\text{th}}\right). \tag{7}$$

The BS transmission power P_k and detector threshold τ_{th} are designed to guarantee maximum error probabilities $p_{\text{md},i}, p_{\text{fa},i} \leq p_e$ across users (this can be achieved via appropriate beam design, see [16]), which yields

$$\tau_{\text{th}} = -\ln(p_e), \tag{8}$$
$$P_k \geq \frac{N_0 W_{\text{tot}} \ell(d_i)}{\pi^2 \|\mathbf{s}_k\|_2^2} \left[\frac{\ln(p_e)}{\ln(1-p_e)} - 1\right] |\mathcal{B}_k|, \ \forall i \in \{1, \dots, M\}.$$

Equivalently, we can express the energy $E_k \triangleq T_{\text{sy}} P_k \|\mathbf{s}_k\|^2$ as

$$E_k \geq \phi_s |\mathcal{B}_k| \tag{9}$$

where T_{sy} is the symbol duration; ϕ_s is the energy/rad^2 to guarantee the required detection performance among all UEs,

$$\phi_s \triangleq \frac{N_0 W_{\text{tot}} T_{\text{sy}}}{\pi^2} \left[\frac{\ln(p_e)}{\ln(1-p_e)} - 1\right] \cdot \ell(d_{\max}). \tag{10}$$

In the rest of the paper, we enforce equality in (9) for the purpose of energy-efficient beam-alignment design, and assume that $p_{\text{md},i} = p_{\text{fa},i} = p_e, \forall i$. Note that this is the worst-case scenario; in fact, in practice, an UE closer to the BS may experience a lower mis-detection probability $p_{\mathrm{md},i} < p_e$ as a result of $\ell(d_i) < \ell(d_{\max})$, see (6).

With this notation, we write the detection sequence as

$$\mathbf{u}_i \triangleq \mathbf{c}_i \oplus \mathbf{e}_i, \tag{11}$$

where $\oplus$ denotes entry-wise modulo 2 addition, and $\mathbf{e}_i \in \{0,1\}^L$ is the beam-alignment error sequence of UE$_i$. Due to the i.i.d. Rayleigh fading assumption and to the fact that false-alarm and misdetection errors occur with probability $p_{\mathrm{md},i} = p_{\mathrm{fa},i} = p_e$, independently across slots, it follows that $\mathbf{e}_i$ is independent of $\mathbf{c}_i$, and that errors are i.i.d. across UEs and slots, with probability mass function (pmf)

$$p(\mathbf{e}_i) = p_e^{W(\mathbf{e}_i)}(1-p_e)^{L-W(\mathbf{e}_i)}, \tag{12}$$

where $W(d) \triangleq \sum_{k=1}^{L} d_k$ is the Hamming weight of $\mathbf{d} \in \{0,1\}^L$.

We now design a coded beam-alignment strategy, robust to detection errors. If UE$_i$ was provided with the error-free detection sequence $\mathbf{c}_i$, it could infer the support of $\boldsymbol{\theta}_i$ relative to the beam sequence $\{\mathcal{B}_k, k{=}1,\dots,L\}$ to be

$$\boldsymbol{\theta}_i \in \mathcal{U}_{\mathbf{c}_i} \triangleq \cap_{k=1}^{L} \mathcal{B}_k^{c_k^{(i)}}, \tag{13}$$

where we have defined

$$\mathcal{B}_k^c = \begin{cases} \mathcal{B}_k & c = 1, \\ [-\frac{\pi}{2}, \frac{\pi}{2}]^2 \setminus \mathcal{B}_k & c = 0. \end{cases} \tag{14}$$

In fact, $c_k^{(i)}{=}1 \Leftrightarrow \boldsymbol{\theta}_i \in \mathcal{B}_k$ and $c_k^{(i)}{=}0 \Leftrightarrow \boldsymbol{\theta}_i \in [-\frac{\pi}{2}, \frac{\pi}{2}]^2 \setminus \mathcal{B}_k$, yielding (13) when considering the entire sequence $\mathbf{c}_i$. We let $\mathcal{C}$ be the set of all possible error-free detection sequences with non-empty beam support, i.e.

$$\mathcal{C} \triangleq \{\mathbf{c} \in \{0,1\}^L : \mathcal{U}_{\mathbf{c}} \neq \emptyset\}, \tag{15}$$

and $\mathcal{G}$ be the corresponding beam-support,

$$\mathcal{G} \triangleq \{\mathcal{U}_{\mathbf{c}} : \mathbf{c} \in \mathcal{C}\}. \tag{16}$$

Note that $(\mathcal{C}, \mathcal{G})$ are uniquely defined by the beam sequence $\{\mathcal{B}_k, k{=}1,\dots,L\}$. Likewise, $\{\mathcal{B}_k, k{=}1,\dots,L\}$ is uniquely defined by a specific choice of $(\mathcal{C}, \mathcal{G})$, as can be seen by letting

$$\mathcal{B}_k \equiv \cup_{\mathbf{c} \in \mathcal{C}: c_k = 1} \mathcal{U}_{\mathbf{c}},\ \mathcal{U}_{\mathbf{c}} \in \mathcal{G}. \tag{17}$$

Therefore, the problem of finding the optimal beam sequence, $\{\mathcal{B}_k, k = 1,\dots,L\}$ is equivalent to that of finding the sets $\mathcal{C}$ and $\mathcal{G}$. However, a joint optimization over $\mathcal{C}$ and $\mathcal{G}$ is intractable due to the combinatorial nature of the problem and lack of convexity. Therefore, we resort to selecting $\mathcal{C}$ and $\mathcal{G}$ independently, where $\mathcal{C}$ is chosen from a predefined codebook with the desired error correction capability and $\mathcal{G}$ is designed to optimize energy efficiency.

Error Correction and Scheduling : One way to choose $\mathcal{C}$ would be as all possible binary sequences of length L, $\mathcal{C} \equiv \{0,1\}^L$. However, a single error during the beam-alignment phase would result in an incorrect selection of the communication beam. For instance, in the case $L{=}3$, if the error-free codeword is $\mathbf{c}_i{=}[1,1,1]$ (and thus $\boldsymbol{\theta}_i \in \mathcal{U}_{[1,1,1]}$) but UE$_i$ detects $\mathbf{u}_i{=}[1,0,1]$, then it will incorrectly infer that $\boldsymbol{\theta}_i \in \mathcal{U}_{[1,0,1]}$, resulting in outage in the data communication phase.

In order to compensate for detection errors, we endow $\mathcal{C}$ with error correction capabilities up to ε errors, e.g., using Hamming codes. Therefore, at the end of the beam-alignment phase, each UE applies the decoding function $f : \{0,1\}^L \to \mathcal{C}$ to the detection sequence $\mathbf{u}_i$. In this paper, we use the minimum Hamming distance criterion to design $f(\cdot)$, i.e.,

$$f(\mathbf{u}) \triangleq \arg\min_{\mathbf{c} \in \mathcal{C}} \|\mathbf{u} - \mathbf{c}\|_2^2. \tag{18}$$

After decoding, each UE feeds back to the BS the ID of its corrected sequence $\hat{\mathbf{c}}_i \triangleq f(\mathbf{u}_i)$, $\forall i \in \{1,2,\dots,M\}$. We assume that the feedback signals are received without errors at the BS, which thus infers that

$$\boldsymbol{\theta}_i \in \mathcal{U}_{f(\mathbf{u}_i)}, \tag{19}$$

where $\mathcal{U}_{\mathbf{d}}$ is defined in (13). Given $f(\mathbf{u}_i)$, the BS allocates the communication resources $(\tau_i, \mathcal{B}_{\mathrm{d},i}, P_i, R_i)$ to UE$_i$ during the data communication phase, denoting the allocated time, BS transmission power and rate, and communication beam. In this paper, we assume a TDMA strategy, i.e., $\tau_i = T_d/M$, $\forall i \in \{1,2,\dots,M\}$. The beam pair $\mathcal{B}_{\mathrm{d},i} \equiv \mathcal{B}_{\mathrm{t},i}^{\mathrm{d}} \times \mathcal{B}_{\mathrm{r},i}^{\mathrm{d}}$ is chosen as

$$\mathcal{B}_{\mathrm{d},i} \equiv \mathcal{B}_{\mathrm{d}}(\mathbf{u}_i) = \mathcal{U}_{f(\mathbf{u}_i)}. \tag{20}$$

Note that, due to the error correction capability endowed in the design of $\mathcal{C}$, if less than (or equal to) ε errors have been introduced in the beam-alignment phase, then $f(\mathbf{u}_i) = \mathbf{c}_i$, and thus correct alignment is achieved in the data communication phase ($\mathcal{B}_{\mathrm{d},i} \equiv \mathcal{U}_{\mathbf{c}_i}$); otherwise, if $f(\mathbf{u}_i) \neq \mathbf{c}_i$, then the data communication beam is not aligned with the AoD/AoA, and outage occurs ($\mathcal{B}_{\mathrm{d},i} \cap \mathcal{U}_{\mathbf{c}_i} \equiv \emptyset$). The resulting mis-alignment probability of UE$_i$ can then be bounded as

$$p_{\mathrm{ma},i}(\mathcal{B}_{\mathrm{d}}) \leq \mathbb{P}(W(\mathbf{e}_i) > \varepsilon) = \sum_{l=\varepsilon+1}^{L} \binom{L}{l} p_e^l (1-p_e)^{L-l}, \tag{21}$$

as per the error model (12). Note that this is a function of ϕ_s via (10), duration L of beam-alignment and number of correctable errors ε (i.e., choice of the error correction codebook $\mathcal{C}$). However, it is independent of the beam-alignment sequence $\mathcal{B}_k, k \in \mathcal{I}_s$. Therefore, the optimization over ϕ_s, L, $\mathcal{C}$ and $\mathcal{B}_k, k \in \mathcal{I}_s$ can be decoupled: ϕ_s, L, $\mathcal{C}$ can be chosen to achieve a target mis-alignment performance $p_{\mathrm{ma},i} \leq p_{\mathrm{ma}}^{\max}, \forall i$, whereas $\mathcal{B}_k, k \in \mathcal{I}_s$ is optimized to achieve energy-efficient design. This optimization is developed in the next section.

Data Communication: In the data communication phase, the BS transmits to UE$_i$ in the assigned TDMA slot using power P_i and rate R_i. These are designed to satisfy a maximum outage probability $p_{\mathrm{out}}(P_i, R_i) \leq \rho$, with no CSI at the transmitter ($h_k^{(i)}$ unknown at BS), and a minimum rate constraint $R_{\min,i}$ of UE$_i$ over the frame. In case of mis-alignment, data communication is in outage, see (21). We now consider the case of alignment, i.e., $f(\mathbf{u}_i){=}\mathbf{c}_i$ and

$\boldsymbol{\theta}_i{\in}\mathcal{B}_{\rm d}(\mathbf{u}_i)$. In this case, the instantaneous signal-to-noise ratio (SNR) during the data communication slots associated with UE$_i$ is

$$\mathsf{SNR}_k^{(i)} = \frac{\pi^2\gamma_k^{(i)}P_i}{N_0 W_{\rm tot}|\mathcal{B}_{\rm d}(\mathbf{u}_i)|}, \tag{22}$$

where $\gamma_k^{(i)}{\triangleq}|h_k^{(i)}|^2$. The outage probability is then given by

$$\begin{aligned}p_{\rm out}(P_i,R_i) &= \mathbb{P}(W_{\rm tot}\log_2(1+\mathsf{SNR}_k^{(i)}) \le R_i|\mathbf{u}_i)\\ &= 1-\exp\left(-(2^{\frac{R_i}{W_{\rm tot}}}-1)\frac{\ell(d_i)N_0W_{\rm tot}}{P_i\pi^2}|B_{\rm d}(\mathbf{u}_i)|\right).\end{aligned}$$

To meet the minimum rate constraint of UE$_i$ over the frame, we enforce $R_i = \frac{T_{\rm fr}}{\tau_i}R_{\min,i}$. To enforce $p_{\rm out}(P_i,R_i)\le\rho$,[1] we find the power P_i and the energy $E_i \triangleq P_i\tau_i$ as

$$E_i = \phi_{{\rm d},i}|\mathcal{B}_{\rm d}(\mathbf{u}_i)|, \tag{23}$$

where $\phi_{{\rm d},i}$ is the minimum energy/rad^2 required to meet the rate requirement of UE$_i$ with outage probability ρ, given by

$$\phi_{{\rm d},i} \triangleq \frac{\tau_i\ell(d_i)N_0W_{\rm tot}\left[2^{\frac{T_{\rm fr}R_{\min,i}}{\tau_i W_{\rm tot}}}-1\right]}{\pi^2\ln(1/(1-\rho))}. \tag{24}$$

III. Optimization Problem

The optimum beam-alignment design seeks to minimize the average power consumption $\bar{P}_{\rm avg}(\mathcal{B})$ of the BS, over the beam-sequence $\mathcal{B}{=}\{\mathcal{B}_k, k\in\mathcal{I}_s\}$ in the beam-alignment phase, i.e.,

$$\mathbf{P1}:\ \mathcal{B}^* = \arg\min_{\mathcal{B}} \bar{P}_{\rm avg}(\mathcal{B}), \tag{25}$$

where, using (9) and (23), $\bar{P}_{\rm avg}(\mathcal{B})$ is given by

$$\bar{P}_{\rm avg}(\mathcal{B}) = \frac{1}{T_{\rm fr}}\mathbb{E}\left[\sum_{k=1}^{L}\phi_s|\mathcal{B}_k| + \sum_{i=1}^{M}\phi_{d,i}|\mathcal{B}_{\rm d}(\mathbf{u}_i)|\right], \tag{26}$$

with $\mathcal{B}_{\rm d}(\mathbf{u}_i)$ given by (20). The expectation is over the detected and error-free sequences $\{(\mathbf{u}_i,\mathbf{c}_i), i=1,\dots,M\}$.

Using (17) and (20), we can express the beam-alignment and data communication beams as

$$\mathcal{B}_k = \cup_{\mathbf{d}\in\mathcal{C}:d_k=1}\mathcal{U}_{\mathbf{d}},\quad \mathcal{B}_{\rm d}(\mathbf{u}_i) = \mathcal{U}_{f(\mathbf{u}_i)}. \tag{27}$$

In fact, $\mathcal{U}_{\hat{\mathbf{c}}_i}$ represents the estimated support of the AoD/AoA of UE$_i$, when it detects the error corrected sequence $\hat{\mathbf{c}}_i{=}f(\mathbf{u}_i)$. Note that $\{\mathcal{U}_{\mathbf{d}}:\mathbf{d}{\in}\mathcal{C}\}$ forms a partition of the AoD/AoA space $[-\pi/2,\pi/2]^2$. In fact, using (13), the fact that $\cap_{k=1}^{L}\mathcal{B}_k^{d_k}\equiv\emptyset,\forall\mathbf{d}\notin\mathcal{C}$, and the set definition (14), we can show that

$$\cup_{\mathbf{d}\in\mathcal{C}}\mathcal{U}_{\mathbf{d}} \equiv \cup_{\mathbf{d}\in\{0,1\}^L}\cap_{k=1}^{L}\mathcal{B}_k^{d_k} = [-\pi/2,\pi/2]^2, \tag{28}$$

$$\mathcal{U}_{\mathbf{d}_1}\cap\mathcal{U}_{\mathbf{d}_2} \equiv \cap_{k=1}^{L}[\mathcal{B}_k^{d_{1,k}}\cap\mathcal{B}_k^{d_{2,k}}] \equiv \emptyset,\quad \forall\mathbf{d}_1\ne\mathbf{d}_2. \tag{29}$$

Therefore, letting $\omega_{\mathbf{d}}\triangleq|\mathcal{U}_{\mathbf{d}}|$ be the beamwidth of the sector $\mathcal{U}_{\mathbf{d}}$ and using (27), we can rewrite the average power as

$$\begin{aligned}\bar{P}_{\rm avg}(\omega) &{=} \frac{1}{T_{\rm fr}}\mathbb{E}\Bigg[\sum_{k=1}^{L}\phi_s\left|\cup_{\mathbf{d}\in\mathcal{C}:d_k=1}\mathcal{U}_{\mathbf{d}}\right|\\ &+\sum_{i=1}^{M}\phi_{d,i}\Big\{|\mathcal{U}_{\mathbf{c}_i}|\chi(W(\mathbf{e}_i){\le}\varepsilon)+\left|\mathcal{U}_{f(\mathbf{c}_i\oplus\mathbf{e}_i)}\right|\chi(W(\mathbf{e}_i){>}\varepsilon)\Big\}\Bigg]\\ &\overset{(a)}{=}\frac{1}{T_{\rm fr}}\mathbb{E}\Bigg[\sum_{k=1}^{L}\phi_s\sum_{\mathbf{d}\in\mathcal{C}:d_k=1}\omega_{\mathbf{d}}\\ &+\sum_{i=1}^{M}\phi_{d,i}\Big\{\omega_{\mathbf{c}_i}\chi(W(\mathbf{e}_i)\le\varepsilon)+\omega_{f(\mathbf{c}_i\oplus\mathbf{e}_i)}\chi(W(\mathbf{e}_i)>\varepsilon)\Big\}\Bigg],\end{aligned}$$

where in (a) we used the facts that $\{\mathcal{U}_{\mathbf{c}}{:}\mathbf{c}{\in}\{0,1\}^L\}$ is a partition of $[-\pi/2,\pi/2)^2$ and that, if fewer than ε errors occur in the beam-alignment phase, then the support of $\boldsymbol{\theta}_i$ is detected correctly. Note that, since the AoD/AoA pair $\boldsymbol{\theta}_i$ is uniformly distributed in the space $[-\pi/2,\pi/2)^2$, the probability of occurrence of the error-free sequence $\mathbf{c}_i{=}\mathbf{x}$ is

$$\mathbb{P}(\mathbf{c}_i=\mathbf{x}) = \mathbb{P}(\boldsymbol{\theta}_i\in\cap_{k=1}^{L}\mathcal{B}_k^{x_k}) = \mathbb{P}(\boldsymbol{\theta}_i\in\mathcal{U}_{\mathbf{x}}) = \frac{\omega_{\mathbf{x}}}{\pi^2}, \tag{30}$$

while the error sequence $\mathbf{e}_i,\forall\mathbf{e}_i\in\{0,1\}^L$ follows the pmf $p(\mathbf{e}_i)$ given in (12). This leads to

$$\begin{aligned}\bar{P}_{\rm avg}(\omega) &= \frac{1}{T_{\rm fr}}\Bigg[\phi_s\sum_{\mathbf{d}\in\mathcal{C}}W(\mathbf{d})\omega_{\mathbf{d}} \qquad (31)\\ &+\frac{M\bar{\phi}_d}{\pi^2}\sum_{\mathbf{c}\in\mathcal{C}}\Big\{\omega_{\mathbf{c}}^2\ \mathbb{P}(W(\mathbf{e})\le\varepsilon)+\sum_{\mathbf{e}\in\{0,1\}^L:W(\mathbf{e})>\epsilon}\omega_{f(\mathbf{c}\oplus\mathbf{e})}\omega_{\mathbf{c}}p(\mathbf{e})\Big\}\Bigg],\end{aligned}$$

where we used the fact that

$$\sum_{k=1}^{L}\sum_{\mathbf{d}\in\mathcal{C}:d_k=1}\omega_{\mathbf{d}} = \sum_{\mathbf{d}\in\mathcal{C}}\sum_{k=1}^{L}\chi(d_k=1)\omega_{\mathbf{d}} = \sum_{\mathbf{d}\in\mathcal{C}}W(\mathbf{d})\omega_{\mathbf{d}},$$

and we have defined $\bar{\phi}_d \triangleq \frac{1}{M}\sum_{i=1}^{M}\phi_{d,i}$. Thus, the optimization problem **P1** can be restated as that of optimizing the "beamwidths" $\omega_{\mathbf{d}},\mathbf{d}\in\mathcal{C}$. The sequence of beams with desired beamwidth solution of this optimization problem can then be obtained via (27), where $|\mathcal{U}_{\mathbf{d}}|=\omega_{\mathbf{d}}$. Note that $\omega_{\mathbf{d}}\triangleq|\mathcal{U}_{\mathbf{d}}|$ needs to satisfy the constraint $\sum_{\mathbf{d}\in\mathcal{C}}\omega_{\mathbf{d}}=\pi^2$, since $\{\mathcal{U}_{\mathbf{d}},\mathbf{d}\in\mathcal{C}\}$ is a partition of $[-\pi/2,\pi/2]^2$.

However, it can be shown that the cost function $\bar{P}_{\rm avg}(\omega)$ is non-convex with respect to ω, due to the quadratic terms $\omega_{f(\mathbf{c}\oplus\mathbf{e})}\omega_{\mathbf{c}}$ appearing in (31). In order to overcome this limitation, we propose to upper bound (31) by a convex function. To determine this upper bound, note that the partition constraint $\sum_{\mathbf{d}\in\mathcal{C}}\omega_{\mathbf{d}}=\pi^2$ and $\omega_{\mathbf{d}}\ge0,\forall\mathbf{d}\in\mathcal{C}$ imply that $\omega_{f(\mathbf{c}\oplus\mathbf{e})}\le\pi^2$. Thus, we upper bound (31) as

$$\begin{aligned}\bar{P}_{\rm avg}(\omega) &\le \frac{1}{T_{\rm fr}}\Bigg[\phi_s\sum_{\mathbf{d}\in\mathcal{C}}W(\mathbf{d})\omega_{\mathbf{d}} \qquad (32)\\ &+\frac{M\bar{\phi}_d}{\pi^2}\sum_{\mathbf{c}\in\mathcal{C}}\Big\{\mathbb{P}(W(\mathbf{e}){\le}\varepsilon)(\omega_{\mathbf{c}}^2-\pi^2\omega_{\mathbf{c}}){+}\pi^2\omega_{\mathbf{c}}\Big\}\Bigg] \triangleq \hat{P}_{\rm avg}(\omega).\end{aligned}$$

[1]Note that the overall outage probability including mis-alignment is given by $p_{\rm ma}^{\max}+(1-p_{\rm ma}^{\max})\rho$.

Note that, if the probability of incurring more than ε errors is made sufficiently small (by appropriately choosing the error correction code $\mathcal{C}$), say $\mathbb{P}(W(\mathbf{e})>\varepsilon)\leq\delta\ll 1$, then we can bound the gap $\hat{P}_{\text{avg}}(\omega)-\bar{P}_{\text{avg}}(\omega)$ by

$$0\leq \hat{P}_{\text{avg}}(\omega)-\bar{P}_{\text{avg}}(\omega)\leq \frac{M\bar{\phi}_d\pi^2}{T_{\text{fr}}}\delta, \tag{33}$$

Thus, we consider the minimization of the upper bound $\hat{P}_{\text{avg}}(\omega)$ instead of the original function $\bar{P}_{\text{avg}}(\omega)$, yielding the optimization problem

$$\mathbf{P2}:\ \ \omega^*=\arg\min_{\omega\geq 0}\hat{P}_{\text{avg}}(\omega)\ \text{s.t.}\ \sum_{\mathbf{d}\in\mathcal{C}}\omega_{\mathbf{d}}=\pi^2, \tag{34}$$

We now study the optimization problem $\mathbf{P2}$. Note that this is a convex quadratic problem with respect to $\omega_{\mathbf{c}}:\mathbf{d}\in\mathcal{C}$. The dual function associated with $\mathbf{P2}$ is given by

$$g(\mu)=\min_{\omega\geq 0}\hat{P}_{\text{avg}}(\omega)-\mu\left(\sum_{\mathbf{d}\in\mathcal{C}}\omega_{\mathbf{d}}-\pi^2\right),$$

whose minimizer yields the "water-filling" solution

$$\omega_{\mathbf{d}}^*=\frac{\pi^2\phi_s}{2\mathbb{P}(W(\mathbf{e})\leq\varepsilon)M\bar{\phi}_d}\Big[\lambda-W(\mathbf{d})\Big]^+. \tag{35}$$

The dual variable λ is chosen so as to satisfy the constraint

$$\sum_{\mathbf{d}\in\mathcal{C}}\omega_{\mathbf{d}}^*=\pi^2, \tag{36}$$

or equivalently, as the unique solver of

$$h(\lambda)=\frac{\phi_s}{2\mathbb{P}(W(\mathbf{e})\leq\varepsilon)M\bar{\phi}_d}\sum_{w=0}^{L}n_w\left[\lambda-w\right]^+=1, \tag{37}$$

where $n_w\triangleq\sum_{\mathbf{c}\in\mathcal{C}}\chi\left(W(\mathbf{c})=w\right)$ is the number of codewords in the codebook $\mathcal{C}$ with Hamming weight equal to w.

The optimal dual variable λ^* can be found using the bisection method over the interval $[\lambda_{\min},\lambda_{\max}]$. In fact, $h(\lambda)$ is a non-decreasing function of $\lambda>0$, with $h(0)=0$ and, using the fact that $[\lambda-w]^+\leq\lambda$, we find that

$$h(\lambda)\leq\frac{\phi_s}{2\mathbb{P}(W(\mathbf{e})\leq\varepsilon)M\bar{\phi}_d}\lambda|\mathcal{C}|,$$

where $|\mathcal{C}|$ is the cardinality of $\mathcal{C}$, hence $\lambda^*\geq\frac{2M\bar{\phi}_dP(W(\mathbf{e})\leq\varepsilon)}{|\mathcal{C}|\phi_s}$. Moreover, by denoting $\overline{W}\triangleq\frac{1}{|\mathcal{C}|}\sum_{w=0}^{L}n_w w$ as the average weight of the codewords in $\mathcal{C}$, we observe that

$$\sum_{w=0}^{L}n_w\left[\lambda-w\right]=[\lambda-\overline{W}]|\mathcal{C}|\leq\frac{2M\bar{\phi}_dP(W(\mathbf{e})\leq\varepsilon)}{\phi_s}h(\lambda),$$

thus implying the following upper and lower bounds to λ^*,

$$\lambda_{\min}\triangleq\frac{2M\bar{\phi}_dP(W(\mathbf{e})\leq\varepsilon)}{|\mathcal{C}|\phi_s}\leq\lambda^*\leq\lambda_{\min}+\overline{W}\triangleq\lambda_{\max}.$$

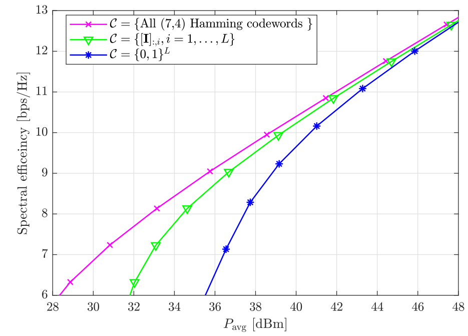

Fig. 2: Spectral Efficiency versus average power consumption.

IV. Numerical Results

In this section, we compare the performance of the proposed scheme with other schemes. We use Monte-Carlo simulation with 10^5 iterations for each simulation point. The common simulation parameters used are as follows: T_{fr}=20ms, T=10μs, [Number of BS antennas]=64, [Number of UE antennas] = 1, [BS-UE separation]=10m, N_0= − 173dBm, W_{tot}=500MHz, [carrier frequency]=30GHz, ϕ_s=6dBm, and ρ=10^{-3}. Moreover, we use the beamforming algorithm in [17] to generate the beamforming codebook. With these values, we have observed numerically that the probability of detection errors is in the range $p_e\in[0.1,0.3]$, due not only to noise and the Rayleigh fading channel, but also to sidelobes, which are not accounted for in the hypothesis testing problem (3). Thus, we set p_e=0.3 to capture this more realistic scenario.

In Fig. 2, we depict the spectral efficiency (Throughput/W_{tot}) versus the average power consumption. The curves correspond to three different choices of the codebook $\mathcal{C}$: the Hamming codebook $\mathcal{C}$=(7,4), representing the proposed coded energy-efficient scheme, with error correction capability up to ε=1 errors; $\mathcal{C}$=$\{[\mathbf{I}]_{:,i}, i$=$1,\dots,L\}$, representing the exhaustive search scheme, where $[\mathbf{I}]_{:,i}$ denotes the ith column of the $L\times L$ identity matrix $\mathbf{I}$; and $\mathcal{C}=\{0,1\}^L$, representing the scheme with no error correction capabilities (uncoded). We use L=16 for the exhaustive search scheme, and L=7 for the coded and uncoded schemes. In the figure, we observe that the proposed scheme using (7,4) Hamming codebook outperforms the other two schemes, thanks to its error correction capabilities, with a performance gain up to 4dB over exhaustive and 8dB over the uncoded scheme. Surprisingly, the exhaustive scheme exhibits superior performance compared to the uncoded scheme, despite its more significant time overhead (L=16 vs L=7). This can be attributed to the fact that the codewords in the exhaustive codebook exhibit a minimum Hamming distance of 2, whereas the uncoded codebook exhibits minimum Hamming distance equal to 1, and is thus more susceptible to detection errors during the beam-alignment phase.

V. Conclusions

In this paper, we have designed a coded energy-efficient beam-alignment. The scheme minimizes power consumption and uses an error correction code to recover from detection errors introduced during beam-alignment. We compare our proposed scheme with energy-efficient uncoded beam-alignment and exhaustive search, demonstrating its superior performance.

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