Quantum Phase Transition in Strongly-Correlated Cavity Polaritons

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***Abstract*—We study the quantum critical behavior in a multiconnected Jaynes-Cummings lattice using the density-matrix renormalization group method, where cavity polaritons exhibit a Mott-insulator-to-superfluid phase transition. We calculate the phase boundaries and the quantum critical points.**

***Index Terms*—multiconnected Jaynes-Cummings lattice, quantum phase transition, quantum critical points**

I. Introduction

Enormous progress has been achieved in the study of scalable solid-state quantum systems, such as superconducting devices and semiconductor photonic devices [1], [2]. At the same time, these devices have also been exploited to emulate many-body phenomena that are difficult to solve classically [3]. Among these efforts, one particularly interesting endeavor is the study of strongly-interacting photons and polaritons [4], [5]. By coupling a cavity to a nonlinear medium, such as a qubit, an effective nonlinear interaction between polariton excitations can be created, which can be related to the onsite Hubbard interaction in the Bose-Hubbard model [6]. In the coupled cavity array (CCA) models [7]–[10], photons can hop between adjacent cavities, and the competition between the hopping and the effective nonlinear interaction results in quantum phase transitions between the Mott-insulating (MI) phase with localized polariton excitations and the superfluid (SF) phase with long-range spatial correlation. Photon blockade and dynamical quantum phase transition have been investigated in the CCA [11]–[13].

In recent works [14]–[17], a multiconnected Jaynes-Cummings (MCJC) lattice has been studied, where qubits and cavities are connected alternately. In contrast to the CCA, no direct hopping exists between neighboring cavities in the MCJC. A MCJC lattice can be realized with superconducting resonator cavities coupled to superconducting qubits or with optical nanocavities coupled to quantum dots [1], [2]. In particular, the MCJC can be constructed by connecting Xmon qubits to superconducting resonators, given the rich connectivity of the superconducting circuits [18], [19]. Using exact diagonalization [14], [15], it was shown that the MI-to-SF phase transition can occur due to the competition between the qubit-cavity couplings when the cavity polaritons are at integer fillings.

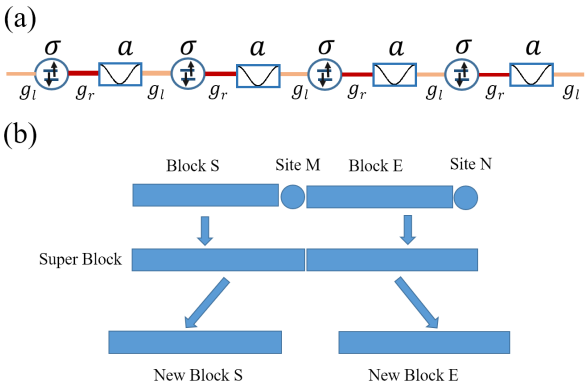

Fig. 1. (a) The schematic of a 1D MCJC. The circles (σ) represent the qubits and the blocks (a) represent the cavities with qubit-cavity couplings g_r and g_l. (b) The DMRG algorithm in our calculation.

Here we study the quantum critical behavior of a one-dimensional (1D) MCJC lattice at both integer and half fillings using the density-matrix renormalization group (DMRG) method [20], [21]. This method has previously been used to study the Bose-Hubbard and the CCA models [22]–[24]. We obtain the phase boundary and the critical point of the phase transitions [25].

II. Multiconnected Jaynes-Cummings Lattice

We consider a one-dimensional (1D) MCJC lattice composed of alternately connected qubits and cavities [14], as shown in Fig. 1(a). Each qubit in this model is coupled to two adjacent cavities. The total Hamiltonian of this model can be written as ($\hbar = 1$) $H_t = \sum_i H_{0i} + H_{li} + H_{ri}$ with the uncoupled Hamiltonian $H_{0i} = (\omega_z/2)\sigma^z_{2i-1} + \omega_c a^\dagger_{2i} a_{2i}$, the coupling Hamiltonian $H_{li} = g_l(\sigma^+_{2i-1}a_{2i-2} + a^\dagger_{2i-2}\sigma^-_{2i-1})$ between the qubit at site $2i-1$ and the cavity at site $2i-2$, and the coupling Hamiltonian $H_{ri} = g_r(\sigma^+_{2i-1}a_{2i} + a^\dagger_{2i}\sigma^-_{2i-1})$ between the qubit at site $2i-1$ and the cavity at site $2i$. Here ω_z is the energy splitting of the qubits, ω_c is the frequency of the cavity modes, $\sigma_i^{z,+,-}$ are the Pauli matrices, a_i is the annihilation operator of cavity modes, g_l (g_r) is the coupling constant between a qubit and a cavity to its left (right) hand side. We assume periodic boundary condition

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in our calculation. In this model, in contrast to the CCA model, the cavities are not directly coupled to each other. The unique geometry of this model renders a symmetry between the coupling constants g_l and g_r, i.e., the energy spectrum is unchanged when we swap these two coupling constants.

When $g_l = 0$ ($g_r = 0$), the MCJC lattice becomes isolated unit cells with one qubit and one cavity governed by the well-known Jaynes-Cummings (JC) model [26]. The JC model is exactly solvable with the polariton eigenstates $|n, \alpha\rangle$, where $n > 0$ is the number of excitations in the coupled system and $\alpha = +\ (-)$ refers to the upper (lower) polariton state. We use $|0, -\rangle$ to denote the lowest state of the JC model.

Below we analyze the behavior of the MCJC lattice at finite g_l with the polariton mapping technique [9]. We define a polariton operator $p^i_{n\alpha} \equiv |0, -\rangle_i\langle n, \alpha|$, which annihilates the $|n, \alpha\rangle$ polariton state at the ith unit cell. The total Hamiltonian, H_t, can be represented in terms of the polariton operators. For example, at zero detuning $\omega_z = \omega_c$,

$$\begin{aligned} H_t &= \sum_{i\alpha n} \left[(n-1/2)\omega_c + \alpha\sqrt{n}g_r\right] p^{i\dagger}_{n\alpha} p^i_{n\alpha} \\ &\quad + g_l \sum_{inm} \sum_{\alpha\alpha'\beta\beta'} k^n_{\alpha\alpha'} t^m_{\beta\beta'} V^{\alpha\alpha'\beta\beta'}_{imn} \end{aligned} \tag{1}$$

with the hopping term

$$V^{\alpha\alpha'\beta\beta'}_{imn} = p^{i\dagger}_{n\alpha} p^{(i-1)\dagger}_{(m-1)\beta'} p^{i-1}_{m\beta} p^i_{(n-1)\alpha'} + h.c.. \tag{2}$$

The first term in (1) describes the local polariton spectrum with a nonlinear term resembling an onsite interaction, which can result in a MI phase with localized polariton excitations. The second term in Eq. (1) describes the hopping of a polariton from site $(i-1)$ to site i, which can result in a superfluid phase with long-range spatial correlation. The competition between these two terms can lead to a transition between the MI and SF phases at integer filling. At $g_l = 0$ ($g_r = 0$), the system is dominated by the onsite interaction and is in the MI phase. When increasing g_l, especially when g_l becomes comparable to g_r, the hopping term can lower the energy due to double occupancy and drive the system into the SF phase.

III. Numerical Results

To confirm our analysis in the above section, we use a finite-lattice DMRG algorithm to study the quantum critical behavior of the MCJC lattice [20], [21]. The algorithm is illustrated in Fig. 1(b). At first step, we have two basic blocks labelled as S and E, respectively, with known eignestates. In the middle (end), there is a single cell that we call M (N). We can solve the eigenstates of the coupled blocks S and M (E and N) and truncate these eigenstates to form a new basis. With this procedure, we effectively merge S and M (E and N) into a new S (E). Conmtinuing this procedure, we can calculation the ground state of 1D systems with high accuracy. In our calculation, we use both infinite and finite DMRG algorithms to achieve high accuracy.

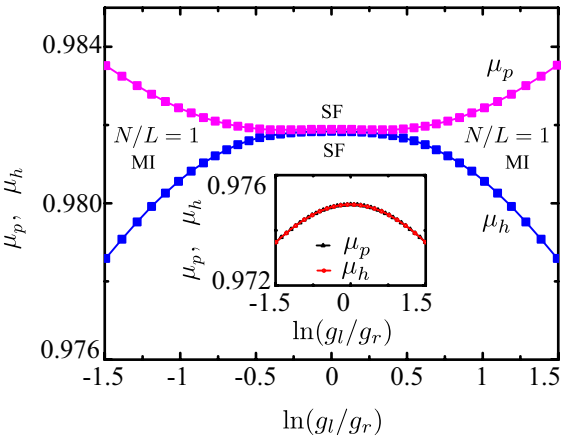

Fig. 2. The chemical potentials μ_p and μ_h versus $\ln(g_l/g_r)$ for the MCJC lattice with $(g_r + g_l)/2\pi = 300$ MHz. The main plot (inset) is at the integer filling $N/L = 1$ (half integer filling $N/L = 1/2$).

A. Phase boundaries

The phase boundaries between the MI and the SF phases can be determined from the chemical potentials

$$\mu_p(N, L) = E_L(N+1) - E_L(N), \tag{3}$$
$$\mu_h(N, L) = E_L(N) - E_L(N-1), \tag{4}$$

where $E_L(N)$ is the ground state energy of a MCJC with L unit cells and N polariton excitations, and μ_p (μ_h) is the chemical potential for adding (removing) one polariton [23]. In Fig. 2, we plot the extrapolated chemical potentials, μ_p and μ_h as functions of the logarithmic ratio $\ln(g_l/g_r)$. At the integer filling $N/L = 1$, our result shows that $\mu_p = \mu_h$ in the regime $g_l \sim g_r$ (small $|\ln(g_l/g_r)|$). As the ratio $|\ln(g_l/g_r)|$ increases, a finite difference appears between μ_p and μ_h, corresponding to a finite energy gap for adding or removing a polariton [23]. This indicates the transition from the SF phase in the small $|\ln(g_l/g_r)|$ regime to the MI phase in the large $|\ln(g_l/g_r)|$ regime. Between μ_p and μ_h, the polariton density remains a constant ($N/L = 1$) with zero compressibility. Our result also shows that there are two quantum critical points due to the symmetry between the couplings g_l and g_r. These two critical points correspond to $g_l/g_r = \beta_0$ and $g_l/g_r = 1/\beta_0$, respectively.

The inset of Fig. 2 shows μ_p and μ_h versus $\ln(g_l/g_r)$ at the half integer filling $N/L = \frac{1}{2}$. We find that $\mu_p = \mu_h$ within numerical error in all parameter regimes, and hence there won't be a phase transition at half filling.

B. Quantum critical points

To accurately determine the position of the quantum critical points, we calculate the single-particle density matrices

$$\Gamma_q(i-j) = \langle \sigma^+_{2i-1}\sigma^-_{2j-1}\rangle, \tag{5}$$
$$\Gamma_r(i-j) = \langle a^\dagger_{2i} a_{2j}\rangle. \tag{6}$$

For the 1D MCJC lattice, the low-energy excitations are Luttinger liquids and the single-particle density matrices in the

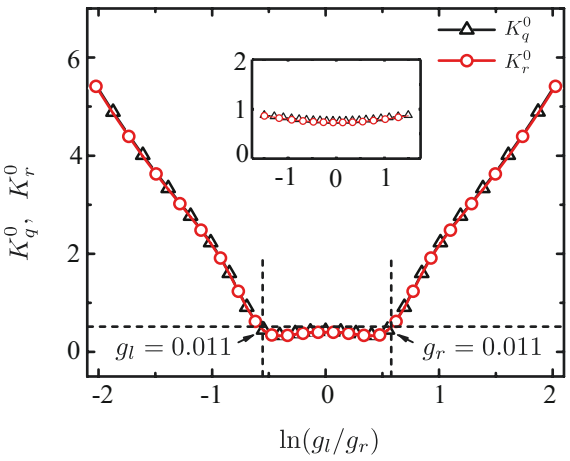

Fig. 3. The Luttinger parameters K_q^0 and K_r^0 versus $\ln(g_l/g_r)$ with $(g_r + g_l)/2\pi = 300$ MHz. The dashed line corresponds to $K_\alpha^0 = 1/2$. The main plot (inset) is at the integer filling $N/L = 1$ (half integer filling $N/L = 1/2$).

superfluid phase obey the relation $\Gamma_\alpha(i-j) \propto |i-j|^{-K_\alpha/2}$ for $\alpha = q, r$ and $|i-j| \to \infty$ [27], where K_q (K_r) is the Luttinger parameters for the qubits (cavities). Our numerical calculation of the single-particle density matrices confirms this power-law dependence. We also know that in the thermodynamic limit with $L \to \infty$, the Luttinger parameters $K_q^0 = K_r^0 = 1/2$ at the quantum critical point for the integer filling $N/L = 1$ [22]. To obtain the Luttinger parameters in the thermodynamic limit, we conduct an extrapolation of the finite lattice results to the $L \to \infty$ limit. The result of K_q^0 and K_r^0 is shown in Fig. 3 as functions of $\ln(g_l/g_r)$. From the result of K_q^0 and K_r^0, we accurately determine the values of the quantum critical points to be $\beta_0 = g_l/g_r \approx 0.579$ and $1/\beta_0 = g_l/g_r \approx 1.727$. Meanwhile, the Luttinger parameters at the half filling $N/L = 1/2$ satisfy $1/2 < K_\alpha^0 < 2$, as shown in the inset of Fig. 3, indicating no phase transition in the whole parameter regime.

IV. Conclusions

To conclude, we study the quantum critical behaviors of the 1D MCJC lattice with the DMRG method. Using the polariton representation, we show that the MCJC lattice can exhibit a MI-to-SF quantum phase transition. We use the DMRG method to calculate the phase boundaries and the quantum critical points. Our result indicates that the qubit-cavity system can be a powerful platform to study strongly-correlated effects in cavity polaritons.

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References

[1] M. H. Devoret and R. J. Schoelkopf, Science **339**, 1169 (2013).
[2] D. D. Awschalom, L. C. Bassett, A. S. Dzurak, E. L. Hu, and J. R. Petta, Science **339**, 1174 (2013).
[3] V. M. Stojanović, M. Vanević, E. Demler, and L. Tian, Phys. Rev. B **89**, 144508 (2014).
[4] M. J. Hartmann, F. Brandão, and M. Plenio, Laser Photonics Rev. **2**, 527 (2008).
[5] A. A. Houck, H. E. Türeci, and J. Koch, Nature Phys. **8**, 292 (2012).
[6] M. P. A. Fisher, P. B. Weichman, G. Grinstein, and D. S. Fisher, Phys. Rev. B **40**, 546 (1989).
[7] M. J. Hartmann, F. G. S. L. Brandão, and M. B. Plenio, Nature Phys. **2**, 849 (2006).
[8] A. D. Greentree, C. Tahan, J. H Cole, and L. C L Hollenberg, Nature Phys.**2**, 856 (2006).
[9] D. G. Angelakis, M. F. Santos, and S. Bose, Phys. Rev. A **76**, 031805(R) (2007).
[10] N. Na, S. Utsunomiya, L. Tian, and Y. Yamamoto, Phys. Rev. A **77**, 031803(R) (2008).
[11] A. J. Hoffman, S. J. Srinivasan, S. Schmidt, L. Spietz, J. Aumentado, H. E. Türeci, and A. A. Houck, Phys. Rev. Lett. **107**, 053602 (2011).
[12] F. Nissen, S. Schmidt, M. Biondi, G. Blatter, H. E. Türeci, J. Keeling, Phys. Rev. Lett. **108**, 233603 (2012).
[13] M. Fitzpatrick, N. Sundaresan, A. C. Y. Li, J. Koch, and A. A. Houck, eprint arXiv 1607.06895.
[14] K. Seo and L. Tian, Phys. Rev. B **91**, 195439 (2015).
[15] K. Seo and L. Tian, SCPMA **58**, 070302 (2015).
[16] Y. Qiu, W. Xiong, L. Tian, and J. Q. You , Phys. Rev. A **89**, 042321 (2014).
[17] A. Kurcz, A. Bermudez, and J. J. García-Ripoll, Phys. Rev. Lett. **112**, 180405 (2014).
[18] R. Barends, J. Kelly, A. Megrant, D. Sank, E. Jeffrey, Y. Chen, Y. Yin, B. Chiaro, J. Mutus, C. Neill, P. O'Malley, P. Roushan, J. Wenner, T. C. White, A. N. Cleland, and J. M. Martinis, Phys. Rev. Lett. **111**, 080502 (2013).
[19] Y. Chen, C. Neill, P. Roushan, N. Leung, M. Fang, R. Barends, J. Kelly, B. Campbell, Z. Chen, B. Chiaro, A. Dunsworth, E. Jeffrey, A. Megrant, J. Y. Mutus, P. J. J. O'Malley, C. M. Quintana, D. Sank, A. Vainsencher, J. Wenner, T. C. White, Michael R. Geller, A. N. Cleland, and J. M. Martinis, Phys. Rev. Lett. **113**, 220502 (2014).
[20] S. R. White, Phys. Rev. Lett. **69**, 2863 (1992).
[21] S. R. White, Phys. Rev. B **48**, 10345 (1993).
[22] T. D. Kühner and H. Monien, Phys. Rev. B **58**, 14741(R) (1998).
[23] T. D. Kühner, S. R. White, and H. Monien, Phys. Rev. B **61**, 12474 (2000).
[24] D. Rossini and R. Fazio, Phys. Rev. Lett. **99**, 186401 (2007).
[25] J. Xue, K. Seo. L, Tian and T. Xiang, Phys.Rev.B **96**, 174502 (2017).
[26] E. Jaynes and F. Cummings, Proc. IEEE **51**, 89 (1963).
[27] F.D.M. Haldane, Phys. Rev. Lett. **47**, 1840 (1981).