Optimal Interactive Energy Efficient Beam-Alignment for Millimeter-Wave Networks

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***Abstract*—Millimeter-wave communications rely on narrow-beam transmissions to cope with the strong signal attenuation at these frequencies, thus demanding precise alignment between transmitter and receiver. The beam-alignment procedure may create a significant overhead, thus potentially offsetting the benefits of using narrow communication beams. In this paper, an *optimal interactive* energy-efficient beam-alignment protocol is designed, that optimizes over the sequence of beams and the duration of beam-alignment, based on feedback received from the user-end (thus, interactive), so as to support a minimum communication rate. It is proved that a *fixed-length fractional search* method that decouples the beam-alignment of base-station and user-end is optimal. Numerical results using analog beams demonstrate the superior performance of our proposed *fractional search*: the *conventional* and *interactive exhaustive search* methods, and the *bisection search* method incur, respectively, 7.5dB, 14dB and 4dB of additional power consumption compared to the proposed fractional search.**

***Index Terms*—Millimeter-wave, beam-alignment, Markov decision process**

I. Introduction

Millimeter-wave (mm-wave) technology has emerged as a promising solution to enable multi-Gbps communication, thanks to abundant bandwidth availability [2]. However, signal propagation at these frequencies poses several challenges to the design of future communication systems supporting high throughput and high mobility, due to high isotropic path loss and sensitivity to blockages [3]. To compensate the propagation loss [2], mm-wave systems are expected to leverage narrow-beam communications, by using large antenna arrays at both base stations (BSs) and user-ends (UEs).

However, narrow beams are susceptible to frequent beam mis-alignment due to mobility or blockage, which necessitate the use of beam-alignment protocols. Maintaining beam-alignment between transmitter and receiver can be challenging, especially in mobile scenarios, since this protocol may consume time, frequency and energy resources, thus potentially offsetting the benefits of mm-wave directionality. Therefore, it is imperative to design schemes to mitigate its overhead.

Beam-alignment in mm-wave has been a subject of intensive research, due to its importance in mm-wave communications. The simplest and yet most popular scheme is the so-called *exhaustive* search [4], which sequentially scans through all possible BS-UE beam pairs and selects the one with maximum signal power. A version of this scheme has been adopted in existing mm-wave standards including IEEE 802.15.3c [5] and IEEE 802.11ad [6]. An interactive version of exhaustive search has been proposed in [7], wherein the beam-alignment phase is terminated once the power of the received beacon exceeds a certain threshold. The second popular scheme is *iterative* search [8], where scanning is first performed using wider beams followed by refinement using narrow beams. The most popular special case of iterative search is referred to as *bisection search* [9]: at each slot, the uncertainty region of the angle of arrival / angle of departure (AoA/AoD) is divided into two sectors of equal area; these are scanned sequentially, followed by reporting of the sector with maximum signal strength, which becomes the new region of uncertainty for the next stage. In the aforementioned papers, the optimality of the corresponding search scheme is not established. To address it, in our previous works [10]–[13] we designed optimal beam-alignment protocols. In [10], [12], we design a throughput-optimal beam-alignment scheme for a single UE and two-UEs cases, respectively, and we proved optimality of a *bisection search*. However, the model therein does not consider the energy cost of beam-alignment, which may be significant when targeting high detection accuracy. In [11], we focus on the energy-efficient design, and prove optimality of a *fractional search*. In [13], we account for the UE mobility by widening the BS beam to mitigate the uncertainty on the UE position, and optimize the trade-off between data communication and beam-sweeping using an exhaustive beam-alignment procedure. However, in [10]–[13], optimal design is carried out under restrictive assumptions that the UE receives isotropically, and that the duration of beam-alignment is deterministic. In practice, the BS may decide to switch to data communication upon finding a strong beam, as in [7], and narrow beams should be used at *both* BS and UE to fully leverage the beamforming gain. Thus, to the best of our knowledge, the optimization of *joint* and *interactive* beam-alignment at both BS and UE is still an open problem.

In this paper, we address these open questions and consider a more flexible model than our aforementioned papers [10]–[13], by allowing the BS to adapt the duration of the beam-alignment procedure based on feedback from the UE (hence, *interactive*), and *joint* beam-alignment at both BS and UE. Our design considers optimization over beam-alignment beams and duration, with the goal of minimizing the power consumption, under a constraint on the minimum downlink transmission rate. Based on a Markov decision process (MDP) formulation, and assuming error free detection and feedback, we prove that *fixed-duration* beam-alignment is indeed optimal; additionally, we prove the optimality of a *fractional search* policy that decouples the BS and UE beam-alignment over subsequent stages, and provide the optimal design in closed-form. We show numerically using analog beams [14] that for spectral efficiency of 15bps/Hz, the *conventional* and *interactive exhaustive search* methods, and the *bisection search* method incur, respectively, 7.5dB, 14dB and 4dB of additional power consumption compared to the proposed fractional search.

The rest of the paper is organized as follows. In Sec. II, we provide the system model. In Sec. III, we describe the problem formulation and carry out the analysis, followed by numerical results in Sec. IV. In Sec. V, we provide concluding remarks.

An extended version of this paper appears on IEEE TWC [1].

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This research has been funded by NSF under grant CNS-1642982.

II. System Model

We consider a downlink scenario in a mm-wave cellular system with one BS and one UE, both equipped with uniform linear arrays (ULAs) with M_t and M_r antennas, respectively. The UE is located at a distance $d \leq d_{\max}$ from the BS, within its coverage area of radius $d_{\max}$. We consider a time-slotted system, with frames of duration T_fr[s], each composed of N slots of duration $T{=}T_\text{fr}/N$[s], indexed by $\mathcal{I}\equiv\{0,1,\ldots,N{-}1\}$.

We assume a single path between the BS and the UE, whose channel gain, angle of departure (AoD) and angle of arrival (AoA) are denoted by γ, θ_t and θ_r, respectively. We assume that these parameters remain fixed over the duration of one frame, and that γ is known at the start of the frame, initially acquired via channel sounding. Additionally, we do not consider blockage occurring within the frame, as it occurs at longer time-scales than the frame duration, determined by the geometry of the environment and mobility of users [15]. We let $\boldsymbol{\theta} = (\theta_\text{t}, \theta_\text{r})$, and assume that $\boldsymbol{\theta}$ is uniformly distributed in the angular support $\mathcal{U}_0 = \mathcal{U}_{\text{t},0} \times \mathcal{U}_{\text{r},0}$, where $\mathcal{U}_{\text{t},0}, \mathcal{U}_{\text{r},0} \subseteq [-\pi, \pi]$; these sets reflect the availability of prior information on AoD and AoA acquired through previous beam-alignment phases, or based on geometric constraints (*e.g.*, buildings blocking the signal in certain directions).

In this paper, we use the *sectored antenna* model [16] to approximate the BS and UE beam-forming gains, represented in Fig. 1. Under this model, the beam-forming gain at the BS (x = t) and UE (x = r) are given by

$$G_\text{x}(\mathcal{B}_\text{x}, \theta_\text{x}) = \frac{2\pi}{|\mathcal{B}_\text{x}|}\, \chi_{\mathcal{B}_\text{x}}(\theta_\text{x}), \qquad \text{x} \in \{\text{t}, \text{r}\}, \tag{1}$$

where $\mathcal{B}_\text{t} \subseteq (-\pi, \pi]$ is the range of AoD covered by the BS beamforming codeword; $\mathcal{B}_\text{r} \subseteq (-\pi, \pi]$ is the range of AoA covered by the UE beamforming codeword; $\chi_\mathcal{A}(\theta)$ is the indicator function of the event $\theta \in \mathcal{A}$, and $|\mathcal{A}|$ is the measure of $\mathcal{A}$. Under this model, the signal model is expressed as

$$y_k = h\sqrt{P_k G_\text{t}(\mathcal{B}_\text{t}, \theta_\text{t}) G_\text{r}(\mathcal{B}_\text{r}, \theta_\text{r})}\, s_k + w_k, \tag{2}$$

where y_k is the signal received at the UE, $w_k \sim \mathcal{CN}(0, N_0 W_\text{tot})$ is AWGN noise with one-sided power spectral density N_0, over the signal bandwidth W_tot, s_k is the transmitted symbol, with $\mathbb{E}[|s_k|^2] = 1$, P_k is the transmit power, h is the fading coefficient, with gain $|h|^2 {=} \gamma$. Hereafter, the two sets $\mathcal{B}_\text{t}$ and $\mathcal{B}_\text{r}$ will be referred to as BS and UE beams, respectively. Additionally, we define $\mathcal{B}_k {=} \mathcal{B}_{\text{t},k} \times \mathcal{B}_{\text{r},k}$ as the 2-dimensional (2D) beam support defined by the BS-UE beams.

The entire frame is partitioned into a beam-alignment phase of duration LT [s], executed using 2D beams $\{\mathcal{B}_k, k{=}0, \ldots, L{-}1\}$, followed by a data communication phase of duration $(N{-}L)T$ [s], with beam $\mathcal{B}_L$. The duration L and the 2D beams used during beam-alignment are *dynamically* adjusted based on ACK/NACK feedback received from the UE, as explained below.

A. Beam-alignment

In slot k of the beam-alignment phase, the BS sends a beacon signal of duration $T_B < T$ using the transmit beam $\mathcal{B}_{\text{t},k}$, with power P_k, and the UE receives the signal using the receive beam $\mathcal{B}_{\text{r},k}$, part of our design. If the UE detects the beacon (*i.e.*, the AoD and AoA $\boldsymbol{\theta}$ fall within $\mathcal{B}_k$, or a false-alarm occurs, see [17]), then, in the remaining portion of the slot of duration $T - T_B$, it transmits an acknowledgment (ACK) packet to the BS, denoted as $C_k = \text{ACK}$. Otherwise (the UE does not detect the beacon due to either mis-alignment or mis-detection error), it transmits $C_k = \text{NACK}$. We assume that the ACK/NACK signal from the UE is received perfectly and within the end of the slot by the BS (for instance, by using a conventional microwave control channel [18]).

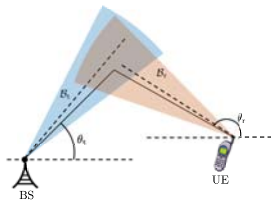

Fig. 1: The beam pattern under the sectored antenna model [16].

For tractability in the design of the beam-alignment protocol, we assume small mis-detection and false-alarm probabilities $p_\text{md}, p_\text{fa} \ll 1$, and neglect the impact of these errors on beam-alignment. Accordingly, the beacon energy satisfies[1]

$$E_k = \phi_s |\mathcal{B}_k|, \tag{3}$$

where ϕ_s is the energy/rad^2 required to achieve the target $p_\text{md}, p_\text{fa} \ll 1$ [17].

B. Data Communication

In slot L (L is part of our design), the BS switches to the data communication phase, of duration $(N-L)T$. During this phase, the BS uses beam $\mathcal{B}_{\text{t},L}$, rate R_L, and transmit power P_L, while the UE processes the received signal using the beam $\mathcal{B}_{\text{r},L}$. Therefore, from (2), the SNR can be expressed as

$$\mathsf{SNR} = \frac{\gamma P_L G_\text{t}(\mathcal{B}_{\text{t},L}, \theta_\text{t}) G_\text{r}(\mathcal{B}_{\text{r},L}, \theta_\text{r})}{N_0 W_\text{tot}} = \frac{(2\pi)^2 \gamma P_L}{N_0 W_\text{tot} |\mathcal{B}_L|} \chi_{\mathcal{B}_L}(\boldsymbol{\theta}).$$

In this paper, we enforce reliability in the data communication phase, i.e., the beams are chosen so as to satisfy

$$\mathbb{P}(\boldsymbol{\theta} \in \mathcal{B}_L | \mathcal{H}^L) = 1, \tag{4}$$

where $\mathcal{H}^k \triangleq \{(\mathcal{B}_0, C_0), \ldots, (\mathcal{B}_{k-1}, C_{k-1})\}$ is the history of 2D beams and feedback until slot k. Therefore,

$$\mathcal{C}(P_L, \mathcal{B}_L) = W_\text{tot} \log_2\left(1 + \frac{(2\pi)^2 \gamma}{N_0 W_\text{tot}} \frac{P_L}{|\mathcal{B}_L|}\right), \tag{5}$$

is the system capacity. The communication parameters are designed so as to support a minimum rate $R_{\min}$ over the entire frame duration. Hence the transmit power and rate (P_L, R_L) must satisfy $\frac{N}{N-L} R_{\min} \leq R_L \leq \mathcal{C}(P_L, \mathcal{B}_L)$. Then, the minimum energy required to support the minimum rate $R_{\min}$ over the data communication phase of duration $(N-L)T$ is given as

$$E_\text{d} \triangleq P_L (N-L) T \geq \phi_d(L) |\mathcal{B}_L|, \tag{6}$$

where $\phi_d(L)$ is the corresponding energy per rad^2, defined as

$$\phi_d(L) \triangleq (2\pi)^{-2} \gamma^{-1} N_0 W_\text{tot} (N-L) T \left(2^{\frac{R_{\min} N}{W_\text{tot}(N-L)}} - 1\right).$$

III. Problem Formulation and Analysis

In this section, we formulate the optimization problem, and characterize it as a Markov decision process (MDP) [19]. The

[1] As per the sectored model, the energy of the signal is spread evenly across the angular directions, so that the energy required to achieve a target SNR at the receiver is proportional to the beamwidth $|\mathcal{B}_k|$.

goal is to minimize the energy consumption at the BS over one frame while satisfying a specified rate requirement $R_{\min}$ at the UE. The MDP is defined over the time horizon $\mathcal{I}$. We model the data communication phase by the absorbing state DC, i.e., once switched to data communication, the system remains in that state until the end of the frame. During beam-alignment, the state at the start of slot k is denoted by the belief f_k over the AoD/AoA $\boldsymbol{\theta}$, given the history $\mathcal{H}^k$, i.e., $f_k(\boldsymbol{\theta})=\mathbb{P}(\boldsymbol{\theta}|\mathcal{H}^k)$. The action $\mathbf{a}_k$ is defined by the pair $(\sigma_k, \mathcal{B}_k)$, where $\sigma_k\in\{\mathrm{BA}, \mathrm{DC}\}$ selects whether to remain in the beam-alignment phase (σ_k=BA) or switch to data communication in slot k (σ_k=DC); $\mathcal{B}_k\triangleq\mathcal{B}_{\mathrm{t},k}\times\mathcal{B}_{\mathrm{r},k}$ is the 2D beam employed to perform the corresponding operation. Note that, if $\sigma_k = \mathrm{DC}$, then the system switches to data communication in slot k (absorbing state DC), hence $L = k$ and $\mathcal{B}_L \supseteq \mathrm{supp}(f_L)$ to provide coverage, see (4); the energy required to perform this operation over the remaining $(N - L)$ slots is $\phi_d(L)|\mathcal{B}_L|$, see (6). On the other hand, if $\sigma_k = \mathrm{BA}$, then the BS performs beam-alignment, with energy cost $\phi_s|\mathcal{B}_k|$, see (3). Thus, we define the energy metric in the beam-alignment state f_k as

$$c_k(\sigma_k, \mathcal{B}_k; f_k)=\Big[\phi_s\chi(\sigma_k{=}\mathrm{BA})+\phi_d(k)\chi(\sigma_k{=}\mathrm{DC})\Big]|\mathcal{B}_k|, \quad (7)$$

and that in the data communication state DC as $c_k(\mathrm{DC}) = 0$.[2] The goal is to determine an optimal policy μ which, given the state f_k, selects whether to remain in the beam-alignment phase (σ_k=BA) or switch to data communication (σ_k=DC), so as to minimize the overall energy consumption over one frame. The optimization problem is stated as

$$\begin{aligned}&\mathbf{OPT}: \min_{\mu}\ \mathbb{E}_\mu\left[\sum_{k=0}^{N-1} c_k(\sigma_k, \mathcal{B}_k; f_k)\middle| f_0,\right]\\ &\text{s.t. } \mathcal{B}_k \supseteq \mathrm{supp}(f_k) \text{ if } \sigma_k = \mathrm{DC},\end{aligned} \quad (8)$$

where $f_0(\boldsymbol{\theta}) = \mathrm{Uniform}(\mathcal{U}_{\mathrm{t},0} \times \mathcal{U}_{\mathrm{r},0})$ denotes the prior belief over $\boldsymbol{\theta}$, and (8) enforces the reliability constraint for the data communication phase. In [10], we have proved that, for the sectored antenna model with no false-alarm and mis-detection errors, $\mathcal{U}_k \triangleq \mathrm{supp}(f_k)$, is a sufficient statistics for optimal control. Using this fact, we can simplify the state space as

$$\mathcal{S} \equiv \{\mathcal{U} : \mathcal{U} \subseteq [-\pi, \pi]^2\} \cup \{\mathrm{DC}\}. \quad (9)$$

Additionally, it can be shown by induction on k that $f_k(\boldsymbol{\theta}) = \mathrm{Uniform}(\mathcal{U}_k)$. Hence, the transitions from state $\mathcal{U}_k$ during the beam-alignment action $(\mathrm{BA}, \mathcal{B}_k)$, along with their probabilities, are given by [10]

$$\mathcal{U}_{k+1}=\begin{cases}\mathcal{U}_k \cap \mathcal{B}_k, & C_k = \mathrm{ACK},\ \text{w.p. } \frac{|\mathcal{B}_k\cap\mathcal{U}_k|}{|\mathcal{U}_k|}\\ \mathcal{U}_k \setminus \mathcal{B}_k, & C_k = \mathrm{NACK},\ \text{w.p. } 1 - \frac{|\mathcal{B}_k\cap\mathcal{U}_k|}{|\mathcal{U}_k|}.\end{cases} \quad (10)$$

In fact, $\boldsymbol{\theta}\in\mathcal{U}_k$ since $\mathcal{U}_k$=supp(f_k). If an ACK is received (with probability $\mathbb{P}(\boldsymbol{\theta}\in\mathcal{B}_k|\mathcal{U}_k)=|\mathcal{B}_k \cap \mathcal{U}_k|/|\mathcal{U}_k|$, owing to $f_k(\boldsymbol{\theta})$=Uniform$(\mathcal{U}_k)$), then the BS infers that $\boldsymbol{\theta}\in\mathcal{B}_k$, otherwise $\boldsymbol{\theta}\notin\mathcal{B}_k$ and a NACK is received. Instead, if the data communication action $(\mathrm{DC}, \mathcal{B}_k)$ is selected, with $\mathcal{B}_k\supseteq\mathcal{U}_k$ to provide coverage, the system transfers to the absorbing state DC.

The problem **OPT** can be solved via dynamic programming (DP). The cost-to-go function from the absorbing state DC is $V_k^*(\mathrm{DC}) = 0$, since the energy cost of data communication is counted upon switching to the data communication phase, see (7). In slot k, state $\mathcal{U}_k$ (beam-alignment phase), the cost-to-go function optimized over the action $(\sigma_k, \mathcal{B}_k)$ is computed as

$$\begin{aligned}V_k^*(\mathcal{U}_k) = \min\Bigg\{&\min_{\mathcal{B}_k=\mathcal{B}_{\mathrm{t},k}\times\mathcal{B}_{\mathrm{r},k}\supseteq\mathcal{U}_k} \phi_d(k)|\mathcal{B}_k|,\\ &\min_{\mathcal{B}_k=\mathcal{B}_{\mathrm{t},k}\times\mathcal{B}_{\mathrm{r},k}}\Big[\phi_s|\mathcal{B}_k|+\frac{|\mathcal{B}_k \cap \mathcal{U}_k|}{|\mathcal{U}_k|}V_{k+1}^*(\mathcal{B}_k \cap \mathcal{U}_k)\\ &+\left(1-\frac{|\mathcal{B}_k \cap \mathcal{U}_k|}{|\mathcal{U}_k|}\right)V_{k+1}^*(\mathcal{U}_k \setminus \mathcal{B}_k)\Big]\Bigg\},\end{aligned} \quad (11)$$

initialized as $V_N^*(\mathcal{U}_N) = \infty$, where the outer minimization is over the actions "switch to data communication" or "perform beam-alignment" in slot k. The inner minimization represents an optimization over the corresponding beam $\mathcal{B}_k$.

This optimization is non trivial, since $\mathcal{B}_k$ needs to satisfy "rectangular" constraints $\mathcal{B}_k$=$\mathcal{B}_{\mathrm{t},k}\times\mathcal{B}_{\mathrm{r},k}$. To solve the problem, we consider the extended beam space $\mathcal{B}_k\subseteq[-\pi, \pi]^2$, so that $\mathcal{B}_k$ can take *any* shape. The optimal data communication beam then becomes $\mathcal{B}_k$=$\mathcal{U}_k$, and that for beam-alignment needs to satisfy $\mathcal{B}_k\subseteq\mathcal{U}_k$ to save energy, so that (11) becomes

$$\begin{aligned}\hat{V}_k(\mathcal{U}_k) = \min\Bigg\{&\phi_d(k)|\mathcal{U}_k|, \min_{\mathcal{B}_k\subseteq\mathcal{U}_k}\Big[\phi_s|\mathcal{B}_k|\\ &+\frac{|\mathcal{B}_k|}{|\mathcal{U}_k|}\hat{V}_{k+1}(\mathcal{B}_k)+\left(1-\frac{|\mathcal{B}_k|}{|\mathcal{U}_k|}\right)\hat{V}_{k+1}(\mathcal{U}_k \setminus \mathcal{B}_k)\Big]\Bigg\},\end{aligned} \quad (12)$$

initialized as $\hat{V}_N(\mathcal{U}_N)$=∞. By optimizing over a larger action space, we achieve a lower bound to the value function, hence $\hat{V}_k(\mathcal{U}_k) \leq V_k^*(\mathcal{U}_k)$. In Sec. III-B, we will prove the existence of a sequence of beams with "rectangular" constraints that indeed achieve this lower bound, and thus are optimal.

A. Optimality of fractional search with deterministic duration

Note that the proposed protocol is *interactive*, so that the duration of the beam-alignment phase, $L \in \mathcal{I}$, is possibly a random variable, function of the realization of the beam-alignment process. For example, the BS may decide to switch to data communication if a sufficiently narrow beam is found. Although it may seem intuitive that L should indeed be random, in this section we will show that, instead, the optimal L is *deterministic*. Additionally, we prove the optimality of a *fractional search method*, which dictates the optimal beam in the beam-alignment phase. To this end, we define $v_k(\mathcal{U}_k)\triangleq\hat{V}_k(\mathcal{U}_k)/|\mathcal{U}_k|$ initialized as $v_N(\mathcal{U}_N) = \infty$, yielding

$$\begin{aligned}v_k(\mathcal{U}_k) = \min\Bigg\{&\phi_d(k), \min_{\mathcal{B}_k\subseteq\mathcal{U}_k}\Big[\phi_s\frac{|\mathcal{B}_k|}{|\mathcal{U}_k|}+\frac{|\mathcal{B}_k|^2}{|\mathcal{U}_k|^2}v_{k+1}(\mathcal{B}_{\mathrm{t},k})\\ &+\left(1-\frac{|\mathcal{B}_k|}{|\mathcal{U}_k|}\right)^2 v_{k+1}(\mathcal{U}_k \setminus \mathcal{B}_k)\Big]\Bigg\}.\end{aligned} \quad (13)$$

Using this fact, we find that $v_{N-1}(\mathcal{U}_{N-1})$=$\phi_d(N-1)$, which is *independent* of $\mathcal{U}_{N-1}$. By induction on k, it is then straightforward to see that $v_k^*(\mathcal{U}_k)$ is *independent* of $\mathcal{U}_k$. We thus define $v_k^* \triangleq v_k(\mathcal{U}_k)$, $\forall\mathcal{U}_k$, which is recursively updated as

$$v_k^*=\min\Big\{\phi_d(k), \min_{x\in[0,1]}\phi_s x+\Big[x^2+(1-x)^2\Big]v_{k+1}^*\Big\}, \quad (14)$$

where x replaces $\frac{|\mathcal{B}_k|}{|\mathcal{U}_k|}$ in (13).

[2]The energy cost of data communication is $\phi_d(L)|\mathcal{B}_L|$ and is counted when switching to data communication, see (7).

The value of x achieving the minimum in (14) is given by

$$x_k^* = \frac{|\mathcal{B}_k|}{|\mathcal{U}_k|} = \frac{1}{2}\left(1 - \frac{\phi_s}{2v_{k+1}^*}\right)^+, \quad (15)$$

yielding

$$v_k^* = \min\left\{\phi_d(k), v_{k+1}^* - \frac{[(2v_{k+1}^* - \phi_s)^+]^2}{8v_{k+1}^*}\right\}. \quad (16)$$

From this decomposition, we infer that:

1) Given v_k^* and x_k^*, the original value function is obtained as $\hat{V}_k(\mathcal{U}_k)=v_k^*|\mathcal{U}_k|$. If, at time k, $\phi_d(k) < v_{k+1}^* - \frac{[(2v_{k+1}^*-\phi_s)^+]^2}{8v_{k+1}^*}$, then it is optimal to switch to data communication in the remaining $N-k$ slots. Otherwise, it is optimal to perform beam-alignment.
2) In this case, the optimal beam-alignment beam satisfies

$$\mathcal{B}_k \subset \mathcal{U}_k, \ \ |\mathcal{B}_k| = x_k^*|\mathcal{U}_k|. \quad (17)$$

3) Finally, since the time to switch to data communication is solely based on $\{v_k^*\}$, but not on the actual value of $\mathcal{U}_k$, it follows that the duration of the beam-alignment phase is *deterministic*, and its duration L^* is obtained as

$$L^* = \min\left\{k : \phi_d(k) < v_{k+1}^* - \frac{[(2v_{k+1}^* - \phi_s)^+]^2}{8v_{k+1}^*}\right\}.$$

These results are elaborated in the following theorem.

Theorem 1. *Let* $L_{\min} = \min\left\{L : \phi_d(L) > \frac{\phi_s}{2}\right\}$ *and let, for* $L_{\min} \le L < N$, $v_L^{(L)} = \phi_d(L)$ *and*

$$v_k^{(L)} = v_{k+1}^{(L)} - \frac{(2v_{k+1}^{(L)} - \phi_s)^2}{8v_{k+1}^{(L)}}, \ k < L. \quad (18)$$

Then, the beam-alignment phase has deterministic *duration*

$$L^* = \underset{L\in\{0\}\cup\{L_{\min},\dots,N-1\}}{\operatorname{argmin}} v_0^{(L)}. \quad (19)$$

For $0 \le k < L^*$ *(beam-alignment phase), the 2D beam for beam-alignment is optimal iff*

$$\mathcal{B}_k \subset \mathcal{U}_k, \ \ |\mathcal{B}_k| = \frac{1}{2}\rho_k|\mathcal{U}_k|, \quad (20)$$

where $\rho_k \in (0,1)$ *is the* fractional search *parameter,*

$$\begin{cases} \rho_{L^*-1} = 1 - \frac{\phi_s}{2v_{L^*}^{(L^*)}}, \\ \rho_k = \frac{2-\rho_{k+1}}{2-\rho_{k+1}^2}\rho_{k+1}, \ k < L^* - 1. \end{cases} \quad (21)$$

For $k \ge L^*$, *the data communication phase occurs with rate* $R_{L^*} = \frac{NR_{\min}}{N-L^*}$, *and 2D beam* $\mathcal{B}_k = \mathcal{U}_k$.

Proof. Since the beam-alignment duration is deterministic, as previously discussed, we consider the optimization of the beam-alignment protocol with fixed duration L, and then optimize over L to minimize energy expenditure. For a given $L \in \mathcal{I}$, the DP updates are obtained by adapting (14) to this case (so that the outer minimization disappears), yielding

$$\begin{cases} v_L^{(L)} = \phi_d(L), \\ v_k^{(L)} = g_k(x_k), \ k < L, \ \text{where} \\ g_k(x) \triangleq \phi_s x + \left[x^2 + (1-x)^2\right] v_{k+1}^{(L)}, \\ x_k = \arg\min_{x\in[0,1]} g_k(x). \end{cases} \quad (22)$$

Since the goal is to minimize energy consumption, the optimal L is $L^* = \arg\min_L v_0^{(L)}$. We now prove that $0<L<L_{\min}$ is suboptimal, so that this optimization can be restricted as in (19). Let $0<L<L_{\min}$, so that $v_L^{(L)} \le \phi_s/2$. Note that $v_k^{(L)}$ is a non-decreasing function of k. In fact, $v_k^{(L)} \le g_k(0) = v_{k+1}^{(L)}$. Then, it follows that $v_k^{(L)} \le \phi_s/2, \forall k$, hence $x_k=0, \forall k$, yielding $v_0^{(L)} = v_L^{(L)}$ by induction. However, $v_L^{(L)}$ is an increasing function of L (it is more energy efficient to spread transmissions over a longer interval), hence $v_0^{(L)} > v_0^{(0)}$ and such L is suboptimal. This proves that any $0<L<L_{\min}$ is suboptimal.

We now prove the updates for $L \ge L_{\min}$, i.e., $v_L^{(L)} > \phi_s/2$. By induction, we can show that $v_k^{(L)} > \phi_s/2, \forall k$. In fact, let $v_{k+1}^{(L)} > \phi_s/2$ for some $k < L$. Then, $v_k^{(L)} = \min_{x\in[0,1]} g_k(x)$, minimized at $x_k = \frac{1}{2}\left(1 - \phi_s/(2v_{k+1}^{(L)})\right)$, so that $v_k^{(L)} = g_k(x_k)$, as expressed in (18). This recursion is an increasing function of $v_{k+1}^{(L)}$, yielding $v_k^{(L)} > \phi_s/2$, thus proving the induction. It follows that $x_k = \frac{1}{2}\left(1 - \phi_s/(2v_{k+1}^{(L)})\right)$, yielding the recursion given by (18). The fractional search parameter ρ_k is finally obtained as $\rho_k = 2x_k$, by substituting $v_{k+1}^{(L)} = \frac{\phi_s}{2(1-\rho_k)}$ into the recursion (18) to find a recursive expression of ρ_k from ρ_{k+1}, yielding (21). These fractional values are used in (17) to obtain (20). The theorem is thus proved. ■

B. Decoupled BS and UE Beam-Alignment

In the previous section, we proved the optimality of fractional search which uses the 2D beam $\mathcal{B}_k$ which can take any shape, provided that the beam support $\mathcal{B}_k$ measures a fraction $\rho_k/2$ of the belief support $\mathcal{U}_k$. However, actual beams should satisfy the rectangular constraint $\mathcal{B}_k = \mathcal{B}_{\mathrm{t},k} \times \mathcal{B}_{\mathrm{r},k}$, and therefore, it is not immediate to see that the optimal scheme outlined in Theorem 1 is attainable by any feasible scheme. Indeed, in this section we prove that there exists a feasible beam design attaining optimality. The proposed beam design decouples the beam-alignment of BS and UE. To this end, we define the support of the marginal belief with respect to θ_{x}, $\mathrm{x} \in \{\mathrm{t}, \mathrm{r}\}$ as $\mathcal{U}_{\mathrm{x},k} \equiv \operatorname{supp}(f_{\mathrm{x},k})$. In *BS beam-alignment*, the 2D beam is chosen as $\mathcal{B}_k = \mathcal{B}_{\mathrm{t},k} \times \mathcal{U}_{\mathrm{r},k}$, where $\mathcal{B}_{\mathrm{t},k} \subset \mathcal{U}_{\mathrm{t},k}$. On the other hand, in *UE beam-alignment*, the 2D beam is chosen as $\mathcal{B}_k = \mathcal{U}_{\mathrm{t},k} \times \mathcal{B}_{\mathrm{r},k}$, where $\mathcal{B}_{\mathrm{r},k} \subset \mathcal{U}_{\mathrm{r},k}$. We introduce the variable $\xi_k \in \{1,2\}$ to distinguish between BS beam-alignment ($\xi_k = 1$) or UE beam-alignment ($\xi_k = 2$).

We now define a policy μ that uses this decoupled beam-alignment, and prove its optimality.

Theorem 2. *Let* L^* *and* $\{\rho_k : k=0,\dots,L^*-1\}$ *be given as in Theorem 1. Then, policy* μ *defined as follows is optimal. In slots* $k = L^*, \dots, N$, *policy* μ *performs data communication with rate* $R_{L^*} = \frac{NR_{\min}}{N-L^*}$ *and beams*

$$\mathcal{B}_{\mathrm{t},L^*} = \mathcal{U}_{\mathrm{t},L^*}, \ \mathcal{B}_{\mathrm{r},L^*} = \mathcal{U}_{\mathrm{r},L^*}. \quad (23)$$

In slots $0 \le k < L^*$, *it performs beam-alignment with beams*

$$\begin{cases} \mathcal{B}_{\mathrm{t},k} \subset \mathcal{U}_{\mathrm{t},k}, \mathcal{B}_{\mathrm{r},k} = \mathcal{U}_{\mathrm{r},k}, \ |\mathcal{B}_{\mathrm{t},k}| = \frac{1}{2}\rho_k|\mathcal{U}_{\mathrm{t},k}|, \ \textit{if } \xi_k = 1 \\ \mathcal{B}_{\mathrm{t},k} = \mathcal{U}_{\mathrm{t},k}, \mathcal{B}_{\mathrm{r},k} \subset \mathcal{U}_{\mathrm{r},k}, \ |\mathcal{B}_{\mathrm{r},k}| = \frac{1}{2}\rho_k|\mathcal{U}_{\mathrm{r},k}|, \ \textit{if } \xi_k = 2, \end{cases} \quad (24)$$

where $\xi_k \in \{1,2\}$ *is chosen arbitrarily.*

Proof. Note that, if this policy satisfies the condition $\mathcal{B}_k \equiv \mathcal{B}_{\mathrm{t},k} \times \mathcal{B}_{\mathrm{r},k} \subseteq \mathcal{U}_k \equiv \operatorname{supp}(f_k)$, then it satisfies all the conditions of Theorem 1, hence it is optimal. By using (10) and induction, it is straightforward to show that $\mathcal{U}_k = \mathcal{U}_{t,k} \times \mathcal{U}_{r,k}$, implying that beams given by (24) indeed satisfy these conditions. Thus, we have proved the theorem. ■

IV. Numerical Results

In this section, we demonstrate the performance of the proposed *decoupled fractional search* (DFS) scheme and compare it with the *bisection search* (BiS) algorithm [9] and two variants of *exhaustive search*. For BiS, since in each beam-alignment slot two sectors are scanned (each of duration T_B), the total duration of the beam-alignment phase is $(2T_B + T_F)L$ [s], where T_F is the feedback time. In *conventional exhaustive search* (CES), the BS-UE scan exhaustively the entire beam space. In the BS beam-alignment sub-phase, the BS searches over $N_B^{(BS)}$ beams covering the AoD space, while the UE receives isotropically; in the second UE beam-alignment sub-phase, the BS transmits using the best beam found in the first sub-phase, whereas the UE searches exhaustively over $N_B^{(UE)}$ beams covering the AoA space. Since the UE reports the best beam at the end of each sub-phase, the total duration of the beam-alignment phase is $[N_B^{(BS)} + N_B^{(UE)}]T_B + 2T_F$. On the other hand, in the *interactive exhaustive search* (IES) method, the UE reports the feedback at the end of each beam-alignment slot, and each beam-alignment sub-phase terminates upon receiving an ACK from the UE. Since the BS awaits for feedback at the end of each beam, the duration of the beam-alignment is $(T_B + T_F)[\hat{N}_B^{(BS)} + \hat{N}_B^{(UE)}]$, where $\hat{N}_B \leq N_B$ is the number of beams scanned until receiving an ACK.

We use the following parameters: [carrier frequency]=30GHz, d=10m, [path loss exponent]=2, $T_{\rm fr}$=20ms, T_B=50μs, T=100μs, $|\mathcal{U}_0|$=$[\pi]^2$, N_0=-173dBm, $W_{\rm tot}$=500MHz, $M_{\rm t}$=$M_{\rm r}$=128. We set $p_{\rm fa}$=$p_{\rm md}$=10^{-5}, hence ϕ_s=-94dBm. For BiS and DFS we optimize L over $L \leq L_{\max}$=10; for CES and IES, we use $N_B^{(BS)}$=$N_B^{(UE)}$=32.

In Fig. 2, we plot the results of a Monte-Carlo simulation with analog beams generated using the algorithm in [14]. The performance gap between the analytical and the simulation-based curves for DFS is attributed to the fact that the beams used in the simulation have non-zero side-lobe gain and non-uniform main-lobe gain, as opposed to the "sectored" beams used in the analytical model. This results in false-alarm, misdetection errors, and leakage, which lead to some performance degradation. *However, the simulation is in line with the analytical curve, and exhibits superior performance compared to the other schemes, thus demonstrating that the analysis using the sectored gain model provides useful insights for practical design.* For instance, to achieve a spectral efficiency of 15bps/Hz, BiS [9] requires 4dB more average power than DFS, mainly due to the time and energy overhead of scanning two sectors in each beam-alignment slot, whereas IES and CES require 7.5dB and 14dB more power, respectively. The performance degradation of IES and CES is due to the exhaustive search of the best sector, which demands a huge time overhead. Indeed, IES outperforms CES since it stops beam-alignment once a strong beam is detected.

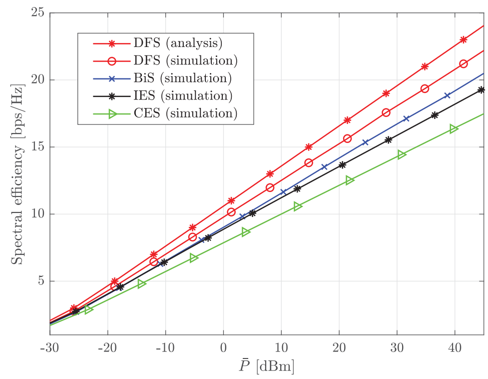

Fig. 2: Spectral efficiency versus average power consumption.

V. Conclusions

In this paper, we design an energy efficient interactive beam-alignment scheme that jointly optimizes over BS and UE beams, and beam-alignment duration. Based upon an MDP formulation, assuming error free detection, we prove the optimality of a *fixed-length fractional* search policy with *decoupled* beam-alignment at BS and UE. We compare our proposed scheme with *exhaustive* and *bisection* search policies using analog beams, demonstrating superior performance.

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