A New Dimensionless Number for Thermoelectric Generator Performance

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Abstract

Thermoelectric generators (TEG) convert heat into electricity and offer a green option for renewable energy generation. The Thomson effect has been proven to impact the overall performance of TEGs negatively but is ignored in the widely-used formula for TEG maximum efficiency calculation. In this study, a new dimensionless number called the Ury number (Ur) is introduced to capture the influence of the Thomson effect on TEG power output and maximum efficiency. Ur is defined as the ratio between the Thomson coefficient and the Seebeck coefficient. It is found that as Ur increases both TEG output power and maximum efficiency decrease. Ur combined with the device figure of merit, ZT, can better dictate the overall performance of a TEG.

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Introduction

Thermoelectricity relies on three main principles the Seebeck, Peltier, and Thomson effects that were discovered between 1821 and 1851[1]. Thermoelectric generator (TEG) devices, which directly convert heat into electricity, are composed of many p-type and n-type semiconductors connected in series by conducting strips that are mainly made of copper. As opposed to the conventional power systems, TEGs possess some advantages including environmental friendliness with zero CO_2 emission, solid-state devices without moving parts, zero noise produced during operation, and scalable from a small to a giant heat source. Maximizing the performances of a TEG can be overwhelming because it requires the optimization of many parameters including geometry of heat sinks [2], allocation of heat transfer areas of heat sinks [3-5] , dimension of TEG legs [6, 7], number of TEG couples [8], and slenderness ratio ((the geometric factor ratio of the n-type to that of the p-type elements)[9]. Nondimensionalization has been performed routinely [10, 11] to reduce the number of design parameters. However, the Thomson effect, which was found to be an essential phenomenon that negatively impacts the maximum power and efficiency [12, 13], was not included in the conventional nondimensionalization.

In this work, a new dimensionless number called the Ury number (Ur) is introduced to capture the influence of the Thomson effect. Ur is defined as the ratio between the Thomson coefficient to the Seebeck coefficient. The maximum efficiency and power are redefined with respect to the device figure of merit, ZT, temperature ratio, θ, and Ur. It is shown that both the maximum efficiency and power reduce as Ur increases. In the absence of the Thomson effect (Ur=0), the maximum efficiency reverts to the well-known formula for TEG maximum efficiency. Compared with Ur=0, the optimal external load for maximum power stays the same while the optimal external load for the maximum efficiency slightly shifts to a smaller value.

This article is organized as follows. A mathematical analysis of the governing equations of a TEG is first performed to obtain the equations for the maximum power and maximum efficiency of a TEG. Next, the maximum power and maximum efficiency are optimized through nondimensionalization with various values including the figure of merit of the device, ZT, the temperature ratio between the cold side and hot side temperatures, θ, and Ur. Finally, the newly derived dimensionless power is validated with experimental data and a detailed discussion on the influence of Ur on the maximum power and maximum efficiency for various θ and ZT values is presented in the results and discussion section.

Nomenclature

A: Area (m^2)

I: Electrical current (A)

K: Thermal conductance (WK^{-1})

k: Thermal conductivity ($Wm^{-1}K^{-1}$)

L: Length (m)

N: Number of pairs of p-type and n-type semiconductors

Q: Heat flow rate (W)

R: Electrical resistance (Ω), Thermal resistance (KW^{-1})

S: Seebeck Coefficient (VK^{-1})

s: Entropy (JK^{-1})

T: Temperature (K)

x: Coordinate

Ur: Ury number

W: Power output (W)

ZT: Figure of merit of the TEG

Greek Letters:

η: Efficiency

μ: Thomson coefficient (VK^{-1})

θ: the ratio between the cold side and hot side temperature

ρ: Electrical resistivity (Ωm)

$\dot{\sigma}$: Rate of entropy generation (WK^{-1})

Subscripts:

c: Cold junction

eff: Effective properties

h: Hot junction

in: Input

j: Joule heat

out: Output

n: N-type semiconductor leg

p: P-type semiconductor leg

th: thermal

Method

Mathematical Analysis

Figure 1 shows the schematic of a 1-D TEG along with the thermal resistance network. The thermal resistance of the p-type and n-type legs has been considered. Heat loss due to radiation, conduction, and convection within the air gap of TEGs was found to have much less impact on TEG performance as compared with Thomson effect[14] and is, therefore, neglected in this study. The temperature dependent thermoelectric (TE) material properties, including the Seebeck coefficient, Thompson coefficient, thermal conductivity, and electrical resistivity, are integrated over the temperature gradient and normalized over the temperature gradient to obtain the effective TE properties as follow:

$$S_{p,eff} = \frac{\int_{T_c}^{T_h} S_p\,(T)\;dT}{T_h - T_c} \tag{1}$$

$$S_{n,eff} = \frac{\int_{T_c}^{T_h} S_n(T)\;dT}{T_h - T_c} \tag{2}$$

$$\mu_{p,eff} = \frac{\int_{T_c}^{T_h} \mu_p(T)\;dT}{T_h - T_c} \tag{3}$$

$$\mu_{n,eff} = \frac{\int_{T_c}^{T_h} \mu_n(T)\;dT}{T_h - T_c} \tag{4}$$

Where μ is the Thomson coefficient obtained through the second Kelvin relationship [15-18] as shown in Equation (5). A constant Seebeck coefficient leads to a zero Thomson coefficient.

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$$\mu = T\frac{dS}{dT} \tag{5}$$

$$k_{p,eff} = \frac{\int_{T_c}^{T_h} k_p(T)\ dT}{T_h - T_c} \tag{6}$$

$$k_{n,eff} = \frac{\int_{T_c}^{T_h} k_n(T)\ dT}{T_h - T_c} \tag{7}$$

$$\rho_{p,eff} = \frac{\int_{T_c}^{T_h} \rho_p(T)\ dT}{T_h - T_c} \tag{8}$$

$$\rho_{n,eff} = \frac{\int_{T_c}^{T_h} \rho_n(T)\ dT}{T_h - T_c} \tag{9}$$

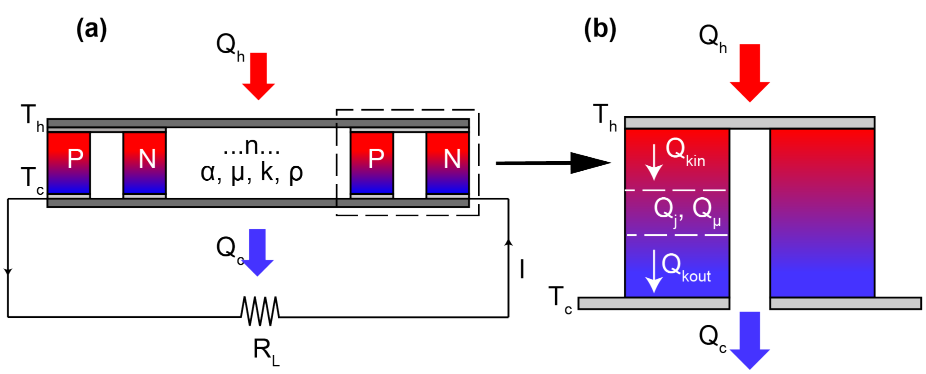

Figure 1: (a) 1-D schematic of a multi-element TEG and (b) heat transfer within a TEG single pair

Figure 1 (a) shows a one-dimensional schematic of a TEG composed of multiple couples of p-type and n-type semiconductors connected electrically in series through strips made of copper and sandwiched between two plates usually made of Alumina. Figure 1(b) depicts a single TEG couple extracted to show the heat transfer within TEG. When energy balance is performed on the control volume within the single TEG, the heat absorbed at the hot side and heat rejected at the cold side are both found to be as follow:

$$Q_h = SIT_h - 0.5I^2R - 0.5I\mu(T_h - T_c) + K(T_h - T_c) \quad (10)$$

$$Q_c = SIT_c + 0.5I^2R + 0.5I\mu(T_h - T_c) + K(T_h - T_c) \quad (11)$$

where the Seebeck coefficient S, Thompson coefficient μ, electrical resistance R, and thermal conductance K are respectively defined as:

$$S = S_{p,eff} - S_{n,eff} \quad (12)$$

$$\mu = \mu_{p,eff} - \mu_{n,eff} \quad (13)$$

$$R = \frac{\rho_{p,eff} L_p}{A_p} + \frac{\rho_{n,eff} L_n}{A_n} \quad (14)$$

$$K = \frac{k_{p,eff} A_p}{L_p} + \frac{k_{n,eff} A_n}{L_n} \quad (15)$$

TEG produces power through the temperature gradient maintained along the p-type and n-type semiconductors which qualify it to be a heat engine. Figure 2 depicts the TEG as a heat engine that generates power by acquiring heat from the hot reservoir and dumping the heat that cannot be transformed into power to the cold reservoir.

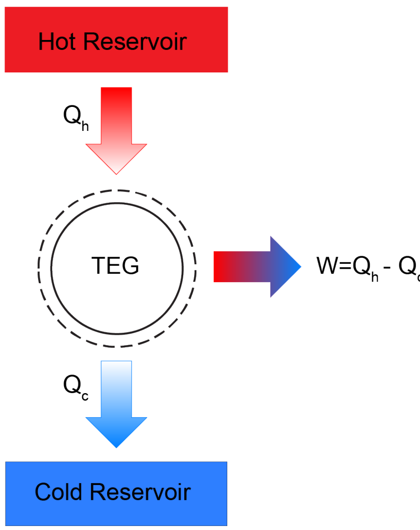

Figure 2: TEG undergoing a power cycle by exchanging heat transfer between two reservoirs

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The entropy rate balance of the TEG system represented in figure 4 is:

$$\frac{ds}{dt} = \sum_j \frac{Q_j}{T_j} + \dot{\sigma} \tag{16}$$

For a steady state, the entropy balance becomes:

$$\dot{\sigma} = -\frac{Q_h}{T_h} + \frac{Q_c}{T_c} \tag{17}$$

TEG power generation is expressed in terms of entropy as follow:

$$W = Q_h - Q_c = \left(1 - \frac{T_c}{T_h}\right) Q_h - T_c \dot{\sigma} \tag{18}$$

Also, the power generated by the TEG can be expressed as the difference between the heat absorbed and heat rejected as shown in the following expression

$$W = Q_h - Q_c = SI(T_h - T_c) - I^2 R - \mu I (T_h - T_c) \tag{19}$$

When the power is divided by the current, the following total voltage expression is obtained

$$V = S(T_h - T_c) - IR - \mu(T_h - T_c) \tag{20}$$

Where the expressions on the right side of Equation (20) are respectively the voltage induced by the Seebeck effect, the voltage generated across the p and n-type legs as described by the Ohm's Law, and the voltage produced through the Thomson effect.

Also, the power can be expressed in terms of the external resistance as follow

$$W = Q_h - Q_c = I^2 R_L \tag{21}$$

where the current I is a function of the Seebeck coefficient S, the Thomson coefficient μ, the hot and cold side temperatures, T_h and T_c, the electrical resistance R, and the external load resistance R_L, as shown in Equation (22).

$$I = \frac{(S - \mu)(T_h - T_c)}{R_L + R} \tag{22}$$

Substituting Equation (22) into Equation (21),

$$W = \frac{(S - \mu)^2 (T_h - T_c)^2}{(R_L + R)^2} R_L \tag{23}$$

The efficiency of a TEG is given by

$$\eta = \frac{W}{Q_h} = \frac{I^2 R_L}{S I T_h - 0.5 I^2 R - 0.5 I \mu (T_h - T_c) + K(T_h - T_c)} \quad (24)$$

Substituting Equation (22) in Equation (24), the efficiency becomes a

$$\eta = \frac{(S-\mu)(T_h - T_c) R_L}{S T_h (S-\mu)(R_L + R) - 0.5(S-\mu)^2 (T_h - T_c) R - 0.5\mu(S-\mu)(T_h - T_c)(R_L + R) + K(R_L + R)^2} \quad (25)$$

Power and efficiency optimization

Power maximization

The optimum external load resistance, R_L, that yields the maximum power of a given TEG at fixed hot and cold junction temperatures can be obtained as follow:

$$\frac{\partial W}{\partial R_L} = 0 \quad (26)$$

Determining the optimum value of the external resistance to maximize the power leads to:

$$(R_L)_{opt} = R \quad (27)$$

The maximum power occurs when the external resistance is equal to the internal resistance of the TEG and replacing Equation (27) into Equation (23) yields to the maximum power expressed as:

$$W_{max} = \frac{(S-\mu)^2 (T_h - T_c)^2}{4R} \quad (28)$$

Nondimensionalizing the above equation leads to the following:

$$W^*_{max} = \frac{W_{max}}{KT_c} = \frac{(1-\theta)^2 ZT}{2(1+\theta)\theta}(1-Ur)^2 \quad (29)$$

where Ur and θ are respectively defined as follow:

$$Ur = \frac{\mu}{S} \quad (30)$$

$$\theta = \frac{T_c}{T_h} \quad (31)$$

and ZT is the figure of merit of a TEG device evaluated at the average temperature between the hot and cold side temperatures

$$ZT = \frac{S^2}{RK}\left(\frac{T_h + T_c}{2}\right) \quad (32)$$

Equation (29) is the dimensionless number that evaluates the effect of Ur, θ, and ZT on the maximum power at optimum R_L. To evaluate the effect of R_L on the overall power, a new dimensionless power that contains a dimensionless external resistance would be needed.

If R^* defines the dimensionless external electrical resistance as follow:

$$R^* = \frac{R_L}{R} \quad (33)$$

A dimensionless power is obtained as:

$$W^* = \frac{W}{KT_c} = \frac{2ZT(1-\theta)^2(1-Ur)^2}{\theta(1+\theta)(R^*+1)^2}R^* \quad (34)$$

However, the value of the maximum power depends on ZT, Ur, and θ.

Efficiency maximization

The optimum external load resistance, R_L, that yields to the maximum efficiency of a given TEG at fixed hot and cold junction temperatures can be obtained from

$$\frac{\partial \eta}{\partial R_L} = 0 \quad (35)$$

For maximum efficiency, the obtained result from Equation (35) shows that the external resistance needs to be equal to:

$$(R_L)_{opt} = R\sqrt{1 + ZT - ZT \times Ur} \quad (36)$$

Note that the optimum external resistance in Equation (36) that maximizes the efficiency is different from the one that maximizes the power, Equation (27). Therefore, maximum power and efficiency cannot occur at the same optimum external resistance.

Replacing Equation (36) into Equation (25) and nondimensionalizing the obtained expression provides the maximum TEG efficiency as depicted in Equation (38)

$$\eta_{max} = \left. \frac{(S-\mu)^2(T_h-T_c)R_L}{ST_h(S-\mu)(R_L+R) - 0.5(S-\mu)^2(T_h-T_c)R - 0.5\mu(S-\mu)(T_h-T_c)(R_L+R) + K(R_L+R)^2} \right|_{R_L=R_{L,opt}} \quad (37)$$

$$\eta^*_{max} = \frac{2ZT(1-Ur)^2(1-\theta)\psi}{2ZT(1-Ur)(1+\psi) - ZT(1-Ur)^2(1-\theta) - ZT * Ur(1-Ur)(1-\theta)(1+\psi) + (1+\theta)(1+\psi)^2} \quad (38)$$

where

$$\psi = \sqrt{1 + ZT - ZT \times Ur} \quad (39)$$

To evaluate the effect of the external resistance on the overall efficiency a new dimensionless efficiency is introduced using a dimensionless external resistance R^*. Incorporating R^* in the nondimensionalization of the efficiency expression defined in Equation (25), yields to:

$$\eta^* = \frac{(1-Ur)(1-\theta)R^*}{(R^*+1)\left(1 - \frac{0.5(1-Ur)(1-\theta)}{(R^*+1)} - 0.5Ur(1-\theta) + \frac{(1+\theta)(R^*+1)}{2ZT(1-Ur)}\right)} \quad (40)$$

Results and discussion

The accuracy of the dimensionless expressions introduced in this study is first evaluated by comparing theoretical results with experimental data reported in [19]. In reality, the Ury number varies between 0-1, i.e., 0<Ur<1. For power generation to occur, a positive voltage needs to be inducted, which means that the voltage induced by the Seebeck effect needs to be higher than the sum of the voltage generated by the Thomson effect and Ohm's Law as shown in Equation (20). Therefore, Ur is always smaller than 1. Ur=0 occurs when a constant Seebeck coefficient is assumed in the analysis which leads to a Thomson coefficient equating to zero according to Equation (5). Consequently, TEG power and efficiency are overestimated due to the omission of the Thomson coefficient in TEG power and efficiency expressions, The experimental investigation was performed on TEGs made of bismuth telluride[20]. A TEG made of bismuth telluride typically operates at a temperature range of 298K-353K. The temperature dependent Seebeck and Thompson coefficients of bismuth telluride are:

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$$S_p = (22224 + 930.6T - 0.9905T^2) * 10^{-9}(VK^{-1}) \qquad (41)$$

$$S_n = -(22224 + 930.6T - 0.9905T^2) * 10^{-9}(VK^{-1}) \qquad (42)$$

$$\mu_p = T\frac{dS_p}{dT}\ (VK^{-1}) \qquad (43)$$

$$\mu_n = T\frac{dS_n}{dT}\ (VK^{-1}) \qquad (44)$$

For a better approximation of the Seebeck and Thompson coefficients, they are both integrated and normalized over the temperature gradient

$$S_{p,eff} = \frac{\int_{298}^{353} S_p\,(T)\,dT}{353 - 298} = 2.2 \times 10^{-4}\ (VK^{-1}) \qquad (45)$$

$$S_{n.eff} = \frac{\int_{298}^{353} S_n\,(T)\,dT}{353 - 298} = -2.2 \times 10^{-4}(VK^{-1}) \qquad (46)$$

$$\mu_{p,eff} = \frac{\int_{298}^{353} \mu_p\,(T)\,dT}{353 - 298} = 9.3 \times 10^{-5}\,(VK^{-1}) \tag{47}$$

$$\mu_{n,eff} = \frac{\int_{298}^{353} \mu_n\,(T)\,dT}{353 - 298} = -9.3 \times 10^{-5}(VK^{-1}) \tag{48}$$

$$S = S_{p,eff} - S_{n,eff} = 4.4 \times 10^{-4}\;\,(VK^{-1}) \tag{49}$$

$$\mu = \mu_{p,eff} - \mu_{n,eff} = 18.6 \times 10^{-5}\,(VK^{-1}) \tag{50}$$

$$Ur = \frac{\mu}{S} = 0.42 \tag{51}$$

The calculated Ur value of 0.42 is far from zero and needs to be included in both maximum and efficiency expressions to accurately represents both TEG power and efficiency. To obtain the dimensionless power data from the experiment, the measured power was divided by the product of the thermal conductance and measured cold side temperature. The theoretical dimensionless power was calculated using Equation (29) where Ur, θ, and ZT were calculated from Equation (30), (31), and (32), respectively. Table (1) contains TEG dimensions and operating temperature ranges used in calculating θ and ZT.

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Table 1: TEG Dimensions and operating temperature ranges

Material	Module Dimensions(mm)	Pellet Dimension (mm)	Couple Number	T_h(°C)	T_c(°C)
Bi_2Te_3	$62 \times 62 \times 5.2$	$4.5 \times 4.5 \times 2.5$	49	$100 - 200$	37

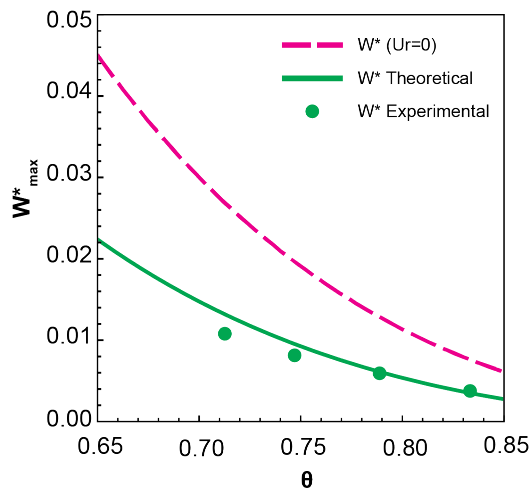

Figure 3: Theoretical and experimental dimensionless power output of bismuth telluride thermoelectric generator device. At θ= {0.698851, 0.73253, 0.773537, 0.817204}, the corresponding Ur= {0.317917, 0.343264, 0.370038, 0.394554}

Both theoretical and experimental dimensionless powers as a function of temperature ratio are plotted in figure 3, and the dimensionless power derived in this study agrees well with the experimental results. As θ decreases, which means the temperature gradient increases, the theoretical curve becomes slightly different from the experimental results because the average material properties used in the Ur and ZT calculations deviate more from the actual temperature-dependent material properties at larger temperature gradients. The power production is significantly overestimated at Ur=0 due to the omission of the Thomson effect. Because the Thompson effect induces negative voltage, it reduces the overall TEG voltage across the p-type and n-type semiconductors as shown in Equation (20). As a result, TEG power decreases. Figure 3 shows how the inclusion of Ur into the dimensionless maximum power provides a much better

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agreement between the theoretical result and experimental data. Therefore, Ur is a crucial number for accurate prediction of TEG performance before actual experimental data collection.

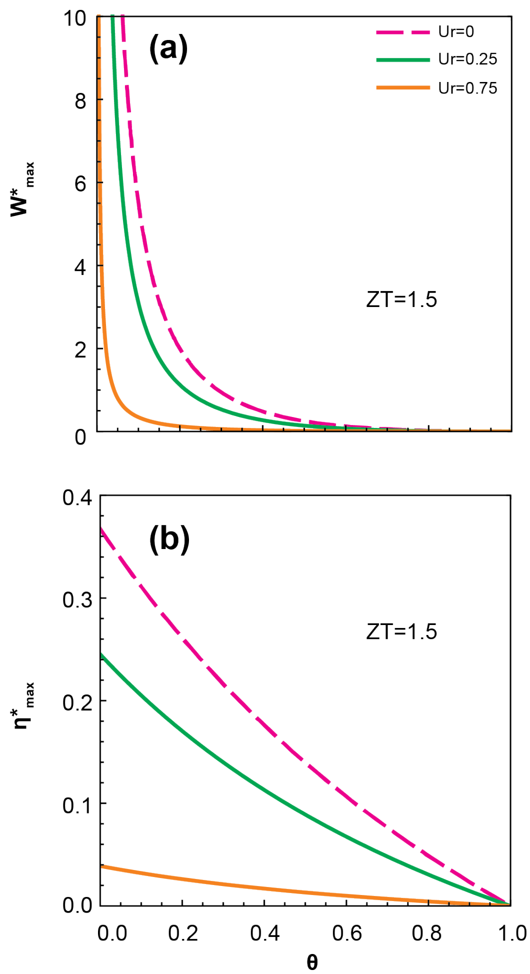

Figure 4: As a function θ, for different Ur values and at ZT of 1.5 (a) Variation of the maximum power (b)Variation of the maximum efficiency

Figure 4 (a) shows the variation of the dimensionless maximum power with respect to θ for various Ur numbers at ZT=1.5. All the dimensionless maximum power curves decrease as θ increases due to lower power generation resulting from lower temperature gradients. Referring back to Figure (2); a more significant temperature gradient means that TEG receives more amount of heat from the hot reservoir and rejects less amount of heat to the cold reservoir, which generates a more substantial power. Figure 4 (b) shows the maximum dimensionless efficiency with respect to θ for different values of Ur. For a given Ur, the maximum efficiency reduces with increasing θ because the maximum power decreases with decreasing temperature gradient. At the same θ, the maximum efficiency decreases as Ur increases due to the Thomson effect. The negative voltage generated by the Thomson effect can be transformed into heat and dissipated out of the p-type and n-type pellets. When part of the heat acquired from the hot reservoir becomes lost to the environment along pellets, it reduces the amount of heat that could have been transformed into useful TEG power and consequently decreases the efficiency. The smaller decay slope of the larger Ur curves indicates that the efficiency becomes less sensitive to operating θ when the Thomson effect is introduced. When Ur=0, both curve falls on top of the calculated curve in Ref. [10], where the Thomson effect was not taken into account. Because the Thomson effect lowers the power output and efficiency for various θ, devising ways to reduce the Thomson effect will lead to higher TEG power and efficiency.

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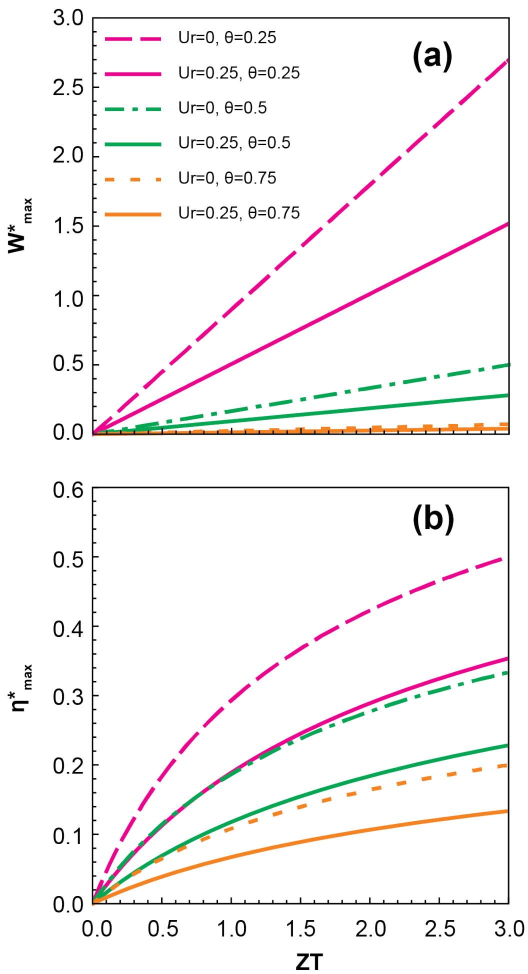

Figure 5: As function of ZT and for different θ and Ur values (a) Variation of the maximum power and (b) Variation of the maximum efficiency

Figure 5 (a) shows the variation of the dimensionless maximum power with respect to ZT for different θs and Urs. The dimensionless maximum power increases linearly with increasing ZT for all the cases. This linear behavior is guaranteed by Equation (29). At given θ and Ur, the dimensionless maximum power is a linear function of ZT with a constant coefficient of

$\frac{(1-\theta)^2}{2(1+\theta)\theta}(1-Ur)^2$. When Ur=0, the curves in Figure 5 (a) become the same as the ones in[10]. The figure of merit, ZT, indicates the ability of a TEG in effectively transforming heat supplied form the hot reservoir into power, and a higher ZT lead to higher TEG power and efficiency. A high ZT values requires a large Seebeck coefficient that ties to higher positive voltage which contributes to higher TEG power and efficiency, a small thermal conductance to maintain large temperature gradients and reduce heat loss, and a small electric resistance to increase electric current. At a given θ, the maximum power output of Ur=0.25 is much lower than that with Ur=0. At lower θ, the difference between Ur=0.25 and Ur=0 is more significant. In other words, power reduction by Thomson effect is more significant at larger temperature gradients because of larger Thomson coefficient. Figure 5 (b) shows the maximum dimensionless efficiency as a function of ZT for different θs and Ur numbers. The maximum efficiency increases with increasing ZT at given Ur and θ, and it will approach a plateau. At a given θ, the maximum efficiency is reduced as Ur increases. This reiterates how the Thomson effect significantly reduces the efficiency of a TEG.

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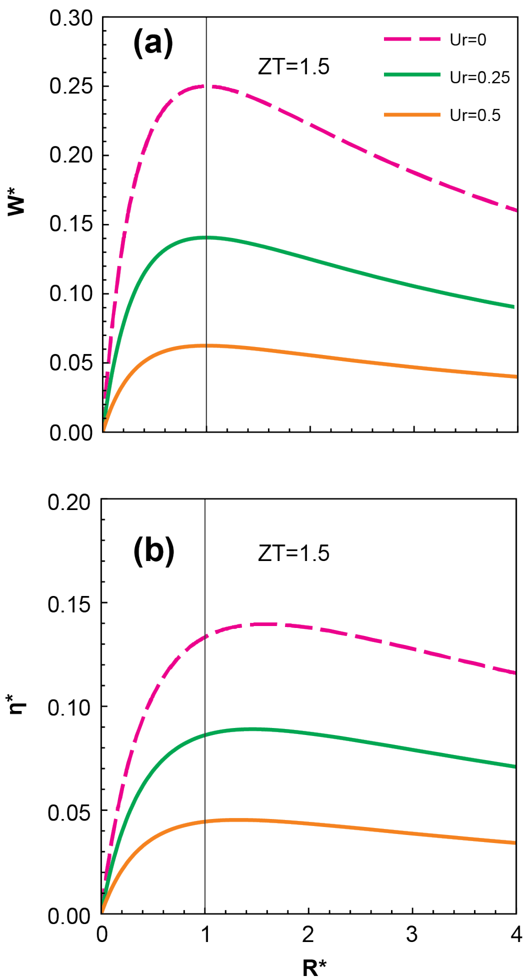

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319 *Figure 6: As a function of R *, at a ϑ of 0.5, and for various Ur values (a) Dimensionless power and (b) Dimensionless efficiency*

320 Figure 6 (a) and (b) shows the variation of the dimensionless power W^* and efficiency η^*as a
321 function of the dimensionless external resistance R^*. As Ur increases, both power and efficiency
322 are reduced due to the Thomson physical phenomenon.

The optimum R^* for maximum power is always 1 as opposed to the optimum R^* for maximum efficiency that occurs at a value of $R^*\sqrt{1 + ZT - Ur \times ZT}$. Taking the case when Ur=0.25, the optimum R^* for maximum efficiency is 1.32. Note that when dealing with a TEG composed of multiple pellets, the electrical resistance adds up leading to a significant difference between the external resistance that will maximizes the power and efficiency. For example, if TEG internal electric resistance of multiple pellets is 20Ω, the optimum external resistance for power will remain 20Ω while 26.4Ω is required for maximum efficiency. The dimensional electrical resistance is an excellent parameter to guide TEG design towards maximization of power or efficiency.

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Conclusion

Nondimensionalization of the output power and the efficiency of a TEG is revisited with the inclusion of the Thomson effect. The Ury number (Ur) that describes the ratio between Thomson and Seebeck coefficients is introduced. Ur is the first dimensionless number reported in the literature that quantifies the relative importance of the Thomson effect. When Ur is nonzero, the maximum power output and efficiency are reduced from those with Ur=0. The findings obtained from this study can be summarized as follow:

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- The negative impact of the Thomson effect on TEG power and efficiency is captured by reformulating the maximum power and thermoelectric efficiency formula via the introduction of a new dimensionless number called the Ury number (Ur).
- The theoretical results are validated with experimental data, and it is found that without the inclusion of Thomson coefficient through the Ury number, TEG power can be significantly overestimated, which suggests that Ury is a crucial number in predicting accurate TEG experimental data.

- The maximum power occurs when the external resistance is equal to the internal resistance of the TEG while the maximum efficiency happens when the external resistance is equivalent to $(R_L)_{opt} = R\sqrt{1 + ZT - Ur \times ZT}$.
- The revised dimensionless maximum power and efficiency are now functions of both ZT and Ur, which better agreement with the experimental data and developing strategies to reduce the Thomson effect will lead to TEG better performances.

Acknowledgments

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