The Degrees of Freedom of MIMO Relay under Coherence Diversity

Fan Zhang and Aria Nosratinia
Department of Electrical Engineering, University of Texas at Dallas, TX, USA.
Email: fxz131330@utdallas.edu;aria@utdallas.edu

***Abstract*—This paper studies the MIMO relay with non-identical link coherence times, a condition that is denoted *coherence diversity*. This can occur, e.g., when the nodes do not all have the same mobility, or the scatterers around some nodes have different mobility than others. Obviously this condition occurs in practice therefore the model is well motivated, but the performance of a relay under such conditions has not been studied to date. This paper calculates achievable degrees of freedom under this condition. Since different coherence times have a prominent impact on channel training, channel state is not made available to the decoder for free, and all channel training resources are accounted for in the calculations. A product superposition technique is employed at the source that allows a more efficient usage of degrees of freedom when the relay and the destination have different training requirements. Following the analysis of a representative example, a general analysis is provided and varying configurations of coherence times are studied. Numerical results demonstrate the gains of the proposed approach under coherence disparity.**

I. Introduction

In wireless networks, mobility of nodes as well as scattering environments often produce unequal link coherence times. In the two-user broadcast channel with unequal coherence times, [1], [2] demonstrated gains over TDMA transmission. This result was extended to K users and staggered coherent time blocks in [3], [4]. Inner and outer bounds for multiple access channel with unequal coherence times was calculated in [3].

This paper studies the effect of unequal link coherence times on the degrees of freedom (DoF) of the MIMO relay channel. The closest results in the literature involve the block fading relay channel under *identical* link coherence times, among them: the diversity-multiplexing trade-off of 3-node relay without direct link [5], outage performance of the two-way relay channel [6], achievable rate of buffer-aided relay without direct link [7] and with direct link [8], achievable rate of buffer-aided diamond relay network with inter-relay interference [9] and achievable rate region of bidirectional buffer-aided relay [10].

For the MIMO relay with coherence diversity, we assume there is no free channel state information (CSI) at the receivers, since unequal coherence times impact channel training and assuming free CSI will distort and obscure important features of the problem. In addition, no channel state information is assumed at transmitters. We propose a product superposition transmission strategy at the source, which was first introduced in [1] for two-user broadcast channel. Product superposition is a technique that allows efficient utilization of channel degrees of freedom under coherence disparity, and we use it when the links from the source to the relay and the destination have unequal coherence times.

We begin by proving that under identical coherence times, the relay cannot provide any DoF gains over the direct link alone. This result is used as a reference. When coherence times are unequal, we start with a representative example to show that disparity in coherence time enables DoF gains over conventional transmission. Then the result is extended to more general coherence time configurations. Numerical examples shed further light on the results.

This work was made possible in part by the grants 1527598 and 1718551 from the National Science Foundation.

II. System Model

Consider a MIMO Gaussian relay in full-duplex mode. The source and destination are equipped with N_S and N_D antennas, respectively. The relay has N_R antennas and uses $n_r \leq N_R$ antennas for transmitting. The received signal at the relay and destination are:

$$\mathbf{y}_R = \mathbf{H}_{SR}\mathbf{x}_S + \mathbf{w}_R \tag{1}$$
$$\mathbf{y}_D = \mathbf{H}_{SD}\mathbf{x}_S + \mathbf{H}_{RD}\mathbf{x}_R + \mathbf{w}_D, \tag{2}$$

where $\mathbf{x}_S$ and $\mathbf{x}_R$ are signals transmitted from the source and relay. $\mathbf{w}_R$ and $\mathbf{w}_D$ are i.i.d. white Gaussian noise and $\mathbf{H}_{SR}$, $\mathbf{H}_{RD}$ and $\mathbf{H}_{SD}$ are channel gain matrices whose entries are i.i.d. Gaussian. Channel gain entries and noise components are zero-mean and have unit variance. Channel gains experience block fading, remaining constant during the coherence intervals which are, respectively, of T_{SR},T_{RD} and T_{SD}, satisfying $T_{SR} \geq 2\max(N_S, N_R)$, $T_{RD} \geq 2\max(N_R, N_D)$ and $T_{SD} \geq 2\max(N_S, N_D)$, but changing independently across blocks [11]. The source and relay obey power constraints $E[\text{tr}(\mathbf{x}_S\mathbf{x}_S')] \leq \rho$ and $E[\text{tr}(\mathbf{x}_R\mathbf{x}_R')] \leq \rho$. We assume there is no free channel state information at the destination and no CSIT at the source or relay.

The source sends messages to the destination with rate $R(\rho)$ at ρ signal-to-noise ratio. The degrees of freedom at the destination achieving rate $R(\rho)$ is defined as

$$d = \lim_{\rho\to\infty} \frac{R(\rho)}{\log(\rho)}. \tag{3}$$

III. Relay Channel with Coherence Diversity

We first show when the coherence times of all links are identical, the relay cannot provide any DoF gains. Then we analyze the scenarios where the coherence times are unequal.

A. Relay with Identical Coherence Times

Theorem 1. *In a relay described in section* (II)*, if $T_{SD} = T_{SR} = T_{RD} = T$, the relay does not provide DoF gains over direct link, which achieves the following DoF:*

$$d = \min(N_S, N_D)(1 - \frac{\min(N_S, N_D)}{T}). \tag{4}$$

Proof. From the cut-set bound, we know

$$R \leq \min\{\mathrm{I}(\mathbf{x}_S; \mathbf{y}_R, \mathbf{y}_S | \mathbf{x}_R), \mathrm{I}(\mathbf{x}_S, \mathbf{x}_R; \mathbf{y}_D)\} \tag{5}$$

If $N_S \leq N_D$, consider the inequality in the cut-set bound, $R \leq \mathrm{I}(\mathbf{x}_S; \mathbf{y}_R, \mathbf{y}_S | \mathbf{x}_R)$. Because $T_{SD} = T_{SR} = T$ and there is no CSIT, the right hand side in the inequality is upper bounded by the capacity of a point-to-point channel having N_S transmit antennas and $(N_D + N_R)$ receive antennas with coherence time T, which is $N_S(1 - \frac{N_S}{T}) \log \rho + o(\log \rho)$. Then we have

$$d \leq N_S(1 - \frac{N_S}{T}), \tag{6}$$

which can be achieved by the direct link.

If $N_S \geq N_D$, consider the inequality, $R \leq \mathrm{I}(\mathbf{x}_S, \mathbf{x}_R; \mathbf{y}_D)$. Since $T_{SD} = T_{SR} = T$, the right hand side in the inequality is upper bounded by the capacity of a point-to-point channel with $(N_D + N_R)$ transmit antennas and N_D receive antennas with coherence time T, whose capacity is $N_D(1 - \frac{N_D}{T}) \log \rho + o(\log \rho)$. Then we have

$$d \leq N_D(1 - \frac{N_D}{T}), \tag{7}$$

and this DoF can be achieved by the direct link. This completes the proof. □

B. Example for Unequal Coherence Times

When links have unequal coherence times, **Theorem** 1 shows if we design the signal as if they had the same coherence time(the shortest), the relay cannot provide DoF gains. In this section, we first consider an example. The source and relay are equipped with 2 antennas and the destination is equipped with 3 antennas. The coherence times of the three links are as follows: $T_{SR} = \infty$, i.e., the source-relay channel is static, therefore the cost of training over this link is amortized over a large number of samples and we can assume the relay knows $\mathbf{H}_{SR}$. Furthermore we asssume $T_{SD} = T_{RD} = 8$.

The source uses product superposition, sending

$$\mathbf{X}_S = \mathbf{X}_u[\mathbf{I}_2 \ \mathbf{0}_{2\times 1} \ \mathbf{X}_d], \tag{8}$$

where $\mathbf{X}_u \in \mathbb{C}^{2\times 2}$ and $\mathbf{X}_d \in \mathbb{C}^{2\times 5}$.

At the relay, the received signal is

$$\mathbf{Y}_R = \mathbf{H}_{SR}\mathbf{X}_S + \mathbf{W}_R \tag{9}$$
$$= \mathbf{H}_{SR}\mathbf{X}_u[\mathbf{I}_2 \ \mathbf{0}_{2\times 1} \ \mathbf{X}_d] + \mathbf{W}_R. \tag{10}$$

The received signal at the first two time slots is

$$\mathbf{Y}'_R = \mathbf{H}_{SR}\mathbf{X}_u + \mathbf{W}'_R. \tag{11}$$

The relay knows $\mathbf{H}_{SR}$ and decodes $\mathbf{X}_u$. Assume the signal decoded by the relay in the previous block is $\mathbf{X}'_u$ and the two rows of $\mathbf{X}'_u$ are $\mathbf{x}'_1, \mathbf{x}'_2 \in \mathbb{C}^{1\times 2}$.

The relay uses one antenna for transmission and sends

$$\mathbf{X}_R = [\mathbf{0}_{1\times 2} \ 1 \ \mathbf{x}'_1 \ \mathbf{x}'_2 \ 0] \in \mathbb{C}^{1\times 8}. \tag{12}$$

Now the received signal at the destination is

$$\mathbf{Y}_D = \mathbf{H}_{SD}\mathbf{X}_S + \mathbf{H}_{RD}\mathbf{X}_R + \mathbf{W}_D \tag{13}$$
$$= [\mathbf{H}_{SD} \ \mathbf{H}_{RD}] \begin{bmatrix} \mathbf{X}_S \\ \mathbf{X}_R \end{bmatrix} + \mathbf{W}_D \tag{14}$$
$$= [\mathbf{H}_{SD} \ \mathbf{H}_{RD}] \begin{bmatrix} \mathbf{X}_u[\mathbf{I}_2 \ \mathbf{0}_{2\times 1} \ \mathbf{X}_d] \\ \mathbf{0}_{1\times 2} \ 1 \ \mathbf{x}'_1 \ \mathbf{x}'_2 \ 0 \end{bmatrix} + \mathbf{W}_D \tag{15}$$
$$= [\mathbf{H}_{SD}\mathbf{X}_u \ \mathbf{H}_{RD}][\mathbf{I}_3 \ \mathbf{X}_D] + \mathbf{W}_D, \tag{16}$$

where

$$\mathbf{X}_D = \begin{bmatrix} \mathbf{X}_d \\ \mathbf{x}'_1 \ \mathbf{x}'_2 \ 0 \end{bmatrix}. \tag{17}$$

The destination estimates the equivalent channel $\mathbf{H}_D = [\mathbf{H}_{SD}\mathbf{X}_u \ \mathbf{H}_{RD}]$ in the first three time slots and decodes $\mathbf{X}_d, \mathbf{x}'_1$ and $\mathbf{x}'_2$. In this proposed scheme, the destination can achieve the DoF $d = (2 \times 5 + 2 \times 1 \times 2)/8 = 1.75$. If we assume all links have coherence time $T = 8$, the DoF we can achieve is $d' = 2 \times (8 - 2)/8 = 1.5$.

This representative example shows when the three links have non-identical coherence times, DoF gains can be achieved via the proposed scheme.

C. $\mathbf{T}_{SR} = \infty$, $\mathbf{T}_{SD} = \mathbf{T}_{RD}$

In this section, the result above is extended to a more general scenario.

Theorem 2. *In a relay described in section* (II)*, if $T_{SR} = \infty$, $T_{SD} = T_{SR} = T$ and $N_S = N_R < N_D$, the following degree of freedom is achievable:*

$$d = \frac{1}{T} \max_{n_r} \min\{d_1 + d_2, d_1 + d_3\}, \tag{18}$$

where

$$d_1 = N_S(T - n_r - N_S), \tag{19}$$
$$d_2 = n_r(T - n_r - N_S), \tag{20}$$
$$d_3 = N_S^2, \tag{21}$$

which is greater than the direct link can achieve.

Proof. The source sends the product superposition signal:

$$\mathbf{X}_S = \mathbf{X}_u[\mathbf{I}_{N_S} \ \mathbf{0}_{N_S\times n_r} \ \mathbf{X}_d], \tag{22}$$

where $n_r \leq \min\{N_S, N_D - N_S\}$,$\mathbf{X}_u \in \mathbb{C}^{N_S\times N_S}$ and $\mathbf{X}_d \in \mathbb{C}^{N_S\times(T - n_r - N_S)}$.

At the relay, the received signal is

$$\mathbf{Y}_R = \mathbf{H}_{SR}\mathbf{X}_S + \mathbf{W}_R \tag{23}$$

$$= \mathbf{H}_{SR}\mathbf{X}_u[\mathbf{I}_{N_S}\ \mathbf{0}_{N_S\times n_r}\ \mathbf{X}_d] + \mathbf{W}_R. \quad (24)$$

The received signal at the first N_S time slots is

$$\mathbf{Y}'_R = \mathbf{H}_{SR}\mathbf{X}_u + \mathbf{W}'_R. \quad (25)$$

The relay knows $\mathbf{H}_{SR}$ and decodes $\mathbf{X}_u$. Assume the message decoded by the relay in the previous block is $\mathbf{X}'_u$. The relay uses n_r antennas for transmission, sending

$$\mathbf{X}_R = [\mathbf{0}_{n_r\times N_S}\ \mathbf{I}_{n_r}\ \mathbf{X}_{r,d}] \in \mathbb{C}^{n_r\times T}, \quad (26)$$

where $\mathbf{X}_{r,d} \in \mathbb{C}^{n_r\times(T-n_r-N_S)}$.

The received signal at the destination is

$$\begin{aligned}
\mathbf{Y}_D &= \mathbf{H}_{SD}\mathbf{X}_S + \mathbf{H}_{RD}\mathbf{X}_R + \mathbf{W}_D && (27)\\
&= [\mathbf{H}_{SD}\ \mathbf{H}_{RD}]\begin{bmatrix}\mathbf{X}_S\\ \mathbf{X}_R\end{bmatrix} + \mathbf{W}_D && (28)\\
&= [\mathbf{H}_{SD}\ \mathbf{H}_{RD}]\begin{bmatrix}\mathbf{X}_u[\mathbf{I}_{N_S}\ \mathbf{0}_{N_S\times n_r}\ \mathbf{X}_d]\\ \mathbf{0}_{n_r\times N_S}\ \mathbf{I}_{n_r}\ \mathbf{X}_{r,d}\end{bmatrix} + \mathbf{W}_D && (29)\\
&= [\mathbf{H}_{SD}\mathbf{X}_u\ \mathbf{H}_{RD}][\mathbf{I}_{(N_S+n_r)}\ \mathbf{X}_D] + \mathbf{W}_D, && (30)
\end{aligned}$$

where

$$\mathbf{X}_D = \begin{bmatrix}\mathbf{X}_d\\ \mathbf{X}_{r,d}\end{bmatrix}. \quad (31)$$

The destination estimates the equivalent channel $\mathbf{H}_D = [\mathbf{H}_{SD}\mathbf{X}_u\ \mathbf{H}_{RD}]$ during the first $(N_S + n_r)$ time slots and then decodes $\mathbf{X}_D$. At the destination, the messages decoded have two parts: $\mathbf{X}_d$ from the source and $\mathbf{X}_{r,d}$ from the relay, which provide DoF $d_1 = N_S(T - n_r - N_S)$ and $d_2 = n_r(T - n_r - N_S)$. The message in $\mathbf{X}_{r,d}$ is from $\mathbf{X}'_u$. The DoF of $\mathbf{X}'_u$ is $d_3 = N_S^2$. The message sent by the relay is less than it decodes. Thus the DoF the destination can achieve is $\frac{1}{T}\min\{d_1+d_2, d_1+d_3\}$ in (18).

The direct link can achieve the following DoF: $d' = \frac{N_S}{T} \times (T - N_S)$. Now we prove d is always greater than d'. Set $n_r = 1$. If $d_3 \le d_2$, the DoF achieved by the proposed scheme is

$$d(1) = \frac{1}{T}(N_S(T-1-N_S) + N_S^2) = \frac{N_S}{T}(T-1). \quad (32)$$

Obviously, $d \ge d(1) \ge d'$; if $d_2 \le d_3$, the DoF achieved is

$$\begin{aligned}
d(1) &= \frac{1}{T}(N_S(T-1-N_S) + (T-1-N_S)) && (33)\\
&= \frac{N_S+1}{T}(T-1-N_S). && (34)
\end{aligned}$$

Because $T \ge 2N_D \ge 2N_S + 2$,

$$d \ge d(1) = \frac{N_S+1}{T}(T-1-N_S) > \frac{N_S}{T}(T-N_S) = d'. \quad (35)$$

This completes the proof. □

D. $T_{SR} = KT_{SD} = KT_{RD}$

In this section, we assume the relay does not know $\mathbf{H}_{SR}$.

Theorem 3. *In a relay described in section* (II)*, if $T_{SR} = KT_{SD} = KT_{RD} = KT, K \in \mathcal{Z}$. and $N_S = N_R < N_D$, the following degree of freedom is achievable:*

$$\begin{aligned}
d = \frac{1}{KT}&(N_S(T-N_S)\\
&+ (K-1)\max_{n_r}\min\{d_1+d_2, d_1+d_3\}). && (36)
\end{aligned}$$

where

$$\begin{aligned}
d_1 &= N_S(T-n_r-N_S), && (37)\\
d_2 &= n_r(T-n_r-N_S), && (38)\\
d_3 &= N_S^2, && (39)
\end{aligned}$$

which is greater than the direct link can achieve.

Proof. We set the length of the transmit block to KT and divide the transmit block into K sub-blocks with length T. During the first sub-block, the source sends the signal

$$\mathbf{X}_S^1 = [\mathbf{I}_{N_S}\ \mathbf{X}_d^1]. \quad (40)$$

where $\mathbf{X}_d^1 \in \mathbb{C}^{N_S\times(T-N_S)}$. The relay estimates $\mathbf{H}_{SR}$ during the first N_S time slots and sends nothing. The destination can decode $\mathbf{X}_d^1$ without the interference form the relay.

In the next $(K-1)$ sub-blocks, the source sends

$$\mathbf{X}_S^k = \mathbf{X}_u^k[\mathbf{I}_{N_S}\ \mathbf{0}_{N_S\times n_r}\ \mathbf{X}_d^k], k = 2,\dots,K, \quad (41)$$

where $n_r \le \min\{N_S, N_D - N_S\}$,$\mathbf{X}_u^k \in \mathbb{C}^{N_S\times N_S}$ and $\mathbf{X}_d^k \in \mathbb{C}^{N_S\times(T-n_r-N_S)}$.

The received signal at the relay is

$$\mathbf{Y}_R^k = \mathbf{H}_{SR}\mathbf{X}_u^k[\mathbf{I}_{N_S}\ \mathbf{0}_{N_S\times n_r}\ \mathbf{X}_d^k]. \quad (42)$$

The relay knows $\mathbf{H}_{SR}$ and decodes $\mathbf{X}_u^k$ and uses n_r transmit antennas, sending

$$\mathbf{X}_R^k = [\mathbf{0}_{n_r\times N_S}\ \mathbf{I}_{n_r}\ \mathbf{X}_{r,d}^k] \in \mathbb{C}^{n_r\times T}, \quad (43)$$

where $\mathbf{X}_{r,d}^k \in \mathbb{C}^{n_r\times(T-n_r-N_S)}$. The received signal at the destination is:

$$\begin{aligned}
\mathbf{Y}_D^k &= \mathbf{H}_{SD}^k\mathbf{X}_S^k + \mathbf{H}_{RD}^k\mathbf{X}_R^k + \mathbf{W}_D^k && (44)\\
&= [\mathbf{H}_{SD}^k\mathbf{X}_u^k\ \mathbf{H}_{RD}^k][\mathbf{I}_{(N_S+n_r)}\ \mathbf{X}_D^k] + \mathbf{W}_D^k, && (45)
\end{aligned}$$

where

$$\mathbf{X}_D^k = \begin{bmatrix}\mathbf{X}_d^k\\ \mathbf{X}_{r,d}^k\end{bmatrix}. \quad (46)$$

The destination estimates the equivalent channel $\mathbf{H}_D^k = [\mathbf{H}_{SD}^k\mathbf{X}_u^k\ \mathbf{H}_{RD}^k]$ during the first $(N_S + n_r)$ time slots , and decodes $\mathbf{X}_D^k$. Messages decoded at the destination have two parts: $\mathbf{X}_d^k$ and $\mathbf{X}_{r,d}^k$, which provides DoF $d_1 = N_S(T - n_r - N_S)$ and $d_2 = n_r(T - n_r - N_S)$. The DoF of $\mathbf{X}_u^{k,\prime}$ is $d_3 = N_S^2$. Thus the DoF the destination can achieve is (36). Consider the direct link from the source to the destination, the DoF it can achieve is $d' = \frac{N_S}{T} \times (T - N_S)$. From the same analysis in the proof for Theorem 2, we can prove that $\max_{n_r}\min\{d_1+d_2, d_1+d_3\}) > N_S(T-N_S)$, then we have $d > d'$. This completes the proof. □

E. $T_{SR}=\infty$,$T_{SD}\neq T_{RD}$

Next we consider the case where the relay knows $\mathbf{H}_{SR}$ and the coherence times of the source-destination link and the relay-destination link are unequal. Assume the source and the relay are equipped with $N_S=N_R$ antennas and the destination is equipped with $N_D>N_S$ antennas.

1) $T_{RD}=KT_{SD}$: In this section we assume $T_{RD}=KT_{SD}$. The length of the transmit block is KT, divided into K sub-blocks with length T.

During the first sub-block, the source sends the signal

$$\mathbf{X}_S^1=\mathbf{X}_u^1[\mathbf{I}_{N_S}\ \mathbf{0}_{N_S\times n_r}\ \mathbf{X}_d^1], \tag{47}$$

where $n_r\leq\min\{N_S,N_D-N_S\}$,$\mathbf{X}_u^1\in\mathbb{C}^{N_S\times N_S}$ and $\mathbf{X}_d^1\in\mathbb{C}^{N_S\times(T-n_r-N_S)}$.

The relay decodes $\mathbf{X}_u^1$ and uses n_r transmit antennas and sends

$$\mathbf{X}_R^1=[\mathbf{0}_{n_r\times N_S}\ \mathbf{I}_{n_r}\ \mathbf{X}_{r,d}^1]\in\mathbb{C}^{n_r\times T}, \tag{48}$$

where $\mathbf{X}_{r,d}^1\in\mathbb{C}^{n_r\times(T-n_r-N_S)}$. The received signal at the destination is

$$\begin{aligned}\mathbf{Y}_D^1&=\mathbf{H}_{SD}^1\mathbf{X}_S^1+\mathbf{H}_{RD}\mathbf{X}_R^1+\mathbf{W}_D^1 &(49)\\ &=[\mathbf{H}_{SD}^1\mathbf{X}_u^1\ \mathbf{H}_{RD}][\mathbf{I}_{(N_S+n_r)}\ \mathbf{X}_D^1]+\mathbf{W}_D^1, &(50)\end{aligned}$$

where

$$\mathbf{X}_D^1=\begin{bmatrix}\mathbf{X}_d^1\\ \mathbf{X}_{r,d}^1\end{bmatrix}. \tag{51}$$

In the first sub-block, $\mathbf{X}_d^1$, $\mathbf{X}_u^1$ and $\mathbf{X}_{r,d}^1$ provide $d_1=N_S(T-n_r-N_S)$, $d_2=n_r(T-n_r-N_S)$ and $d_3=N_S^2$ DoF.

During the next $(K-1)$ sub-blocks, the source sends the signal

$$\mathbf{X}_S^k=\mathbf{X}_u^k[\mathbf{I}_{N_S}\ \mathbf{X}_d^k], \tag{52}$$

where $\mathbf{X}_d^k\in\mathbb{C}^{N_S\times(T-n_r-N_S)}$.

The relay uses n_r transmit antennas and sends:

$$\mathbf{X}_R^k=[\mathbf{0}_{n_r\times N_S}\ \mathbf{X}_{r,d}^k]\in\mathbb{C}^{n_r\times T}, \tag{53}$$

where $\mathbf{X}_{r,d}^k\in\mathbb{C}^{n_r\times(T-N_S)}$. The received signal at the destination is

$$\begin{aligned}\mathbf{Y}_D^k&=\mathbf{H}_{SD}^k\mathbf{X}_S^k+\mathbf{H}_{RD}\mathbf{X}_R^k+\mathbf{W}_D^k &(54)\\ &=[\mathbf{H}_{SD}^k\mathbf{X}_u^k\ \mathbf{H}_{RD}][\mathbf{I}_{N_S}\ \mathbf{X}_D^k]+\mathbf{W}_D^k, &(55)\end{aligned}$$

where

$$\mathbf{X}_D^k=\begin{bmatrix}\mathbf{X}_d^k\\ \mathbf{X}_{r,d}^k\end{bmatrix}. \tag{56}$$

During the sub-block k, the destination can decode $\mathbf{X}_D^k$. $\mathbf{X}_d^k$, $\mathbf{X}_u^k$ and $\mathbf{X}_{r,d}^k$ provide $\bar{d}_1=N_S(T-N_S)$,$\bar{d}_2=n_r(T-N_S)$ and $\bar{d}_3=N_S^2$ DoF.

Thus the DoF the destination can achieve is

$$\begin{aligned}d=\frac{1}{KT}\max_{n_r}\min(&d_1+d_2+(K-1)(\bar{d}_1+\bar{d}_2),\\ &d_1+d_3+(K-1)(\bar{d}_1+\bar{d}_3)).\end{aligned} \tag{57}$$

Using reasoning similar to the proof of Theorem 2, it follows that this DoF is greater than what the direct link can achieve.

2) $T_{SD}=KT_{RD}$: In this section we assume $T_{SD}=KT_{RD}=KT$. The length of the transmit block is KT, divided into K sub-blocks with length T.

The source uses product superposition, sending:

$$\mathbf{X}_S=\mathbf{X}_u[\mathbf{X}_v^1\ \mathbf{X}_v^2\ \ldots\ \mathbf{X}_v^K], \tag{58}$$

where $n_r\leq\min\{N_S,N_D-N_S\}$,$\mathbf{X}_u\in\mathbb{C}^{N_S\times N_S}$ and

$$\mathbf{X}_v^1=[\mathbf{I}_{N_S}\ \mathbf{0}_{N_S\times n_r}\ \mathbf{X}_d^1]\in\mathbb{C}^{N_S\times T}, \tag{59}$$

$$\mathbf{X}_v^k=[\mathbf{0}_{N_S\times n_r}\ \mathbf{X}_d^k]\in\mathbb{C}^{N_S\times T}, \tag{60}$$

where $\mathbf{X}_d^1\in\mathbb{C}^{T-n_r-N_s}$ and $\mathbf{X}_d^k\in\mathbb{C}^{T-n_r}$, $k=2,3,\ldots,K$.

At the relay, the received signal is

$$\mathbf{Y}_R=\mathbf{H}_{SR}\mathbf{X}_u[\mathbf{X}_v^1\ \mathbf{X}_v^2\ \ldots\ \mathbf{X}_v^K]+\mathbf{W}_R. \tag{61}$$

The received signal at the first N_S time slots is

$$\mathbf{Y}'_R=\mathbf{H}_{SR}\mathbf{X}_u+\mathbf{W}'_R. \tag{62}$$

The relay knows $\mathbf{H}_{SR}$ and decodes $\mathbf{X}_u$. Assume the message decoded by the relay in the previous block is $\mathbf{X}'_u$.

The relay uses n_r transmit antennas and sends

$$\mathbf{X}_R=[\mathbf{X}_R^1,\mathbf{X}_R^2,\ldots,\mathbf{X}_R^K]\in\mathbb{C}^{n_r\times KT}, \tag{63}$$

where

$$\mathbf{X}_R^1=[\mathbf{0}_{n_r\times N_S}\ \mathbf{I}_{n_r}\ \mathbf{X}_{r,d}^1]\in\mathbb{C}^{n_r\times T}, \tag{64}$$

and

$$\mathbf{X}_R^k=[\mathbf{I}_{n_r}\ \mathbf{X}_{r,d}^k]\in\mathbb{C}^{n_r\times T}, \tag{65}$$

where $\mathbf{X}_{r,d}^1\in\mathbb{C}^{n_r\times(T-n_r-N_S)}$ and $\mathbf{X}_{r,d}^k\in\mathbb{C}^{n_r\times(T-n_r)}$.

In the first sub-block, the received signal at the destination is

$$\begin{aligned}\mathbf{Y}_D^1&=\mathbf{H}_{SD}\mathbf{X}_S^1+\mathbf{H}_{RD}^1\mathbf{X}_R^1+\mathbf{W}_D^1 &(66)\\ &=[\mathbf{H}_{SD}\ \mathbf{H}_{RD}^1]\begin{bmatrix}\mathbf{X}_S^1\\ \mathbf{X}_R^1\end{bmatrix}+\mathbf{W}_D^1 &(67)\\ &=[\mathbf{H}_{SD}\ \mathbf{H}_{RD}^1]\begin{bmatrix}\mathbf{X}_u[\mathbf{I}_{N_S}\ \mathbf{0}_{N_S\times n_r}\ \mathbf{X}_d^1]\\ \mathbf{0}_{n_r\times N_S}\ \mathbf{I}_{n_r}\ \mathbf{X}_{r,d}^1\end{bmatrix}+\mathbf{W}_D^1 &(68)\\ &=[\mathbf{H}_{SD}\mathbf{X}_u\ \mathbf{H}_{RD}^1][\mathbf{I}_{(N_S+n_r)}\ \mathbf{X}_D^1]+\mathbf{W}_D^1, &(69)\end{aligned}$$

where

$$\mathbf{X}_D^1=\begin{bmatrix}\mathbf{X}_d^1\\ \mathbf{X}_{r,d}^1\end{bmatrix}. \tag{70}$$

The destination estimates the equivalent channel $\mathbf{H}_D^1=[\mathbf{H}_{SD}\mathbf{X}_u\ \mathbf{H}_{RD}^1]$ during the first (N_S+n_r) time slots and decodes $\mathbf{X}_D^1$ during the remaining ones.

During the sub-block k, the received signal at the destination is

$$\begin{aligned}\mathbf{Y}_D^k&=\mathbf{H}_{SD}\mathbf{X}_S^k+\mathbf{H}_{RD}^k\mathbf{X}_R^k+\mathbf{W}_D^k &(71)\\ &=[\mathbf{H}_{SD}\ \mathbf{H}_{RD}^k]\begin{bmatrix}\mathbf{X}_u[\mathbf{0}_{N_S\times n_r}\ \mathbf{X}_d^k]\\ \mathbf{I}_{n_r}\ \mathbf{X}_{r,d}^k\end{bmatrix}+\mathbf{W}_D^k &(72)\\ &=[\mathbf{H}_{RD}^k\quad[\mathbf{H}_{SD}\mathbf{X}_u\ \mathbf{H}_{RD}^k]\mathbf{X}_D^k]+\mathbf{W}_D^k, &(73)\end{aligned}$$

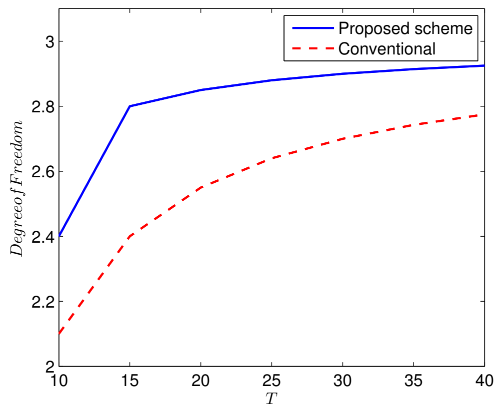

Fig. 1. Degrees of freedom for $T_{SR} = \infty, T_{RD} = T_{SD} = T$

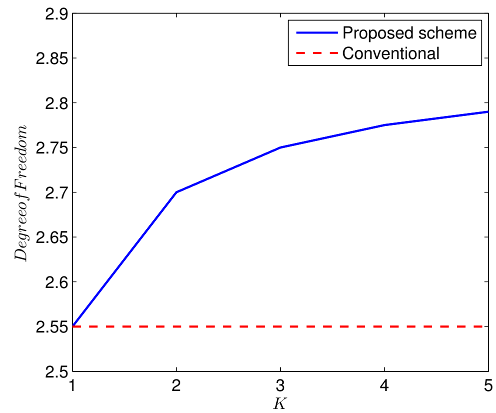

Fig. 2. Degrees of freedom for $T_{SR} = KT_{RD} = KT_{SD} = KT, T = 10$

where

$$\mathbf{X}_D^k = \begin{bmatrix} \mathbf{X}_d^k \\ \mathbf{X}_{r,d}^k \end{bmatrix}. \tag{74}$$

The destination estimates $\mathbf{H}_{RD}^k$ during the first n_r time slots. Because the destination already estimated $\mathbf{H}_{SD}\mathbf{X}_u$, it knows the equivalent channel $\mathbf{H}_D^k = [\mathbf{H}_{SD}\mathbf{X}_u \ \mathbf{H}_{RD}^k]$. During the remaining time slots, the destination decodes $\mathbf{X}_D^k$. $\mathbf{X}_d^1$ provides $d_1 = N_S(T - n_r - N_s)$ DoF, and $\mathbf{X}_d^k$ provides $\bar{d}_1 = N_S(T - n_r)$ DoF. $\mathbf{X}_{r,d}^1$ provides $d_2 = n_r(T - n_r - N_s)$ DoF and $\mathbf{X}_{r,d}^k$ provides $\bar{d}_2 = n_r(T - n_r)$ DoF. Thus the destination can achieve the following DoF:

$$\begin{aligned} d = \frac{1}{KT} \max_{n_r} \min(&d_1 + (K-1)\bar{d}_1 + d_3, \\ &d_1 + (K-1)\bar{d}_1 + d_2 + (K-1)\bar{d}_2). \end{aligned} \tag{75}$$

However, in this case, there is no guarantee that the proposed scheme can obtain DoF gains over the direct link. Whether the DoF gain exists depends on the coherence time configurations. One example is when $N_S = N_R = 3, N_D = 5, T = 6, K = 2$, $d = \frac{7}{3}, d' = \frac{9}{4}$, the proposed scheme has DoF gains; when $N_S = N_R = 3, N_D = 5, T = 4, K = 3$, $d = 2, d' = \frac{9}{4}$, the proposed scheme provides no DoF gains.

IV. Numerical Results

In this section, we compare the performance of the proposed scheme with conventional transmission demonstrating the gains in degrees of freedom. We set the number of antennas $N_S = N_R = 3$ and $N_D = 5$. In Fig 1, we consider the case where $T_{SR} = \infty, T_{RD} = T_{SD} = T$. We can see the proposed scheme has a significant DoF gain over the conventional transmission. In Fig 2, we consider the case where $T_{SR} = KT_{RD} = KT_{SD} = KT, T = 10$ for different K. When $K = 1$, i.e., all links have identical coherence time, there is no DoF gain by the proposed scheme; when K grows, the gain achieved by the proposed scheme increases.

V. Conclusion

This paper studies the MIMO relay with coherence diversity. The main contribution of this paper to demonstrate new gains in the relay channel under this scenario, and propose and analyze a transmission scheme achieving these gains.

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