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Peer-to-Peer Energy Trading in DC Packetized Power Microgrids

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***Abstract*—As distributed energy resources (DERs) are widely deployed, DC packetized power microgrids have been considered as a promising solution to incorporate DERs effectively and steadily. In this paper, we consider a DC packetized power microgrid, where the energy is dispatched in the form of power packets with the assist of a power router. However, the benefits of the microgrid can only be realized when energy subscribers (ESs) equipped with DERs actively participate in the energy market. Therefore, peer-to-peer (P2P) energy trading is necessary in the DC packetized power microgrid to encourage the usage of DERs. Different from P2P energy trading in AC microgrids, the dispatching capability of the router needs to be considered in DC microgrids, which will complicate the trading problem. To tackle this challenge, we formulate the P2P trading problem as an auction game, in which the demander ESs submit bids to compete for power packets, and a controller decides the energy allocation and power packet scheduling. Analysis of the proposed scheme is provided, and its effectiveness is validated through simulation.**

***Index Terms*—Peer-to-peer energy trading, DC packetized power microgrid, pricing, iterative auction**

NOMENCLATURE

$\boldsymbol{a}^s$	Energy allocation from supplier ESs to demander ESs
$\boldsymbol{a}^u$	Energy allocation from the UG to demander ESs
$\boldsymbol{a}^{s,y}$	Energy allocation from supplier ESs to demander ESs in the y-th iteration
$\boldsymbol{a}^{u,y}$	Energy allocation from the UG to demander ESs in the y-th iteration
$\boldsymbol{b}$	Unit bidding prices
$\boldsymbol{b}^y$	Unit bidding prices in the y-th auction iteration
$\boldsymbol{s}^j$	Vector of desired supplier ESs of demander ES *j*
$\boldsymbol{X}$	Matrix of power packet scheduling
$\epsilon_{i,j}$	Transmission loss factor between ES *i* and ES *j*
γ	Price ratio
$\mathbb{E}[\cdot]$	Expectation operator
$\mathbb{P}[\cdot]$	Probability operator
$\mathcal{G}$	Set of demander ESs who choose not to participate in the trading in this cycle
$\mathcal{I}$	Set of supplier ESs
$\mathcal{J}$	Set of demander ESs
$\mathcal{K}$	Set of power channels
$\mathcal{M}^u$	Set of unscheduled power packets
$\mathcal{M}^{uu}$	Set of unscheduled and unconflict power packets
$\mathcal{M}_k$	Set of power packets to be scheduled on power channel k
$\mathcal{N}$	Set of time slots in a transmission step
ν	maximum unit valuation among all the demander ESs
Φ	Sum of revenues
$\phi(\cdot)$	Expected unit payment
Φ^{max}	maximum sum of revenues
Φ^{min}	minimum sum of revenues
π^a	Average unit price
π^s	minimum selling price of supplier ESs
π^u	minimum selling price of the UG
Ψ	Efficiency
σ	Increased value of unit bidding price in an iteration
a_j^s	Energy allocation from supplier ESs to demander ES *j*
a_j^s	Energy allocation from the UG to demander ES *j*
$a_j^{s,y}$	Energy allocation from supplier ESs to demander ES *j* in the y-th iteration
$a_j^{u,y}$	Energy allocation from the UG to demander ES *j* in the y-th iteration
b_j	Unit bidding price of demander ES *j*
e_i^e	Export energy of supplier ES *i*
e_j^r	Demanded energy of demander ES *j*
$e^{d,max}$	Maximum demanded energy of an ES
$e^{s,max}$	Maximum supplied energy of an ES
F	Distribution of unit valuation
f	Density of unit valuation
I	Number of supplier ESs
J	Number of demander ESs
K	Number of power channels
l_k^c	Number of occupied time slots on power channel k
L^{max}	maximum number of time slots of a packet
l_m	Number of time slots of packet *m*
N	Number of time slots in a transmission step

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p_i^e	Export power of ES i
$p_{i,j}^l$	Power transmission loss between ES i and ES j
$p_{i,j}^r$	Received power of ES j from ES i
p^{max}	maximum export power of a packet
p^{min}	minimum export power of a packet
r^*	Optimal reserve price
T	Duration of a time slot
t_m^f	Duration of the footer of packet m
t_m^h	Duration of the header of packet m
t_m^P	Duration of the maximum payload of packet m
t_m^p	Duration of the payload of packet m
t_m	Duration of packet m
u_j	Utility of demander ES j
v_j	Unit valuation of demander ES j
x_{ijkn}	Scheduling of packet from ES i to ES j in the n-th time slot over power channel k
y	Index of auction iterations

I. Introduction

Distributed energy resources (DERs) such as photovoltaic cells and wind generators, are considered as promising alternatives to traditional central electricity generation using fossil and nuclear fuels [1]. However, the increasing penetration of DERs will bring challenges to the steady and efficient operations of current power grids due to the intermittent nature of DERs [2]. To address this issue, the microgrid has been proposed as a new paradigm to define the operation of distributed electricity generation [3]. Consisting of a cluster of locally-controlled sources, loads, and batteries, the microgrid can independently perform coordination control, frequency synchronization, and transmission management [4], and thus alleviate the demand load fluctuation to the utility grid (UG). Moreover, since most DERs have the DC output, DC packetized power microgrids have been considered as an effective solution to integrate DERs due to its potential benefits, such as avoiding generator synchronization and reducing AC-DC conversion loss [5].

The concept of packetized power was proposed in [6], where electricity was treated as a power packet tagged with the information. This is different from typical power systems where energy and information are transmitted separately [7]. Following this pioneer work, [8] designed power routers which could forward power packets labeled with IP information to specific users accordingly. By adopting time division multiplexing (TDM) technique, power packets with different power could be delivered over the same power line. The power packet transmission was also verified in this work. The authors in [9] considered a local area packetized power network, where all the energy subscribers (ESs) were connected by a multi-channel power router which can dispatch multiple power packets simultaneously. This work was extended in [10], where ESs were connected by multiple power routers, and power packets can be transmitted through different energy routers.

However, the benefits of DC packetized power microgrid can only be realized if the ESs equipped with DERs are fully integrated into the energy market. To this end, peer-to-peer (P2P) energy trading, where ESs can trade energy with one another, is considered as a potential tool to promote the usage of DERs [11]. With P2P energy trading, demander ESs are encouraged to purchase energy from other ESs at a lower price, and thus their dependence on the UG is reduced [12]. Also, the development of P2P energy trading has the potential to benefit the supplier ESs by earning revenues, reducing transmission loss, and lowering their dependency on the UG [13].

In this paper, we consider P2P energy trading in a DC packetized power microgrid. Demander ESs can directly buy energy from either supplier ESs or the UG. As the prices offered by supplier ESs are lower than the UG, demander ESs prefer to trade with supplier ESs. Nevertheless, since the energy from supplier ESs is limited, demander ESs need to compete for the energy from supplier ESs. Besides, during peak hours, a large number of power packets may cause congestion in the power router because the dispatching capability of the router is limited by the number of its power channels where power packets are delivered over in a TDM manner. Consequently, the number of traded power packets needs to be restricted to avoid the congestion, which will complicate the trading problem.

To tackle this challenge, we formulate the trading problem as an auction game, where a centralized controller is the auctioneer, and demander ESs are the bidders. In the auction game, demander ESs compete for power packets by submitting bids to maximize their own utilities, and the controller decides the energy allocation and power packet scheduling to maximize the overall benefits[1]. To solve the auction problem efficiently, we propose an iterative auction scheme, where the controller and demander ESs iteratively optimize their own objectives.

In the literature, existing works have discussed various P2P energy trading schemes in typical smart grids. These energy trading schemes can be broadly categorized into two types: the centralized schemes [14]-[16] and decentralized schemes [17]-[19]. In the centralized schemes, a single objective is optimized because there is only one energy user or all the energy users has to follow the trading orders of a centralized controller [20]. The authors in [14] considered the energy exchange between two microgrids under the regulation of a controller, and proposed a centralized scheme to minimize the overall cost. In [15], a hybrid optimization approach was adopted by the controller to minimize the cost for electricity generation. The work in [16] investigated the trade between the grid and an end-user equipped with DERs, and proposed a stochastic algorithm to maximize the profit of the end-user.

Different from the centralized schemes, in the decentralized schemes,each ES tries to maximize its own utility without considering the utilities of other ESs and the condition of the power grid, and the trading results are obtained through the interactions among ESs [21]. The authors in [17] considered an energy market where energy suppliers and users could trade directly, and analysed the trading cooperation between them using the coalitional game. The work [18] investigated the energy trading between the plug-in hybrid electric vehicles

[1]Since the controller has all the trading information, it can allocate energy to part of demander ESs to avoid the congestion when the number of power packets is large.

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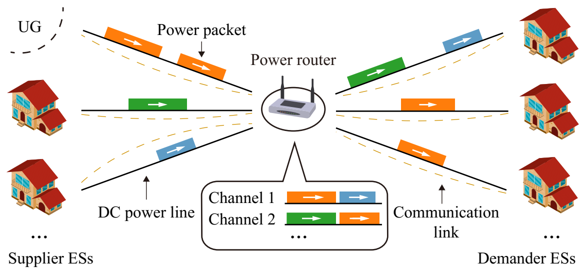

Fig. 1. Diagram of a DC packetized power microgrid.

and microgrids, and a Stackelberg game was utilized to model the trading process. In [19], electricity exchange among plug-in hybrid vehicles was considered, where each vehicle was self-interested and maximized its own profit.

However, the aforementioned schemes cannot be applied to P2P energy trading in the DC packetized power microgrid. In typical power grids, electricity is transmitted in AC form, while in the DC packetized power microgrid, the transmission mechanism is completely different. Since power packets are transmitted over power channels, a limited number of power channels will lead to severe congestion in the router when there are a large of number of power packets. This circumstance will not occur in typical power grids. To avoid the congestion, the trading scheme in DC microgrids have to take the power packet scheduling in the power router into consideration, making the trading scheme more difficult to design. To the best of of our knowledge, P2P energy trading in DC packetzied power microgrid has not been investigated. Therefore, our contributions can be summarized below.

- We propose a power packet trading protocol that divides the timeline into trading cycles that contain registration, auction, and transmission steps to regulate P2P energy trading in DC microgrids efficiently.
- We formulate the P2P energy trading problem, in which the controller maximizes the sum of revenues, and each demander ES maximizes its own utility. To solve this problem efficiently, we propose an iterative auction scheme together with the strategies for the controller and demander ESs.
- The simulation results demonstrate the effectiveness of the proposed iterative auction scheme in realizing an efficient P2P energy trading.

The rest of this paper is organized as follows. In Section II, we introduce the system model of the DC packetized power microgrid. A trading protocol is introduced in Section III. In Section IV, we formulate a P2P energy trading problem. An auction scheme is proposed in Section V to solve the formulated problem. The analysis of the proposed scheme is provided in Section VI. Simulation results and discussions are presented in Section VII. Finally, we draw the conclusions in Section VIII.

II. System Model

As shown in Fig. 1, the DC packetized power microgrid consists of a core power router, a UG and a group of

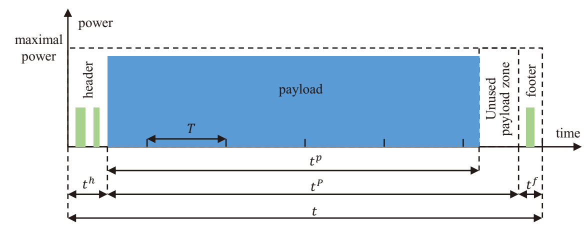

Fig. 2. Structure of a power packet.

geographically nearby ESs equipped with DERs and batteries[2]. The UG and all the ESs are linked to the power router via DC power lines and communication links. A centralized controller equipped in the power router regulates the P2P energy trading and transmission centrally by communicating with ESs and the UG through communication links. Electricity in the microgrid is transmitted in the form of power packets over power lines. In the following, we will elaborate on the power packet and power router, respectively.

A. Power Packet

The structure of a power packet is presented in Fig. 2, where the information is transmitted together with the electricity on the power line. It is composed of a header, a payload and a footer. The header contains the address information of its destination. The footer functions as the end mark of the power packet. Different from the packets in communication networks, the payload of a power packet carries the energy to be transmitted.

Because of the TDM transmission, the timeline is divided into time slots, and the time duration of each time slot is T. Let t_m denote the duration of power packet m, and we have $t_m = l_m T$, where l_m is a positive integer with $l_m \leq L^{max}$. Here, L^{max} is the maximum number of time slots of a power packet. Let t_m^h and t_m^f denote the duration of the header and the footer, respectively. Besides, define t_m^p and t_m^P as the duration and the maximum duration for the payload, respectively. To realize efficient transmission, the unused payload zone needs to be less than one slot, i.e., $t_m^P - t_m^p < T$. Thus, a standard power packet m needs to satisfy $t_m = t_m^h + t_m^P + t_m^f$ and $0 \leq t_m^P - t_m^p < T$.

The power levels of power packets are related to the types of DERs. Let p_i^e denote the export power of power packet from ES i. Because of the resistances on the power lines, the received power of the same power packet is lower than the export power. Let $p_{i,j}^r$ and $p_{i,j}^l$ denote the received power of ES j from ES i and the power transmission loss between ES i and ES j, respectively. They satisfy

$$p_i^e = p_{i,j}^r + p_{i,j}^l. \tag{1}$$

[2]Electricity generated by DERs or bought from other ESs will be stored in the batteries first, and then be sold to other ESs or consumed. In the P2P trading, supplier ESs will only sell the energy stored in the battery to guarantee the successful power packet dispatching, and demander ESs only need to confirm that the capacities left in the batteries are enough to store the received power packets. Therefore, constraints on power generation/consumption are not directly related to the trading and are ignored.

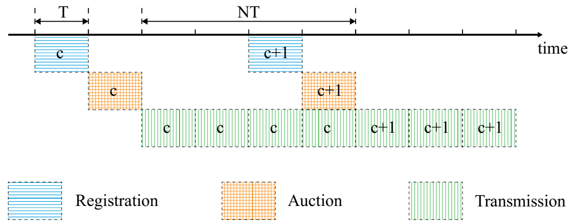

Fig. 3. Power packet trading protocol.

We utilize $\epsilon_{i,j}$ to denote the transmission loss factor between ES i and ES j, which is defined as

$$\epsilon_{i,j} = \frac{p^l_{i,j}}{p^e_i}, \tag{2}$$

and we have

$$p^r_{i,j} = p^e_i(1 - \epsilon_{i,j}). \tag{3}$$

To utilize the capacity of power lines efficiently, the export power of a power packet needs to be larger than a predefined threshold. In addition, the export power of a power packet cannot be larger than the capacity of power lines for the sake of safety [22]. Let p^{max} and p^{min} be the maximum and the minimum export power requirement for a power packet, respectively. Therefore, we have

$$p^{min} \le p^e_i \le p^{max}. \tag{4}$$

B. Power Router

Each ES connected to the router is assigned a unique IP address. The power packet received by the power router will be sent to the target demander ES according to the tagged address. The power router can dispatch multiple power packets from or to different ESs simultaneously, while it cannot dispatch multiple power packets from or to the same ES at the same time because the router links to each ES with only one power line.

We use *power channel* to describe the power lines in the router to transmit power packets [23]. We assume that there are K power channels, denoted by $\mathcal{K} = \{1, \ldots, K\}$. Each power channel operates in a TDM manner, and only one power packet can be delivered on a power channel at a time. Therefore, K power channels can support the simultaneous transmission of at most K power packets.

III. Power Packet Trading Protocol

In this section, the power packet trading protocol is proposed. The timeline of a day is divided into trading cycles. Each trading cycle consists of three steps: registration, auction, and transmission steps. In the registration step, each ES registers as a demander ES or supplier ES. Then, in the auction step, demander ESs will bid for power packets from supplier ESs or the UG, and the controller will decide the energy allocation and power packet scheduling. Finally, the power packets traded in the auction step will be transmitted according to the auction results in the transmission step.

As shown in Fig. 3, the registration, auction, and transmission steps in the same cycle are executed sequentially. The registration and auction steps occupy one time slot, respectively, while the transmission step occupies N time slots. Let $\mathcal{N} = \{1, \ldots, N\}$ denote the time slots in one transmission step.

Registration: Each ES needs to register as a demander ES or supplier ES in the first time slot of each trading cycle. Let $\mathcal{I} = \{0, 1, \ldots, I\}$ denote the set of supplier ESs, and let $\mathcal{J} = \{1, \ldots, J\}$ denote the set of demander ESs. Here, we use 0 to represent the UG. Each supplier ES is required to report the power and energy of its export power packet to the controller. Note that the amount of export energy should be lower than the amount of energy stored in the batteries for every supplier ES. Let e^e_i denote the export energy of supplier ES i. The controller will calculate the received energy of power packets for each demander ES and inform each demander ES of it. Each demander ES will select its desired power packets and bid for them in the next step.

Auction: At first, the demander ESs will decide whether to participate in the auction game in this trading cycle or not. Then, the controller and demander ESs will iteratively optimize their objectives. Specifically, in each iteration, the controller will update the energy allocation and power packet scheduling results according to the bidding prices submitted in the last iteration. This trading result will be sent to each demander ESs and they will update their bids accordingly. The detailed auction mechanism will be presented in Section V. After the auction step terminates, the trading results containing energy allocation and power packet scheduling will be sent to each ES and the UG.

Transmission: Before transmission, the supplier ESs and the UG will tag the IP information to their power packets according to the energy allocation. Then, these power packets will be sent to the power router at the scheduled time and transmitted over the allocated power channel according to the scheduling table. Finally, the power router will deliver these power packets according to their tagged IP addresses.

IV. Problem Formulation

In this section, we first introduce the utility model of demander ESs, and then formulate the trading problem.

A. Utility Model

We assume that each demander ES requires a fixed amount of energy to satisfy its own energy demand. Let e^r_j denote the demanded energy of demander ES j. A demander ES can purchase power packets from either supplier ESs or the UG. Let $\boldsymbol{s}^j = [s^j_i], i \in \mathcal{I}/\{0\}$ denote the vector of desired supplier ESs of demander ES j. Here, $s^j_i = 1$ implies that demander ES j wants the power packet from supplier ES i, and otherwise $s^j_i = 0$. Therefore, we have $e^r_j = \sum_{i \in \mathcal{I}/\{0\}} s^j_i e^e_i$. Whether the demanded energy is supplied by supplier ESs or the UG is decided by the controller. We use $\boldsymbol{a}^s = [a^s_j]$ to denote the energy allocation from supplier ESs to demander ESs, and

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$\boldsymbol{a}^u = [a_j^u]$ to denote the energy allocation from the UG to demander ESs. Specifically, we have

$$a_j^s = \begin{cases} 1, & \text{the controller allocates energy from} \\ & \text{supplier ESs to demander ES } j, \\ 0, & \text{otherwise,} \end{cases} \tag{5}$$

and

$$a_j^u = \begin{cases} 1, & \text{the controller allocates energy from} \\ & \text{the UG to demander ES } j, \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

The utility of a demander ES is defined as its valuation minus the payment in the auction. For demander ES *j*, its valuation is defined as the welfare for its demanded energy e_j^r, and we assume that its valuation is independent of the valuations of others. Let v_j denote the unit valuation of demander ES *j*. The payment in the auction is equal to the bidding price submitted by demander ES *j*. Let $\boldsymbol{b} = [b_j]$ denote the unit bidding prices of the demander ESs, where b_j denotes the unit bidding price of demander ES *j*. Therefore, the utility of demander ES *j* can be given by

$$u_j = (v_j - b_j)e_j^r(a_j^s + a_j^u). \tag{7}$$

B. Problem Formulation

There are two kinds of agents in the P2P energy trading, i.e., demander ESs and the controller. The rational demander ESs aim to maximize their own utilities, while the controller's objective is to maximize the sum of revenues Φ. Therefore, the trading problem can be divided into two parts at the demander ES and controller sides.

1) Demander ES Side: For demander ES *j*, it maximizes its utility u_j by selecting the optimal unit bidding price b_j. Therefore, its trading problem can be written as

$$\mathrm{P}_{ES}: \max_{b_j} \ (v_j - b_j)e_j^r(a_j^s + a_j^u) \tag{8a}$$

$$s.t. \ b_j \le v_j, \tag{8b}$$

where constraint (8b) ensures the individual rationality of the demander ESs. As a rational demander ES, it will not submit a unit bidding price higher than its unit utility to obtain a negative utility because zero utility can be obtained if it chooses not to participate in the auction.

2) Controller Side: The objective of the controller is to maximize the sum of revenues by allocating and scheduling power packets. Since the sum of revenues is equal to the sum of the bidding prices, its objective can be written as

$$\max_{\boldsymbol{a}^s, \boldsymbol{a}^u, \boldsymbol{X}} \sum_{j \in \mathcal{J}} b_j e_j^r (a_j^s + a_j^u) \tag{9}$$

where $\boldsymbol{X} = [x_{ijkn}], i \in \mathcal{I}, j \in \mathcal{J}, k \in \mathcal{K}, n \in \mathcal{N}$ denotes the power packet scheduling result decided by the controller. Specifically, we have

$$x_{ijkn} = \begin{cases} 1, & \text{the power packet from ESs } i \text{ to } j \text{ occupies} \\ & \text{the } n\text{-th time slot over power channel } k, \\ 0, & \text{otherwise.} \end{cases} \tag{10}$$

Moreover, the controller also faces the following constraints:

Allocation constraint: As each demander ES can only receive power packets from either the supplier ESs or the UG, we have

$$a_j^s + a_j^u \le 1, \forall j \in \mathcal{J}. \tag{11}$$

Supply constraint: Since each supplier ES can sell the energy to at most one demander ES, we have

$$\sum_{j \in \mathcal{J}} a_j^s s_i^j \le 1, \forall i \in \mathcal{I}/\{0\}. \tag{12}$$

It is worthwhile to mention that the UG can support multiple demander ESs because the energy supplied by it is sufficient.

Price constraint: The supplier ESs will sell energy only when the unit bidding price is not lower than π^s. That is,

$$(b_j - \pi^s)a_j^s \ge 0, \forall j \in \mathcal{J}. \tag{13}$$

Similarly, the UG will sell energy only when the unit bidding price is not lower than π^u. Therefore, we have

$$(b_j - \pi^u)a_j^u \ge 0, \forall j \in \mathcal{J}. \tag{14}$$

In general, to promote the utilization of the energy generated by local supplier ESs, the unit prices offered by the supplier ESs are lower than that offered by the UG. Therefore, we have $\pi^u \ge \pi^s$.

Scheduling constraint: Considering the dispatching capability of the power router is limited, we need to assure that the power packets traded in an auction step should be scheduled in the following transmission step. Thus, we have

$$\sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} x_{ijkn} = l_{ij} s_i^j a_j^s, \forall j \in \mathcal{J}, \forall i \in \mathcal{I}/\{0\}, \tag{15}$$

$$\sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} x_{ijkn} = l_{ij} a_j^u, \forall j \in \mathcal{J}, i = 0, \tag{16}$$

where l_{ij} is the number of time slots of the power packet from supplier ES *i* to demander ES *j*.

Transmission constraint: One power packet can only be transmitted over at most one power channel, i.e.,

$$\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} x_{ijkn} \le 1, \forall k \in \mathcal{K}, n \in \mathcal{N}. \tag{17}$$

Dispatching constraint: Since the power router cannot dispatch multiple power packets to the same demander ES, at most one power channel can by occupied by each demander ES in every time slot, i.e.,

$$\sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} x_{ijkn} \le 1, \forall i \in \mathcal{I}, n \in \mathcal{N}. \tag{18}$$

Continuity constraint: A power packet is the minimum unit for energy transmission, and thus it needs to be transmitted continuously, i.e.,

$$\exists n_0, k, \prod_{n=n_0}^{n_0+l_{ij}} x_{ijkn} = 1, \forall i \in \mathcal{I}, j \in \mathcal{J}. \tag{19}$$

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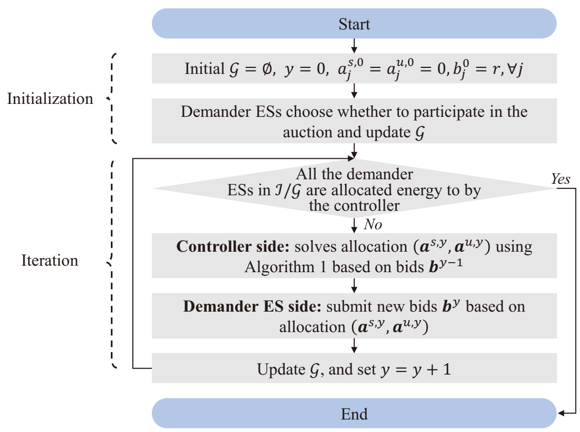

Fig. 4. Iterative auction for P2P energy trading.

Therefore, the trading problem at the controller side can be written by

$$
\begin{aligned}
\mathrm{P}_{Con}: \max_{\boldsymbol{a}^s,\boldsymbol{a}^u,\boldsymbol{X}} & \sum_{j\in\mathcal{J}} b_j e_j^r (a_j^s + a_j^u) \qquad (20)\\
s.t.\ & (11)\text{-}(19),\\
& a_j^s, a_j^u, x_{ijkn} \in \{0,1\}.
\end{aligned}
$$

V. Iterative Auction for P2P Energy Trading

Since the objectives of the demander ESs and the controller are interactive, we introduce the iterative auction scheme [24] to model their decision-making processes. In this auction game, we maximize the sum of revenues in (20) in consideration of the selfish objective in (8a) for each demander ES. The auction consists of two phases: initialization and iteration phases, as shown in Fig. 4.

In the initialization phase, variables $\mathcal{G}$, y, $\boldsymbol{a}^{s,0}$, $\boldsymbol{a}^{u,0}$ and $\boldsymbol{b}^0$ are initialized first, where $\mathcal{G}$ is the set of demander ESs who do not participate in the auction in the trading cycle. Note that the controller will set a unit reserve price r, and each demander ES is required to start bidding from this unit price. Next, demander ESs will decide whether to participate in the auction or not. According to constraint (8b), the demander ES will not participate in the auction if its unit valuation is lower than r. Otherwise, it will participate in the auction in this cycle. Demander ESs who do not participate in the auction in this cycle will be added into set $\mathcal{G}$.

In the iteration phase, demander ESs and the controller will iteratively maximize their own objectives. Specifically, in the y-th iteration, the controller first maximizes the sum of revenues (20) by optimizing $(\boldsymbol{a}^{s,y},\boldsymbol{a}^{u,y})$ based on the bidding prices $\boldsymbol{b}^{y-1}$ in the last iteration. Then, demander ESs who are not allocated energy to in this iteration need to submit new bids to maximize their own utilities. The iteration will terminate until the controller allocates energy to all the demander ESs in $\mathcal{J}/\mathcal{G}$. In the following, we will provide the strategies in both the demander ES side and the controller side.

Algorithm 1: Energy Allocation and Power Packet Scheduling Algorithm

Input: Unit bidding prices $\boldsymbol{b}^{y-1}$;
Output: Allocation $(\boldsymbol{a}^{1,y},\boldsymbol{a}^{2,y})$;
Set success flag $f=0$, and length $l=KN$;
while $l>0$ *and* $f=0$ **do**
  Solve allocation $(\boldsymbol{a}^{1,y},\boldsymbol{a}^{2,y})$ using the energy allocation (EA) algorithm based on l;
  Examine allocation $(\boldsymbol{a}^{1,y},\boldsymbol{a}^{2,y})$ using the power packet scheduling (PPS) algorithm, and derive f;
  $l=l-1$;
end

A. Strategy at the Demander ES Side

In this part, we analyze the strategies of demander ESs. For demander ES j who has not been allocated energy to, it is provided with two options. The first option is to increase its unit bidding price from b_j^{y-1} to $b_j^y = b_j^{y-1}+\sigma$, where σ is a positive constant. The second one is to give up its trading opportunity in this trading cycle. Demander ESs who give up bidding will be added into set $\mathcal{G}$. The dominant strategies of demander ESs are shown by the following proposition:

Proposition 1: For demander ES j with $a_j^{s,y}+a_j^{u,y}=0$, if $b_j^{y-1}+\sigma<v_j$, its dominant strategy is to increase its unit bidding price from b_j^{y-1} to $b_j^y=b_j^{y-1}+\sigma$. If $b_j^{y-1}+\sigma>v_j$, its dominant strategy is to give up its bidding opportunity in this trading cycle.

Proof: If $b_j^{y-1}+\sigma<v_j$, increasing its bidding price may bring a positive utility to demander ES j, whereas the utility obtained by the second option is 0. Similarly, if $b_j^{y-1}+\sigma>v_j$, demander ES j may obtain a negative utility if it increases the price, which is smaller than the utility obtained by the second option. ■

B. Strategy at the Controller Side

Problem (P_{Con}) is an integer nonlinear program, which has been proved to be NP-hard [25]. To solve this problem efficiently, we propose the energy allocation and power packet scheduling (AS) algorithm. First, we neglect the power packet scheduling constraints (15)-(19), and thus problem (P_{Con}) is transformed into an allocation subproblem (P_A) which is only related to variables $\boldsymbol{a}^s$ and $\boldsymbol{a}^u$. To accelerate the convergence, we add a linear constraint to problem (P_A), where the sum of lengths of all the allocated power packets can not exceed l. Thus, problem (P_A) can be formulated as:

$$
\begin{aligned}
\mathrm{P}_A: \max_{\boldsymbol{a}^s,\boldsymbol{a}^u} & \sum_{j\in\mathcal{J}} b_j e_j^r (a_j^s + a_j^u) \qquad (21)\\
s.t.\ & (11)\text{-}(14),\\
& \sum_{j\in\mathcal{J}}\sum_{i\in\mathcal{I}} l_{ij}(a_j^s+a_j^u) \le l.
\end{aligned}
$$

Since problem (P_A) is a binary linear program, we introduce the energy allocation (EA) algorithm based on the branch-and-bound method [26] to solve the allocation $(\boldsymbol{a}^s,\boldsymbol{a}^u)$. Then, we

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examine whether the allocation $(\boldsymbol{a}^s, \boldsymbol{a}^u)$ satisfies the power packet scheduling constraints (15)-(19) using the power packet scheduling (PPS) algorithm. The AS algorithm will terminate if all the allocated packets are scheduled successfully using the PPS algorithm. Otherwise, the AS algorithm will gradually decrease the value of l until the allocation $(\boldsymbol{a}^s, \boldsymbol{a}^u)$ generated by the EA algorithm can be scheduled successfully. The EA algorithm and the PPS algorithm are elaborated on as follows:

1) Energy Allocation Algorithm: The basic idea of the EA algorithm is to repeatedly partition the feasible solution region into smaller ones, and eliminate some regions to accelerate the calculation. The algorithm will generate many subproblems by partitioning problems, and the algorithm will terminate until all the generated problems are solved. Let Φ denote the final result of (20), which is initialized as $-\infty$. In the following, we will clarify how to calculate the bound and conduct branching.

Bound Calculation: At first, we relax the binary variables in vectors $\boldsymbol{a}^s$ and $\boldsymbol{a}^u$ to continuous values in $[0, 1]$ to derive the relaxation problem (RP$_A$) of problem (P$_A$). Since the problem (RP$_A$) is a convex optimization problem, we use the interior-point methods [27] to solve this problem. Let $(\boldsymbol{a}^s_R, \boldsymbol{a}^u_R)$ denote the solution of the relaxation problem (RP$_A$) using the interior-point methods, and let Φ^u denote the sum of revenues with solution $(\boldsymbol{a}^s_R, \boldsymbol{a}^u_R)$. Since the optimal solution of problem (P$_A$) is a feasible solution of problem (RP$_A$), Φ^u can be viewed as an upper bound of problem (P$_A$).

Variable Branching: There are two cases in this step.

Case 1: If all the elements in $\boldsymbol{a}^s_R$ and $\boldsymbol{a}^u_R$ are integers and $\Phi^u > \Phi$, the algorithm will set $\Phi = \Phi^u$ and the output $\boldsymbol{a}^{1,y} = \boldsymbol{a}^s_R, \boldsymbol{a}^{2,y} = \boldsymbol{a}^u_R$.

Case 2: If $\Phi^u > \Phi$, the algorithm will partition regions of non-integer elements in $\boldsymbol{a}^s_R$ and $\boldsymbol{a}^u_R$. Specifically, the algorithm will choose the first non-integer element a^s_j or a^u_j in $\boldsymbol{a}^s_R$ and $\boldsymbol{a}^u_R$ and partition its region to generate two subproblems, i.e., problem (P$_A$) adding constraint $a^s_j = 1$ or $a^s_j = 0$.

2) Power Packet Scheduling Algorithm: In this step, we propose the PPS algorithm to check whether all the allocated power packets can be scheduled. If all the power packets can be scheduled, it will return 1; otherwise it will return 0. The AS algorithm will not end until the PPS algorithm returns 1.

Let $\mathcal{M}^u$ denote the set of unscheduled power packets. At first, all the power packets allocated to demander ESs will be added into the set $\mathcal{M}^u$. When scheduling power packets in $\mathcal{M}^u$, the algorithm needs to confirm that power packets allocated to the same demander ESs cannot be transmitted simultaneously according to constraint (18). We call that power packet m_i **conflicts** with power packet m_j if they are allocated to the same demander ES. Therefore, the power packet scheduling can be viewed as a series of unscheduled and unconflict power packet scheduling, and we can eliminate constraint (18) when scheduling.

To find a set $\mathcal{M}^{uu}$ of unscheduled and unconflict power packets, we need to confirm that each packet in $\mathcal{M}^{uu}$ will not conflict with other power packets in this set and the scheduled power packets. Specifically, let l^c_k denote the number of occupied time slots on power channel k, and let $l^{min} = \min_{k \in \mathcal{K}} l^c_k$. Let $\mathcal{M}'$ denote the set of power packets which occupy the time slots with $n > l^{min}$. A power packet in $\mathcal{M}^{uu}$ needs to avoid conflicts with power packets in both $\mathcal{M}^{uu}/\{m\}$ and $\mathcal{M}'$.

Algorithm 2: Power Packet Scheduling Algorithm

Input: Allocation set $\boldsymbol{a}^{1,y}, \boldsymbol{a}^{2,y}$;
Output: Success flag f;
Set $\mathcal{M}^u$ based on $(\boldsymbol{a}^{1,y}, \boldsymbol{a}^{2,y})$, and set $f = 1$ and $l_k = 0, \forall k$;
while $\mathcal{M}^u \neq \varnothing$ **do**
  Find a set $\mathcal{M}^{uu}$ of unscheduled and unconflicted packets in $\mathcal{M}^u$, and set $\mathcal{M}^u = \mathcal{M}^u / \mathcal{M}^{uu}$;
  Sort packets in $\mathcal{M}^{uu}$ in descending order of their lengths;
  for *power packet* $m \in \mathcal{M}^{uu}$ **do**
    Choose power channel k with the lowest l_k;
    if $l_k + l_m \leq N$ **then**
      Schedule power packet m over power channel k;
      Set $l_k = l_k + l_m$;
    else
      $f = 0$, return;
    end
  end
end

To efficiently schedule power packets in $\mathcal{M}^{uu}$, the maximum number of the occupied time slots among all the power channels is minimized, which is shown as:

$$\min_{\mathcal{M}_1, \ldots, \mathcal{M}_K} \max_{k \in \mathcal{K}} \left(l^c_k + \sum_{m \in \mathcal{M}_k} l_m \right) \tag{22}$$

where $\mathcal{M}_k$ denotes the set of power packets to be scheduled on power channel k, and we have $\mathcal{M}_1 \cup \ldots \mathcal{M}_K = \mathcal{M}^{uu}$. Since problem (22) is a NP-hard problem [28], we propose the longest power packet length algorithm to solve it efficiently [29]. Specifically, we first sort all the power packets in $\mathcal{M}^{uu}$ in a descending order and then schedule these power packets successively according to this order. When scheduling each power packet, we first select the power channel with the minimum occupied number of time slots and then schedule this power packet over the selected power channel. In this way, the time slots in different power channels can be utilized evenly and efficiently, and thus, more power packets can be scheduled over the power channels.

VI. Performance Analysis

In this section, we first provide the convergence and complexity of the auction scheme, and then analyze the optimal reserve price r^* for the auction scheme.

A. Convergence

Proposition 2: The proposed iterative auction scheme converges after at most $J((\lfloor \nu - \pi^s \rfloor)/\sigma + 1)$ iterations, where parameter ν denotes the maximum unit valuation among all the demander ESs.

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Proof: Since ν is the maximum unit valuation, a demander ES can increase its unit bidding price for at most $(\lfloor \nu - \pi^s \rfloor)/\sigma$ times. If a demander ES is not allocated energy within $(\lfloor \nu - \pi^s \rfloor)/\sigma + 1$ iterations, it will give up bidding in this trading cycle. According to the auction scheme, in each iteration, at least one demander ES will increase its unit bidding price or give up bidding if the auction does not terminate after this iteration. Therefore, after $J((\lfloor \nu - \pi^s \rfloor)/\sigma + 1)$ iterations, all the demander ESs will give up bidding in this cycle, and the termination condition will be satisfied. ■

B. Complexity

After providing the convergence of the auction scheme, we analyze the complexity in each auction iteration.

1) The Complexity at the Controller Side: In each iteration of the AS algorithm, the EA algorithm and the PPS algorithm are executed once, respectively. Therefore, the complexity of the AS algorithm depends on the number of iterations and the complexity of the EA algorithm and the PPS algorithm.

As parameter l is initialized to KN and will be reduced by 1 in each iteration, the AS algorithm has at most KN iterations. Besides, since the complexity of the interior-point methods is $O((J^2 + IJ)\log J)$ [30], the complexity of the EA algorithm is $O(4^J(J^2 + IJ)\log J)$ [31]. Note that the size of the DC packetized power microgrid is moderate. Therefore, the proposed EA algorithm is still efficient in practice. As for the PPS algorithm, its complexity can be provided by the following proposition.

Proposition 3: Suppose I and J are the input sizes of the algorithm, the complexity of the PPS algorithm is $O((IJ)^3)$.

Proof: The complexity of the PPS algorithm is related to the number of iterations and the complexity in each iteration. There are at most IJ iterations in the PPS algorithm because the number of power packets is no more than IJ, and at least one power packet is scheduled in each iteration. In each iteration, to find a set of unscheduled and unconflicted power packets $\mathcal{M}^{uu}$, we can select one unscheduled power packet from every demander ES and compare it with the scheduled power packets. Thus, its complexity is $O(J)$. As for power packet sorting, since $|\mathcal{M}^{uu}| < IJ$, we can assume its complexity is $O((IJ)^2)$. The complexity to schedule power packets in $\mathcal{M}^{uu}$ is $O(IJ)$ because $|\mathcal{M}^{uu}| < IJ$. Therefore, the time complexity of the PPS algorithm is $O((IJ)^3)$. ■

2) The Complexity at the Demander ES Side: Since each demander ES submits its bid independently, different demander ESs can submit new bids simultaneously. Note that each demander ES only needs to compare the bidding price with its valuation in order to submit new bids. Therefore, the complexity at the demander ES side is $O(1)$.

C. Optimal Reserve Price

In this subsection, we provide the analysis of the optimal reserve price r^*. We assume $r \in [\pi^s, \pi^u]$ in the following analysis. Before the analysis, the symmetric model of demander ESs is introduced first [32]. Assume that the unit valuation of each demander ES is independently and identically distributed (i.i.d.) and follows the distribution F in $[0, \nu]$, and its demanded energy is also i.i.d. Let f denote the density function of the unit valuation.

The optimal reserve price for the worst-case sum of revenues is π^s. This is because in the worst case all the demander ESs has unit valuation r, and $r > \pi^s$ may lead to the sum of revenues being 0. The analysis of the optimal reserve price for the expected sum of revenues $\mathbb{E}[\Phi]$ is more complicated. In the following we analyze the optimal reserve price for the expected sum of revenues in two special cases.

1) Large Number of Supplier ESs: In this case, we assume the number of supplier ESs is sufficiently large, and therefore every demander ES will choose different supplier ESs to satisfy its own demand. Assume that the number of power channels is also sufficient large, and the router can support the transmission of all the packets required by demander ESs. In this case, $\mathbb{E}[\Phi]$ can be given by the following proposition:

Proposition 4: The expected sum of revenues in this case is $\mathbb{E}[\Phi] = rJ\mathbb{E}[e_j^r](1 - F(r))$.

Proof: For every demander ES, if its unit valuation is not less than the reserve price r, it can purchase energy from supplier ESs at the unit price r. Therefore, the expected revenue generated by one demander ESs is $r\mathbb{E}[e_j^r]\mathbb{P}[v_j \geq r]$, and the expected revenue generated by all the demander ESs is $rJ\mathbb{E}[e_j^r]\mathbb{P}[v_j \geq r] = rJ\mathbb{E}[e_j^r](1 - F(r))$. ■

This implies that the optimal reserve price r^* is determined by the distribution function F. For example, let $F(r) = \frac{r}{\nu}$. If $\nu > 2\pi^s$, we have $r^* = \nu/2 > \pi^s$. However, if $\nu \leq 2\pi^s$, the optimal reserve price r^* is equal to π^s.

2) One Supplier ES: There is only one supplier ES in this case, and every demander ES desires to buy the power packet from this supplier ES. We assume that the router can satisfy the transmission requirements of all the demander ESs. We also assume that σ approaches to 0 so that the bidding prices can increase continuously. The following proposition provides the expected sum of revenues in this case:

Proposition 5: The expected sum of revenues in this case is

$$\mathbb{E}[\Phi] = J\mathbb{E}[e_j^r](rG(r)(F(\pi^u) - F(r)) + \pi^u(1 - F(\pi^u)) + \int_r^{\pi^u} (F(\pi^u) - F(u))ug(u)du) \tag{23}$$

where $G(r) = F(r)^{J-1}$ and $g(r) = G'(r)$.

Proof: See Appendix A. ■

When J is large, $G(r)$ and $g(u)$ are close to 0, and thus all the terms containing $G(r)$ and $g(u)$ can be neglected. Therefore, the effect of reserve price on the sum of revenues can also be neglected.

D. Efficiency

Besides sum of revenues, efficiency Ψ is another criterion to evaluate the performance of the auction scheme [32]. The efficiency is defined as the ratio of social welfare of the auction scheme to the maximum social welfare, where social welfare is defined as sum of utilities and revenues. However, in this paper, it is difficult to obtain the efficiency of the proposed auction mechanism in general case because it is complicated to calculate the maximum social welfare. Even we assume that the valuations of demander ESs is known and the power packet congestion is neglected, optimal energy allocation with the maximum social welfare is still an integer nonlinear program, which is NP-hard.

Instead, the expected efficiency $\mathbb{E}[\Psi]$ of the proposed auction mechanism in two special cases can be obtained, which is elaborated as follows:

1) Large number of Supplier ESs: In this case, $\mathbb{E}[\Psi]$ is given by the following proposition:

Proposition 6: The expected efficiency in this case is

$$\mathbb{E}[\Psi] = \frac{(1-F(r))\int_r^\nu vf(v)dv}{(1-F(\pi^s))\int_{\pi^s}^\nu vf(v)dv}. \tag{24}$$

Proof: Since every demander ES with valuation no less than r can purchase energy, the expected utility of demander ES j is $\mathbb{E}[e_j^r]\mathbb{P}[v_j \geq r](\int_r^\nu vf(v)dv - r) = \mathbb{E}[e_j^r](1-F(r))(\int_r^\nu vf(v)dv - r)$, and the corresponding expected revenue is $r\mathbb{E}[e_j^r](1-F(r))$. Thus, the expected social welfare of the proposed auction scheme is $\mathbb{E}[e_j^r](1-F(r))\int_r^\nu vf(v)dv$. Since $(1-F(r))$ and $\int_r^\nu vf(v)dv$ increase when r decreases, the expected social welfare is maximized when $r=\pi^s$. Therefore, the expected efficiency is $\frac{(1-F(r))\int_r^\nu vf(v)dv}{(1-F(\pi^s))\int_{\pi^s}^\nu vf(v)dv}$. ■

According to propositions 4 and 6, we can infer that the expected revenues and efficiency cannot be maximized simultaneously for some distribution functions.

2) One Supplier ES: In this case, $\mathbb{E}[\Psi]$ is given by the following proposition:

Proposition 7: The expected efficiency in this is 1.

Proof: The expected social welfare is maximized when the power packet is allocated to the demander ES with the maximum valuation. Besides, according to the auction mechanism, every demander ES will increase its bidding price to bid for the power packet until only one demander ES is left. Finally, the demander ES with the maximum valuation will get the power packet in the proposed auction scheme. Therefore, the expected efficiency is 1. ■

VII. Simulation Results

In this section, we present the performance of the proposed iterative auction (PI) scheme and also provide the simulation results of the optimal reserve price. To begin with, the settings in the simulation are introduced. We consider a DC packetized power microgrid presented in Fig. 1, where each ES possesses a load, a battery and an energy generation system with DERs. We assume that the capacity of each ES's battery is $100KWh$. Since the electricity generation and consumption follow different changing patterns, the stored energy in the batteries will fluctuate over time. To satisfy its own energy demand, we assume that each ES will keep its battery at least half-full. As a result, the ESs with stored energy above $50KWh$ will sell the surplus energy, while the ESs with stored electricity below $50KWh$ will buy power packets to at least reach the $50KWh$ threshold. Therefore, the maximum supplied energy of a supplier ES $e^{s,max}$ is set to $25KWh$, and the maximum demanded energy of a demander ES $e^{d,max}$ is set to $75KWh$.

The duration of a time slot T is set to 3 minutes, and we assume that a trading cycle contains 20 time slots. As for supplier ESs, we assume that each supplier ES will sell one power packet in a trading cycle. As the amount of stored energy in the batteries varies, power packets from different supplier ESs can be different. We assume that the export power of a power packet p^e is within $[50,100]$ KW, and its occupied time slots l is within $[1,5]$. Note that the time lengths of header l^h and footer l^f are neglected because they are about tens of microseconds [8]. For the same power packet, the received power for a demander ES is slightly lower that the export power because of the transmission loss, while the length of the power packet remains unchanged. Since the transmission loss from an ES to the router is proportional to the transmission distance, the transmission loss of different power packets can also be different. We assume that the transmission loss between ESs and the router is within $[0,0.05]$, and we ignore the transmission loss in the router.

In terms of demander ESs, we assume that their unit valuations follow uniform distribution in $[0,5]$. Since the maximum demanded energy is $75KWh$ and the maximum supplied energy is $25KWh$, we assume that a demander ES can purchase at most 3 power packets. The minimum selling price of supplier ESs π^s is set to be 1, and the minimum selling price of the UG π^u is set to be 4. The constant σ is 0.1. The parameters are listed in Table I.

TABLE I
Simulation Parameters

Parameters	Values
Maximum supplied energy of an ES $e^{s,max}$	$25KWh$
Maximum demanded energy of an ES $e^{d,max}$	$75KWh$
Duration of a time slot T	$3min$
Number of time slots in a transmission step N	20
Minimum export power of a power packet p^{min}	$50KW$
Maximum export power of a power packet p^{max}	$100KW$
Minimum number of time slots of a power packet l	1
Maximum number of time slots of a power packet l	5
Range of the transmission loss factor ϵ	$[0,0.1]$
Minimum selling price of supplier ESs π^s	1
Maximum unit valuation among all the demander ESs ν	5
Maximum number of packets a demander ES can purchase	3
Minimum selling price of the UG π^u	4
Increased value of unit bidding price in an iteration σ	0.1

A. Performance Analysis

To evaluate the performance of the PI scheme, we give the performance obtained by the following three schemes.

Optimal (OPT) scheme: The exhaustive search approach is applied in this algorithm, and therefore the optimal solution with the maximum sum of revenues can be solved in each auction iteration.

ES First (ESF) scheme: This algorithm first allocates power packets to demander ESs in the descending order of the bidding prices, and then schedules these power packets over the power channels. When allocating power packets, this algorithm always tries to first allocate power packets from supplier ESs.

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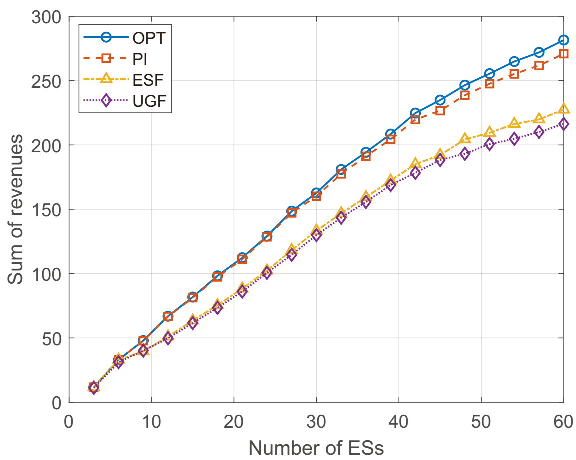

Fig. 5. The sum of revenues Φ versus the number of ESs $I + J$.

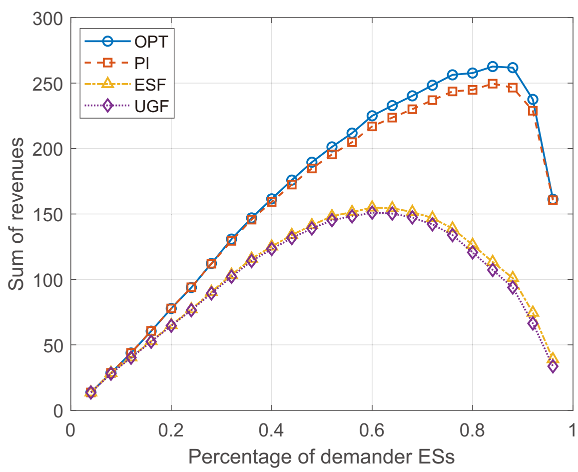

Fig. 6. The sum of revenues Φ versus the percentage of demander ESs $J/(I + J)$.

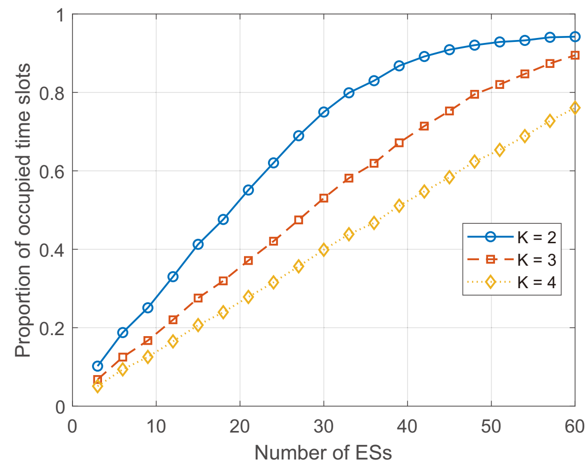

Fig. 7. Proportion of occupied time slots p^o versus number of ESs $I + J$.

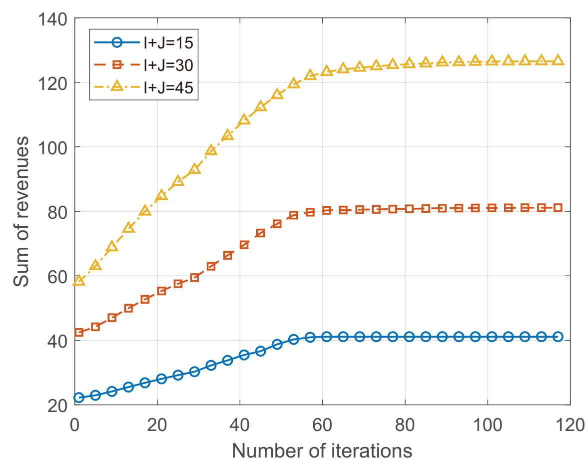

Fig. 8. The number of iterations N^I versus the number of ESs $I + J$.

UG First (UGF) scheme: Different from the ESF scheme, this algorithm first allocates power packets from the UG to demander ESs. If their unit bidding prices are less than π^u, the algorithm will then allocate power packets from supplier ESs to them.

First, we evaluate the performance of the PI scheme for different sizes of microgrids. Fig. 5 shows the sum of revenues Φ versus the number of ESs $I + J$ with $K = 2$. It is shown that Φ obtained by the PI scheme is close to that obtained by the OPT scheme and is higher than those obtained by the ESF and UGF schemes, which has shown the effectiveness of the proposed scheme.

Next, we show the performance of the PI scheme for different percentage of demander ESs. Due to the variation of stored electricity, the percentages of supplier ESs or demander ESs will also vary in different trading cycles. Fig. 6 presents the sum of revenues Φ versus the percentage of demander ESs $J/(I + J)$ when the number of ESs in the microgrid is 25. Similar to the results observed from Fig. 5, Φ obtained by the PI scheme is still higher than those obtained by the ESF and UGF schemes and is close to that obtained by the OPT scheme. We can also observe that the sum of revenues first increases and then decreases when the percentage of demander ESs increases, and there exists an optimal percentage corresponding to the maximum sum of revenues.

In Fig. 7, we illustrate the proportion of the occupied time slots p^o versus the number of ESs $I + J$. We can observe that utilization of time slots increase with the number of ESs, while the proportion of occupied time slots is always lower than 1. This verifies that the PI scheme can alleviate the power packet congestion when the number of power packets is large. We can also observe that the utilization of time slots decreases when the number of power channels increases, which implies that selecting a proper number of power channels is important to guarantee the system utilization efficiency.

Fig. 8 illustrates the number of iterations N^I versus the number of ESs $I + J$. It is shown that the sum of revenues converges to a constant when the number of iterations increases. In addition, we can also observe that the convergence rate decreases when the number of ESs increases, which is consistent with the results in Section VI.

Fig. 9 shows the running time t^R versus the number of ESs $I + J$. We can observe that the running time increases with the number of ESs. This indicates that the complexity increases when the number of ESs increases, which is consistent with

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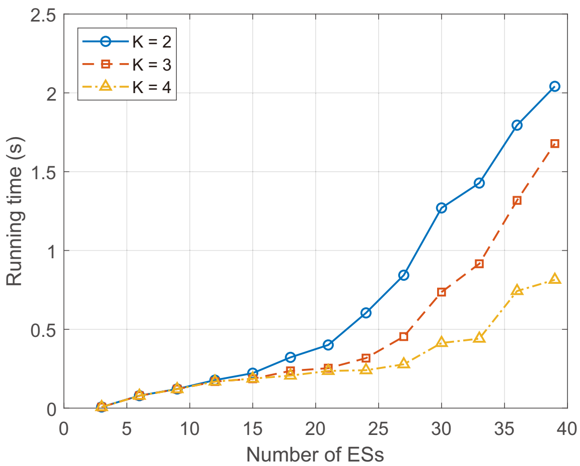

Fig. 9. Running time t^R versus the number of ESs $I + J$.

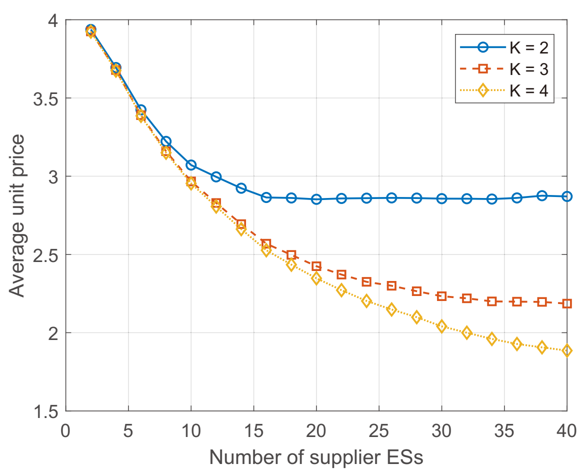

Fig. 10. The average unit price π^a versus the number of supplier ESs I.

the results in Section VI. We can also observe that the running time decreases when the number of power channels increases, which implies that increasing the number of power channels can decrease the average computational complexity.

Fig. 10 shows the average unit price of power packets π^a versus the number of supplier ESs I when the number of demander ESs is 20. We can observe that π^a decreases when I and K increase, which implies that the average unit price can be reduced when more supplier ESs and power channels are involved in the energy trading, and therefore demander ESs are expected to obtain higher utilities. We can also observe that π^a remains constant with the increase of I when $K = 2$ and $I > 20$. This is because the controller restricts the number of traded power packets in order to avoid the congestion when the number of supplier ESs is sufficiently large.

Fig. 11 shows the number of demander ESs J versus the minimum price of the UG π^u with $I = 40$, $J = 20$, and $K = 4$. The demander ESs in the figure are divided into two types: demander ESs trading with supplier ESs and demander ESs trading with the UG. We can observe that when π^u increases, the number of demander ESs trading with the UG decreases, while the number of demander ESs trading with the supplier ESs increases. This is because more demander ESs

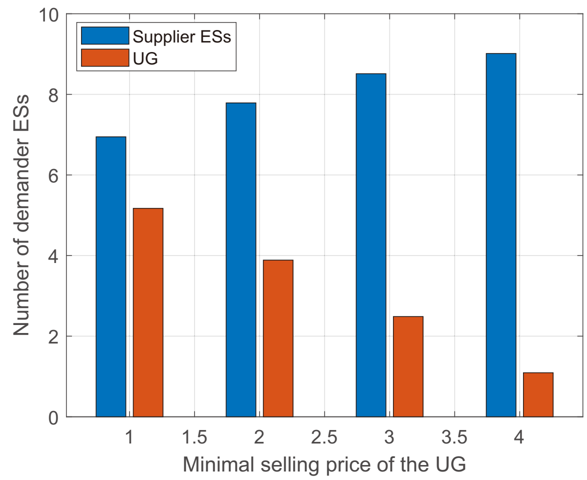

Fig. 11. The number of demander ESs J versus the minimum selling price of the UG π^u.

will purchase energy from supplier ESs which provide lower unit selling prices. Since the increased number of demander ESs trading with supplier ESs is smaller than the decreased number of demander ESs trading with the UG, we can infer that the total number of demander ESs trading with ESs and the UG decreases when π^u increases.

B. Reserve Price Analysis

In this subsection, we provide the simulation results for the reserve price. The reserve price is important because setting proper reserve price can increase the sum of revenues. As discussed in Section VI-C1, the expected sum of revenues is related to the reserve price. This implies that the controller can optimize its objective by setting a suitable reserve price. In the following, we will analyze the optimal reserve price and its effect on the sum of revenues.

Fig. 12 depicts the sum of revenues Φ versus the reserve price r with $I = 2000$, 100, 10 and 1 when the number of demander ESs is 20. We also provide the theoretical values of sum of revenues in Section VI-C with $I = \infty$ and $I = 1$. It can be observed that the simulation values converge to the theoretical values with $I = \infty$ when I increases, and converge to the theoretical values with $I = 1$ when I decreases, which is consistent with the propositions 4 and 5 in Section VI-C.

Fig. 13 presents the optimal reserve price r^* versus the number of supplier ESs I when $J = 20$ and K is sufficient. We can observe that when I increases, the value of optimal reserve price becomes closer to 2.5, which is equal to $\nu/2$. This implies that the optimal reserve price converges to $\nu/2$ when the number of supplier ESs increases.

To evaluate the effect of the reserve price r on the sum of revenues Φ, we define a price ratio γ as

$$\gamma = \frac{\Phi^{max}}{\Phi^{min}}, \tag{25}$$

where Φ^{max} denotes the maximum sum of revenues, and Φ^{min} denotes the minimum sum of revenues when $r \in [\pi^s, \pi^u]$. A larger γ implies that the variation of reserve price has a stronger influence on the sum of revenues. Fig. 14 shows the

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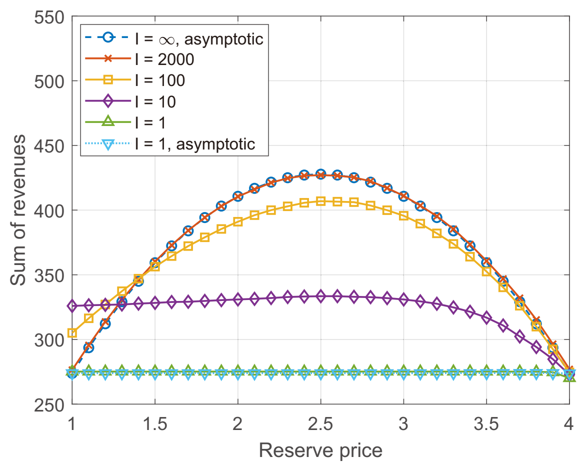

Fig. 12. The sum of revenues Φ versus the reserve price r.

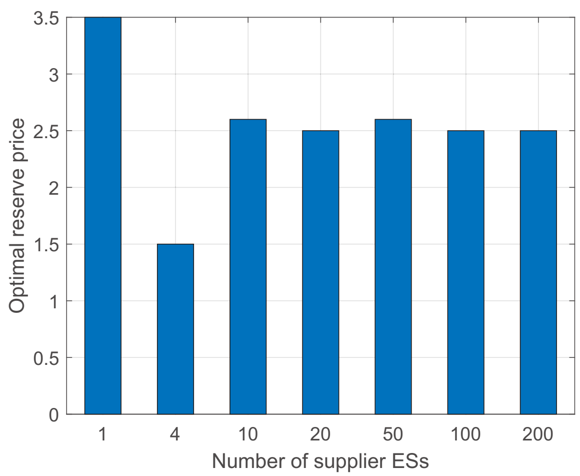

Fig. 13. The optimal reserve price r^* versus the number of supplier ESs I.

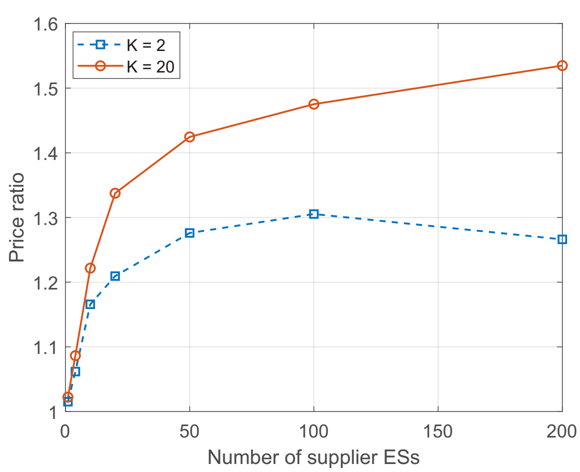

Fig. 14. The price ratio γ versus the number of supplier ESs I.

price ratio γ versus the number of supplier ESs when the number of demander ESs is 20 and $r \in [1, 4]$. We can observe that γ is close to 1 when $I = 1$, which is consistent with the discussions in Section VI-C2. Besides, we can observe that γ increases with I and K, indicating that the effect of reserve price on the sum of revenues increases when the numbers of supplier ESs and power channels increase. We can also observe that when $K = 2$ and $I > 100$, γ decreases as I grows. This is because the limited number of power channels restricts the increase of the maximum sum of revenues.

VIII. Conclusion

In this paper, we have studied P2P energy trading in a DC packetized power microgrid. We have proposed a power packet trading protocol and formulated a P2P energy trading problem. An iterative auction scheme has been introduced to solve the trading problem, and the optimal strategies of the controller and the demander ESs have been discussed. We have analysed the convergence and the complexity of the auction scheme. The effect of reserve price on the sum of revenues has also been discussed theoretically. We have obtained two conclusions from the simulation analysis. First, when there are sufficient power channels, the average unit price of the energy decreases as the number of supplier ESs and power channels increase. Second, the number of demander ESs trading with the UG and supplier ESs decreases as the minimum selling price of the UG grows.

Appendix A
Proof of Proposition 5

Let v denote the unit valuation of a demander ES. If $v < r$, its expected unit payment $\phi(v)$ is 0 because it will give up bidding in this trading cycle. If $r \le v < \pi^u$, this demander ES will obtain the power packet only if the unit valuations of other demander ESs are lower than v, and the unit payment of this demander ES is related to the highest unit valuation of other demander ESs. Let u denote the highest unit valuation of the other $J-1$ demander ESs, where u follows the distribution $G(u) = F(u)^{J-1}$, and the density of u is $g(u) = G'(u)$. If $u < r$, the expected unit payment is r. If $u > r$, the expected unit payment is u. Thus, the expected unit payment $\phi(v)$ with $r \le v < \pi^u$ is $rG(r) + \int_r^v ug(u)du$. If $v > \pi^u$, the expected unit payment $\phi(v)$ is π^u. Therefore, we have

$$\phi(v) = \begin{cases} 0, & v < r, \\ rG(r) + \int_r^v ug(u)du, & r \le v < \pi^u, \\ \pi^u, & v > \pi^u. \end{cases} \tag{26}$$

As the unit valuation of a demander ES is private, the expected unit payment of a demander ES with unknown valuation is

$$\begin{aligned}\phi(V) =& \int_0^{\nu} \phi(v) f(v) dv \\ =& \int_r^{\pi^u} \left(rG(r) + \int_r^{v} u g(u) du \right) f(v) dv + \int_{\pi^u}^{\nu} \pi^u f(v) dv \\ =& rG(r) \int_r^{\pi^u} f(v) dv + \pi^u \int_{\pi^u}^{\nu} f(v) dv \\ &+ \int_r^{\pi^u} \int_r^{v} u g(u) du f(v) dv \\ =& rG(r)(F(\pi^u) - F(r)) + \pi^u (1 - F(\pi^u)) \\ &+ \int_r^{\pi^u} \left(\int_u^{\pi^u} f(v) dv \right) u g(u) du \\ =& rG(r)(F(\pi^u) - F(r)) + \pi^u (1 - F(\pi^u)) \\ &+ \int_r^{\pi^u} (F(\pi^u) - F(u)) u g(u) du. \end{aligned} \tag{27}$$

Therefore, the expected sum of revenues is

$$\begin{aligned}\mathbb{E}[\Phi] =& J\mathbb{E}[e_j^r](rG(r)(F(\pi^u) - F(r)) + \pi^u(1 - F(\pi^u)) \\ &+ \int_r^{\pi^u} (F(\pi^u) - F(u)) u g(u) du). \end{aligned} \tag{28}$$

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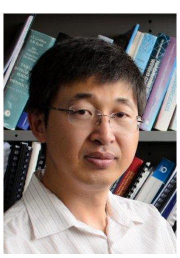

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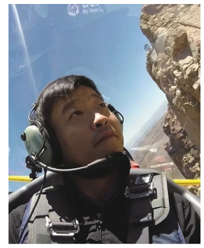

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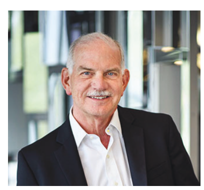

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