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Supply Chain Contracts That Prevent Information Leakage

Yiwei Chen,[a] Özalp Özer[b]

[a] Carl H. Lindner College of Business, University of Cincinnati, Cincinnati, Ohio 45221; [b] Jindal School of Management, The University of Texas at Dallas, Richardson, Texas 75080

Contact: chen3yw@ucmail.uc.edu, https://orcid.org/0000-0001-6243-868X (YC); oozer@utdallas.edu, https://orcid.org/0000-0001-5552-0968 (ÖÖ)

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Abstract. This paper determines categories of contracts that facilitate vertical information sharing in a supply chain while precluding horizontal information leakage among competing newsvendors. We consider a supply chain in which retailers replenish inventory from a common supplier to satisfy uncertain demand and are engaged in newsvendor competition. Each retailer has imperfect demand information. Yet one of the retailers (the incumbent) has a more accurate demand forecast than the other (the entrant). Information leakage among such competing retailers precludes vertical information sharing and is often the reason for many retailers to abandon collaborative forecast-sharing initiatives, leading to suboptimized supply chains. We show that whether a contract can prevent information leakage depends on how the inventory risk (i.e., cost of supply–demand mismatch) is allocated among the supplier and retailers in conjunction with the allocation of profits. We categorize contracts according to how they allocate inventory risk among firms when compared with a wholesale-price contract. This comparison yields four mutually exclusive and collectively exhaustive categories of contracts. A *downside-protection* contract is one that effectively reduces retailers' cost of excess inventory by shifting some of their overage cost to the supplier. Examples of such contracts include *buy-back* and *revenue-sharing* contracts. An *upside-protection* contract is one that effectively increases retailers' cost of inventory shortage by shifting some of the supplier's underage cost to retailers. Examples of such contracts include *penalty* and *rebate* contracts. A *two-sided protection* contract combines the properties of the previous two categories. A *no-protection contract* is one that fails to shift firms' cost of inventory shortage or excess from one to the other. Examples of such contracts include *wholesale-price* and *two-part tariff* contracts. We show that no-protection contracts, which are extensively used in practice, cannot prevent information leakage, whereas others may do so. We also show that preventing information leakage could be costly for the supply chain (i.e., low channel efficiency). We conclude by illustrating how our unified framework to study a variety of contracts can enable a firm to determine the best-performing contract (among many) that precludes information leakage while almost coordinating the channel. For example, we show why buy-back contracts perform significantly better than revenue-sharing or rebate contracts.

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Keywords: supply chain risk management • information leakage • newsvendor competition • contract categorization • buy-back • revenue-sharing • rebate • two-part tariff • wholesale-price

1. Introduction

This paper determines supply chain contracts that can prevent information leakage among competing retailers (an incumbent and an entrant) who source from a common supplier to fulfill uncertain market demand. The retailers face the classic newsvendor problem in that each must order from the supplier and stock inventory before the sales season, during which inventory replenishment is not possible. In deciding how much inventory to stock, each retailer maximizes her own expected revenue minus the expected cost of supply–demand mismatch (i.e., cost of overage and underage). Customers who do not find the product available at their preferred retailer may visit the other retailer, and hence, the retailers compete to satisfy uncertain demand. To complicate matters, one of the retailers (the incumbent) often has private demand information that neither the other retailer (entrant) nor the supplier possesses. This informational advantage benefits the incumbent in deciding how much inventory to stock and in capturing the entrant's market share. However, if the incumbent transfers this information to the supplier, the supplier may find it profitable to leak the incumbent's private forecast information to the entrant; inducing a more competitive environment and a higher order quantity by mitigating demand forecast uncertainty. As a result, the incumbent could lose her informational advantage and face fierce competition with her rival. Therefore, such *information leakage* deters firms from vertically sharing information in horizontal competition and even from participating in collaborative forecast-sharing initiatives, leading to suboptimized supply chain relationships. Our goal in this paper is to identify contracts that can prevent information leakage, characterize properties of information leakage prevention conditions, and quantify the resulting supply chain profits.

Demand information leakage among competing retailers in a supply chain negatively affects profitability of firms from various industries. Several related anecdotes have been recorded over the past two decades. For example, Anand and Goyal (2009) highlight the case of Liz Clairborne, an apparel supplier currently owned by JCPenney. The company faced stiff resistance from its retailers to share their private demand information, fearing the company would leak such valuable information to competing retailers (Salmon and Blasberg 1997). In addition, the authors report more recent cases involving such companies as Walmart and Newbury Comic. Similarly, Kong et al. (2013) report United Kingdom apparel retailers' reluctance to share their point-of-sale data with their supplier, owing to concerns that the supplier might leak such sensitive information to competing retailers (Adewole 2005). Furthermore, these authors discuss why a specialized toy retailer, which is able to identify market trends earlier than discount retailers, may be unwilling to share her demand information with her supplier.

The literature is abundant with examples of the perils of vertical information sharing in various industries from apparel and toy retailers to high technology. The majority of these industry examples have also been used repeatedly in the operations management literature to motivate the classic newsvendor problem (e.g., an environment in which a retailer often has one replenishment opportunity to stock inventory and satisfy uncertain demand for a short life-cycle product that requires relatively long replenishment leadtimes). In such markets, retailers' order quantities would not necessarily affect market price of a product. To begin with, the supplier (e.g., Liz Claiborne) often sets the price, and hence prices do not differ much across retailers. Therefore, unlike the previous literature on this topic, we analyze information leakage among competing newsvendors and their common supplier.

This paper shows why and how firms in the aforementioned supply chains need to jointly optimize material flow (i.e., stocking decisions and hence allocation of inventory risk in the face of uncertain demand and competition) and information flow (i.e., sharing of demand information vertically) as well as financial flow (i.e., allocation of profits). We show how some contracts that are primarily used for optimizing *operational imperatives* (e.g., how much to order and stock) affect firms' *informational imperatives* (e.g., whether to share private demand information vertically). We also show ignoring operational imperatives and optimizing *financial imperatives* alone also leads to deficient information flows (e.g., information leakage) and suboptimal performance (e.g., loss of profits and low channel efficiency). To quantify and understand the joint role of these three flows on supply chain performance, we provide a unified framework that encompasses a wide range of contracts extensively studied in the literature (e.g., wholesale-price, buy-back, rebate and revenue-sharing contracts).

Our analysis reveals that information leakage is due to the expected supply–demand mismatch cost. In other words, supply–demand mismatch drives firms' information imperatives in addition to their well-known role in replenishment decisions. We also show that contracts designed only to distribute financial flows without proper distribution of inventory risk among firms cannot prevent information leakage in a supply chain. We show that allocation of inventory risk (i.e., cost of supply–demand mismatch) in conjunction with allocation of financial flows is necessary to prevent information leakage. In addition, we illustrate the properties of contracts (i.e., sufficient conditions) that prevent information leakage, and quantify the resulting order quantities and profits.

This study also enables us to categorize contracts in terms of how the supply chain's inventory risk is shared among the supplier and retailers compared with the wholesale-price contract. We define a contract as a *downside-protection* contract if it effectively reduces retailers' cost of excess inventory (downside risk due to potentially low demand) by shifting some of their overage cost to the supplier. Such a shift encourages retailers to carry additional inventory because they are now less concerned about potentially observing a low demand, hence protecting the supply chain for a possible downside risk (low demand). Buy-back and revenue-sharing contracts fall within this category. We define a contract as an *upside-protection* contract if it effectively increases retailers' cost of inventory shortage (upside risk due to potentially high demand) by shifting some of the supplier's underage cost to retailers. Such a shift encourages retailers to carry additional inventory as well, but this time it is because retailers are more concerned about potentially observing a high demand, hence protecting the supply chain for upside risk (high demand). Penalty and rebate contracts fall within this category. We define a contract as a *two-sided protection* contract if the contract increases retailers' cost of inventory shortage while reducing their cost of excess inventory by reallocating inventory risk among firms. Combinations of some rebate and buy-back contracts fall within this category. Finally, we define a contract as a *no-protection* contract if it is only designed to reallocate financial flows without shifting any party's cost of inventory shortage or cost of excess inventory. Wholesale-price and two-part tariff contracts fall within this category. Together these four categories of contracts are mutually exclusive and collectively exhaustive (i.e., they cover a large class of supply chain contracts).

We show that no-protection contracts cannot prevent information leakage, whereas the other three categories may do so. We show how much each retailer needs to optimally order in such a competitive market. We quantify the resulting supply chain profits and channel efficiency and show that preventing information leakage with certain contracts (e.g., a two-sided protection contract) can result in low channel efficiency (i.e., firms leaving money on the table). We illustrate how our unified framework to study a variety of contracts can help a firm to select the best-performing contract (among those that preclude information leakage) and almost coordinate the channel.

The rest of this paper is organized as follows. In Section 2, we review the relevant literature. In Section 3, we introduce the model and formally define the aforementioned four categories of contracts. In Section 4, we establish two benchmarks that are used in establishing nonleakage conditions. In Section 5, we provide the necessary and sufficient nonleakage conditions to prevent information leakage. In Section 6, we quantify the nonleakage conditions and the resulting profits. In Section 7, we conclude.

2. Literature Review

Three streams of literature informed and inspired our research: information leakage in horizontal competition, vertical information sharing in competing supply chains, and newsvendor competition.

The first stream of literature studies information leakage in horizontal competition (Li 2002; Zhang 2002; Li and Zhang 2008; Anand and Goyal 2009; Kong et al. 2013, 2016). These studies analyze retailers engaged in Cournot or Bertrand competition and source from a common supplier. Each retailer is endowed with private demand information. In Li (2002), Zhang (2002), and Li and Zhang (2008), each retailer decides whether to share her private demand information (i) with the supplier only, (ii) with the supplier and retailers who share information, or (iii) with everyone (i.e., make their demand information public) *before* the supplier determines the wholesale price. The authors show that the wholesale price can signal private information to other retailers that do not participate in information sharing. Hence, even if a retailer decides not to disclose her information to other retailers and the supplier agrees not to share her information with nonparticipating retailers (under confidentiality agreements), the retailer's private information can still be leaked indirectly. Anand and Goyal (2009) and Kong et al. (2013) deviate from the aforementioned studies in two important aspects. First, they consider a model in which one of the two retailers—the incumbent—receives her private demand information *after* the parties agree on the contractual terms, which specify the financial and product delivery terms. Second, the supplier actively decides whether to leak the incumbent retailer's private information to the other retailer (entrant). Anand and Goyal (2009) show that wholesale-price contracts cannot prevent information leakage. In other words, the supplier would have incentive to leak the incumbent's proprietary information about the market to the entrant, and hence would do so when it is profitable. Kong et al. (2013) show that revenue-sharing contracts can prevent such information leakage for limited but not all market environments.

The present paper analyzes a sequence of events similar to that of Anand and Goyal (2009) and Kong et al. (2013) because we also would like to capture the impact of obtaining new and private demand information after parties agree on establishing a supply chain by agreeing on the terms of trade. However, we deviate from their work in three important dimensions. First, these authors study market environments in which one retailer (the incumbent) has perfect demand information, that is, knows demand precisely. Our model considers a general market environment that allows retailers' demand forecast to be imperfect, including that of the incumbent. Second, they assume that retailers are engaged in Cournot competition. Thus, in their model, retailers' total order quantity perfectly matches market demand for the product. By contrast, our retailers are engaged in newsvendor competition. Therefore, our model can be used to study those industries in which supply and demand mismatches are inherent. Finally, Anand and Goyal (2009) and Kong et al. (2013) investigate the roles of one specific type of contract, namely the wholesale-price contract and revenue-sharing contract, respectively. We show that a single contract type (including the revenue-sharing contract) cannot guarantee to prevent information leakage for all possible market environments. We provide a general approach that considers a number of practically implemented and well-studied contracts, including wholesale-price and revenue-sharing contracts, in a unified framework, allowing us to identify contracts that can prevent information leakage for a variety of market environments. In fact, we also observe that buy-back contracts perform significantly better than revenue-sharing contracts in that they can prevent information leakage for a wide range of market environments while nearly coordinating the supply chain.

The second stream of literature investigates vertical information sharing in competing supply chains (Ha and Tong 2008, Ha et al. 2011, and Shamir and Shin 2015 and references therein). These papers consider competition among two supply chains, each of which consists of one supplier and one retailer. Retailers from different supply chains are engaged in Cournot competition, and they may be endowed with private and superior demand forecast information that the supplier does not have. Ha and Tong (2008) and Ha et al. (2011) explore whether each supplier has incentive to invest in acquiring the retailer's private forecast information. Shamir and Shin (2015) show that if one retailer has superior demand forecast information that neither the supplier nor the competing retailer has, then by making this superior forecast information publicly available to both the supplier and competitor, this retailer could credibly share the forecast information with her supplier. Although this literature is not about information leakage, we find it useful in informing our research. We consider a supply chain in which two competing retailers are served by one common supplier, rather than two separate suppliers, and hence the possibility of information leakage horizontally.

The third stream of literature introduces and studies the newsvendor competition model (Parlar 1988, Lippman and McCardle 1997, Netessine and Rudi 2003). This model is used in characterizing horizontal competition in industries that face the possibility of supply and demand mismatch. In a newsvendor competition model, each retailer decides on the order quantity to stock before observing the uncertain and exogenously specified demand to maximize expected profit net of the expected cost of overage and underage. Each retailer first satisfies her own demand as much as possible from her stock. The retailer may either run out of inventory with unmet demand or have excess inventory. Next, some of those customers who do not find the product available at their preferred retailer may visit other retailers. If that retailer also does not have enough stock, then demand is lost. Hence, the retailers compete to satisfy uncertain demand. These authors study a variety of settings (centralized, decentralized, and different demand allocation mechanisms) and resulting equilibrium outcomes. All demand information is common knowledge in this literature. The present paper uses a similar newsvendor competition model and builds on this literature by incorporating a different information scenario to study the information leakage problem.

We refer the reader to Kaya and Özer (2012) for a recent review of the wide range of practically implemented supply chain contracts. To the best of our knowledge, they are the first to propose a contract categorization approach that categorizes a large number of contracts in terms of how the supply chain's excess inventory and inventory shortage risks are shared among the members of a supply chain. The authors use this contract categorization to study how each category of contracts helps to coordinate decisions in a supply chain and achieves channel coordination. A similar categorization enables us to determine contracts that can prevent information leakage.

3. Model Framework

Here, we first model the interaction between the two competing retailers and their supplier, the uncertain demand representing the general market condition as perceived by the retailers, and provide the resulting profits. We introduce the model without the need to specify the contract type. Next, we illustrate how the model encompasses a wide variety of contract types by providing specific examples. Finally, we introduce contract classification, which proves useful in determining properties of contracts that prevent information leakage.

3.1. Sequence of Events and Expected Profits

Consider a supply chain with one supplier (referred to as "*he*," indexed by s) and two competing retailers (each is referred to as "*she*") who sell substitutable products. Each retailer is endowed with uncertain demand during a sales season. Each retailer's demand state is either high or low and is possibly correlated with the other retailer's demand state. One of the retailers ("the incumbent," indexed by i) learns of her own demand state $t_i \in \{H, L\}$ before the sales season commences because of her superior relationship with end customers. However, the other retailer ("the entrant," indexed by e) does not know her own demand state $t_e \in \{H, L\}$. When the incumbent's demand state is $t_i \in \{H, L\}$, the incumbent's demand is given by $D_i = \mu^i_{t_i} + \eta\epsilon$. The term $\mu^i_{t_i}$ represents the incumbent's mean demand in state t_i, with the property that $\mu^i_H > \mu^i_L$. The term $\eta\epsilon$ represents the unpredictable demand shock on the incumbent's market. The random variable ϵ represents unsystematic total market uncertainty and has a zero mean strictly increasing cumulative distribution function $F(\cdot)$ and a probability distribution function $f(\cdot)$ supported on $[\underline{\epsilon}, \bar{\epsilon}]$. The term $\eta \in (0, 1)$ represents the impact of the unsystematic total market uncertainty on the incumbent's market. Similarly, when the entrant's demand state is $t_e \in \{H, L\}$, her demand is given by $D_e = \mu^e_{t_e} + (1 - \eta)\epsilon$, with the property $\mu^e_H > \mu^e_L$. The term $1 - \eta$ represents the impact of the unsystematic total market uncertainty on the entrant's demand.

The prior belief is such that the incumbent's demand is in a high state $t_i = H$ with probability $\lambda \in (0, 1)$, or in a low state $t_i = L$ with probability $1 - \lambda$. We remark that the incumbent knows her demand state with certainty. However, the entrant only knows that the incumbent's demand state is high with probability λ. The retailers' demand states are correlated. If the incumbent's demand state is high, $t_i = H$, the entrant's demand is in a high state $t_e = H$ with probability $\lambda' \in [0, 1]$, or in a low state $t_e = L$ with probability $1 - \lambda'$. If the incumbent's demand state is low, $t_i = L$, the entrant's demand is in a low state $t_e = L$ with probability 1. In other words, the incumbent has deeper understanding about the product, the fashion trend, and the market condition than the entrant has. The incumbent's superior knowledge of the market and well-operated established business model allow her to attract more customers than the entrant does. Therefore, when the incumbent's market condition is good ($t_i = H$), the entrant's market condition may or may not be good. When the incumbent's market condition is bad ($t_i = L$), the entrant's market condition cannot be better. We provide the glossary of notation in Appendix A.

The sequence of events is as follows. (1) The supplier and the two retailers agree on the contract T that specifies the financial and product delivery terms. (2) The incumbent observes her demand state $t_i \in \{H, L\}$ and orders q_i units from the supplier to maximize her expected profit, anticipating that the supplier may leak this order information to the entrant. (3) The supplier decides whether to leak the incumbent's order quantity to the entrant. (4) The entrant orders q_e units from the supplier, taking into account the leaked information (if leaked). Hence, the interaction between the incumbent and the supplier/entrant leads to a signaling game. (5) The supplier produces at a per-unit cost c and delivers q_i units to the incumbent and q_e units to the entrant. (6) Demand at both retailers is realized.

Retailers fulfill demand according to the following rule. Each retailer first fulfills her own demand (primary source of demand) from her available inventory. Next, if retailer $l \in \{i, e\}$ fails to fulfill all her demand D_l, then a fraction $a_{lk} \in [0, 1]$ of retailer l's unmet demand spills over and visits retailer $k \neq l$. Retailer k fulfills demand spilled from retailer l (secondary source of demand) as much as possible. Therefore, retailer k's *total effective demand* is given by

$$\bar{D}_k = D_k + a_{lk}(D_l - q_l)^+,$$

which depends on the other retailer l's order quantity q_l. We drop q_l from the argument of retailer k's total effective demand function for expositional clarity. Instead, we explicitly state this dependency in the text when needed.

The retail price for fulfilling one unit of demand is $p > c$. On the basis of the terms of the contract T, money and products are transferred. Unsold product has no salvage value. All information except two retailer demand states is common knowledge. Our competing newsvendor model is similar to that of Parlar (1988), Lippman and McCardle (1997), and Netessine and Rudi (2003).

Below, we introduce the resulting profit functions in their most general form, which does not require us to specify the contract terms. This general form enables us to determine key characteristics of contracts that affect the equilibrium strategies. Later in this section, we illustrate that many widely used and carefully studied supply chain contracts (such as buy-back and revenue-sharing contracts) can be represented by this general form.

Conditional on the incumbent's demand state $t_i \in \{H, L\}$, the integrated firm's total expected profit does not depend on the contract T and is given by

$$\Pi^I_{t_i}(q_i, q_e) = (c^I_u + c^I_o)\mathbb{E}_{t_e}\big[\mathbb{E}_\epsilon[\min\{\bar{D}_i, q_i\} + \min\{\bar{D}_e, q_e\}]\,|t_i\big] - c^I_o(q_i + q_e), \quad (1)$$

where the expectation $\mathbb{E}_{t_e}[\cdot]$ is with respect to the entrant's demand state t_e, the expectation $\mathbb{E}_\epsilon[\cdot]$ is with respect to ϵ, and $c^I_u = p - c$ and $c^I_o = c$ are the integrated firm's unit underage and overage costs, respectively.

Conditional on the incumbent's demand state $t_i \in \{H, L\}$, the incumbent's expected profit depends on the contract T and is given by

$$\Pi^i_{t_i}(q_i, q_e; T) = (c^r_u + c^r_o)\mathbb{E}_{t_e}\big[\mathbb{E}_\epsilon[\min\{\bar{D}_i, q_i\}]\,|t_i\big] - c^r_o q_i - h^i, \quad (2)$$

where $c^r_u > 0$ and $c^r_o > 0$ are retailers' unit underage and overage costs, respectively.[1] A positive (negative) h^i represents a *fixed* fee paid (received) by the incumbent to (from) the supplier; that is, money transfer that is independent of order quantities and the incumbent's demand state. The underage and overage costs depend only on the contract T, and the fixed fee may depend on both the contract T and the ex ante specified values of possible demand realizations.[2] We drop T from the argument of these cost functions for expositional clarity. Instead, we explicitly state this dependency in the text when needed.

Similarly, conditional on the incumbent's demand state $t_i \in \{H, L\}$, the entrant's expected profit depends on the contract T and is given by

$$\Pi^e_{t_i}(q_i, q_e; T) = (c^r_u + c^r_o)\mathbb{E}_{t_e}\big[\mathbb{E}_\epsilon[\min\{\bar{D}_e, q_e\}]\,|t_i\big] - c^r_o q_e - h^e, \quad (3)$$

where h^e represents the fixed fee transferred between the entrant and the supplier.

Conditional on the incumbent's demand state $t_i \in \{H, L\}$, the supplier's expected profit is given by

$$\begin{aligned}\Pi^s_{t_i}(q_i, q_e; T) &= \Pi^I_{t_i}(q_i, q_e) - \Pi^i_{t_i}(q_i, q_e; T) - \Pi^e_{t_i}(q_i, q_e; T)\\ &= (c^s_u + c^s_o)\mathbb{E}_{t_e}\big[\mathbb{E}_\epsilon[\min\{\bar{D}_i, q_i\} + \min\{\bar{D}_e, q_e\}]\,|t_i\big] - c^s_o(q_i + q_e) + h^i + h^e,\end{aligned} \quad (4)$$

where $c^s_u \triangleq c^I_u - c^r_u$ is the supplier's unit underage cost and $c^s_o \triangleq c^I_o - c^r_o$ is the supplier's unit overage cost. We assume $c^s_u, c^s_o \neq 0$ simultaneously.[3] The supplier's unit underage and overage costs depend on the contract T (i.e., c^s_u and c^s_o depend on the contractual terms) and could be positive or negative, unlike retailers' unit underage and overage costs.

3.2. Contract Examples

Next, we provide examples to illustrate how one can derive the profit functions under various contracts from the aforementioned general profit functions.

Wholesale-Price Contract (w): Each retailer pays the supplier $w \in (c, p)$ for each unit ordered. The retailers' and the supplier's profits are given by Equations (2)–(4), where $h^i = 0$, $h^e = 0$, $c_u^r = p - w$, $c_o^r = w$, $c_u^s = w - c$, and $c_o^s = -(w - c)$.

Two-Part Tariff Contract (w, F_i, F_e): The supplier charges the incumbent and entrant upfront lump-sum payments F_i and F_e, respectively, in addition to a per-unit wholesale price w. The retailers' and the supplier's profits are given by Equations (2)–(4), where $h^i = F_i$, $h^e = F_e$, $c_u^r = p - w$, $c_o^r = w$, $c_u^s = w - c$, and $c_o^s = -(w - c)$.

Buy-Back Contract (w, b): Each retailer pays the supplier $w \in (c, p)$ for each unit ordered. At the end of the sales season, the supplier buys back unsold inventories from retailers at unit price $b \in (0, w)$. The retailers' and the supplier's profits are given by Equations (2)–(4), where $h^i = 0$, $h^e = 0$, $c_u^r = p - w$, $c_o^r = w - b$, $c_u^s = w - c$, and $c_o^s = b - (w - c)$.

Revenue-Sharing Contract (w, f): Each retailer pays the supplier w for each unit ordered. At the end of the sales season, each retailer keeps $f \in (0, 1)$ fraction of her sales revenue and transfers $1 - f$ fraction of her sales revenue to the supplier. The terms of the contract are such that $w \in ((c - (1 - f)p)^+, fp)$, because otherwise retailers would make negative profits if $w \geq fp$, retailers would order infinite amount if $w \leq 0$, and the supplier would make negative profit if $w \leq c - (1 - f)p$. The retailers' and the supplier's profits are given by Equations (2)–(4), where $h^i = 0$, $h^e = 0$, $c_u^r = fp - w$, $c_o^r = w$, $c_u^s = (1 - f)p + w - c$, and $c_o^s = -(w - c)$.

Penalty Contract (w, u): Each retailer pays the supplier $w > c$ for each unit ordered. At the end of the sales season, the supplier charges each retailer a unit penalty price u for each unit of unsatisfied demand in the retailer's own market. For the retailer who also fulfills the other retailer's demand after fulfilling her own demand, the supplier rewards her for the demand she fulfills for the other retailer at a unit reward price $u > 0$. The terms of the contract are such that $u > (w - p)^+$, because otherwise, retailers' profits are maximized by ordering no item. The retailers' and the supplier's profits are given by Equations (2)–(4), where $h^i = u\mathbb{E}_{t_i}[\mu_{t_i}^{i}]$, $h^e = u\mathbb{E}_{t_e}[\mu_{t_e}^{e}]$, $c_u^r = p - w + u$, $c_o^r = w$, $c_u^s = w - u - c$, and $c_o^s = -(w - c)$.

Rebate Contract (w, r, K): Each retailer $k \in \{i, e\}$ pays the supplier $w > c$ for each unit ordered and her fixed fee[4] K^k. At the end of the sales season, the supplier offers retailers a rebate reward $r > 0$ for each sold item. The terms of the contract are such that $r > (w - p)^+$, because otherwise, retailers would make negative profit. The retailers' and the supplier's profits are given by Equations (2)–(4), where $h^i = K^i$, $h^e = K^e$, $c_u^r = p - w + r$, $c_o^r = w$, $c_u^s = w - r - c$, and $c_o^s = -(w - c)$.

3.3. Contract Classification

Our goal is to determine the properties of a range of contracts for which the supplier does not leak the incumbent's order quantity information to the entrant. We categorize contracts under four categories based on how the inventory risks are shared between the supplier and retailers. This categorization plays a key role in determining the conditions that affect equilibrium outcomes.

Downside-Protection Contracts: A contract belongs to this category if the retailer's per-unit overage cost with this contract is smaller than her per-unit overage cost with a wholesale-price contract while her per-unit underage cost remains the same. In other words, compared with the wholesale-price contract, contracts in this category effectively reduce a retailer's cost of excess inventory (downside risk due to facing potentially low demand) by shifting the cost of excess inventory to the supplier. Because having excess inventory becomes less costly, these contracts provide incentive for retailers to order and carry more inventory. Therefore, these contracts protect the supply chain against downside risk. Note that $c_u^s = c_u^I - c_u^r$ and $c_o^s = c_o^I - c_o^r$. Hence, these contracts increase the supplier's overage cost while keeping his underage cost unchanged. In addition, note that under the wholesale-price contract, $c_u^s > 0$ and $c_u^s + c_o^s = 0$. Hence, all contracts in this category satisfy the following properties: $c_u^s > 0$ and $c_u^s + c_o^s > 0$, in addition to $c_u^r > 0$ and $c_o^r > 0$. Buy-back and revenue-sharing contracts[5] target at mitigating retailers' downside risk. Thus, these contracts are downside-protection contracts.

Upside-Protection Contracts: A contract belongs to this category if the retailer's per-unit underage cost with this contract is larger than her per-unit underage cost with a wholesale-price contract while her per-unit overage cost remains the same. In other words, compared with the wholesale-price contract, contracts in this category effectively increase a retailer's cost of inventory shortage (upside risk due to facing potentially high demand) by shifting the cost of insufficient inventory to the retailer. Because having inventory shortage becomes more costly, these contracts provide the incentive for retailers to order and carry more inventory. Therefore, these contracts protect the supply chain against upside risk. Note that $c_u^s = c_u^I - c_u^r$ and $c_o^s = c_o^I - c_o^r$. Hence, these contracts reduce the supplier's underage cost while keeping his overage cost unchanged. Recall that the wholesale-price contract has properties such that $c_o^s < 0$ and $c_u^s + c_o^s = 0$. Hence, all contracts in this category satisfy the following properties: $c_o^s < 0$ and $c_u^s + c_o^s < 0$, in addition to $c_u^r > 0$ and $c_o^r > 0$. Penalty and rebate contracts target at increasing retailers' upside risk. Thus, these contracts are upside-protection contracts.

Two-Sided Protection Contracts: A contract belongs to this category if, with this contract, the *retailer*'s per-unit underage cost is larger and her per-unit overage cost is smaller than, respectively, her per-unit underage and overage costs with a wholesale-price contract. In other words, compared with a wholesale-price contract, contracts in this category simultaneously increase a retailer's cost of inventory shortage (upside risk) while reducing her cost of excess inventory (downside risk) by reallocating cost of demand-supply mismatch between the retailer and the supplier. Because having inventory shortage becomes more costly and having excess inventory becomes less costly, these contracts provide incentive for retailers to order and carry more inventory. Therefore, these contracts protect the supply chain against both upside and downside risks. Note that under any wholesale-price contract, $c_u^s > 0$, $c_o^s < 0$, and $c_u^s + c_o^s = 0$. Hence, all contracts in this category satisfy the following properties: $c_u^s \leq 0$, $c_o^s \geq 0$, and $c_u^s, c_o^s \neq 0$ simultaneously, in addition to $c_u^r > 0$ and $c_o^r > 0$. Combinations of some rebate and buy-back contracts studied in Taylor (2002) fall within the two-sided protection category.

No-Protection Contracts: A contract belongs to this category if the *retailer*'s per-unit underage and overage costs with this contract remain the same as her per-unit underage and overage costs with a wholesale-price contract, respectively. In other words, this category of contracts does not shift inventory risk (cost of overage or shortage due to uncertain demand) from one firm to another. Therefore, these contracts do not protect the supply chain against either the upside or the downside risk. Such contracts are primarily used to transfer and share profits (i.e., rearrange financial flows) without changing inventory risk profiles. Note that $c_u^s = c_u^I - c_u^r$ and $c_o^s = c_o^I - c_o^r$. Recall that the wholesale-price contract has properties such that $c_u^s > 0$, $c_o^s < 0$, and $c_u^s + c_o^s = 0$. Hence, all contracts in this category satisfy the following properties: $c_u^s > 0$, $c_o^s < 0$, and $c_u^s + c_o^s = 0$, in addition to $c_u^r > 0$ and $c_o^r > 0$. Wholesale-price and two-part tariff contracts neither increase retailers' upside risk nor decrease retailers' downside risk. Hence, they belong to the category of no-protection contracts.

The four categories of contracts defined above are collectively exhaustive and mutually exclusive within the class of contracts that do not depend on possible demand scenarios defined ex ante. Any such supply chain contract falls within exactly one of these four categories. Subsequent sections clarify why we introduce these four categories of contracts. Briefly, we show that the supplier always has incentive to leak information (i.e., nonleakage equilibrium does not exist) under a no-protection contract. In contrast, the supplier may have incentive not to leak (i.e., nonleakage equilibrium exists) only when firms use a downside-protection, an upside-protection, or a two-sided protection contract. In other words, we also show that contracts designed only to distribute financial flows (e.g., allocation of profits) without proper allocation of inventory risk among firms cannot prevent information leakage in a supply chain. In addition, we derive the necessary and sufficient nonleakage conditions for each category and provide transparent insights on the profit implications of these conditions and resulting contracts. To establish these results, we first analyze two benchmarks.

4. Benchmark Analysis

We consider two benchmarks by fixing the supplier's decision. The first benchmark is obtained by assuming that the supplier *never* leaks the incumbent's order quantity information to the entrant. The second benchmark is obtained by assuming that the supplier *always* leaks this information.[6] A scenario can be imagined in which strict contract terms are put in place to prevent any leakage (corresponding to the first benchmark) or another scenario in which all three firms know the incumbent's demand state (corresponding to the second benchmark). In the first benchmark, the retailers' equilibrium order quantities correspond to those under the equilibrium in which the supplier has no incentive to leak the incumbent's order quantity to the entrant. In the second benchmark, the retailers' equilibrium order quantities correspond to those under the equilibrium in which the supplier has incentive to leak. We use these benchmarks to determine the nonleakage equilibrium conditions in the subsequent sections.

We denote the critical ratio for retailers as

$$z_r \triangleq \frac{c_u^r}{c_u^r + c_o^r}.$$

The retailers' critical ratio depends on the contract T, which we drop from the argument of this ratio for expositional clarity. The following lemma is used later to derive and characterize equilibrium order quantities and the incentive compatibility conditions. We provide all proofs in Appendix B.

Lemma 1. *Under a contract T, suppose the incumbent's demand state is $t_i \in \{H, L\}$; then,*

1. *The incumbent's best response to the entrant's order quantity q_e, $q_{it_i}^*(q_e; z_r) = \arg\max_{q_i \geq 0} \Pi_{t_i}^i(q_i, q_e; T)$, is decreasing*[7] *in q_e and greater than $\mu_{t_i}^i + \eta F^{-1}(z_r)$.*

The entrant's best response to the incumbent's order quantity q_i, $q_e^*(q_i, \lambda; z_r) = \arg\max_{q_e \geq 0} \lambda \Pi_H^e(q_i, q_e; T) + (1-\lambda)\Pi_L^e(q_i, q_e; T) \triangleq \arg\max_{q_e \geq 0} \Pi^e(q_i, q_e, \lambda; T)$, is decreasing in q_i, *increasing in* μ_H^i *and* λ*, respectively, and greater than* $\mu_L^e + (1-\eta)F^{-1}(z_r)$.

2. *The incumbent's expected profit,* $\Pi_{t_i}^i(q_i, q_e; T)$*, is increasing in* $q_i \in [0, q_{it_i}^*(q_e; z_r)]$ *and decreasing in* $q_e \in \mathbb{R}_+$.

The entrant's expected profit, $\Pi^e(q_i, q_e, \lambda; T)$*, is increasing in* $q_e \in [0, q_e^*(q_i, \lambda; z_r)]$ *and decreasing in* $q_i \in \mathbb{R}_+$.

3. *Under full information and Nash game: suppose the entrant precisely knows the demand state* $t_i \in \{H, L\}$ *and both retailers place their orders with the supplier simultaneously; then, there exists a Nash equilibrium* $(q_{it_i}^F(z_r), q_{et_i}^F(z_r))$*, where*

$$q_{iH}^F(z_r) = q_{iH}^*(q_{eH}^F(z_r); z_r) \text{ and } q_{eH}^F(z_r) = q_e^*(q_{iH}^F(z_r), 1; z_r),$$
$$q_{iL}^F(z_r) = q_{iL}^*(q_{eL}^F(z_r); z_r) = \mu_L^i + \eta F^{-1}(z_r) \text{ and}$$
$$q_{eL}^F(z_r) = q_e^*(q_{iL}^F(z_r), 0; z_r) = \mu_L^e + (1-\eta)F^{-1}(z_r),$$

and has the following properties: $q_{iH}^F(z_r) > q_{iL}^F(z_r)$ *and* $q_{eH}^F(z_r) > q_{eL}^F(z_r)$.

4. *Under full information and Stackelberg game: suppose the entrant precisely knows the demand state* t_i *and the entrant places an order after observing the incumbent's order quantity* q_i*; then, the incumbent's equilibrium order quantity,* $q_{it_i}^S(z_r) \triangleq \arg\max_{q_i \geq 0} \Pi_{t_i}^i(q_i, q_e^*(q_i, \mathbf{1}\{t_i = H\}; z_r); T)$*, has the properties that* $q_{iH}^S(z_r) \geq q_{iH}^F(z_r)$ *and* $q_{iL}^S(z_r) = q_{iL}^F(z_r)$*, and the entrant's equilibrium order quantity,* $q_{et_i}^S(z_r) \triangleq q_e^*(q_{it_i}^S(z_r), \mathbf{1}\{t_i = H\}; z_r)$*, has the property that* $q_{eH}^S(z_r) \in (q_{eL}^F(z_r), q_{eH}^F(z_r)]$ *and* $q_{eL}^S(z_r) = q_{eL}^F(z_r)$.

Lemma 1 characterizes retailers' best responses to each other, their resulting profits, and their equilibrium order quantities. It shows that the retailers' critical ratio is a sufficient statistic that determines retailers' best responses to each other's order quantities. Part 1 characterizes each retailer's best response order quantity. Notice first that when one of the retailers (incumbent or entrant) orders more, the other retailer orders less as a response. With a higher level of inventory, each retailer is more likely to satisfy her own demand, resulting in fewer spillovers to the other retailer who, as a result, optimally orders less from the supplier. However, each retailer orders at least a minimum quantity regardless of how much the other retailer orders. For example, the incumbent optimally orders a minimum of $\mu_{t_i}^i + \eta F^{-1}(z_r)$ units. This amount is the optimal newsvendor order quantity when the incumbent faces only her primary source of demand D_i. Had the incumbent not received any spillover demand from the entrant (e.g., $a_{ie} = 0$), this minimum order would have been the incumbent's optimal order quantity. Hence, the incumbent orders and stocks enough inventory to hedge against at least her own uncertain demand plus some extra to satisfy possible spillover demand from the entrant. Similarly, the entrant's minimum order quantity is $\mu_L^e + (1-\eta)F^{-1}(z_r)$. In addition, the entrant optimally orders more when the incumbent faces a large market size (or when the entrant believes the incumbent's market size is more likely to be in a high demand state) because the incumbent's unsatisfied demand is more likely to spill over to the entrant in that case.

Part 2 shows that each retailer's profit decreases if the other retailer orders more from the supplier because each would face lower demand from the secondary demand source (i.e., the other retailer's unfilled demand). Each retailer would increase her optimal order quantity, increasing their corresponding profits, until they both reach to their corresponding best response order quantity as specified in part 1. These observations partially show why an incumbent retailer would like her order quantity (or knowledge of her demand state) to be kept confidential and not leaked to the entrant retailer.

Part 3 shows what happens when both retailers order from the supplier without observing each other's order decisions and the incumbent's demand state is public knowledge (i.e., under full information). When the entrant knows the incumbent's demand state to be low $t_i = L$, then a Nash equilibrium would be to order and hedge against only one's own uncertain demand (primary demand source). In other words, in the low demand state, the retailers do not order extra in the hopes of fulfilling other's unsatisfied demand.[8] However, when the incumbent's demand state is high $t_i = H$, they both order more than what is optimal for their respective primary demand source, to compete on satisfying unfilled (i.e., spillover) demand from the other retailer.

Part 4 shows what happens when the entrant observes the incumbent's order quantity (or when the supplier always leaks this information) before the entrant places an order with the supplier. When the entrant knows the incumbent's demand state to be low $t_i = L$, then a subgame perfect equilibrium would be to order and hedge against only one's own uncertain demand (primary demand source), as in part 3. In other words, in the low demand state, the retailers do not order extra in the hopes of fulfilling other's unsatisfied demand. However, when the entrant knows the incumbent's demand state is high $t_i = H$, both retailers order more than what is optimal for their respective primary demand source and compete on satisfying the secondary demand source (i.e., spill over demand from the other retailer). Unlike the equilibrium under the Nash game in part 3, however, the incumbent competes even more aggressively in this case. In particular, the incumbent uses her first mover advantage and orders a quantity higher than what she would have ordered under a simultaneous move game (i.e., $q_{iH}^S(z_r) \geq q_{iH}^F(z_r)$) to deter the entrant from taking advantage of the market information that the entrant obtains in a sequential move game (i.e., the incumbent's order quantity). By doing so, the incumbent makes it difficult for the entrant to compete and go after the incumbent's unsatisfied demand. As a result, the entrant's secondary source for demand shrinks (i.e., less spillover demand from the incumbent compared with a simultaneous move game). Hence, the entrant as the second mover optimally orders a quantity that is less than what she would have ordered under a simultaneous move game (i.e., $q^S_{eH}(z_r) \leq q^F_{eH}(z_r)$).

4.1. Benchmark I: Supplier Never Leaks Information

Suppose the supplier never leaks the incumbent's order quantity to the entrant. This benchmark helps us characterize the retailers' order quantities in the nonleakage equilibrium, if it exists, in which the supplier does not leak the incumbent's order quantity to the entrant. As a result, the entrant cannot infer the incumbent's demand state from the incumbent's order quantity. Hence, the entrant's order quantity does not depend on the incumbent's demand state. The incumbent and the entrant play a simultaneous move game with asymmetric information in the incumbent's demand state. Lemma 1 part 3 (i.e., Nash game with full information) helps us characterize Bayesian Nash equilibrium for this game with asymmetric information (as stated in the following theorem).

Theorem 1. *Under a contract T, suppose the supplier never leaks information; then, we have a Bayesian Nash equilibrium, as the solution to the following equations*

$$q^N_{it_i}(z_r) = \arg\max_{q_i \geq 0} \Pi^i_{t_i}(q_i, q^N_e(z_r); T) \text{ when } t_i \in \{H, L\}, \text{ and}$$

$$q^N_e(z_r) = \arg\max_{q_e \geq 0} \lambda \Pi^e_H(q^N_{iH}(z_r), q_e; T) + (1-\lambda)\Pi^e_L(q^N_{iL}(z_r), q_e; T),$$

which has the following properties:

1. $q^N_{iH}(z_r) > q^F_{iH}(z_r)$ *and* $q^N_{iL}(z_r) = q^F_{iL}(z_r)$;
2. $q^N_e(z_r) \in (q^F_{eL}(z_r), q^F_{eH}(z_r))$.

Recall from Lemma 1 part 3 that if the entrant were to know the incumbent's demand state, then her equilibrium order quantity would be $q^F_{et_i}(z_r)$ when $t_i \in \{H, L\}$. However, the entrant cannot make an ordering decision based on the incumbent's demand state because the supplier commits not to leak the incumbent's order quantity to the entrant. Thus, the entrant optimally balances between the possibility of the realization of the incumbent's high and low demand states and chooses an intermediate order quantity $q^N_e(z_r)$, which falls between her optimal order quantities under a full information case, that is, between $(q^F_{eL}(z_r), q^F_{eH}(z_r))$. Hence, anticipating the entrant's order strategy, when the demand state is high, the incumbent orders more than her order quantity under full information, $q^F_{iH}(z_r)$, because the entrant orders $q^F_{eH}(z_r) - q^N_e(z_r)$ units less than optimal had the entrant known the incumbent's demand state is high. In other words, the incumbent knows she is more likely to face high unfilled demand (spillovers) from the entrant. Hence, she orders and stocks more than what she would have ordered under full information ($q^N_{iH}(z_r) > q^F_{iH}(z_r)$) to capture this extra flow of demand due to the entrant's informational disadvantage. However, when the demand state is low, without knowing this information, the entrant orders more than needed. This higher order quantity will likely result in fewer spillover demands to the incumbent. Hence, anticipating this outcome, the incumbent optimally orders her minimal best response order quantity $q^F_{iL}(z_r)$, which, as we know from Lemma 1 part 3, is the order quantity to hedge against only the incumbent's uncertain primary demand source. Therefore, if the supplier does not leak the incumbent's order information, then the incumbent orders at least what she would order under the full information scenario. However, the entrant ends up ordering less from the supplier when incumbent's demand state is high. Note that if the supplier were to decide when to leak the incumbent's high demand state to the entrant, he would be trading off the benefit of leaking this information (i.e., higher order quantity from the entrant) versus the cost (i.e., lower order quantity from the incumbent). We will revisit this trade-off later to obtain nonleakage equilibrium conditions. Note also that, similar to Lemma 1, the retailers' critical ratio is a sufficient statistic that determines nonleakage equilibrium order quantities.

4.2. Benchmark II: Supplier Always Leaks Information

Suppose the supplier always leaks the incumbent's order information to the entrant. In this case, the entrant forms a posterior belief about the incumbent's demand state and determines her best response order quantity on the basis of the incumbent's order quantity and her updated belief. This benchmark characterizes a perfect Bayesian equilibrium (PBE) that includes two retailers' equilibrium order quantities and how the entrant updates her belief about the incumbent's demand state. Lemma 1 part 4 (i.e., Stackelberg game with full information) helps us characterize the resulting equilibriums for this game with asymmetric (i.e., incomplete) information.

Theorem 2. *Under a contract T, suppose the supplier always leaks information; then, PBE exists, with the following results. There exists $\underline{\mu}^i_H \geq \mu^i_L$ and $\underline{\lambda}(\mu^i_H) \in (0,1]$ for $\mu^i_H \in (\mu^i_L, \underline{\mu}^i_H)$, with the property that $\underline{\lambda}(\mu^i_H)$ is decreasing in $\mu^i_H \in (\mu^i_L, \underline{\mu}^i_H)$, such that*[9]

1. *(Separating equilibrium) When $\mu^i_H \geq \underline{\mu}^i_H$,*
   a. *The incumbent's order quantity is given by*

$$q^A_{it_i}(z_r) = \begin{cases} q^S_{iH}(z_r) & \text{when } t_i = H, \\ q^F_{iL}(z_r) & \text{when } t_i = L. \end{cases}$$

b. *The entrant's posterior belief on the probability that the incumbent's demand state is high after being informed with the incumbent's order quantity q_i is given by*

$$\lambda_e(q_i; z_r) = \begin{cases} 1 & \text{when } q_i \neq q_{iL}^F(z_r), \\ 0 & \text{when } q_i = q_{iL}^F(z_r). \end{cases}$$

c. *The entrant's best response to the incumbent's order quantity q_i is*

$$q_e^A(q_i; z_r) = \begin{cases} q_e^*(q_i, 1; z_r) & \text{when } q_i \neq q_{iL}^F(z_r), \\ q_e^*(q_i, 0; z_r) & \text{when } q_i = q_{iL}^F(z_r). \end{cases}$$

In the PBE, the incumbent's order quantity is $q_{it_i}^A(z_r)$, and the entrant's order quantity is

$$q_e^A(z_r) = \begin{cases} q_{eH}^S(z_r) & \text{when } q_{it_i}^A(z_r) = q_{iH}^S(z_r), \\ q_{eL}^F(z_r) & \text{when } q_{it_i}^A(z_r) = q_{iL}^F(z_r). \end{cases}$$

2. *(Pooling equilibrium) When $\mu_H^i \in (\mu_L^i, \underline{\mu}_H^i)$ and $\lambda \in (0, \underline{\lambda}(\mu_H^i))$,*

a. *The incumbent's order quantity is given by*

$$q_{it_i}^A(z_r) = q_{iL}^F(z_r), \ \forall\ t_i \in \{H, L\}.$$

b. *The entrant's posterior belief on the probability that the incumbent's demand state is high after being informed with the incumbent's order quantity q_i is given by*

$$\lambda_e(q_i; z_r) = \begin{cases} 1 & \text{when } q_i \neq q_{iL}^F(z_r), \\ \lambda & \text{when } q_i = q_{iL}^F(z_r). \end{cases}$$

c. *The entrant's best response to the incumbent's order quantity q_i is*

$$q_e^A(q_i; z_r) = \begin{cases} q_e^*(q_i, 1; z_r) & \text{when } q_i \neq q_{iL}^F(z_r), \\ q_e^*(q_i, \lambda; z_r) & \text{when } q_i = q_{iL}^F(z_r). \end{cases}$$

In the PBE, the incumbent's order quantity is $q_{iL}^F(z_r)$, and the entrant's order quantity is

$$q_e^A(z_r) = q_e^*(q_{iL}^F(z_r), \lambda; z_r),$$

which is greater than $q_{eL}^F(z_r)$.

3. *(Semiseparating equilibrium) When $\mu_H^i \in (\mu_L^i, \underline{\mu}_H^i)$ and $\lambda \in [\underline{\lambda}(\mu_H^i), 1)$,*

a. *The incumbent's order quantity is given by*

$$q_{it_i}^A(z_r) = \begin{cases} q_{iH}^S(z_r) \text{ w.p. } p^{SS} \text{ and } q_{iL}^F(z_r) \text{ w.p. } 1 - p^{SS} & \\ \qquad \text{when } t_i = H, \\ q_{iL}^F(z_r) \qquad \text{when } t_i = L, \end{cases}$$

where p^{SS} is a solution to the following equation

$$\Pi_H^i(q_{iH}^S(z_r), q_e^*(q_{iH}^S(z_r), 1; z_r); T) = \Pi_H^i\left(q_{iL}^F(z_r), q_e^*\left(q_{iL}^F(z_r), \frac{\lambda(1 - p^{SS})}{\lambda(1 - p^{SS}) + (1 - \lambda)}; z_r\right); T\right).$$

b. *The entrant's posterior belief on the probability that the incumbent's demand state is high after being informed with the incumbent's order quantity q_i is given by*

$$\lambda_e(q_i; z_r) = \begin{cases} 1 & \text{when } q_i \neq q_{iL}^F(z_r), \\ \bar{\lambda} & \text{when } q_i = q_{iL}^F(z_r), \end{cases}$$

where $\bar{\lambda} \triangleq \frac{\lambda(1-p^{SS})}{\lambda(1-p^{SS})+(1-\lambda)}$ and is strictly smaller than λ.

c. *The entrant's best response to the incumbent's order quantity q_i is*

$$q_e^A(q_i; z_r) = \begin{cases} q_e^*(q_i, 1; z_r) & \text{when } q_i \neq q_{iL}^F(z_r), \\ q_e^*(q_i, \bar{\lambda}; z_r) & \text{when } q_i = q_{iL}^F(z_r). \end{cases}$$

In the PBE, the incumbent's order quantity is $q_{it_i}^A(z_r)$, and the entrant's order quantity is

$$q_e^A(z_r) = \begin{cases} q_{eH}^S(z_r) & \text{when } q_{it_i}^A(z_r) = q_{iH}^S(z_r), \\ q_e^*(q_{iL}^F(z_r), \bar{\lambda}; z_r) & \text{when } q_{it_i}^A(z_r) = q_{iL}^F(z_r), \end{cases}$$

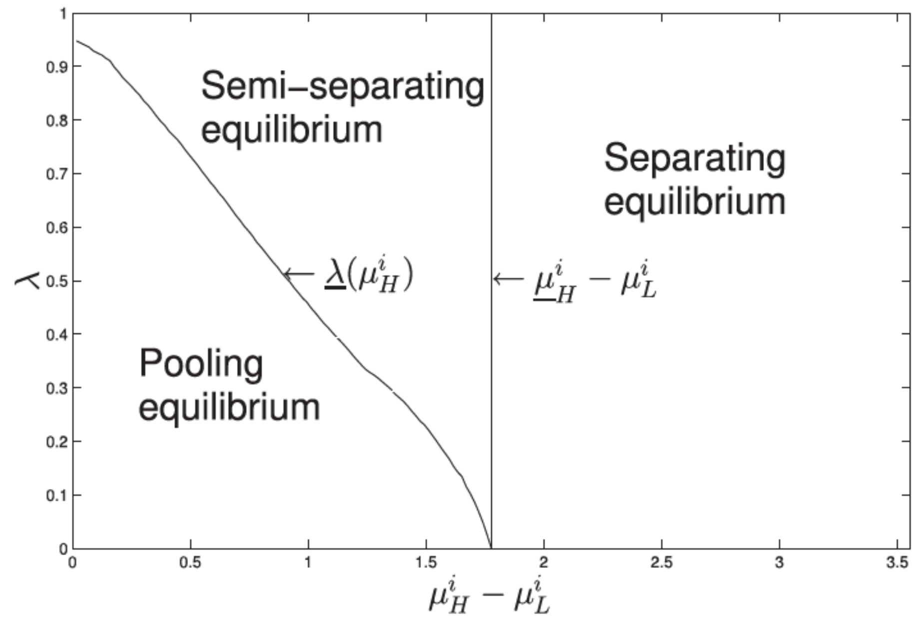
which is greater than $q_{eL}^F(z_r)$.

4. *These equilibriums survive the intuitive criterion.*

The theorem above shows that three types of equilibriums emerge when the supplier always leaks the information. When the incumbent's demand state is low, she has no incentive to pretend to be in the high demand state, that is, she always orders $q_{iL}^A(z_r) = q_{iL}^F(z_r)$ in all equilibriums [as shown in parts 1(a), 2(a), and 3(a)]. When the incumbent's demand state is high, she chooses between her equilibrium order quantities in her high and low demand states, $q_{iH}^S(z_r)$ and $q_{iL}^F(z_r)$, respectively. In this case, however, the incumbent can mislead the entrant to believe the demand state to be low by ordering $q_{iL}^F(z_r)$ units. Her incentive to mislead the entrant depends on two factors related to the market environment: (i) the difference between the incumbent's high and low demand states $\mu_H^i - \mu_L^i$ and (ii) the entrant's prior belief on the probability that the incumbent is facing a high demand state, λ. These two factors, therefore, affect the incumbent's ordering strategy, leading to three types of PBE: separating equilibrium, pooling equilibrium, and semiseparating equilibrium. Figure 1 illustrates how the market environment (specified by $\mu_H^i - \mu_L^i$ and λ) defines the equilibrium type.

Consider Theorem 2 part 1 the first market scenario in which the difference between the incumbent's high and low demand states is large ($\mu_H^i - \mu_L^i \geq \underline{\mu}_H^i - \mu_L^i$). When the incumbent's demand state is high, suppose the incumbent pretends to be in the low demand state by ordering $q_{iL}^F(z_r)$ and thus inducing the entrant to order less. In this case, the entrant stocks insufficient quantities, and as a result her unsatisfied demand flows to the incumbent, generating a larger secondary source of demand for the incumbent. However, to signal low demand the incumbent also ordered too little, and in doing so she even fails to fully satisfy her own primary

Figure 1. Three Types of PBE When $\mu_H^e = 10$, $\mu_L^e = 5$, $\lambda' = 0.5$, $a_{ie} = 0.8$, $a_{ei} = 0.8$, $\epsilon \sim U[-6, 6]$, $z_r = 0.7$



source of demand (because she placed an order to hedge against only a low demand state scenario) let alone take advantage of the large spillover from the entrant. Therefore, when the difference between the incumbent's high and low demand states is high, the incumbent has incentive to truthfully reveal her demand state by ordering $q_{iH}^S(z_r)$. As a result, we have a separating equilibrium in which the incumbent truthfully reveals her demand state and the entrant places her order under full information.

Consider Theorem 2 part 2 the second market scenario in which the difference between the incumbent's high and low demand states is small ($\mu_H^i - \mu_L^i < \underline{\mu}_H^i - \mu_L^i$) and the entrant's prior belief on the probability that the incumbent is facing a high demand state is low ($\lambda < \underline{\lambda}(\mu_H^i)$, or in other words, the entrant believes that the incumbent is highly likely to face a low demand state). When the incumbent's demand state is high, suppose the incumbent signals the entrant her high demand state by ordering more than her pooling equilibrium order quantity $q_{iL}^F(z_r)$. In this case the entrant updates her belief and assumes that the incumbent is facing a high demand state with probability one. Hence, the entrant also stocks more inventory to satisfy her primary source of demand. As a result, the entrant would have fewer unsatisfied demand, reducing the secondary source of demand for the incumbent. Hence, the incumbent has an incentive to order $q_{iL}^F(z_r)$ (and signal low demand) even when she faces a high demand, to increase her secondary source of demand (i.e., spillover demand from the entrant). However, the incumbent faces a trade off. By ordering more than her pooling equilibrium order quantity $q_{iL}^F(z_r)$ (and as a result signaling and possibly revealing her true high demand state), the incumbent could better serve her own primary source of demand. Note, however, that when the difference between the incumbent's high and low demand states is small, the expected cost of not satisfying some of her own primary source of demand can be dominated by the expected benefit of misleading the entrant and as a result satisfying a large secondary source of demand spilled over from the entrant. This later benefit dominates the cost especially when the entrant initially believes the incumbent is unlikely to face a high demand state (i.e., $\lambda < \underline{\lambda}(\mu_H^i)$). In this case, the incumbent benefits if the entrant does not update her prior belief about the probability of the incumbent having a high demand state. Hence, the incumbent optimally orders the same quantity $q_{iL}^F(z_r)$ regardless of her demand state. Therefore, the entrant cannot update her belief after observing the incumbent's order quantity. Hence, this market scenario leads to a pooling equilibrium in which neither the supplier nor the entrant can learn the incumbent's demand state. Notice also that in this pooling equilibrium, the incumbent orders the same amount as in the separating equilibrium with a low incumbent-demand state, which is less than what she would have ordered in the separating equilibrium with a high incumbent-demand state (i.e., $q_{it_i}^A(z_r) = q_{iL}^F(z_r) < q_{iH}^S(z_r)$). However, the entrant orders more than what she would have ordered in the separating case with a low incumbent-demand state (i.e., $q_e^A(z_r) > q_{eL}^F(z_r)$). This outcome is because the entrant does not know the incumbent's demand state in the pooling equilibrium but only believes the incumbent faces a high demand state with probability λ. So, she orders a little more than the separating equilibrium order quantity to hedge against the possibility that the incumbent is actually facing a high demand state.

Consider now Theorem 2 part 3 the third market scenario in which the difference between the incumbent's high and low demand states is small ($\mu_H^i - \mu_L^i < \underline{\mu}_H^i - \mu_L^i$), but unlike in part 2, the entrant's prior belief on the probability that the incumbent is facing a high demand state is high ($\lambda \geq \underline{\lambda}(\mu_H^i)$). When the incumbent's demand state is high, the incumbent faces a trade-off in signaling her demand state. Specifically by signaling and possibly deceiving the entrant that she is in the low demand state (i.e., by ordering from the supplier the pooling quantity $q_{iL}^F(z_r)$), the incumbent may increase her secondary source of demand (because she may induce the entrant to order less). However, by doing so, she may end up satisfying less of her own primary source of demand (because she orders and stocks less inventory than what is optimal for a high demand state). Unlike in the previous case, the entrant's prior belief on the probability that the incumbent is facing a high demand state is high. When the incumbent orders the same quantity $q_{iL}^F(z_r)$ regardless of her demand state (i.e., in a pooling ordering strategy), the entrant cannot update her belief. As a result, the entrant continues to believe that the incumbent faces a high demand state and orders a high quantity. Hence, this deception through always pooling does not substantially increase the incumbent's secondary source of demand. Instead, the incumbent can randomize her ordering strategy (i.e., her signal) when she faces a high demand state, that is, order $q_{iH}^S(z_r)$ with probability p^{SS} and order $q_{iL}^F(z_r)$ with probability $1 - p^{SS}$. In doing so, the incumbent garbles her signal. As a result, the entrant cannot perfectly infer from the incumbent's order quantity whether she is facing a low demand state. This garbling (i.e., sometimes sending a signal by placing a low order quantity $q_{iL}^F(z_r)$ and suggesting as though the incumbent is facing a low demand state) causes the entrant to update her belief and reduce the probability that the incumbent is facing a high demand state from λ down to $\bar{\lambda}$. As a result, the entrant orders less than what she would have ordered had she known the true state of the demand. Hence, by garbling her message in the high demand state (and deceiving the entrant), the incumbent faces more secondary demand from the entrant. In summary, when the incumbent places an order and signals her high demand state truthfully, the entrant learns the incumbent's demand state and orders $q_{eH}^S(z_r)$ to satisfy customers in a high-demand state environment. In this case, all parties have full information (similar to the separating equilibrium). However, when the incumbent orders $q_{iL}^F(z_r)$ units, the entrant cannot perfectly infer the incumbent's demand state. Hence, the entrant updates her belief and orders $q_e^A(z_r) = q_e^*(q_{iL}^F(z_r), \bar{\lambda}; z_r)$ units. In this case, only the incumbent knows her true demand state (similar to the pooling equilibrium). Hence, this market scenario leads to a semiseparating equilibrium.

Notice also that in this semiseparating equilibrium, the incumbent on average orders less than what she would have ordered in the separating equilibrium (i.e., when the incumbent faces the high demand state, she sometimes orders less to signal low demand state and deceive the entrant). In contrast, the entrant orders more than what she would have ordered in the separating equilibrium (i.e., when the incumbent faces the low demand state, the entrant cannot perfectly infer the incumbent's low demand state and orders more than the $q_{eL}^F(z_r)$).

Finally, we remark that all three types of equilibriums characterized above survive the intuitive criterion, which we discuss further in Appendix D.

5. Contract Properties for the Existence of Nonleakage Equilibrium

This section provides the necessary and sufficient nonleakage conditions for a wide range of contracts. In addition, we show that no-protection contracts fail to satisfy these conditions, whereas downside-protection, or upside-protection, or two-sided protection contracts can.

5.1. Necessary and Sufficient Nonleakage Conditions

Nonleakage equilibrium exists if and only if the following two conditions jointly hold: (i) the supplier has no incentive to leak the incumbent's nonleakage equilibrium order quantity $q_{it_i}^N(z_r)$ to the entrant, and (ii) the incumbent has no incentive to deviate from this equilibrium order quantity. We refer to the first condition as the supplier's incentive compatibility condition and the second one as the incumbent's incentive compatibility condition.

Theorem 3. *A contract T prevents information leakage if and only if both the supplier's and the incumbent's incentive compatibility conditions hold:*

1. *(Supplier's incentive compatibility condition) When the incumbent orders $q_{it_i}^N(z_r)$ for $t_i \in \{H, L\}$, the supplier has no incentive to leak this information to the entrant:*

$$\Pi_{t_i}^s(q_{it_i}^N(z_r), q_e^A(q_{it_i}^N(z_r); z_r); T) \leq \Pi_{t_i}^s(q_{it_i}^N(z_r), q_e^N(z_r); T), \text{ when } t_i \in \{H, L\}, \quad (5)$$

where $q_e^A(q_{iH}^N(z_r); z_r) \geq q_e^N(z_r)$.

2. *(Incumbent's incentive compatibility condition) When the incumbent's demand state is $t_i \in \{H, L\}$, for the incumbent's any order quantity $q_i \neq q_{it_i}^N(z_r)$, either the supplier has no incentive to leak this information to the entrant, $\Pi_{t_i}^s(q_i, q_e^A(q_i; z_r); T) \leq \Pi_{t_i}^s(q_i, q_e^N(z_r); T)$, or the supplier has incentive to leak this information to the entrant, but the incumbent cannot make more profit by deviating from the nonleakage equilibrium order quantity $q_{it_i}^N(z_r)$, $\Pi_{t_i}^i(q_i, q_e^A(q_i; z_r); T) \leq \Pi_{t_i}^i(q_{it_i}^N(z_r), q_e^N(z_r); T)$.*

Part 1 establishes the necessary conditions for the supplier not to leak information. By not leaking the incumbent's order quantity information to the entrant, the supplier earns at least as much as what he would have earned had he leaked this information. Note that the supplier's incentive compatibility conditions need to hold in the incumbent's *both* high and low demand states. Suppose this is not the case and only one condition holds. For example, suppose the supplier has no incentive to leak when the incumbent's demand state is low but has incentive to leak when the incumbent's demand state is high, so as to be more profitable by inducing the entrant to order more. When the incumbent's demand state is low, although the supplier does not leak, the entrant can correctly infer that the incumbent's demand state is low because she knows that the supplier would have leaked this information had the incumbent's demand state been high. Not leaking this information is a perfect signal that the incumbent's demand state is low. Hence, the incentive compatibility conditions need to hold in the incumbent's both demand states. Part 2 gives conditions under which the incumbent has no reason to deviate from the nonleakage equilibrium because doing so does not help her to collect more profit.

We note that Theorem 3 (and comparing Equations (4) and (5)) also reveals that information leakage is due to supply–demand mismatch cost. Under any contract T, the fixed fee transferred between each retailer $k \in \{i,e\}$ and the supplier, h^k, only affects the allocation of financial flows between the retailer and the supplier. However, this fixed fee does not affect whether the nonleakage conditions (Theorem 3) hold. This result identifies a new consequence of supply–demand mismatch—it affects firms' information-sharing prerogatives, besides affecting inventory-related costs.

Corollary 1. *When the incumbent's high demand state is sufficiently high,* $\mu^i_H \geq \underline{\mu}^i_H$*, and the entrant's demand state is perfectly and positively correlated with the incumbent's demand state,* $\lambda' = 1$*, the necessary and sufficient condition for a contract* T *to prevent information leakage is only the supplier's incentive compatibility condition.*

The corollary shows that in such a market environment, the entrant would correctly infer both her and the incumbent's demand states if the supplier leaked the incumbent's nonleakage equilibrium order quantity. The incumbent has no reason to deviate from the nonleakage equilibrium as long as the supplier's incentive compatibility conditions hold. To examine whether a contract supports the nonleakage equilibrium, it is necessary and sufficient to only examine whether the supplier's incentive compatibility conditions (5) hold in both the incumbent's high and low demand states.

5.2. Why Is Reallocating Inventory Risk Among Retailers and Supplier Necessary?

Here, we show why reallocating inventory risk among supplier and the retailers is necessary for a contract to prevent information leakage.

Theorem 4. *Nonleakage equilibrium does not exist under any no-protection contract (e.g., the wholesale price and two-part tariff contracts) in a supply chain with competing newsvendors.*

Recall that under a no-protection contract, the supplier is penalized from the supply chain's inventory shortage (i.e., $c^s_u > 0$) and is rewarded from the supply chain's excess inventory (i.e., $c^s_o = -c^s_u < 0$). Hence, under a no-protection contract, the supplier always has incentive to induce retailers to order more. As a result, when the incumbent's demand state is high, the supplier always has incentive to leak the incumbent's order quantity information to the entrant and benefit from the entrant ordering additional quantity.

No-protection contracts are designed to only enable profit sharing (i.e., rearranging financial flows) among firms in the supply chain. However, they do not change each firm's inventory risk profile. Therefore, a contract that only distributes financial flows without proper distribution of inventory risk among firms cannot prevent information leakage. For example, wholesale-price contracts and two-part tariff contracts cannot prevent information leakage in a supply chain with competing newsvendors. Two-part tariff contracts are often extensively used in practice because they achieve channel coordination in a variety of channel settings and are simple to administer. However, the above result identifies a negative consequence of using such contracts—they cannot prevent information leakage.

To prevent information leakage, we need contracts that facilitate allocation of inventory risk among firms in a supply chain, such that the supplier's incentive to induce retailers to order as much as possible can be eliminated. There are three possible ways to do so. One is to design contracts that shift retailers' cost of excess inventory to the supplier while keeping each party's cost of inventory shortage unchanged, such that (i) the supplier is not rewarded by the supply chain's excess inventory but incurs a cost from it (i.e., $c^s_o > 0$); and (ii) the supplier still incurs cost from the supply chain's inventory shortage (i.e., $c^s_u > 0$). These contracts are essentially downside-protection contracts with $c^s_o > 0$. The second possible way to prevent information leakage is to design contracts that shift the supplier's cost of inventory shortage to retailers while keeping each party's cost of excess inventory unchanged, such that (i) the supplier is not penalized from the supply chain's inventory shortage but is rewarded from it (i.e., $c^s_u < 0$); and (ii) the supplier is still rewarded from the supply chain's excess inventory (i.e., $c^s_o < 0$). These contracts are essentially upside-protection contracts with $c_u^s < 0$. The third possible way is to design contracts that simultaneously shift the supplier's cost of inventory shortage to retailers and shift retailers' cost of excess inventory to the supplier, such that $c_o^s \geq 0$ and $c_u^s \leq 0$.[10] These contracts are essentially two-sided protection contracts. The following theorem shows that only these three categories of contracts may lead to nonleakage equilibrium.

Theorem 5. *Nonleakage equilibrium exists only if the contract is a downside-protection contract with $c_o^s > 0$, or an upside-protection contract with $c_u^s < 0$, or a two-sided protection contract.*

A downside-protection contract with $c_o^s > 0$ shifts retailers' downside risk (due to facing low demand) to the supplier and penalizes the supplier for the supply chain's excess inventory. Hence, when the incumbent's demand state is high, a downside-protection contract can deter the supplier from leaking the incumbent's demand information and inducing the entrant to order more by increasing the supplier's expected cost of excess supply chain inventory. By contrast, the upside-protection contract with $c_u^s < 0$ shifts the supplier's upside risk (due to facing high demand) to retailers and rewards the supplier for the supply chain's inventory shortage. Hence, when the incumbent's demand state is high, an upside-protection contract can deter the supplier from leaking the incumbent's demand information and inducing the entrant to order more. A two-sided protection contract both shifts the supplier's upside risk (due to facing high demand) to retailers and shifts retailers' downside risk (due to facing low demand) to the supplier. In addition, a two-sided protection contract rewards the supplier for the supply chain's inventory shortage and penalizes the supplier for the supply chain's excess inventory. Hence, when the incumbent's demand state is high, a two-sided protection contract can deter the supplier from leaking the incumbent's demand information and inducing the entrant to order more by simultaneously increasing the supplier's expected reward of supply chain inventory shortage and his expected cost of excess supply chain inventory. Downside-protection, upside-protection, and two-sided protection contracts have the functionality to distribute inventory risk among firms, in addition to distributing financial flows. These results show that a contract needs to be able to redistribute inventory risk to prevent information leakage.

5.3. Further Characterization of Nonleakage Conditions

The supplier's profit in Equation (4) consists of two parts: fixed revenue, including transfer payments received from retailers, minus the variable of expected inventory cost due to possible shortage or excess (e.g., expected cost of supply–demand mismatch). The supplier's decision to leak depends only on the variable expected inventory cost (i.e., it is independent of the revenue). Specifically, this decision depends on how the supply chain's excess inventory and inventory shortage change if the supplier leaks the incumbent's order quantity information to the entrant. The next lemma characterizes the change in total excess inventory and unmet demand as a result of information leakage for a market environment with $\mu_H^i \geq \underline{\mu}_H^i$ and $\lambda' = 1$ (i.e., in such a market, the entrant can correctly infer the incumbent's demand states if the supplier were to leak the incumbent's nonleakage equilibrium order quantity).

Lemma 2. *We define $\Delta O_{t_i}(z_r)$ and $\Delta U_{t_i}(z_r)$ as the changes in the supply chain's excess inventory and unmet demand, respectively, due to information leakage when the incumbent's demand state is $t_i \in \{H, L\}$. We have $\Delta O_H(z_r) > 0$ and $\Delta U_H(z_r) < 0$ when the incumbent's demand state is high or $\Delta O_L(z_r) < 0$ and $\Delta U_L(z_r) > 0$ when the incumbent's demand state is low.*

When the incumbent's demand state is high, the entrant would order more if she were informed of the incumbent's order quantity. Such information leakage would lead to an increase in the supply chain's total inventory ($\Delta O_H(z_r) > 0$) and a decrease in the expected amount of unsatisfied demand ($\Delta U_H(z_r) < 0$). When the incumbent's demand state is low, the entrant would order less if she were informed of the incumbent's order quantity. Such leakage would lead to a decrease in the supply chain's total inventory and an increase in the supply chain's expected inventory shortage. We note that the magnitudes of such an excess and shortage due to leakage are functions of retailers' critical ratio z_r, because retailers face customers, not the supplier.

Theorem 6. *We define $\gamma_{t_i}(z_r) \triangleq \left|\frac{\Delta O_{t_i}(z_r)}{\Delta U_{t_i}(z_r)}\right|$ for $t_i \in \{H, L\}$. Then,*

1. *A downside-protection contract supports nonleakage equilibrium if and only if $c_u^s/c_o^s \leq \gamma_H(z_r)$ and $c_u^s/c_o^s \geq \gamma_L(z_r)$.*
2. *An upside-protection contract supports nonleakage equilibrium if and only if $c_u^s/c_o^s \geq \gamma_H(z_r)$ and $c_u^s/c_o^s \leq \gamma_L(z_r)$.*
3. *When $a_{ie} = a_{ei} = 1$, we have that every downside- (upside-) protection contract that supports nonleakage equilibrium has $z_r \in (0, z_I)$ ($z_r \in (z_I, 1)$).*

Parts 1 and 2 show that information leakage critically depends on the ratio of the change in excess inventory to the change in unsatisfied demand due to information leakage, that is, $\gamma_{t_i}(z_r)$. We refer to this ratio as the supply chain's *critical leakage ratio*, which is in closed form for normally distributed demand, and can be calculated easily using a simple spreadsheet. Similarly, given the terms of a downside contract or an upside contract, the supplier's cost ratio c_u^s/c_o^s can be calculated and it can be verified quickly whether the proposed contract can stop information leakage by checking the conditions in the above theorem. Hence, the cost ratio and critical leakage ratio are sufficient statistics that determine whether a contract supports nonleakage equilibrium.

Part 3 shows that in a very competitive market (i.e., when all unmet demand of a retailer flows to the other retailer), these contracts cannot prevent information leakage and at the same time coordinate the supply chain (i.e., $z_r \neq z_I$). To prevent information leakage, supply chain efficiency needs to be sacrificed to reward the supplier for being tight-lipped about the incumbent's demand information. With a downside-protection contract, avoiding information leakage leads the supply chain to carry less inventory than system optimal. By contrast, with an upside-protection contract, avoiding information leakage leads the supply chain to carry more inventory than system optimal. We consider other market conditions and two-sided protection contracts in the next section.

6. Quantifying Nonleakage Conditions and Supply Chain Profits

We first quantify and illustrate the necessary and sufficient conditions that prevent information leakage in a supply chain. Next we investigate how a contract that prevents information leakage affects supply chain profits. To do so, we focus on one representative contract from each of the three categories of contracts, namely buy-back contracts, rebate contracts, and the combination of the two. All our discussions continue to hold for other contracts. For each firm in the supply chain $k \in \{i, e, s\}$ and a given contract T, when the incumbent's demand state is $t_i \in \{H, L\}$, the expected profit is given by

$$\Pi^k_{t_i l}(T) \triangleq \begin{cases} \Pi^k_{t_i}(q^N_{it_i}(z_r), q^N_e(z_r); T) & \text{when } l = N, \\ \Pi^k_{t_i}(q^A_{it_i}(z_r), q^A_e(q^A_{it_i}(z_r); z_r); T) & \text{when } l = A, \end{cases}$$

where N refers to Benchmark I in Section 4.1 (i.e., the supplier never leaks the information) and A refers to Benchmark II in Section 4.2 (i.e., the supplier always leaks). The supply chain's total expected profit is given by $\Pi^I_{t_i l}(T) = \Pi^i_{t_i l}(T) + \Pi^e_{t_i l}(T) + \Pi^s_{t_i l}(T)$. The integrated firm's optimal expected profit, which does not depend on a contract, is given by

$$\Pi^{I,*}_{t_i} \triangleq \max_{q_i, q_e \geq 0} \Pi^I_{t_i}(q_i, q_e), \text{ for } t_i \in \{H, L\}.$$

In what follows, we first illustrate in a figure (e.g., Figure 2) the range of contract parameters that can be used to prevent information leakage. Next, we pick five contracts (within this figure) that prevent information leakage and report firms' corresponding expected profits in a table (e.g., Table 1). We also report the total supply chain's expected profit in the nonleakage equilibrium and compare that profit with the integrated firm's optimal expected profit. This comparison helps us quantify the expected cost of preventing information leakage to the supply chain (and also the expected value of demand information to the incumbent). We set $\mu^i_H = 12$, $\mu^i_L = 10$, $\mu^e_H = 8$, $\mu^e_L = 6$, $\lambda = 0.5$, $\lambda' = 0.9$, $\eta = 0.6$, $p = 1$, $c = 0.9$, $a_{ie} = 0.9$, $a_{ei} = 0.9$, and ϵ that is uniformly distributed on $[-3, 3]$ to define our base case market environment. We also discuss how the strength of competition among retailers affects outcomes by analyzing variants of the base case market environment.

6.1. Buy-Back Contracts

The shaded area in Figure 2 represents all buy-back contracts that prevent information leakage (i.e., all buy-back contracts that satisfy the conditions in Theorem 3). Recall that the main driver of information leakage is the supply–demand mismatch. We observe that for a given wholesale price w, to prevent information leakage, buy-back price b can neither be too low nor too high (i.e., $b \in [b_l, b_h]$). The lower curve represents the lowest buy-back price b_l that allows for no information leakage when the incumbent's demand state is *high*. For any buy-back contract that falls on this curve, the supplier is indifferent to leak because his additional expected benefit from a reduction in inventory-shortage cost due to leaking the incumbent's high demand state to the entrant is completely offset by his additional expected cost of excess inventory. For any buy-back price with $b' > b_l$, the supplier incurs more downside risk, that is, he is penalized more from each unsold product. Hence, under higher buy-back prices, the supplier has less incentive to leak the incumbent's high-demand state information to induce the entrant to order more.

Figure 2. Buy-Back Contracts That Prevent Information Leakage

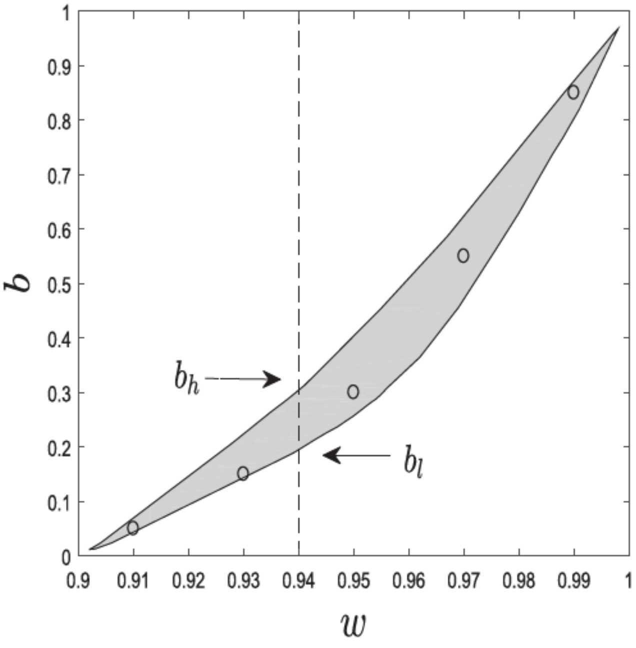


Table 1. Expected Profits Under Buy-Back Contracts That Prevent Information Leakage

$T=(w,b)$	t_i	$\Pi^i_{t_i l}(T)$	$\Pi^e_{t_i l}(T)$	$\Pi^s_{t_i l}(T)$	$\Pi^I_{t_i l}(T)$	$\Pi^I_{t_i l}(T)/\Pi^{I,*}_{t_i}$
(0.91,0.05)	H	1.04	0.47	0.15	1.66	99.39%
	L	0.76	0.44	0.12	1.32	99.58%
	$\mathbb{E}_{t_i}[\cdot]$	0.90	0.46	0.14	1.49	99.47%
(0.93,0.15)	H	0.78	0.35	0.52	1.65	98.79%
	L	0.57	0.33	0.42	1.32	99.58%
	$\mathbb{E}_{t_i}[\cdot]$	0.68	0.34	0.47	1.49	99.14%
(0.95,0.30)	H	0.61	0.28	0.77	1.66	99.39%
	L	0.45	0.26	0.62	1.33	99.95%
	$\mathbb{E}_{t_i}[\cdot]$	0.53	0.27	0.70	1.49	99.64%
(0.97,0.55)	H	0.34	0.15	1.17	1.66	99.39%
	L	0.25	0.14	0.93	1.32	99.58%
	$\mathbb{E}_{t_i}[\cdot]$	0.30	0.15	1.05	1.49	99.47%
(0.99,0.85)	H	0.11	0.05	1.49	1.65	98.79%
	L	0.08	0.05	1.19	1.32	99.58%
	$\mathbb{E}_{t_i}[\cdot]$	0.10	0.05	1.34	1.49	99.14%

However, this buy-back price must also be set such that $b' < b_h$ for the supplier not to leak the information when the incumbent's demand state is *low*. As the buy-back price increases, the supplier incurs more downside risk, that is, he is penalized more from each unsold product. Hence, under higher buy-back prices, the supplier has more incentive to leak the incumbent's low-demand state information to induce the entrant to order less. The upper boundary represents the price above which the benefit of leaking this information (hence a reduction in expected cost of inventory excess) surpasses the incremental expected cost of inventory shortage. Therefore, preventing information leakage requires the buy-back price to be neither too low nor too high.

Next, we analyze the impact of buy-back contracts on each firm's expected profit. Among buy-back contracts that fall in the shaded area of Figure 2, we randomly pick five of them (marked as circles) and report the resulting profits in Table 1. We observe that all leakage-proof buy-back contracts listed in Table 1 capture at least 99.14% of the integrated firm's optimal expected profit. We also remark that among all contracts that fall into the shaded area in Figure 2, a buy-back contract with (0.91, 0.06) results in the lowest channel efficiency of 99.10%. These observations show that a buy-back contract can both prevent information leakage and almost coordinate the supply chain (i.e., preventing information leakage results in a small cost).

6.2. Rebate Contracts

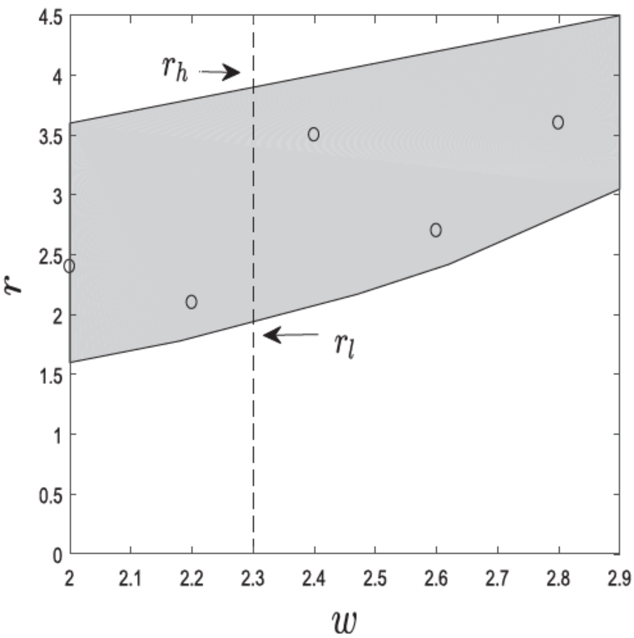
The shaded area in Figure 3 represents all rebate contracts that prevent information leakage. For a rebate contract (w,r,K), the fixed fee term K^k with $k \in \{i,e\}$ only facilitates the reallocation of the supply chain's total profit among firms in the supply chain. It does not affect order quantities and the resulting allocation of inventory risk. Hence, it has no effect on the supplier's leakage decision. Therefore, we drop it from our ensuing discussions. We observe that for a given wholesale price w, to prevent information leakage, rebate reward r can neither be too low nor too high (i.e., $r \in [r_l, r_h]$). The lower curve represents the lowest reward rate r_l that allows for no information leakage when the incumbent's demand state is *high*. For any rebate rate with $r' > r_l$, the supplier incurs less upside risk, that is, he rewards retailers more for each fulfilled demand. Hence, under higher rebate rates, the supplier has less incentive to leak the incumbent's high-demand state information to induce the entrant to order more. However, this rebate rate must also be set such that $r' < r_h$ for the supplier not to leak the information when the incumbent's demand state is *low*. Under higher rebate rates, the supplier has more incentive to leak the incumbent's low-demand state information to induce the entrant to order less. The upper boundary represents the rebate rate above which benefit of leaking this information surpasses the reward loss from the reduction of excess sales to the entrant (hence, excess inventory). Therefore, preventing information leakage requires rebate rate to be neither too low nor too high.

Among the rebate contracts that fall in the shaded area of Figure 3, we randomly pick five of them (marked in circles in Figure 3) and report the resulting profits in Table 2. We observe that a rebate contract that prevents information leakage is far from coordinating the supply chain. Among *all* rebate contracts that prevent information leakage in Figure 3 (represented by the shaded area, including the five contracts marked with circles), the one that achieves the highest total supply chain expected profit, $\mathbb{E}_{t_i}[\Pi^I_{t_i N}(T)]$, is the one with the wholesale price $w = 2.13$ and the rebate rate $r = 1.73$. Under this contract, the firms capture at most 84.79% of the integrated firm's optimal expected profit, $\mathbb{E}_{t_i}[\Pi^{I,*}_{t_i}]$. Hence, a rebate contract that prevents information leakage may result in firms leaving a large amount of profit on the table (i.e., the supply chain profit loss is as high as 15.21%). Preventing information leakage by using a rebate contract is costly in this base case market environment.

6.3. Combination of Buy-Back and Rebate Contracts

We combine the buy-back with rebate contracts to obtain a two-sided contract. Under such a contract (w,b,r,K), each retailer pays the supplier $w > c$ for each unit ordered and a fixed fee K. At the end of the sales season, the supplier buys back unsold inventories from retailers at unit price $b \in (0,w)$ and also offers a rebate reward $r > 0$ for each unit of sales. The retailers' and the supplier's profits are given by Equations (2)–(4), $h^i = K^i$, $h^e = K^e$, $c^r_u = p - w + r$, $c^r_o = w - b$, $c^s_u = w - r - c$, and $c^s_o = b - (w - c)$. Recall from Section 3 that a two-sided protection contract is defined to satisfy conditions $c^s_u \le 0$ and $c^s_0 \ge 0$. Therefore, we set $r \ge w - c$ and $b \ge w - c$.

Figure 3. Rebate Contracts That Prevent Information Leakage



We highlight an important observation. For the buy-back and rebate combined contract (which is a two-sided protection contract), there is no combination of contract parameters that can satisfy the necessary and sufficient conditions to prevent information leakage for the base case market environment. We remark that a downside, upside, or a two-sided contract cannot always be designed to prevent information leakage for every possible market environment. Hence, to prevent information leakage in the base case market environment, one needs to use the buy-back contract (among the three specific contracts considered here). In other words, a specific form of contract (e.g., rebate contract) cannot always be designed to prevent information leakage for all possible market environments. Hence, one needs to have a unified framework to understand how other types of contracts impact information leakage across a variety of market environments. The present paper provides such a framework.

To facilitate the analysis of the nonleakage conditions and the allocation of supply chain profit among firms for buy-back and rebate combined contracts, we consider a different market environment specified by $\mu_H^i = 12$, $\mu_L^i = 10$, $\mu_H^e = 8$, $\mu_L^e = 4$, $\lambda = 0.3$, $\lambda' = 0.6$, $\eta = 0.7$, $p = 1$, $c = 0.6$, $a_{ie} = 0.9$, $a_{ei} = 0.9$, and ϵ that is uniformly distributed on $[-5, 5]$. The shaded area in Figure 4 represents all buy-back and rebate combined contracts that prevent information leakage. The fixed fee K does not affect the allocation of inventory risks among firms and hence does not play a role in the supplier's leakage decision. Therefore, we focus on the effect of r and b for a family of contracts with the same

Table 2. Expected Profits Under Rebate Contracts That Prevent Information Leakage

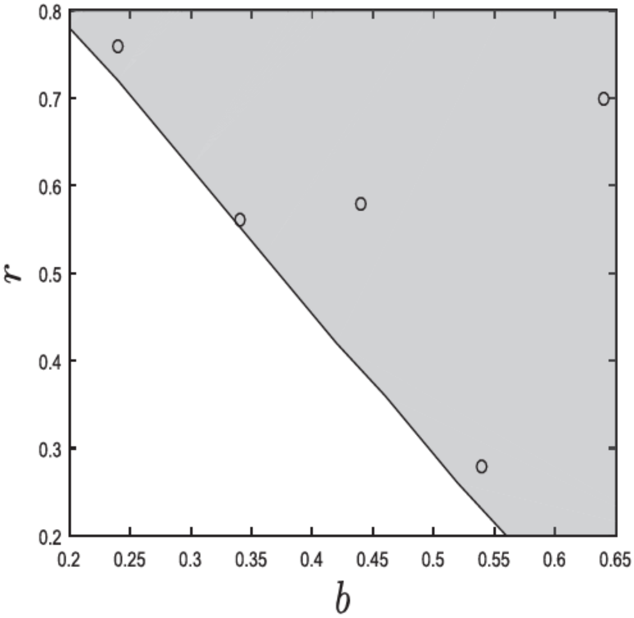
$T = (w, r)$	t_i	$\Pi^i_{t_i l}(T)$	$\Pi^e_{t_i l}(T)$	$\Pi^s_{t_i l}(T)$	$\Pi^I_{t_i l}(T)$	$\Pi^I_{t_i l}(T)/\Pi^{I*}_{t_i}$
(2.00,2.40)	H	0.69	0.55	0.15	1.39	83.22%
	L	0.07	0.08	0.04	0.19	14.33%
	$\mathbb{E}_{t_i}[\cdot]$	0.38	0.32	0.09	0.79	52.74%
(2.20,2.10)	H	0.82	0.58	0.18	1.58	94.60%
	L	0.23	0.21	0.12	0.56	42.25%
	$\mathbb{E}_{t_i}[\cdot]$	0.53	0.40	0.15	1.07	71.43%
(2.40,3.50)	H	0.65	0.37	0.27	1.29	77.24%
	L	0.02	0.01	0.11	0.14	10.56%
	$\mathbb{E}_{t_i}[\cdot]$	0.33	0.19	0.19	0.71	47.73%
(2.60,2.70)	H	0.65	0.37	0.27	1.29	77.24%
	L	0.02	0.01	0.11	0.14	10.56%
	$\mathbb{E}_{t_i}[\cdot]$	0.33	0.19	0.19	0.71	47.73%
(2.80,3.60)	H	0.69	0.66	0.12	1.47	88.01%
	L	0.04	0.06	0.19	0.29	21.88%
	$\mathbb{E}_{t_i}[\cdot]$	0.37	0.36	0.16	0.88	58.75%

wholesale price $w = 0.8$. We highlight four observations. First, from Figure 4, we observe that buy-back and rebate combined contracts with sufficiently high buy-back price b and rebate reward r prevent information leakage. Second, from Table 3, which reports profits under five randomly picked contracts from the shaded area in Figure 4, we observe that a buy-back and rebate combined contract that prevents information leakage is far from coordinating the supply chain. Among *all* contracts that prevent information leakage in Figure 4 (represented by the shaded area), the one that achieves the highest total supply chain expected profit, $\mathbb{E}_{t_i}[\Pi^I_{t_i N}(T)]$, is the one with the buy-back price $b = 0.30$ and the rebate rate $r = 0.62$. Under this contract, the firms capture 88.48% of the integrated firm's optimal expected profit, $\mathbb{E}_{t_i}[\Pi^{I*}_{t_i}]$. Hence, this combined contract that prevents information leakage results in firms leaving significant profit on the table (i.e., the supply chain profit loss is as high as 11.52%).

6.4. Impact of the Strength of Competition

The competition between the incumbent and the entrant is stronger when demand for each retailer is highly correlated (e.g., high λ') and a larger fraction of unfilled demand spills over to the other retailer. The previous analysis was for $\lambda' = 0.9$ (with $a_{ie} = a_{ei} = 0.9$) corresponding to what we refer to as strong competition. Here we focus on two more market environments with a medium strength competition, $\lambda' = 0.5$, and weak competition, $\lambda' = 0.1$, while keeping everything else the same. We also focus on the buy-back contract because it performs better than the others for the base case market environment. We defer the corresponding figures and tables to Appendix C. We highlight three observations. First, in these two market environments, rebate contracts can no longer prevent information leakage, but buy-back contracts can. Second, all our

Figure 4. Buy-Back and Rebate Combined Contracts That Prevent Information Leakage



observations reported for the strong competition case (i.e., $\lambda' = 0.9$) continue to hold under the medium and weak competition market environments. These observations illustrate that buy-back contracts can both prevent information leakage and nearly coordinate the supply chain in a wide range of market environments with different levels of competition.

6.5. Comparison of the Three Contracts in the Same Market Environment

Here we compare the above three categories of contracts in the same base case market environment specified in the beginning of this section. We study the supplier's equilibrium leakage decision and the supply chain's financial performance under the buy-back contract $(w, b) = (0.96, 0.05)$, the rebate contract $(w, r) = (2.76, 2.00)$, and the buy-back and rebate combined contracts $(w, b, r) = (2.80, 0.50, 2.00)$. All these contracts have the same retailers' critical ratio $z_r = 0.08$. Results are reported in Table 4. We observe that the buy-back contract can both prevent information leakage and almost coordinate the supply chain. However, in the same market environment, the rebate contract and the buy-back and rebate combined contract can only coordinate the supply chain at the expense of loosing the incumbent's privacy.

6.6. Summary of Results

Aforementioned analyses show that several categories of contracts can be used to prevent information leakage in a supply chain as long as they fall under one of the three categories of contracts (each having different necessary and sufficient conditions). However, these

Table 3. Expected Profits Under Buy-Back and Rebate Combined Contracts That Prevent Information Leakage

$T = (w, b, r)$	t_i	$\Pi^i_{t_i I}(T)$	$\Pi^e_{t_i I}(T)$	$\Pi^s_{t_i I}(T)$	$\Pi^I_{t_i I}(T)$	$\Pi^I_{t_i I}(T)/\Pi^{I,*}_{t_i}$
(0.80,0.24,0.76)	H	2.96	0.47	2.10	5.53	93.92%
	L	1.27	1.11	1.22	3.60	83.29%
	$\mathbb{E}_{t_i}[\cdot]$	2.12	0.79	1.66	4.57	89.42%
(0.80,0.34,0.56)	H	3.68	0.49	1.41	5.58	94.77%
	L	2.03	0.96	0.68	3.67	84.91%
	$\mathbb{E}_{t_i}[\cdot]$	2.86	0.73	1.05	4.63	90.60%
(0.80,0.44,0.58)	H	1.93	0.85	2.56	5.34	90.70%
	L	1.88	0.60	0.90	3.38	78.20%
	$\mathbb{E}_{t_i}[\cdot]$	1.91	0.73	1.73	4.36	85.41%
(0.80,0.54,0.28)	H	2.53	1.24	1.74	5.51	93.58%
	L	2.17	0.58	0.85	3.60	83.29%
	$\mathbb{E}_{t_i}[\cdot]$	2.35	0.91	1.30	4.56	89.23%
(0.80,0.64,0.70)	H	2.66	0.95	1.02	4.63	78.64%
	L	0.99	0.70	0.53	2.22	51.36%
	$\mathbb{E}_{t_i}[\cdot]$	1.83	0.83	0.77	3.43	67.09%

analyses also show that a downside, upside, or a two-sided protection contract cannot always prevent information leakage for every possible market environment. In addition, these analyses reveal that preventing information leakage (although necessary for firms to participate and work together) could be costly for the supply chain. Hence, these observations together further highlight the importance of having a unified framework to study different contract types and their roles in information leakage. Such a framework enables us to choose a contract that performs the best (i.e., prevents information leakage while not leaving much profit on the table). The present paper provides the framework and conditions to achieve this goal. Finally, we also observe that buy-back contracts prevent information leakage for a wide range of market environments while nearly coordinating the supply chain (leaving little money on the table).

7. Conclusion

In today's global supply chains, many customer-driven firms share a common supplier with their competitors. For example, Intel, a leading semiconductor manufacturer, supplies processors to computer assemblers that sell almost perfectly substitutable products. Various other industries and supply chain settings, as discussed extensively in the literature, face information leakage problem when two competitors' supply chains cross and utilize a common supplier. Hence, an incumbent such as Apple who has proprietary knowledge of the consumer electronics market would be concerned about losing this informational advantage to an entrant. Such concerns often lead a prominent manufacturer/retailer/brand to be very strict with her supplier agreement and choice. Knowing this fact, the supplier may prefer to design an agreement to eliminate such concerns. This paper provides a unified framework to characterize the conditions

Table 4. Expected Profits Under Three Categories of Contracts

T	Supplier's decision	t_i	$\Pi^i_{t_i l}(T)$	$\Pi^e_{t_i l}(T)$	$\Pi^s_{t_i l}(T)$	$\Pi^I_{t_i l}(T)$	$\Pi^I_{t_i l}(T)/\Pi^{I,*}_{t_i}$
Buy-back	Never leak	H	0.45	0.21	1.01	1.67	99.21%
(0.96,0.50)		L	0.33	0.19	0.80	1.33	99.03%
		$\mathbb{E}_{t_i}[\cdot]$	0.39	0.20	0.91	1.50	99.13%
Rebate	Always leak	H	1.23	0.26	0.19	1.67	99.55%
(2.76,2.00)		L	0.51	0.15	0.68	1.33	98.92%
		$\mathbb{E}_{t_i}[\cdot]$	0.87	0.20	0.43	1.50	99.27%
Buy-back and	Always leak	H	0.97	0.46	0.25	1.68	99.42%
rebate combined		L	0.38	0.36	0.60	1.33	99.17%
(2.80,0.50,2.00)		$\mathbb{E}_{t_i}[\cdot]$	0.67	0.41	0.42	1.50	99.31%

required for a wide range of supply chain contracts that can prevent information leakage in such supply chains. The framework can also enable a supplier to choose the best contract (among many) that can prevent information leakage while ensuring high channel efficiency.

Our study also shows that the incentive to leak information is due to how the supply chain's expected supply–demand mismatch costs are distributed among the supplier and retailers. We show that no-protection contracts, such as wholesale-price and two-part tariff contracts, cannot prevent information leakage. These contracts need to be amended in one of the three possible directions to prevent information leakage. The first direction is to amend the wholesale-price contract to shift some of the excess-inventory cost from the retailers to the supplier while keeping each party's inventory-shortage cost unchanged, such that the supplier is punished from the supply chain's unsold inventory and unmet demand. This amendment results in what we define as *downside-protection* contracts. Examples of this category of contracts include *buy-back* and *revenue-sharing* contracts. The second direction is to amend the contract to shift some of the inventory-shortage cost from the supplier to the retailers, while keeping each party's excess-inventory cost unchanged, such that the supplier is rewarded by the supply chain's unsold inventory and unmet demand. This amendment results in what we define as *upside-protection* contracts. Examples of this category of contracts include *penalty* and *rebate* contracts. The third direction is to amend the contract to both shift some of the inventory-shortage cost from the supplier to the retailers and shift some of the excess-inventory cost from the retailers to the supplier, such that the supplier is rewarded by the supply chain's unmet demand but is punished from the supply chain's unsold inventory. This amendment results in what we define as *two-sided protection* contracts. Examples of this category of contracts include combinations of some rebate and buy-back contracts.

The paper also characterizes conditions required for each category of contracts to prevent information leakage. These conditions specify how much inventory overage and underage costs need to be reallocated to prevent information leakage. We also show that a downside, upside, or a two-sided contract cannot always prevent information leakage for every possible market environment. Therefore, studying different forms of contracts under a unified framework is necessary to identify a leakage-proof contract for any possible market environment. The paper provides the framework to achieve such a goal. The paper also investigates the impact of each category of contracts on the supply chain profits. For example, we show that buy-back contracts (a downside-protection contract) are more effective than other contracts because they can prevent vertical information leakage while nearly coordinating the supply chain (i.e., leaving little money on the table) for a wide range of market environment. We believe these results have potential to help firms from various industries to reevaluate whether a preexisting contract used in their respective industry (e.g., rebates in the retail industry) could prevent information leakage, and decide how to amend contracts to prevent information leakage while attaining high supply chain profits.

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Appendix A. Summary of Notation

Demand	Cost parameters
t_i: the incumbent's demand state, $t_i \in \{H, L\}$	c: supplier's unit production cost
t_e: the entrant's demand state, $t_e \in \{H, L\}$	p: retailers' sales price
D_i: the incumbent's demand	c_u^r: retailers' unit underage cost
D_e: the entrant's demand	c_o^r: retailers' unit overage cost
$\mu_{t_i}^i$: the incumbent's mean demand in state t_i	c_u^s: supplier's unit underage cost
$\mu_{t_e}^e$: the entrant's mean demand in state t_i	c_o^s: supplier's unit overage cost
ϵ: noise term of the aggregate demand	c_u^I: integrated firm's unit underage cost
$F(\cdot)$: cumulative distribution function of ϵ	c_o^I: integrated firm's unit overage cost
λ: probability that the incumbent's demand state is high	z_r: retailers' critical ratio
λ': probability that the entrant's demand state is high conditional on that the incumbent's demand state is high	
η: the incumbent's market share on ϵ	
a_{ie}: fraction of the incumbent's unmet demand that flows to the entrant	
a_{ei}: fraction of the entrant's unmet demand that flows to the incumbent	

Decision variables
q_k: retailer k's order quantity, where $k \in \{i, e\}$
$q_{it_i}^*(q_e; z_r)$: the incumbent's best response to the entrant's order quantity q_e, when the incumbent's demand state is t_i
$q_e^*(q_i, \lambda; z_r)$: the entrant's best response to the incumbent's order quantity q_i, under the entrant's belief that the probability that incumbent's demand state is high is λ
$q_{kt_i}^F(z_r)$: retailer k's equilibrium order quantity in the full information scenario when the incumbent's demand state is t_i, where $k \in \{i, e\}$ and $t_i \in \{H, L\}$
$q_{it_i}^S(z_r)$: the incumbent's equilibrium order quantity in the Stackelberg game when the incumbent's demand state is t_i, where $k \in \{i, e\}$ and $t_i \in \{H, L\}$
$q_{it_i}^N(z_r)$: the incumbent's equilibrium order quantity when the incumbent's demand state is t_i given that the supplier never leaks
$q_e^N(z_r)$: the entrant's equilibrium order quantity given that the supplier never leaks
$q_{it_i}^A(z_r)$: the incumbent's equilibrium order quantity when the incumbent's demand state is t_i given that the supplier always leaks
$q_e^A(q_i; z_r)$: the entrant's best response to the incumbent's order quantity q_i in the equilibrium given that the supplier always leaks
$\lambda_e(q_i; z_r)$: the entrant's posterior belief on the probability that the incumbent's demand state is high after being informed with the incumbent's order quantity q_i given that the supplier always leaks

Profit functions
$\Pi_{t_i}^I(q_i, q_e)$: the integrated firm's expected profit when the incumbent's demand state is t_i
$\Pi_{t_i}^k(q_i, q_e; T)$: firm k's expected profit when the incumbent's demand state is t_i, where $k \in \{i, e, s\}$

Appendix B. Proofs

Proof of Lemma 1. For part 1, we begin with proving the properties of the incumbent's best response function. For the incumbent's profit function, when the incumbent's demand state is high, we have

$$\frac{\partial \Pi_H^i(q_i, q_e; T)}{\partial q_i} = -\left(c_u^r + c_o^r\right)\left(\lambda' \mathbb{P}\left(\mu_H^i + \eta\epsilon + a_{ei}\left(\mu_H^e + (1-\eta)\epsilon - q_e\right)^+ \le q_i\right) + (1-\lambda')\mathbb{P}\left(\mu_H^i + \eta\epsilon + a_{ei}\left(\mu_L^e + (1-\eta)\epsilon - q_e\right)^+ \le q_i\right)\right) + c_u^r.$$

Hence, $\frac{\partial^2 \Pi_H^i(q_i, q_e; T)}{\partial q_i \partial q_e} \le 0$ and $\frac{\partial^2 \Pi_H^i(q_i, q_e; T)}{\partial q_i^2} \le 0$.

Following Topkis (1998, theorem 2.8.1), the property that $\frac{\partial^2 \Pi_H^i(q_i, q_e; T)}{\partial q_i \partial q_e} \le 0$ implies that $q_{iH}^*(q_e; z_r)$ is decreasing in q_e.

In addition, we have

$$\begin{aligned}
&\frac{\partial \Pi_H^i(q_i, q_e; T)}{\partial q_i}\Big|_{q_i = \mu_H^i + \eta F^{-1}(z_r)} \\
&= -\left(c_u^r + c_o^r\right)\left(\lambda' \mathbb{P}\left(\eta\left(\epsilon - F^{-1}(z_r)\right) + a_{ei}\left(\mu_H^e + (1-\eta)\epsilon - q_e\right)^+ \le 0\right) + (1-\lambda')\mathbb{P}\left(\eta\left(\epsilon - F^{-1}(z_r)\right) + a_{ei}\left(\mu_L^e + (1-\eta)\epsilon - q_e\right)^+ \le 0\right)\right) + c_u^r \\
&\ge -\left(c_u^r + c_o^r\right)\left(\lambda' \mathbb{P}\left(\eta\left(\epsilon - F^{-1}(z_r)\right) \le 0\right) + (1-\lambda')\mathbb{P}\left(\eta\left(\epsilon - F^{-1}(z_r)\right) \le 0\right)\right) + c_u^r \\
&= -\left(c_u^r + c_o^r\right)\mathbb{P}\left(\epsilon - F^{-1}(z_r) \le 0\right) + c_u^r \\
&= 0.
\end{aligned}$$

Hence, the property that $\frac{\partial^2 \Pi_H^i(q_i, q_e; T)}{\partial q_i^2} \le 0$ implies that $q_{iH}^*(q_e; z_r) \ge \mu_H^i + \eta F^{-1}(z_r)$.

When the incumbent's demand state is low, we have

$$\frac{\partial \Pi^i_L(q_i,q_e;T)}{\partial q_i} = -(c^r_u + c^r_o)\mathbb{P}(\mu^i_L + \eta\epsilon + a_{ei}(\mu^e_L + (1-\eta)\epsilon - q_e)^+ \le q_i) + c^r_u.$$

Hence, $\frac{\partial^2 \Pi^i_L(q_i,q_e;T)}{\partial q_i \partial q_e} \le 0$ and $\frac{\partial^2 \Pi^i_L(q_i,q_e;T)}{\partial q_i^2} \le 0$.

Following Topkis (1998, theorem 2.8.1), the property that $\frac{\partial^2 \Pi^i_L(q_i,q_e;T)}{\partial q_i \partial q_e} \le 0$ implies that $q^*_{iL}(q_e;z_r)$ is decreasing in q_e.

In addition, we have

$$\begin{aligned}\frac{\partial \Pi^i_L(q_i,q_e;T)}{\partial q_i}\Big|_{q_i=\mu^i_L+\eta F^{-1}(z_r)} &= -(c^r_u + c^r_o)\mathbb{P}(\eta(\epsilon - F^{-1}(z_r)) + a_{ei}(\mu^e_L + (1-\eta)\epsilon - q_e)^+ \le 0) + c^r_u \\ &\ge -(c^r_u + c^r_o)\mathbb{P}(\eta(\epsilon - F^{-1}(z_r)) \le 0) + c^r_u \\ &= 0.\end{aligned}$$

Hence, the property that $\frac{\partial^2 \Pi^i_L(q_i,q_e;T)}{\partial q_i^2} \le 0$ implies that $q^*_{iL}(q_e;z_r) \ge \mu^i_L + \eta F^{-1}(z_r)$.

Now, we prove the properties of the entrant's best response function. We have

$$\begin{aligned}\frac{\partial \Pi^e_H(q_i,q_e;T)}{\partial q_e} &= -(c^r_u + c^r_o)(\lambda'\mathbb{P}(\mu^e_H + (1-\eta)\epsilon + a_{ie}(\mu^i_H + \eta\epsilon - q_i)^+ \le q_e) \\ &\quad + (1-\lambda')\mathbb{P}(\mu^e_L + (1-\eta)\epsilon + a_{ie}(\mu^i_H + \eta\epsilon - q_i)^+ \le q_e)) + c^r_u\end{aligned}$$

and

$$\begin{aligned}\frac{\partial \Pi^e_L(q_i,q_e;T)}{\partial q_e} &= -(c^r_u + c^r_o)\mathbb{P}(\mu^e_L + (1-\eta)\epsilon \\ &\quad + a_{ie}(\mu^i_L + \eta\epsilon - q_i)^+ \le q_e) + c^r_u.\end{aligned}$$

Hence, we have

$$\frac{\partial^2 \Pi^e(q_i,q_e,\lambda;T)}{\partial q_e \partial q_i} = \lambda \frac{\partial^2 \Pi^e_H(q_i,q_e;T)}{\partial q_e \partial q_i} + (1-\lambda)\frac{\partial^2 \Pi^e_L(q_i,q_e;T)}{\partial q_e \partial q_i} \le 0.$$

Therefore, following Topkis (1998, theorem 2.8.1), we have that $q^*_e(q_i,\lambda;z_r)$ is decreasing in q_i.

We have

$$\frac{\partial^2 \Pi^e(q_i,q_e,\lambda;T)}{\partial q_e \partial \mu^i_H} = \lambda \frac{\partial^2 \Pi^e_H(q_i,q_e;T)}{\partial q_e \partial \mu^i_H} + (1-\lambda)\frac{\partial^2 \Pi^e_L(q_i,q_e;T)}{\partial q_e \partial \mu^i_H} \ge 0.$$

Therefore, following Topkis (1998, theorem 2.8.1), we have that $q^*_e(q_i,\lambda;z_r)$ is increasing in μ^i_H.

We have

$$\frac{\partial^2 \Pi^e(q_i,q_e,\lambda;T)}{\partial q_e \partial q_i} = \frac{\partial \Pi^e_H(q_i,q_e;T)}{\partial q_e} - \frac{\partial \Pi^e_L(q_i,q_e;T)}{\partial q_e} \ge 0,$$

where the inequality follows properties that $\mu^i_H > \mu^i_L$ and $\mu^e_H > \mu^e_L$. Therefore, following Topkis (1998, theorem 2.8.1), we have that $q^*_e(q_i,\lambda;z_r)$ is increasing in λ.

In addition, we have

$$\begin{aligned}\frac{\partial \Pi^e_H(q_i,q_e;T)}{\partial q_e}\Big|_{q_e=\mu^e_L+(1-\eta)F^{-1}(z_r)} &= -(c^r_u + c^r_o)(\lambda'\mathbb{P}(\mu^e_H - \mu^e_L + (1-\eta)(\epsilon - F^{-1}(z_r)) \\ &\quad + a_{ie}(\mu^i_H + \eta\epsilon - q_i)^+ \le 0) \\ &\quad + (1-\lambda')\mathbb{P}((1-\eta)(\epsilon - F^{-1}(z_r)) \\ &\quad + a_{ie}(\mu^i_L + \eta\epsilon - q_i)^+ \le 0)) + c^r_u \\ &> -(c^r_u + c^r_o)(\lambda'\mathbb{P}((1-\eta)(\epsilon - F^{-1}(z_r)) \le 0) \\ &\quad + (1-\lambda')\mathbb{P}((1-\eta)(\epsilon - F^{-1}(z_r)) \le 0)) + c^r_u \\ &= -(c^r_u + c^r_o)\mathbb{P}(\epsilon - F^{-1}(z_r) \le 0) + c^r_u \\ &= 0,\end{aligned}$$

where the inequality holds since $\mu^e_H > \mu^e_L$ and $z_r \in (0,1)$ imply $F^{-1}(z_r) \in (\underline{\epsilon}, \bar{\epsilon})$, and

$$\begin{aligned}\frac{\partial \Pi^e_L(q_i,q_e;T)}{\partial q_e}\Big|_{q_e=\mu^e_L+(1-\eta)F^{-1}(z_r)} &= -(c^r_u + c^r_o)\mathbb{P}((1-\eta)(\epsilon - F^{-1}(z_r)) + a_{ie}(\mu^i_L + \eta\epsilon - q_i)^+ \le 0) + c^r_u \\ &\ge -(c^r_u + c^r_o)\mathbb{P}((1-\eta)(\epsilon - F^{-1}(z_r)) \le 0) + c^r_u \\ &= 0.\end{aligned}$$

Therefore, the property that $\frac{\partial^2 \Pi^e(q_i,q_e,\lambda;T)}{\partial q_e^2} = \lambda \frac{\partial^2 \Pi^e_H(q_i,q_e;T)}{\partial q_e^2} + (1-\lambda)\frac{\partial^2 \Pi^e_L(q_i,q_e;T)}{\partial q_e^2} \le 0$ implies that for $\lambda > 0$, $q^*_e(q_i,\lambda;z_r) = \arg\max_{q_e \ge 0} \Pi^e(q_i,q_e,\lambda;T) > \mu^e_L + (1-\eta)F^{-1}(z_r)$, and for $\lambda = 0$, $q^*_e(q_i,0;z_r) \ge \mu^e_L + (1-\eta)F^{-1}(z_r)$.

For part 2, for the incumbent's profit function, when the incumbent's demand state is $t_i \in \{H,L\}$, following the property that $\frac{\partial \Pi^i_{t_i}(q_i,q_e;T)}{\partial q_i} \ge 0$ for $q_i \le q^*_{it_i}(q_e;z_r)$, established above in proving part 1 of this lemma, we have that $\Pi^i_{t_i}(q_i,q_e;T)$ is increasing in $q_i \in [0, q^*_{it_i}(q_e;z_r)]$. In addition, the property that $\Pi^i_{t_i}(q_i,q_e;T)$ is decreasing in $q_e \in \mathbb{R}_+$ holds since the term in the expectation on the R.H.S. of Equation (2), $\min\{\bar{D}_i, q_i\} = \min\{D_i + a_{ei}(D_e - q_e)^+, q_i\}$, is decreasing in q_e.

For the entrant's profit function, following the property that $\frac{\partial \Pi^e(q_i,q_e,\lambda;T)}{\partial q_e} \ge 0$ for $q_e \le q^*_e(q_i,\lambda;z_r)$, established above in proving part 1 of this lemma, we have that $\Pi^e(q_i,q_e,\lambda;T)$ is increasing in $q_e \in [0, q^*_e(q_i,\lambda;z_r)]$. In addition, the property that $\Pi^e(q_i,q_e,\lambda;T)$ is decreasing in $q_i \in \mathbb{R}_+$ holds since the term in the expectation on the R.H.S. of Equation (3), $\min\{\bar{D}_e, q_e\} = \min\{D_e + a_{ie}(D_i - q_i)^+, q_e\}$, is decreasing in q_i.

For part 3, suppose the entrant precisely knows the incumbent's demand state $t_i \in \{H,L\}$; then, $(q^F_{it_i}(z_r), q^F_{et_i}(z_r))$ is a Nash equilibrium if and only if it satisfies the conditions

$$q^F_{it_i}(z_r) = \arg\max_{q_i \ge 0} \Pi^i_{t_i}(q_i, q^F_{et_i}(z_r); T),$$

$$q^F_{et_i}(z_r) = \arg\max_{q_e \ge 0} \Pi^e_{t_i}(q^F_{it_i}(z_r), q_e; T).$$

Following the properties that $\frac{\partial^2\Pi^i_{t_i}(q_i,q_e;T)}{\partial q_i^2}\le 0$ and $\frac{\partial^2\Pi^e_{t_i}(q_i,q_e;T)}{\partial q_e^2}\le 0$, $(q^F_{it_i}(z_r), q^F_{et_i}(z_r))$ needs to be solutions to the following equations

$$\left.\frac{\partial\Pi^i_{t_i}(q_i,q^F_{et_i}(z_r);T)}{\partial q_i}\right|_{q_i=q^F_{it_i}(z_r)}=0,$$
$$\left.\frac{\partial\Pi^e_{t_i}(q^F_{it_i}(z_r),q_e;T)}{\partial q_e}\right|_{q_e=q^F_{et_i}(z_r)}=0.$$

If we rename q_i as q_1 and q_e as q_2, then we can directly repeat the proof of Netessine and Rudi (2003, proposition 4) to get the property that there exists a solution to above equations. Therefore, a Nash equilibrium $(q^F_{it_i}(z_r), q^F_{et_i}(z_r))$ exists.

Now, we show that when the incumbent's demand state is low, one Nash equilibrium takes the form $q^F_{iL}(z_r)=\mu^i_L+\eta F^{-1}(z_r)$ and $q^F_{eL}(z_r)=\mu^e_L+(1-\eta)F^{-1}(z_r)$. This holds since

$$\begin{aligned}&\left.\frac{\partial\Pi^i_L(q_i,q^F_{eL}(z_r);T)}{\partial q_i}\right|_{q_i=q^F_{iL}(z_r)}\\&=-(c^r_u+c^r_o)\mathbb{P}\big(\eta(\epsilon-F^{-1}(z_r))+a_{ei}(1-\eta)(\epsilon-F^{-1}(z_r))^+\le 0\big)+c^r_u\\&=-(c^r_u+c^r_o)\mathbb{P}(\epsilon-F^{-1}(z_r)\le 0)+c^r_u\\&=0,\end{aligned}$$

and

$$\begin{aligned}&\left.\frac{\partial\Pi^e_L(q^F_{iL}(z_r),q_e;T)}{\partial q_e}\right|_{q_e=q^F_{eL}(z_r)}\\&=-(c^r_u+c^r_o)\mathbb{P}\big((1-\eta)(\epsilon-F^{-1}(z_r))+a_{ie}\eta(\epsilon-F^{-1}(z_r))^+\le 0\big)+c^r_u\\&=-(c^r_u+c^r_o)\mathbb{P}(\epsilon-F^{-1}(z_r)\le 0)+c^r_u\\&=0.\end{aligned}$$

In addition, following part 1 in this lemma that $q^*_{iH}(q_e;z_r)\ge\mu^i_H+\eta F^{-1}(z_r)$ and the assumption that $\mu^i_H>\mu^i_L$, we have $q^F_{iH}(z_r)>q^F_{iL}(z_r)$. Following the proof of part 1 in this lemma that $q^*_e(q_i,1;z_r)>\mu^e_L+(1-\eta)F^{-1}(z_r)$, we have $q^F_{eH}(z_r)>q^F_{eL}(z_r)$.

For part 4, for $t_i\in\{H,L\}$ and any $q_i\le q^F_{it_i}(z_r)$, we have $\Pi^i_{t_i}(q_i,q^*_e(q_i,\mathbf{1}\{t_i=H\};z_r);T)\le\Pi^i_{t_i}(q_i,q^*_e(q^F_{it_i}(z_r),\mathbf{1}\{t_i=H\};\ z_r);T)=\Pi^i_{t_i}(q_i,q^F_{et_i}(z_r);T)\le\Pi^i_{t_i}(q^F_{it_i}(z_r),q^F_{et_i}(z_r);T)=\Pi^i_{t_i}(q^F_{it_i}(z_r),q^*_e(q^F_{it_i}(z_r),\mathbf{1}\{t_i=H\};z_r);T)$, where the first inequality follows parts 1 and 3 of this lemma that $q_i\le q^F_{iL}(z_r)=\mu^i_L+\eta F^{-1}(z_r)<\mu^i_H+\eta F^{-1}(z_r)\le q^F_{iH}(z_r)$, part 1 of this lemma that $q^*_e(q_i,\lambda;z_r)$ is decreasing in q_i, and part 2 of this lemma that $\Pi^i_{t_i}(q_i,q_e;T)$ is decreasing in q_e, the first and the second equalities follow the property that $q^*_e(q^F_{it_i}(z_r),\mathbf{1}\{t_i=H\};z_r)=q^F_{et_i}(z_r)$, the second inequality follows the property that $q^F_{it_i}(z_r)=q^*_{it_i}(q^F_{et_i}(z_r);z_r)$. Therefore, $q^S_{it_i}(z_r)\ge q^F_{it_i}(z_r)$.

Now, we analyze the scenario that $t_i=L$. Recall from part 1 of this lemma that $q^*_e(q_i,0;z_r)\ge q^F_{eL}(z_r)$. For any q_i, we have $\Pi^i_L(q_i,q^*_e(q_i,0;z_r);T)\le\Pi^i_L(q_i,q^F_{eL}(z_r);T)\le\Pi^i_L(q^F_{iL}(z_r),q^F_{eL}(z_r);T)=\Pi^i_L(q^F_{iL}(z_r),q^*_e(q^F_{iL}(z_r),0;z_r);T)$, where the first inequality follows part 1 of this lemma that $q^*_e(q_i,0;z_r)\ge q^F_{eL}(z_r)$ and part 2 of this lemma that $\Pi^i_L(q_i,q_e;T)$ is decreasing in q_e, the second inequality follows part 3 of this lemma that $q^F_{iL}(z_r)=q^*_{iL}(q^F_{eL}(z_r);z_r)$, the equality follows part 3 of this lemma that $q^*_e(q^F_{iL}(z_r),0;z_r)=q^F_{eL}(z_r)$. Therefore, $q^S_{iL}(z_r)=q^F_{iL}(z_r)$.

Next, we analyze $q^S_{et_i}(z_r)$. When $t_i=H$, following the proof of part 1 of this lemma, we have $q^S_{eH}(z_r)=q^*_e(q^S_{iH}(z_r),1;z_r)>\mu^e_L+(1-\eta)F^{-1}(z_r)=q^F_{eL}(z_r)$. In addition, following part 1 of this lemma that $q^*_e(q_i,1;z_r)$ is decreasing in q_i and the result proved in this part that $q^S_{iH}(z_r)\ge q^F_{iH}(z_r)$, we have $q^S_{eH}(z_r)=q^*_e(q^S_{iH}(z_r),1;z_r)\le q^*_e(q^F_{iH}(z_r),1;z_r)=q^F_{eH}(z_r)$. When $t_i=L$, following Lemma 1 part 3, we have $q^S_{eL}(z_r)=q^*_e(q^S_{iL}(z_r),0;z_r)=q^*_e(q^F_{iL}(z_r),0;z_r)=q^F_{eL}(z_r)$. □

Proof of Theorem 1. Define the entrant's profit function

$$\Pi^e_N(q_{iH},q_{iL},q_e;T)\triangleq\lambda\Pi^e_H(q_{iH},q_e;T)+(1-\lambda)\Pi^e_L(q_{iL},q_e;T).$$

We have that $(q^N_{iH}(z_r),q^N_{iL}(z_r),q^N_e(z_r))$ is a Bayesian Nash equilibrium if and only if it satisfies the conditions

$$\begin{aligned}q^N_{iH}(z_r)&=\arg\max_{q_{iH}\ge 0}\Pi^i_H(q_{iH},q^N_e(z_r);T),\\q^N_{iL}(z_r)&=\arg\max_{q_{iL}\ge 0}\Pi^i_L(q_{iL},q^N_e(z_r);T),\\q^N_e(z_r)&=\arg\max_{q_e\ge 0}\Pi^e_N(q^N_{iH}(z_r),q^N_{iL}(z_r),q_e;T).\end{aligned}$$

For the incumbent's profit function, when the incumbent's demand state is high, we have

$$\begin{aligned}&\frac{\partial\Pi^i_H(q_{iH},q_e;T)}{\partial q_{iH}}\\&=-(c^r_u+c^r_o)\big(\lambda'\mathbb{P}(\mu^i_H+\eta\epsilon+a_{ei}(\mu^e_H+(1-\eta)\epsilon-q_e)^+\le q_{iH})\\&\quad+(1-\lambda')\mathbb{P}(\mu^i_H+\eta\epsilon+a_{ei}(\mu^e_L+(1-\eta)\epsilon-q_e)^+\le q_{iH})\big)+c^r_u.\end{aligned}$$

Hence, $\frac{\partial\Pi^i_H(q_{iH},q_e;T)}{\partial q_{iH}}$ is decreasing in q_{iH}.

For the incumbent's profit function, when the incumbent's demand state is low, we have

$$\begin{aligned}&\frac{\partial\Pi^i_L(q_{iL},q_e;T)}{\partial q_{iL}}\\&=-(c^r_u+c^r_o)\mathbb{P}(\mu^i_L+\eta\epsilon+a_{ei}(\mu^e_L+(1-\eta)\epsilon-q_e)^+\le q_{iL})+c^r_u.\end{aligned}$$

Hence, $\frac{\partial\Pi^i_L(q_{iL},q_e;T)}{\partial q_{iL}}$ is decreasing in q_{iL}.

For the entrant's profit function, we have

$$\begin{aligned}&\frac{\partial\Pi^e_N(q_{iH},q_{iL},q_e;T)}{\partial q_e}\\&=-(c^r_u+c^r_o)\big(\lambda\lambda'\mathbb{P}(\mu^e_H+(1-\eta)\epsilon+a_{ie}(\mu^i_H+\eta\epsilon-q_{iH})^+\le q_e)\\&\quad+\lambda(1-\lambda')\mathbb{P}(\mu^e_L+(1-\eta)\epsilon+a_{ie}(\mu^i_H+\eta\epsilon-q_{iH})^+\le q_e)\\&\quad+(1-\lambda)\mathbb{P}(\mu^e_L+(1-\eta)\epsilon+a_{ie}(\mu^i_L+\eta\epsilon-q_{iL})^+\le q_e)\big)+c^r_u.\end{aligned}$$

Hence, $\frac{\partial\Pi^e_N(q_{iH},q_{iL},q_e)}{\partial q_e}$ is decreasing in q_e.

Following above properties that $\frac{\partial \Pi_H^i(q_{iH},q_e;T)}{\partial q_{iH}}$ is decreasing in q_{iH}, $\frac{\partial \Pi_L^i(q_{iL},q_e;T)}{\partial q_{iL}}$ is decreasing in q_{iL}, and $\frac{\partial \Pi_N^e(q_{iH},q_{iL},q_e)}{\partial q_e}$ is decreasing in q_e, $(q_{iH}^N(z_r), q_{iL}^N(z_r), q_e^N(z_r))$ needs to be solutions to following equations

$$\left.\frac{\partial \Pi_H^i(q_{iH}, q_e^N(z_r);T)}{\partial q_{iH}}\right|_{q_{iH}=q_{iH}^N(z_r)} = 0,$$
$$\left.\frac{\partial \Pi_L^i(q_{iL}, q_e^N(z_r);T)}{\partial q_{iL}}\right|_{q_{iL}=q_{iL}^N(z_r)} = 0,$$
$$\left.\frac{\partial \Pi_N^e(q_{iH}^N(z_r), q_{iL}^N(z_r), q_e;T)}{\partial q_e}\right|_{q_e=q_e^N(z_r)} = 0.$$

If we rename q_{iH} as q_1, q_{iL} ad q_2, and q_e as q_3, then we can directly repeat the proof of Netessine and Rudi (2003, proposition 4) to get the property that there exists a solution to above equations. Therefore, there exists a Bayesian Nash equilibrium $(q_{iH}^N(z_r), q_{iL}^N(z_r), q_e^N(z_r))$.

Now, we characterize the properties of the Bayesian Nash equilibrium. We proceed in the following steps.

Step 1: we show that $q_e^N(z_r) > q_{eL}^F(z_r)$.

We have

$$\begin{aligned}
&\left.\frac{\partial \Pi_N^e(q_{iH}^N(z_r), q_{iL}^N(z_r), q_e;T)}{\partial q_e}\right|_{q_e=q_{eL}^F(z_r)}\\
&= -(c_u^r + c_o^r)\big(\lambda\lambda'\mathbb{P}\big(\mu_H^e + (1-\eta)\epsilon\\
&\quad + a_{ie}\big(\mu_H^i + \eta\epsilon - q_{iH}^N(z_r)\big)^+ \le q_{eL}^F(z_r)\big)\\
&\quad + \lambda(1-\lambda')\mathbb{P}\big(\mu_L^e + (1-\eta)\epsilon\\
&\quad + a_{ie}\big(\mu_H^i + \eta\epsilon - q_{iH}^N(z_r)\big)^+ \le q_{eL}^F(z_r)\big)\\
&\quad + (1-\lambda)\mathbb{P}\big(\mu_L^e + (1-\eta)\epsilon\\
&\quad + a_{ie}\big(\mu_L^i + \eta\epsilon - q_{iL}^N(z_r)\big)^+ \le q_{eL}^F(z_r)\big)\big) + c_u^r\\
&\ge -(c_u^r + c_o^r)\big(\lambda\lambda'\mathbb{P}\big(\mu_H^e + (1-\eta)\epsilon \le q_{eL}^F(z_r)\big)\\
&\quad + \lambda(1-\lambda')\mathbb{P}\big(\mu_L^e + (1-\eta)\epsilon \le q_{eL}^F(z_r)\big)\\
&\quad + (1-\lambda)\mathbb{P}\big(\mu_L^e + (1-\eta)\epsilon \le q_{eL}^F(z_r)\big)\big) + c_u^r\\
&= -(c_u^r + c_o^r)\big(\lambda\lambda'\mathbb{P}\big(\mu_H^e - \mu_L^e + (1-\eta)\big(\epsilon - F^{-1}(z_r)\big) \le 0\big)\\
&\quad + \lambda(1-\lambda')\mathbb{P}\big((1-\eta)\big(\epsilon - F^{-1}(z_r)\big) \le 0\big)\\
&\quad + (1-\lambda)\mathbb{P}\big((1-\eta)\big(\epsilon - F^{-1}(z_r)\big) \le 0\big)\big) + c_u^r\\
&> -(c_u^r + c_o^r)\big(\lambda\lambda'\mathbb{P}\big((1-\eta)\big(\epsilon - F^{-1}(z_r)\big) \le 0\big)\\
&\quad + \lambda(1-\lambda')\mathbb{P}\big((1-\eta)\big(\epsilon - F^{-1}(z_r)\big) \le 0\big)\\
&\quad + (1-\lambda)\mathbb{P}\big((1-\eta)\big(\epsilon - F^{-1}(z_r)\big) \le 0\big)\big) + c_u^r\\
&= -(c_u^r + c_o^r)\mathbb{P}\big(\epsilon - F^{-1}(z_r) \le 0\big) + c_u^r\\
&= 0.
\end{aligned}$$

Because $\frac{\partial \Pi_N^e(q_{iH},q_{iL},q_e)}{\partial q_e}$ is decreasing in q_e, we have $q_e^N(z_r) > q_{eL}^F(z_r)$.

Step 2: we show that $q_{iL}^N(z_r) = q_{iL}^F(z_r)$.

We have

$$\begin{aligned}
&\left.\frac{\partial \Pi_L^i(q_{iL}, q_e^N(z_r);T)}{\partial q_{iL}}\right|_{q_{iL}=q_{iL}^F(z_r)}\\
&= -(c_u^r + c_o^r)\mathbb{P}\big(\mu_L^i + \eta\epsilon + a_{ei}\big(\mu_L^e + (1-\eta)\epsilon - q_e^N(z_r)\big)^+\\
&\quad \le q_{iL}^F(z_r)\big) + c_u^r\\
&= -(c_u^r + c_o^r)\mathbb{P}\big(\eta\big(\epsilon - F^{-1}(z_r)\big) + a_{ei}\big(\mu_L^e + (1-\eta)\epsilon\\
&\quad - q_e^N(z_r)\big)^+ \le 0\big) + c_u^r\\
&= -(c_u^r + c_o^r)\mathbb{P}\big(\eta\big(\epsilon - F^{-1}(z_r)\big) \le 0\big) + c_u^r\\
&= 0,
\end{aligned}$$

where the third equality follows the property that $q_e^N(z_r) > q_{eL}^F(z_r) = \mu_L^e + (1-\eta)F^{-1}(z_r)$. Therefore, $q_{iL}^N(z_r) = q_{iL}^F(z_r)$.

Step 3: we show that $q_{iH}^N(z_r) > q_{iH}^F(z_r)$ and $q_e^N(z_r) < q_{eH}^F(z_r)$.

Define $\Delta_i \triangleq q_{iH}^F(z_r) - q_{iH}^N(z_r)$ and $\Delta_e \triangleq q_e^N(z_r) - q_{eH}^F(z_r)$. In the Bayesian Nash equilibrium, we have

$$\begin{aligned}
&\left.\frac{\partial \Pi_H^i(q_{iH}, q_e^N(z_r);T)}{\partial q_{iH}}\right|_{q_{iH}=q_{iH}^N(z_r)}\\
&= -(c_u^r + c_o^r)\big(\lambda'\mathbb{P}\big(\mu_H^i + \eta\epsilon + a_{ei}\big(\mu_H^e + (1-\eta)\epsilon - q_e^N(z_r)\big)^+\\
&\quad \le q_{iH}^N(z_r)\big) + (1-\lambda')\mathbb{P}\big(\mu_H^i + \eta\epsilon\\
&\quad + a_{ei}\big(\mu_L^e + (1-\eta)\epsilon - q_e^N(z_r)\big)^+ \le q_{iH}^N(z_r)\big)\big) + c_u^r\\
&= -(c_u^r + c_o^r)\big(\lambda'\mathbb{P}\big(\mu_H^i + \eta\epsilon + a_{ei}\big(\mu_H^e + (1-\eta)\epsilon\\
&\quad - q_{eH}^F(z_r) - \Delta_e\big)^+ \le q_{iH}^F(z_r) - \Delta_i\big)\\
&\quad + (1-\lambda')\mathbb{P}\big(\mu_H^i + \eta\epsilon + a_{ei}\big(\mu_L^e + (1-\eta)\epsilon\\
&\quad - q_{eH}^F(z_r) - \Delta_e\big)^+ \le q_{iH}^F(z_r) - \Delta_i\big)\big) + c_u^r\\
&= 0.
\end{aligned}$$

In the full information scenario that the entrant knows that the incumbent's demand state is high, in the Nash equilibrium, we have

$$\begin{aligned}
&\left.\frac{\partial \Pi_H^i(q_{iH}, q_e^F(z_r);T)}{\partial q_{iH}}\right|_{q_{iH}=q_{iH}^F(z_r)}\\
&= -(c_u^r + c_o^r)\big(\lambda'\mathbb{P}\big(\mu_H^i + \eta\epsilon + a_{ei}\big(\mu_H^e + (1-\eta)\epsilon\\
&\quad - q_{eH}^F(z_r)\big)^+ \le q_{iH}^F(z_r)\big)\\
&\quad + (1-\lambda')\mathbb{P}\big(\mu_H^i + \eta\epsilon + a_{ei}\big(\mu_L^e + (1-\eta)\epsilon - q_{eH}^F(z_r)\big)^+\\
&\quad \le q_{iH}^F(z_r)\big)\big) + c_u^r\\
&= 0.
\end{aligned}$$

Therefore, the two equilibrium conditions above imply that either $\Delta_i = \Delta_e = 0$, or $\Delta_i > 0$ and $\Delta_e > 0$, or $\Delta_i < 0$ and $\Delta_e < 0$.

Now, we prove that $\Delta_i > 0$ and $\Delta_e > 0$. We prove by using the contradiction argument. Suppose $\Delta_i \ge 0$. Following the property that for $a \in (0,1)$ and $\Delta \ge 0$, $(x-\Delta) - a(b-(y+\Delta))^+ \le (x-\Delta) - a(b-y)^+ + a\Delta \le x - a(b-y)^+$, we have $\Delta_e \ge \Delta_i$.

Therefore, we have

$$\begin{aligned}
&\left.\frac{\partial\Pi_N^e(q_{iH}^N(z_r),q_{iL}^N(z_r),q_e;T)}{\partial q_e}\right|_{q_e=q_{eL}^N(z_r)}\\
&=-(c_u^r+c_o^r)\big(\lambda\lambda'\mathbb{P}(\mu_H^e+(1-\eta)\epsilon\\
&\quad+a_{ie}(\mu_H^i+\eta\epsilon-q_{iH}^N(z_r))^+\le q_e^N(z_r))\\
&\quad+\lambda(1-\lambda')\mathbb{P}(\mu_L^e+(1-\eta)\epsilon+a_{ie}(\mu_H^i+\eta\epsilon-q_{iH}^N(z_r))^+\\
&\le q_e^N(z_r))\\
&\quad+(1-\lambda)\mathbb{P}(\mu_L^e+(1-\eta)\epsilon+a_{ie}(\mu_L^i+\eta\epsilon-q_{iL}^N(z_r))^+\\
&\le q_e^N(z_r))\big)+c_u^r\\
&=-(c_u^r+c_o^r)\big(\lambda\lambda'\mathbb{P}(\mu_H^e+(1-\eta)\epsilon\\
&\quad+a_{ie}(\mu_H^i+\eta\epsilon-q_{iH}^F(z_r)+\Delta_i)^+\le q_{eL}^F(z_r)+\Delta_e)\\
&\quad+\lambda(1-\lambda')\mathbb{P}(\mu_L^e+(1-\eta)\epsilon\\
&\quad+a_{ie}(\mu_H^i+\eta\epsilon-q_{iH}^F(z_r)+\Delta_i)^+\le q_{eL}^F(z_r)+\Delta_e)\\
&\quad+(1-\lambda)\mathbb{P}(\mu_L^e+(1-\eta)\epsilon+a_{ie}(\mu_L^i+\eta\epsilon-q_{iL}^F(z_r))^+\\
&\le q_e^N(z_r))\big)+c_u^r\\
&\le-(c_u^r+c_o^r)\big(\lambda\lambda'\mathbb{P}(\mu_H^e+(1-\eta)\epsilon\\
&\quad+a_{ie}(\mu_H^i+\eta\epsilon-q_{iH}^F(z_r))^+\le q_{eL}^F(z_r))\\
&\quad+\lambda(1-\lambda')\mathbb{P}(\mu_L^e+(1-\eta)\epsilon+a_{ie}(\mu_H^i+\eta\epsilon-q_{iH}^F(z_r))^+\\
&\le q_{eL}^F(z_r))+(1-\lambda)\mathbb{P}(\mu_L^e+(1-\eta)\epsilon+a_{ie}(\mu_L^i+\eta\epsilon-q_{iL}^F(z_r))^+\\
&\le q_e^N(z_r))\big)+c_u^r<-(c_u^r+c_o^r)\big(\lambda\lambda'\mathbb{P}(\mu_H^e+(1-\eta)\epsilon\\
&\quad+a_{ie}(\mu_H^i+\eta\epsilon-q_{iH}^F(z_r))^+\le q_{eL}^F(z_r))+\lambda(1-\lambda')\mathbb{P}(\mu_L^e+(1-\eta)\epsilon\\
&\quad+a_{ie}(\mu_H^i+\eta\epsilon-q_{iH}^F(z_r))^+\le q_{eL}^F(z_r))+(1-\lambda)\mathbb{P}(\mu_L^e+(1-\eta)\epsilon\\
&\quad+a_{ie}(\mu_L^i+\eta\epsilon-q_{iL}^F(z_r))^+\le q_{eL}^F(z_r))\big)+c_u^r\\
&=-(c_u^r+c_o^r)(\lambda z_r+(1-\lambda)z_r)+c_u^r=0.
\end{aligned}$$

The first inequality follows the property that for $a\in(0,1)$ and $\Delta_e\ge\Delta_i$, $(x+\Delta_e)-a(b-(y-\Delta_i))^+\ge(x+\Delta_e)-a(b-y)^+-a\Delta_i\ge x-a(b-y)^+$. The second inequality follows the property that $q_e^N(z_r)>q_{eL}^F(z_r)$.

This result contradicts the property that $\left.\frac{\partial\Pi_N^e(q_{iH}^N(z_r),q_{iL}^N(z_r),q_e;T)}{\partial q_e}\right|_{q_e=q_{eL}^N(z_r)}=0$. Therefore, $\Delta_i>0$ and $\Delta_e>0$, i.e., $q_{iH}^N(z_r)>q_{iH}^F(z_r)$ and $q_e^N(z_r)<q_{eH}^F(z_r)$. □

Proof of Theorem 2. The thresholds on the market state and its likelihood, i.e., $\underline{\mu}_H^i$ and $\underline{\lambda}(\mu_H^i)$, are defined as follows:

$$\begin{aligned}
\underline{\mu}_H^i&\triangleq\min\{\mu_H^i\ge\mu_L^i:\Pi_H^i(q_{iH}^S(z_r),q_e^*(q_{iH}^S(z_r),1;z_r);T)\\
&\quad\ge\Pi_H^i(q_{iL}^F(z_r),q_e^*(q_{iL}^F(z_r),0;z_r);T)\},\\
\underline{\lambda}(\mu_H^i)&\triangleq\min\{\lambda_e\in(0,1]:\Pi_H^i(q_{iH}^S(z_r),q_e^*(q_{iH}^S(z_r),1;z_r);T)\\
&\quad=\Pi_H^i(q_{iL}^F(z_r),q_e^*(q_{iL}^F(z_r),\lambda_e;z_r);T)\},\ \forall\mu_H^i\in(\mu_L^i,\underline{\mu}_H^i),
\end{aligned}$$

We begin with proving the following results that are used in this proof.

First, we show that

$$q_{iL}^F(z_r)=\underset{q_i\in[0,q_{iL}^F(z_r)]}{\arg\max}\ \Pi_{t_i}^i(q_i,q_e^*(q_i,\lambda;z_r);T),\forall t_i\in\{H,L\},\lambda\in[0,1].$$

For any $q_i\in[0,q_{iL}^F(z_r)]$, we have $\Pi_{t_i}^i(q_i,q_e^*(q_i,\lambda;z_r);T)\le\Pi_{t_i}^i(q_i,q_e^*(q_{iL}^F(z_r),\lambda;z_r);T)\le\Pi_{t_i}^i(q_{iL}^F(z_r),q_e^*(q_{iL}^F(z_r),\lambda;z_r);T)$, where the first inequality follows the condition $q_i\le q_{iL}^F(z_r)$, Lemma 1 part 1 that $q_e^*(q_i,\lambda;z_r)$ is decreasing in q_i, and Lemma 1 part 2 that $\Pi_{t_i}^i(q_i,q_e;T)$ is decreasing in q_e, the second inequality follows Lemma 1 part 1 that $\Pi_{t_i}^i(q_i,q_e;T)$ is increasing in $q_i\in[0,q_{iL}^F(z_r)]$.

Second, we define

$$\begin{aligned}
\Delta_H^i(\mu_H^i,\lambda)&\triangleq\Pi_H^i(q_{iH}^S(z_r),q_e^*(q_{iH}^S(z_r),1;z_r);T)-\Pi_H^i(q_{iL}^F(z_r),\\
&\quad q_e^*(q_{iL}^F(z_r),\lambda;z_r);T).
\end{aligned}$$

We show that $\Delta_H^i(\mu_H^i,\lambda)$ is increasing in μ_H^i and λ, respectively. Consider $\hat{\mu}_H^i>\mu_H^i$. In the proof below, all notation with 'hat' is associated with the environment that the incumbent's average demand is $\hat{\mu}_H^i$ when her demand state is high. We have

$$\begin{aligned}
&\Delta_H^i(\hat{\mu}_H^i,\lambda)\\
&\quad\ge\hat{\Pi}_H^i(\hat{\mu}_H^i-\mu_H^i+q_{iH}^S(z_r),\hat{q}_e^*(\hat{\mu}_H^i-\mu_H^i+q_{iH}^S(z_r),1;z_r);T)\\
&\qquad-\hat{\Pi}_H^i(q_{iL}^F(z_r),\hat{q}_e^*(q_{iL}^F(z_r),\lambda;z_r);T)\\
&\quad\ge\hat{\Pi}_H^i(\hat{\mu}_H^i-\mu_H^i+q_{iH}^S(z_r),\hat{q}_e^*(\hat{\mu}_H^i-\mu_H^i+q_{iH}^S(z_r),1;z_r);T)\\
&\qquad-\hat{\Pi}_H^i(q_{iL}^F(z_r),q_e^*(q_{iL}^F(z_r),\lambda;z_r);T)\\
&\quad=\hat{\Pi}_H^i(\hat{\mu}_H^i-\mu_H^i+q_{iH}^S(z_r),q_e^*(q_{iH}^S(z_r),1;z_r);T)\\
&\qquad-\hat{\Pi}_H^i(q_{iL}^F(z_r),q_e^*(q_{iL}^F(z_r),\lambda;z_r);T)\\
&\quad=\hat{\mu}_H^i-\mu_H^i+\Pi_H^i(q_{iH}^S(z_r),q_e^*(q_{iH}^S(z_r),1;z_r);T)\\
&\qquad-\hat{\Pi}_H^i(q_{iL}^F(z_r),q_e^*(q_{iL}^F(z_r),\lambda;z_r);T)\\
&\quad\ge\Pi_H^i(q_{iH}^S(z_r),q_e^*(q_{iH}^S(z_r),1;z_r);T)\\
&\qquad-\Pi_H^i(q_{iL}^F(z_r),q_e^*(q_{iL}^F(z_r),\lambda;z_r);T)=\Delta_H^i(\mu_H^i,\lambda),
\end{aligned}$$

where the second inequality follows Lemma 1 part 1 that $q_e^*(q_i,\lambda;z_r)$ is increasing in μ_H^i and Lemma 1 part 2 that $\Pi_H^i(q_i,q_e;T)$ is decreasing in q_e, the first equality follows the property that $\frac{\partial\hat{\Pi}_H^e(\hat{\mu}_H^i-\mu_H^i+q_i,q_e;T)}{\partial q_e}=\frac{\partial\Pi_H^e(q_i,q_e;T)}{\partial q_e}$ and the result that immediately follows this property that $\hat{q}_e^*(\hat{\mu}_H^i-\mu_H^i+q_{iH}^S(z_r),1;z_r)=q_e^*(q_{iH}^S(z_r),1;z_r)$, the second equality follows the property that $\hat{\Pi}_H^i(\hat{\mu}_H^i-\mu_H^i+q_i,q_e;T)=\hat{\mu}_H^i-\mu_H^i+\Pi_H^i(q_i,q_e;T)$, the third inequality follows the property that $\hat{\Pi}_H^i(q_i,q_e;T)\le\hat{\mu}_H^i-\mu_H^i+\Pi_H^i(q_i,q_e;T)$. Therefore, $\Delta_H^i(\mu_H^i,\lambda)$ is increasing in μ_H.

For any $\lambda,\hat{\lambda}\in[0,1]$ with $\lambda<\hat{\lambda}$, we have $\Pi_H^i(q_{iL}^F(z_r),q_e^*(q_{iL}^F(z_r),\hat{\lambda};z_r);T)\le\Pi_H^i(q_{iL}^F(z_r),q_e^*(q_{iL}^F(z_r),\lambda;z_r);T)$, where the inequality follows Lemma 1 part 1 that $q_e^*(q_i,\lambda;z_r)$ is increasing in λ and Lemma 1 part 2 that $\Pi_H^i(q_i,q_e;T)$ is decreasing in q_e. Therefore, $\Delta_H^i(\mu_H^i,\lambda)$ is increasing in λ.

Third, we show that when $\mu_H^i>q_{iL}^F(z_r)-\eta\underline{\epsilon}$, $\Delta_H^i(\mu_H^i,\lambda)\ge0$.

Define

$$\tilde{\Pi}_H^i(q_i;T)\triangleq(c_u^r+c_o^r)\mathbb{E}_\epsilon[\min\{D_i,q_i\}\,|\,t_i=H]-c_o^rq_i-h^i$$

to be the incumbent's expected profit derived from serving her own market when the incumbent's demand state is high. Define $\tilde{q}_{iH}^*(z_r)\triangleq\arg\max_{q_i\ge0}\tilde{\Pi}_H^i(q_i;T)$.

When $\mu_H^i>q_{iL}^F(z_r)-\eta\underline{\epsilon}$, we have

$$\begin{aligned}
\Delta_H^i(\mu_H^i,\lambda)&=\Pi_H^i(q_{iH}^S(z_r),q_e^*(q_{iH}^S(z_r);z_r),1;T)\\
&\quad-\Pi_H^i(q_{iL}^F(z_r),q_e^*(q_{iL}^F(z_r),\lambda;z_r);T)\\
&=\Pi_H^i(q_{iH}^S(z_r),q_e^*(q_{iH}^S(z_r),1;z_r);T)-\tilde{\Pi}_H^i(q_{iL}^F(z_r);T)\\
&\ge\Pi_H^i(\tilde{q}_{iH}^*(z_r),q_e^*(\tilde{q}_{iH}^*(z_r),1;z_r);T)-\tilde{\Pi}_H^i(q_{iL}^F(z_r);T)\\
&\ge\tilde{\Pi}_H^i(\tilde{q}_{iH}^*(z_r);T)-\tilde{\Pi}_H^i(q_{iL}^F(z_r);T)\ge0,
\end{aligned}$$

where the second equality follows the property that the condition $\mu_H^i > q_{iL}^F(z_r) - \eta\underline{\epsilon}$ implies that $q_{iL}^F(z_r) < \mu_H^i + \eta\epsilon$ for all $\epsilon \in [\underline{\epsilon}, \bar{\epsilon}]$, the second inequality follows the property that $\Pi_H^i(q_i, q_e; T) \geq \tilde{\Pi}_H^i(q_i; T)$.

Fourth, following the second property proved in this theorem above that $\Delta_H^i(\mu_H^i, 0)$ is increasing in μ_H^i, and the third property proved in this theorem above that $\Delta_H^i(\mu_H^i, 0) \geq 0$ when μ_H^i is large enough, there exists $\underline{\mu}_H^i \geq \mu_L^i$, such that

$$\underline{\mu}_H^i \triangleq \min\{\mu_H^i \geq \mu_L^i : \Delta_H^i(\mu_H^i, 0) \geq 0\}.$$

Therefore, for any $\mu_H^i \in (\mu_L^i, \underline{\mu}_H^i)$, $\Delta_H^i(\mu_H^i, 0) < 0$, and for any $\mu_H^i \geq \underline{\mu}_H^i$, $\Delta_H^i(\mu_H^i, 0) \geq 0$.

Fifth, following the property that $\Delta_H^i(\mu_H^i, 1) = \Pi_H^i(q_{iH}^S(z_r), q_e^*(q_{iH}^S(z_r), 1; z_r); T) - \Pi_H^i(q_{iL}^F(z_r), q_e^*(q_{iL}^F(z_r), 1; z_r); T) \geq 0$, when $\mu_H^i \in (\mu_L^i, \underline{\mu}_H^i)$, because $\Delta_H^i(\mu_H^i, 0) < 0$, there exists $\underline{\lambda}(\mu_H^i) \in (0, 1]$, such that

$$\underline{\lambda}(\mu_H^i) \triangleq \min\{\lambda_e \in (0, 1] : \Delta_H^i(\mu_H^i, \lambda_e) = 0\}.$$

Therefore, following the second property proved in this theorem above that $\Delta_H^i(\mu_H^i, \lambda_e)$ is increasing in λ_e, we have that for any $\lambda_e < \underline{\lambda}(\mu_H^i)$, $\Delta_H^i(\mu_H^i, \lambda_e) < 0$, and for any $\lambda_e \geq \underline{\lambda}(\mu_H^i)$, $\Delta_H^i(\mu_H^i, \lambda_e) \geq 0$.

In addition, for $\mu_H^i, \hat{\mu}_H^i \in (\mu_L^i, \underline{\mu}_H^i)$ with $\mu_H^i < \hat{\mu}_H^i$, for any $\lambda_e > \underline{\lambda}(\mu_H^i)$, we have $\Delta_H^i(\hat{\mu}_H^i, \lambda_e) \geq \Delta_H^i(\mu_H^i, \underline{\lambda}(\mu_H^i)) \geq 0$, where the first inequality follows the second property proved in this theorem that $\Delta_H^i(\mu_H^i, \lambda_e)$ is increasing in μ_H^i and λ_e, respectively. Therefore, $\underline{\lambda}(\hat{\mu}_H^i) \leq \underline{\lambda}(\mu_H^i)$, i.e., $\underline{\lambda}(\mu_H^i)$ is decreasing in $\mu_H^i \in (\mu_L^i, \underline{\mu}_H^i)$.

Now, we use above results to prove this theorem.

Case 1: we show that when $\mu_H^i \geq \underline{\mu}_H^i$, there exists a separating equilibrium, as characterized in this theorem.

First, when the entrant's posterior belief about the incumbent's demand state is given by $\lambda_e(q_i; z_r) = \begin{cases} 1 & \text{when } q_i \neq q_{iL}^F(z_r) \\ 0 & \text{when } q_i = q_{iL}^F(z_r) \end{cases}$, the entrant's best response is given by $q_e^A(q_i; z_r) = q_e^*(q_i, \lambda_e(q_i; z_r); z_r) = \begin{cases} q_e^*(q_i, 1; z_r) & \text{when } q_i \neq q_{iL}^F(z_r) \\ q_e^*(q_i, 0; z_r) & \text{when } q_i = q_{iL}^F(z_r) \end{cases}$.

Second, we compute the incumbent's equilibrium order quantity given the entrant's best response function $q_e^A(q_i; z_r) = q_e^*(q_i, \lambda_e(q_i; z_r); z_r)$. When the incumbent's demand state is high, we have

$$\begin{aligned}\max_{q_i \geq 0} \Pi_H^i(q_i, q_e^A(q_i; z_r); T) &= \max\Big\{ \max_{q_i > q_{iL}^F(z_r)} \Pi_H^i(q_i, q_e^*(q_i, 1; z_r); T), \\ &\qquad \max_{q_i \in [0, q_{iL}^F(z_r))} \Pi_H^i(q_i, q_e^*(q_i, 0; z_r); T) \Big\} \\ &= \max\{\Pi_H^i(q_{iH}^S(z_r), q_e^*(q_{iH}^S(z_r), 1; z_r); T), \\ &\qquad \Pi_H^i(q_{iL}^F(z_r), q_e^*(q_{iL}^F(z_r), 0; z_r); T)\} \\ &= \Pi_H^i(q_{iH}^S(z_r), q_e^*(q_{iH}^S(z_r), 1; z_r); T),\end{aligned}$$

where the second equality follows the property proved in the first step of this proof that for any $\lambda_e \in [0, 1]$, $q_{iL}^F(z_r) = \arg\max_{q_i \in [0, q_{iL}^F(z_r)]} \Pi_{t_i}^i(q_i, q_e^*(q_i, \lambda_e; z_r); T)$ and Lemma 1 part 4 that $q_{iH}^S(z_r) > q_{iL}^F(z_r)$, the third equality follows the fourth property proved in this theorem above that when $\mu_H^i \geq \underline{\mu}_H^i$, $\Delta_H^i(\mu_H^i, 0) \geq 0$. Therefore, $q_{iH}^A(z_r) = q_{iH}^S(z_r)$.

When the incumbent's demand state is low, for any $q_i \geq 0$, we have $\Pi_L^i(q_i, q_e^A(q_i; z_r); T) = \Pi_L^i(q_i, q_e^*(q_i, \lambda_e(q_i; z_r); z_r); T) \leq \Pi_L^i(q_i, q_{eL}^F(z_r); T) \leq \Pi_L^i(q_{iL}^F(z_r), q_{eL}^F(z_r); T) = \Pi_L^i(q_{iL}^F(z_r), q_e^A(q_{iL}^F(z_r); z_r); T)$, where the first inequality follows Lemma 1 part 1 that $q_e^*(q_i, \lambda_e(q_i; z_r); z_r) \geq q_{eL}^F(z_r)$, and Lemma 1 part 2 that $\Pi_L^i(q_i, q_e; T)$ is decreasing in q_e, the second inequality follows the property that $q_{iL}^F(z_r) = q_{iL}^*(q_{eL}^F(z_r); z_r)$. Therefore, $q_{iL}^A(z_r) = q_{iL}^F(z_r)$.

Third, we verify that the entrant's posterior belief function,

$$\lambda_e(q_i; z_r) = \begin{cases} 1 & \text{when } q_i \neq q_{iL}^F(z_r) \\ 0 & \text{when } q_i = q_{iL}^F(z_r) \end{cases}, \text{ satisfies Bayes' rule.}$$

We have

$$\begin{aligned}\lambda_e(q_{iH}^S(z_r); z_r) &= \frac{\lambda \cdot \text{Prob}\,(q_{it_i}^A(z_r) = q_{iH}^S(z_r)\,|t_i = H)}{\lambda \cdot \text{Prob}\,(q_{it_i}^A(z_r) = q_{iH}^S(z_r)\,|t_i = H) + (1-\lambda)\cdot \text{Prob}\,(q_{it_i}^A(z_r) = q_{iH}^S(z_r)\,|t_i = L)} \\ &= \frac{\lambda}{\lambda} = 1,\end{aligned}$$

and

$$\begin{aligned}\lambda_e(q_{iL}^F(z_r); z_r) &= \frac{\lambda \cdot \text{Prob}\,(q_{it_i}^A(z_r) = q_{iL}^F(z_r)\,|t_i = H)}{\lambda \cdot \text{Prob}\,(q_{it_i}^A(z_r) = q_{iL}^F(z_r)\,|t_i = H) + (1-\lambda)\cdot \text{Prob}\,(q_{it_i}^A(z_r) = q_{iL}^F(z_r)\,|t_i = L)} \\ &= \frac{0}{1-\lambda} = 0.\end{aligned}$$

Case 2: we show that when $\mu_H^i \in (\mu_L^i, \underline{\mu}_H^i)$ and $\lambda \in (0, \underline{\lambda}(\mu_H^i))$, there exists a pooling equilibrium, as characterized in this theorem.

First, when the entrant's posterior belief about the incumbent's demand state is given by $\lambda_e(q_i; z_r) = \begin{cases} 1 & \textit{when } q_i \neq q_{iL}^F(z_r) \\ \lambda & \textit{when } q_i = q_{iL}^F(z_r) \end{cases}$, the entrant's best response is given by $q_e^A(q_i; z_r) = q_e^*(q_i, \lambda_e(q_i; z_r); z_r) = \begin{cases} q_e^*(q_i, 1; z_r) & \text{when } q_i \neq q_{iL}^F(z_r) \\ q_e^*(q_i, \lambda; z_r) & \text{when } q_i = q_{iL}^F(z_r) \end{cases}$.

Second, we compute the incumbent's equilibrium order quantity given the entrant's best response function $q_e^A(q_i; z_r) = q_e^*(q_i, \lambda_e(q_i; z_r); z_r)$. When the incumbent's demand state is high, we have

$$\begin{aligned}&\max_{q_i \geq 0} \Pi_H^i(q_i, q_e^A(q_i; z_r); T) \\ &= \max\Big\{ \max_{q_i > q_{iL}^F(z_r)} \Pi_H^i(q_i, q_e^*(q_i, 1; z_r); T), \\ &\qquad \max_{q_i \in [0, q_{iL}^F(z_r))} \Pi_H^i(q_i, q_e^*(q_i, \lambda; z_r); T) \Big\} \\ &= \max\{\Pi_H^i(q_{iH}^S(z_r), q_e^*(q_{iH}^S(z_r), 1; z_r); T), \\ &\qquad \Pi_H^i(q_{iL}^F(z_r), q_e^*(q_{iL}^F(z_r), \lambda; z_r); T)\} \\ &= \Pi_H^i(q_{iL}^F(z_r), q_e^*(q_{iL}^F(z_r), \lambda; z_r); T),\end{aligned}$$

where the second equality follows the property proved in the first step of this proof that for any $\lambda_e \in [0, 1]$, $q_{iL}^F(z_r) = \arg\max_{q_i \in [0, q_{iL}^F(z_r)]} \Pi_{t_i}^i(q_i, q_e^*(q_i, \lambda_e; z_r); T)$ and Lemma 1 part 4 that $q_{iH}^S(z_r) > q_{iL}^F(z_r)$, the third equality follows the property proved in the fifth step of this proof that when $\mu_H^i \in (\mu_L^i, \underline{\mu}_H^i)$ and $\lambda < \underline{\lambda}(\mu_H^i)$, $\Delta_H^i(\mu_H^i, \lambda) < 0$. Therefore, $q_{iH}^A(z_r) = q_{iL}^F(z_r)$.

When the incumbent's demand state is low, for any $q_i \geq 0$, we have $\Pi_L^i(q_i, q_e^A(q_i; z_r); T) = \Pi_L^i(q_i, q_e^*(q_i, \lambda_e(q_i; z_r); z_r); T) \leq \Pi_L^i(q_i, q_{eL}^F(z_r); T) \leq \Pi_L^i(q_{iL}^F(z_r), q_{eL}^F(z_r); T) = \Pi_L^i(q_{iL}^F(z_r), q_e^*(q_{iL}^F(z_r), \lambda; z_r); T) = \Pi_L^i(q_{iL}^F(z_r), q_e^A(q_{iL}^F(z_r); z_r); T)$, where the first inequality follows Lemma 1 part 1 that $q_e^*(q_i, \lambda_e(q_i; z_r); z_r) \geq q_{eL}^F(z_r)$ and Lemma 1 part 2 that $\Pi_L^i(q_i, q_e; T)$ is decreasing in q_e, the second inequality follows the property that $q_{iL}^F(z_r) = q_{iL}^*(q_{eL}^F(z_r); z_r)$, the second equality follows the property that when $q_{iL}^F(z_r) > \mu_L^i + \eta\epsilon$, $q_e^*(q_{iL}^F(z_r), \lambda; z_r) \geq q_{eL}^F(z_r) > \mu_L^e + (1-\eta)\epsilon$, i.e., the incumbent cannot steal the entrant's demand when the entrant's order quantity is $q_e^*(q_{iL}^F(z_r), \lambda; z_r)$ or $q_{eL}^F(z_r)$. Therefore, $q_{iL}^A(z_r) = q_{iL}^F(z_r)$.

Third, we verify that the entrant's posterior belief function, $\lambda_e(q_i; z_r) = \begin{cases} 1 & \text{when } q_i \neq q_{iL}^F(z_r) \\ \lambda & \text{when } q_i = q_{iL}^F(z_r) \end{cases}$, satisfies Bayes' rule.

We have

$$\lambda_e(q_{iL}^F(z_r); z_r) = \frac{\lambda \cdot \text{Prob}\,(q_{it_i}^A(z_r) = q_{iL}^F(z_r) | t_i = H)}{\lambda \cdot \text{Prob}\,(q_{it_i}^A(z_r) = q_{iL}^F(z_r) | t_i = H) + (1-\lambda) \cdot \text{Prob}\,(q_{it_i}^A(z_r) = q_{iL}^F(z_r) \,|t_i = L)} = \frac{\lambda}{\lambda + (1-\lambda)} = \lambda.$$

Case 3: we show that when $\mu_H^i \in (\mu_L^i, \underline{\mu}_H^i)$ and $\lambda \in [\underline{\lambda}(\mu_H^i), 1)$, there exists a semi-separating equilibrium, as characterized in this theorem. For notational clarity purpose, we define $\lambda^{SS} \triangleq \frac{\lambda(1-p^{SS})}{\lambda(1-p^{SS}) + (1-\lambda)}$.

First, when the entrant's posterior belief about the incumbent's demand state is given by $\lambda_e(q_i; z_r) = \begin{cases} 1 & \text{when } q_i \neq q_{iL}^F(z_r) \\ \lambda^{SS} & \text{when } q_i = q_{iL}^F(z_r) \end{cases}$, the entrant's best response is given by $q_e^A(q_i; z_r) = q_e^*(q_i, \lambda_e(q_i; z_r); z_r) = \begin{cases} q_e^*(q_i, 1; z_r) & \text{when } q_i \neq q_{iL}^F(z_r) \\ q_e^*(q_i, \lambda^{SS}; z_r) & \text{when } q_i = q_{iL}^F(z_r) \end{cases}$.

Second, we compute the incumbent's equilibrium order quantity given the entrant's best response function $q_e^A(q_i; z_r) = q_e^*(q_i, \lambda_e(q_i; z_r); z_r)$. When the incumbent's demand state is high, we have

$$\begin{aligned} &\max_{q_i \geq 0} \Pi_H^i(q_i, q_e^A(q_i; z_r); T) \\ &= \max\left\{ \max_{q_i > q_{iL}^F(z_r)} \Pi_H^i(q_i, q_e^*(q_i, 1; z_r); T), \max_{q_i \in [0, q_{iL}^F(z_r))} \Pi_H^i(q_i, q_e^*(q_i, \lambda^{SS}; z_r); T) \right\} \\ &= \max\{\Pi_H^i(q_{iH}^S(z_r), q_e^*(q_{iH}^S(z_r), 1; z_r); T), \Pi_H^i(q_{iL}^F(z_r), q_e^*(q_{iL}^F(z_r), \lambda^{SS}; z_r); T)\}, \end{aligned}$$

where the second equality follows the property proved in the first step of this proof that for any $\lambda_e \in [0,1]$, $q_{iL}^F(z_r) = \arg\max_{q_i \in [0, q_{iL}^F(z_r)]} \Pi_{t_i}^i(q_i, q_e^*(q_i, \lambda_e; z_r); T)$ and Lemma 1 part 4 that $q_{iH}^S(z_r) > q_{iL}^F(z_r)$. Therefore, $q_{iH}^A(z_r) = \{q_{iH}^S(z_r), q_{iL}^F(z_r)\}$.

When the incumbent's demand state is low, for any $q_i \geq 0$, we have $\Pi_L^i(q_i, q_e^A(q_i; z_r); T) = \Pi_L^i(q_i, q_e^*(q_i, \lambda_e(q_i; z_r); z_r); T) \leq \Pi_L^i(q_i, q_{eL}^F(z_r); T) \leq \Pi_L^i(q_{iL}^F(z_r), q_{eL}^F(z_r); T) = \Pi_L^i(q_{iL}^F(z_r), q_e^*(q_{iL}^F(z_r), \lambda^{SS}; z_r); T) = \Pi_L^i(q_{iL}^F(z_r), q_e^A(q_{iL}^F(z_r); z_r); T)$, where the first inequality follows Lemma 1 part 1 that $q_e^*(q_i, \lambda_e(q_i; z_r); z_r) \geq q_{eL}^F(z_r)$ and Lemma 1 part 2 that $\Pi_L^i(q_i, q_e; T)$ is decreasing in q_e, the second inequality follows the property that $q_{iL}^F(z_r) = q_{iL}^*(q_{eL}^F(z_r); z_r)$, the second equality follows the property that when $q_{iL}^F(z_r) > \mu_L^i + \eta\epsilon$, $q_e^*(q_{iL}^F(z_r), \lambda^{SS}; z_r) \geq q_{eL}^F(z_r) > \mu_L^e + (1-\eta)\epsilon$, i.e., the incumbent cannot steal the entrant's demand when the entrant's order quantity is $q_e^*(q_{iL}^F(z_r), \lambda^{SS}; z_r)$ or $q_{eL}^F(z_r)$. Therefore, $q_{iL}^A(z_r) = q_{iL}^F(z_r)$.

Third, we verify that the entrant's posterior belief function, $\lambda_e(q_i; z_r) = \begin{cases} 1 & \text{when } q_i \neq q_{iL}^F(z_r) \\ \lambda & \text{when } q_i = q_{iL}^F(z_r) \end{cases}$, satisfies Bayes' rule.

We have

$$\lambda_e(q_{iH}^S(z_r); z_r) = \frac{\lambda \cdot \text{Prob}\,(q_{it_i}^A(z_r) = q_{iH}^S(z_r) \,|t_i = H)}{\lambda \cdot \text{Prob}\,(q_{it_i}^A(z_r) = q_{iH}^S(z_r) \,|t_i = H) + (1-\lambda) \cdot \text{Prob}\,(q_{it_i}^A(z_r) = q_{iH}^S(z_r) \,|t_i = L)} = \frac{\lambda p}{\lambda p} = 1,$$

and

$$\lambda_e(q_{iL}^F(z_r); z_r) = \frac{\lambda \cdot \text{Prob}\,(q_{it_i}^A(z_r) = q_{iL}^F(z_r) \,|t_i = H)}{\lambda \cdot \text{Prob}\,(q_{it_i}^A(z_r) = q_{iL}^F(z_r) \,|t_i = H) + (1-\lambda) \cdot \text{Prob}\,(q_{it_i}^A(z_r) = q_{iL}^F(z_r) \,|t_i = L)} = \frac{\lambda(1-p)}{\lambda(1-p) + (1-\lambda)}.$$

Finally, we prove that all three types of equilibriums characterized in Theorem 2 survive the intuitive criterion. Consider the incumbent's any order quantity q_i that is off the equilibrium path, i.e., $q_i \in \mathbb{R}_+ \backslash \{q_{iL}^F(z_r)\} \backslash \{q_{iH}^S(z_r)\}$ for the separating and the semi-separating equilibriums and $q_i \in \mathbb{R}_+ \backslash \{q_{iL}^F(z_r)\}$ for the pooling equilibrium.

Define

$$\tilde{\Pi}_L^i(q_i; T) \triangleq (c_u^r + c_o^r)\mathbb{E}_\epsilon[\min\{D_i, q_i\} \,|t_i = L] - c_o^r q_i - h^i$$

to be the incumbent's expected profit derived from serving her own market when the incumbent's demand state is low.

For any $\lambda' \in [0,1]$, we have

$$\begin{aligned} \Pi_L^i(q_i, q_e^*(q_i, \lambda'; z_r); T) &\leq \Pi_L^i(q_i, q_{eL}^F(z_r); T) < \Pi_L^i(q_{iL}^F(z_r), q_{eL}^F(z_r); T) \\ &= \tilde{\Pi}_L^i(q_{iL}^F(z_r); T) = \Pi_L^i(q_{iL}^F(z_r), q_e^A(q_{iL}^F(z_r); z_r); T), \end{aligned}$$

where the first inequality follows Lemma 1 part 1 that $q_e^*(q_i, \lambda'; z_r) \geq \mu_L^e + (1-\eta)F^{-1}(z_r)$, part 3 that $q_{eL}^F(z_r) = \mu_L^e + (1-\eta)F^{-1}(z_r)$ and part 2 that $\Pi_L^i(q_i, q_e; T)$ is decreasing in q_e, the second inequality follows Lemma 1 part 3, the first equality follows the property that $q_{eL}^F(z_r) < \mu_L^e + (1-\eta)\epsilon$ (equivalently $F^{-1}(z_r) < \epsilon$) implies $q_{iL}^F(z_r) < \mu_L^i + \eta\epsilon$ (equivalently $F^{-1}(z_r) < \epsilon$), the second equality follows the definition of $q_e^A(q_i; z_r)$ given in Theorem 2 that $q_e^A(q_i; z_r) = q_e^*(q_i, \lambda_e(q_i; z_r); z_r)$, Lemma 1 part 1 that $q_e^*(\cdot, \cdot; z_r) \geq \mu_L^e + (1-\eta)\epsilon$ and the argument for the validity of the first equality.

Therefore, the incumbent's any order quantity q_i that is off the equilibrium path is equilibrium dominated when the incumbent's demand state is low.

Recall from Theorem 2 that $\lambda_e(q_i; z_r) = 1$ for all q_i that is off the equilibrium path, i.e., the entrant believes that the probability that the incumbent's demand state is low is zero.

Therefore, all three types of equilibriums characterized in Theorem 2 survive the intuitive criterion. □

Proof of Theorem 3. Given that the incumbent orders $q^N_{it_i}(z_r)$ when $t_i \in \{H, L\}$, the supplier's profit function satisfies the condition

$$\Pi^s_{t_i}(q^N_{it_i}(z_r), q^A_e(q^N_{it_i}(z_r); z_r); T) \le \Pi^s_{t_i}(q^N_{it_i}(z_r), q^N_e(z_r); T), \quad \text{when } t_i \in \{H, L\}.$$

Given that the incumbent's demand state is $t_i \in \{H, L\}$, for the incumbent's any order quantity $q_i \neq q^N_{it_i}(z_r)$, either the incumbent's profit is worse off if she chooses q_i and the supplier leaks this information to the entrant, i.e.,

$$\Pi^i_{t_i}(q_i, q^A_e(q_i; z_r); T) \le \Pi^i_{t_i}(q^N_{it_i}(z_r), q^N_e(z_r); T),$$

or the supplier's profit is worse off if he receives the order quantity information q_i from the incumbent and decides to leak this information to the entrant, i.e.,

$$\Pi^s_{t_i}(q_i, q^A_e(q_i; z_r); T) \le \Pi^s_{t_i}(q_i, q^N_e(z_r); T).$$

Now, we prove the property that $q^A_e(q^N_{iH}(z_r); z_r) \ge q^N_e(z_r)$. We proceed in two steps. First, we have $q^A_e(q^N_{iH}(z_r); z_r) = q^*_e(q^N_{iH}(z_r), 1; z_r)$, which follows Theorem 1 part 1 that $q^N_{iH}(z_r) > q^F_{iH}(z_r)$, Lemma 1 part 3 that $q^F_{iH}(z_r) > q^F_{iL}(z_r)$, and Theorem 2 that $\lambda_e(q_i; z_r) = 1$ when $q_i \neq q^F_{iL}(z_r)$.

Second, we prove the property that $q^*_e(q^N_{iH}(z_r), 1; z_r) \ge q^N_e(z_r)$. We have

$$\begin{aligned}\frac{\partial \Pi^e_H(q_i, q_e; T)}{\partial q_e} = &-(c^r_u + c^r_o)\big(\lambda' \mathbb{P}(\mu^e_H + (1-\eta)\epsilon \\ &+ a_{ie}(\mu^i_H + \eta\epsilon - q_i)^+ \le q_e) \\ &+ (1-\lambda')\mathbb{P}(\mu^e_L + (1-\eta)\epsilon \\ &+ a_{ie}(\mu^i_H + \eta\epsilon - q_i)^+ \le q_e)\big) + c^r_u\end{aligned}$$

and

$$\frac{\partial \Pi^e_L(q_i, q_e; T)}{\partial q_e} = -(c^r_u + c^r_o)\mathbb{P}(\mu^e_L + (1-\eta)\epsilon + a_{ie}(\mu^i_L + \eta\epsilon - q_i)^+ \le q_e) + c^r_u.$$

Hence,

$$\frac{\partial^2 \Pi^e_{t_i}(q_i, q_e; T)}{\partial q_e^2} \le 0, \quad \text{when } t_i \in \{H, L\}. \tag{B.1}$$

Thus, the entrant's best response order quantities $q^N_e(z_r)$, $q^*_e(q^N_{iH}(z_r), 1; z_r)$, and $q^*_e(q^N_{iL}(z_r), 0; z_r)$ satisfy the following first order conditions:

$$\left.\frac{\partial\big(\lambda \Pi^e_H(q^N_{iH}(z_r), q_e; T) + (1-\lambda)\Pi^e_L(q^N_{iL}(z_r), q_e; T)\big)}{\partial q_e}\right|_{q_e = q^N_e(z_r)} = 0 \tag{B.2}$$

and

$$\left.\frac{\partial \Pi^e_H(q^N_{iH}(z_r), q_e; T)}{\partial q_e}\right|_{q_e = q^*_e(q^N_{iH}(z_r), 1; z_r)} = 0 \tag{B.3}$$

and

$$\left.\frac{\partial \Pi^e_L(q^N_{iL}(z_r), q_e; T)}{\partial q_e}\right|_{q_e = q^*_e(q^N_{iL}(z_r), 0; z_r)} = 0. \tag{B.4}$$

We have $q^*_e(q^N_{iL}(z_r), 0; z_r) = q^*_e(q^F_{iL}(z_r), 0; z_r) = q^F_{eL}(z_r) < q^N_e(z_r)$, where the first equality follows Theorem 1 part 1 that $q^N_{iL}(z_r) = q^F_{iL}(z_r)$, the second equality follows Lemma 1 part 3, and the inequality follows Theorem 1 part 2. Thus, this property, Equation (B.1) for $t_i = L$ and Equation (B.4) jointly imply $\frac{\partial \Pi^e_L(q^N_{iL}(z_r), q_e; T)}{\partial q_e}\big|_{q_e = q^N_e(z_r)} \le \frac{\partial \Pi^e_L(q^N_{iL}(z_r), q_e; T)}{\partial q_e}\big|_{q_e = q^F_{eL}(z_r)} = 0$. Hence, this result and Equation (B.2) jointly imply $\frac{\partial \Pi^e_H(q^N_{iH}(z_r), q_e; T)}{\partial q_e}\big|_{q_e = q^N_e(z_r)} \ge 0$. Therefore, Equation (B.1) for $t_i = H$ and Equation (B.3) jointly imply $q^*_e(q^N_{iH}(z_r), 1; z_r) \ge q^N_e(z_r)$. □

Proof of Corollary 1. We denote by Q the set of the incumbent's all possible order quantities with which the supplier does not leak. We consider the following three scenarios.

Scenario 1: When the incumbent's demand state is $t_i \in \{H, L\}$, the incumbent chooses an order quantity $q_i \in Q \backslash \{q^N_{it_i}(z_r)\}$. Since the entrant cannot observe the incumbent's deviation from $q^N_{it_i}(z_r)$, she still orders $q^N_e(z_r)$. Because $q^*_{it_i}(q^N_e(z_r); z_r) = q^N_{it_i}(z_r)$, the incumbent cannot be more profitable if she deviates from $q^N_{it_i}(z_r)$.

Scenario 2: When the incumbent's demand state is high, the incumbent chooses an order quantity $q_i \notin \mathbb{Q}$. We have $\Pi^i_H(q_i, q^A_e(q_i; z_r); T) \le \Pi^i_H(q^A_{iH}(z_r), q^A_e(q^A_{iH}(z_r); z_r); T) = \Pi^i_H(q^S_{iH}(z_r), q^S_{eH}(z_r); T) = \Pi^i_H(q^F_{iH}(z_r), q^F_{eH}(z_r); T) \le \Pi^i_H(q^F_{iH}(z_r), q^N_e(z_r); T) \le \Pi^i_H(q^N_{iH}(z_r), q^N_e(z_r); T)$, where the first equality follows Theorem 2 part 1, the second equality follows Lemma 1 part 4 that when $\lambda' = 1$, $q^S_{iH}(z_r) = q^F_{iH}(z_r)$ and $q^S_{eH}(z_r) = q^F_{eH}(z_r)$, the second inequality follows Theorem 1 that $q^N_e(z_r) < q^F_{eH}(z_r)$ and Lemma 1 part 2 that the incumbent's profit is decreasing in the entrant's order quantity, the third inequality holds since $q^N_{iH}(z_r) = q^*_{iH}(q^N_e(z_r); z_r)$. This result entails that the incumbent cannot be more profitable if she deviates from $q^N_{iH}(z_r)$ when her demand state is high.

Scenario 3: When the incumbent's demand state is low, the incumbent chooses an order quantity $q_i \notin \mathbb{Q}$. For expositional clarity, we introduce

$$\hat{\Pi}^i_L(q_i; T) \triangleq (c^r_u + c^r_o)\mathbb{E}_\epsilon[\min\{D_i, q_i\} \mid t_i = L] - c^r_o q_i - h^i$$

to denote the incumbent's expected profit derived in her own market when her demand state is low. We have $\Pi^i_L(q_i, q^A_e(q_i; z_r); T) \le \Pi^i_L(q^A_{iL}(z_r), q^A_e(q^A_{iL}(z_r); z_r); T) = \Pi^i_L(q^F_{iL}(z_r), q^F_{eL}(z_r); T) = \hat{\Pi}^i_L(q^F_{iL}(z_r); T) = \Pi^i_L(q^F_{iL}(z_r), q^N_e(z_r); T) = \Pi^i_L(q^N_{iL}(z_r), q^N_e(z_r); T)$, where the first equality follows Theorem 2 that $q^A_{iL}(z_r) = q^F_{iL}(z_r)$, and thus $q^A_e(q^A_{iL}(z_r); z_r) = q^A_e(q^F_{iL}(z_r); z_r) = q^F_{eL}(z_r)$, the second equality holds since the property that $\frac{q^F_{iL}(z_r) - \mu^i_L}{q^F_{eL}(z_r) - \mu^e_L} = \frac{\eta}{1-\eta}$ implies that the entrant's unmet demand never flows to the incumbent, the third equality holds since the property that $q^N_e(z_r) > q^F_{eL}(z_r)$ proved in Theorem 1 implies $\frac{q^F_{iL}(z_r) - \mu^i_L}{q^N_e(z_r) - \mu^e_L} < \frac{q^F_{iL}(z_r) - \mu^i_L}{q^F_{eL}(z_r) - \mu^e_L} = \frac{\eta}{1-\eta}$ and this result further implies that the entrant's unmet demand never flows to the incumbent, the fourth equality follows Theorem 1 that $q^N_{iL}(z_r) = q^F_{iL}(z_r)$. This result entails that the incumbent cannot be more profitable if she deviates from $q_{iL}^N(z_r)$ when her demand state is low.

The analyses of the three scenarios above indicate that if the inequalities (5) with $t_i = H$ and L both hold, then the incumbent has no incentive to deviate from the nonleakage equilibrium order quantity $q_{it_i}^N(z_r)$ when $t_i \in \{H, L\}$. □

Proof of Theorem 4. Under a no-protection contract, there exists a wholesale price $w \in (c, p)$, such that $c_u^s = -c_o^s = w - c > 0$. Following Equation (4), the supplier's profit is $\Pi_{t_i}^s(q_i, q_e; T) = (w - c)(q_i + q_e) + h^i + h^e$. Thus,

$$\begin{aligned}\Pi_H^s(q_{iH}^N(z_r), q_e^A(q_{iH}^N(z_r); z_r); T) - \Pi_H^s(q_{iH}^N(z_r), q_e^N(z_r); T) \\ = (w - c)(q_{iH}^N(z_r) + q_e^A(q_{iH}^N(z_r); z_r)) - (w - c)(q_{iH}^N(z_r) + q_e^N(z_r)) \\ = (w - c)(q_e^A(q_{iH}^N(z_r); z_r) - q_e^N(z_r)) \\ \geq 0,\end{aligned}$$

where the inequality follows Theorem 3 part 1 that $q_e^A(q_{iH}^N(z_r); z_r) \geq q_e^N(z_r)$. As a result, nonleakage equilibrium does not exist under a wholesale-price contract. □

Proof of Theorem 5. Suppose $c_u^s \geq 0$, $c_o^s \leq 0$, and c_u^s and c_o^s do not equal zero simultaneously. We show that

$$g(q_i, q_e) \triangleq (c_u^s + c_o^s)\big(\min\{\bar{D}_i, q_i\} + \min\{\bar{D}_e, q_e\}\big) - c_o^s(q_i + q_e)$$

is increasing in $q_e \in \mathbb{R}_+$.

First, $g(q_i, q_e)$ is continuous in q_e.

Second, we show that $g(q_i, q_e)$ is increasing in $q_e \in [0, D_e)$. For $q_e \in [0, D_e)$, we have

$$g(q_i, q_e) = (c_u^s + c_o^s)\big(\min\{D_i + a_{ei}(D_e - q_e), q_i\} + q_e\big) - c_o^s(q_i + q_e).$$

Therefore,

$$\begin{aligned}\frac{\partial g(q_i, q_e)}{\partial q_e} &= (c_u^s + c_o^s)\big(-a_{ei}\mathbf{1}\{D_i + a_{ei}(D_e - q_e) \leq q_i\} + \mathbf{1}\big) - c_o^s \\ &= c_u^s\big(1 - a_{ei}\mathbf{1}\{D_i + a_{ei}(D_e - q_e) \leq q_i\}\big) \\ &\quad - c_o^s a_{ei}\mathbf{1}\{D_i + a_{ei}(D_e - q_e) \leq q_i\} \\ &\geq 0,\end{aligned}$$

where the inequality follows the conditions $c_u^s \geq 0$ and $c_o^s \leq 0$.

Third, we show that $g(q_i, q_e)$ is increasing in $q_e \in [D_e, \infty)$. For $q_e \in [D_e, \infty)$, we have

$$g(q_i, q_e) = (c_u^s + c_o^s)\big(\min\{D_i, q_i\} + \min\{\bar{D}_e, q_e\}\big) - c_o^s(q_i + q_e).$$

Therefore,

$$\begin{aligned}\frac{\partial g(q_i, q_e)}{\partial q_e} &= (c_u^s + c_o^s)\mathbf{1}\{q_e \leq \bar{D}_e\} - c_o^s \\ &= c_u^s\mathbf{1}\{q_e \leq \bar{D}_e\} - c_o^s\big(1 - \mathbf{1}\{q_e \leq \bar{D}_e\}\big) \\ &\geq 0,\end{aligned}$$

where the inequality follows the conditions $c_u^s \geq 0$ and $c_o^s \leq 0$.

Therefore, the three steps above jointly imply that $g(q_i, q_e)$ is increasing in $q_e \in \mathbb{R}_+$. Therefore, $\Pi_{t_i}^s(q_i, q_e; T) = \mathbb{E}_{t_e}[\mathbb{E}_\epsilon[g(q_i, q_e)] | t_i] + h^i + h^e$ is increasing in q_e. As a result, following Theorem 3 part 1 that $q_e^A(q_{iH}^N(z_r); z_r) \geq q_e^N(z_r)$, we have $\Pi_H^s(q_{iH}^N(z_r), q_e^A(q_{iH}^N(z_r); z_r); T) \geq \Pi_H^s(q_{iH}^N(z_r), q_e^N(z_r); T)$, i.e., when the incumbent's demand state is high, the supplier always has incentive to leak.

Hence, if the inequalities (5) with $t_i = H$ holds, then either $c_u^s > 0$ and $c_o^s > 0$, or $c_u^s < 0$ and $c_o^s < 0$, or $c_u^s \leq 0$ and $c_o^s \geq 0$. □

Proof of Lemma 2. We note that

$$\begin{aligned}\Delta O_{t_i}(z_r) &= \mathbb{E}_\epsilon\big[(q_{it_i}^N(z_r) - \bar{D}_i(q_e^A(q_{it_i}^N(z_r); z_r)))^+ + (q_e^A(q_{it_i}^N(z_r); z_r) \\ &\quad - \bar{D}_e(q_{it_i}^N(z_r)))^+ | t_i\big] \\ &\quad - \mathbb{E}_\epsilon\big[(q_{it_i}^N(z_r) - \bar{D}_i(q_e^N(z_r)))^+ + (q_e^N(z_r) \\ &\qquad - \bar{D}_e(q_{it_i}^N(z_r)))^+ | t_i\big], \\ \Delta U_{t_i}(z_r) &= \mathbb{E}_\epsilon\big[(\bar{D}_i(q_e^A(q_{it_i}^N(z_r); z_r)) - q_{it_i}^N(z_r))^+ + (\bar{D}_e(q_{it_i}^N(z_r)) \\ &\quad - q_e^A(q_{it_i}^N(z_r); z_r))^+ | t_i\big] - \mathbb{E}_\epsilon\big[(\bar{D}_i(q_e^N(z_r)) \\ &\quad - q_{it_i}^N(z_r))^+ + (\bar{D}_e(q_{it_i}^N(z_r)) - q_e^N(z_r))^+ | t_i\big].\end{aligned}$$

First, we consider the scenario that the incumbent's demand state is high.

Recall from Theorem 1 part 1 that $q_{iH}^N(z_r) > q_{iH}^F(z_r)$. Hence, if the entrant orders $q_e \leq q_{eH}^F(z_r)$, then the condition $q_e > \mu_H^e + (1 - \eta)\epsilon$ implies $q_{iH}^N(z_r) > \mu_H^i + \eta\epsilon$. Hence, the entrant does not have the chance to use her excess inventory to satisfy the incumbent's unmet demand, i.e., for $q_e \leq q_{eH}^F(z_r)$, $(q_e - \bar{D}_e(q_{iH}^N(z_r)))^+ = (q_e - D_e)^+$.

We have

$$\begin{aligned}\Delta O_H(z_r) &= \mathbb{E}_\epsilon\big[(q_{iH}^N(z_r) - \bar{D}_i(q_e^A(q_{iH}^N(z_r); z_r)))^+ + (q_e^A(q_{iH}^N(z_r); z_r) \\ &\quad - \bar{D}_e(q_{iH}^N(z_r)))^+ | t_i = H\big] \\ &\quad - \mathbb{E}_\epsilon\big[(q_{iH}^N(z_r) - \bar{D}_i(q_e^N(z_r)))^+ + (q_e^N(z_r) \\ &\quad - \bar{D}_e(q_{iH}^N(z_r)))^+ | t_i = H\big] \\ &= \mathbb{E}_\epsilon\big[(q_{iH}^N(z_r) - \bar{D}_i(q_{eH}^F(z_r)))^+ + (q_{eH}^F(z_r) \\ &\quad - \bar{D}_e(q_{iH}^N(z_r)))^+ | t_i = H\big] \\ &\quad - \mathbb{E}_\epsilon\big[(q_{iH}^N(z_r) - \bar{D}_i(q_e^N(z_r)))^+ + (q_e^N(z_r) \\ &\quad - \bar{D}_e(q_{iH}^N(z_r)))^+ | t_i = H\big] \\ &\geq \mathbb{E}_\epsilon\big[(q_{eH}^F(z_r) - \bar{D}_e(q_{iH}^N(z_r)))^+ | t_i = H\big] - \mathbb{E}_\epsilon\big[(q_e^N(z_r) \\ &\quad - \bar{D}_e(q_{iH}^N(z_r)))^+ | t_i = H\big] \\ &= \mathbb{E}_\epsilon\big[(q_{eH}^F(z_r) - D_e)^+ | t_i = H\big] \\ &\qquad - \mathbb{E}_\epsilon\big[(q_e^N(z_r) - D_e)^+ | t_i = H\big] \\ &= \int_{q_e = q_e^N(z_r)}^{q_{eH}^F(z_r)} \mathbb{P}(q_e \geq D_e | t_i = H) dq_e \\ &= \int_{q_e = q_e^N(z_r)}^{q_{eH}^F(z_r)} \mathbb{P}(q_e \geq \mu_H^e + (1 - \eta)\epsilon) dq_e > 0,\end{aligned}$$

where the second equality holds since Theorem 2, and Lemma 1 parts 1–2 imply $q_e^A(q_{iH}^N(z_r); z_r) = q_e^*(q_{iH}^N(z_r), 1; z_r) = q_e^*(q_{iH}^F(z_r), 1; z_r) = q_{eH}^F(z_r)$, the first inequality follows Theorem 1 part 2 that $q_e^N(z_r) < q_{eH}^F(z_r)$ and the property that $(q_i - \bar{D}_i(q_e))^+$ is increasing in q_e, the third equality follows Theorem 1 part 2 that $q_e^N(z_r) < q_{eH}^F(z_r)$ and the property that $(q_e - \bar{D}_e(q_{iH}^N(z_r)))^+ = (q_e - D_e)^+$ when $q_e \leq q_{eH}^F(z_r)$, the second inequality follows Theorem 1 part 2 that $q_e^N(z_r) < q_{eH}^F(z_r)$, Lemma 1 part 3 that $\mathbb{P}(q_{eH}^F(z_r) \geq \mu_H^e + (1 - \eta)\epsilon) = z_r > 0$, and the property that $\mathbb{P}(q_e \geq \mu_H^e + (1 - \eta)\epsilon)$ is continuous in q_e.

We have

$$\begin{aligned}\Delta U_H(z_r) &= \mathbb{E}_\epsilon\big[(\bar{D}_i(q_e^A(q_{iH}^N(z_r);z_r)) - q_{iH}^N(z_r))^+ + (\bar{D}_e(q_{iH}^N(z_r)) \\ &\quad - q_e^A(q_{iH}^N(z_r);z_r))^+ \,|t_i = H\big] \\ &\quad - \mathbb{E}_\epsilon\big[(\bar{D}_i(q_e^N(z_r)) - q_{iH}^N(z_r))^+ + (\bar{D}_e(q_{iH}^N(z_r)) \\ &\quad - q_e^N(z_r))^+ \,|t_i = H\big] \\ &= \mathbb{E}_\epsilon\big[(\bar{D}_i(q_{eH}^F(z_r)) - q_{iH}^N(z_r))^+ + (\bar{D}_e(q_{iH}^N(z_r)) \\ &\quad - q_{eH}^F(z_r))^+ \,|t_i = H\big] \\ &\quad - \mathbb{E}_\epsilon\big[(\bar{D}_i(q_e^N(z_r)) - q_{iH}^N(z_r))^+ + (\bar{D}_e(q_{iH}^N(z_r)) \\ &\quad - q_e^N(z_r))^+ \,|t_i = H\big] \\ &\le \mathbb{E}_\epsilon[(\bar{D}_e(q_{iH}^N(z_r)) - q_{eH}^F(z_r))^+ \,|t_i = H] \\ &\quad - \mathbb{E}_\epsilon[(\bar{D}_e(q_{iH}^N(z_r)) - q_e^N(z_r))^+ \,|t_i = H] \\ &= -\int_{q_e = q_e^N(z_r)}^{q_{eH}^F(z_r)} \mathbb{P}\big(q_e \le \bar{D}_e(q_{iH}^N(z_r))\,|t_i = H\big)dq_e \\ &\le -\int_{q_e = q_e^N(z_r)}^{q_{eH}^F(z_r)} \mathbb{P}\big(q_e \le D_e(q_{iH}^N(z_r))\,|t_i = H\big)dq_e \\ &= -\int_{q_e = q_e^N(z_r)}^{q_{eH}^F(z_r)} \mathbb{P}\big(q_e \le \mu_H^e + (1-\eta)\epsilon\big)dq_e < 0,\end{aligned}$$

where the second equality holds since Theorem 2, and Lemma 1 parts 1–2 imply $q_e^A(q_{iH}^N(z_r);z_r) = q_e^*(q_{iH}^N(z_r),1;z_r) = q_e^*(q_{iH}^F(z_r),1;z_r) = q_{eH}^F(z_r)$, the first inequality follows Theorem 1 part 2 that $q_e^N(z_r) < q_{eH}^F(z_r)$ and the property that $(q_i - \bar{D}_i(q_e))^+$ is increasing in q_e, the second inequality follows Theorem 1 part 2 that $q_e^N(z_r) < q_{eH}^F(z_r)$ and the property that $\bar{D}_e(q_i) \ge D_e$, the third inequality follows Theorem 1 part 2 that $q_e^N(z_r) < q_{eH}^F(z_r)$, Lemma 1 part 3 that $\mathbb{P}(q_{eH}^F(z_r) \le \mu_H^e + (1-\eta)\epsilon) = 1 - z_r > 0$, and the property that $\mathbb{P}(q_e \le \mu_H^e + (1-\eta)\epsilon)$ is continuous in q_e.

Next, we consider the scenario that the incumbent's demand state is low.

Recall from Theorem 1 part 1 that $q_{iL}^N(z_r) = q_{iL}^F(z_r)$. Hence, if the entrant orders $q_e \ge q_{eL}^F(z_r)$, then the condition $q_{iL}^N(z_r) > \mu_L^i + \eta\epsilon$ implies $q_e > \mu_L^e + (1-\eta)\epsilon$. Hence, the entrant does not have the chance to use her excess inventory to satisfy the incumbent's unmet demand, i.e., for $q_e \ge q_{eL}^F(z_r)$, $(q_i - \bar{D}_i(q_e))^+ = (q_i - D_i)^+$. If the entrant orders $q_e = q_{eL}^F(z_r)$, then the condition $q_e > \mu_L^e + (1-\eta)\epsilon$ implies $q_{iL}^N(z_r) > \mu_L^i + \eta\epsilon$. Hence, the incumbent does not have the chance to use her excess inventory to satisfy the entrant's unmet demand. Therefore, $\mathbb{P}(q_{eL}^F(z_r) \ge \bar{D}_e(q_{iL}^N(z_r))\,|t_i = L) = \mathbb{P}(q_{eL}^F(z_r) \ge D_e\,|t_i = L) = z_r$.

We have

$$\begin{aligned}\Delta O_L(z_r) &= \mathbb{E}_\epsilon\big[(q_{iL}^N(z_r) - \bar{D}_i(q_e^A(q_{iL}^N(z_r);z_r)))^+ + (q_e^A(q_{iL}^N(z_r);z_r) \\ &\quad - \bar{D}_e(q_{iL}^N(z_r)))^+ \,|t_i = L\big] \\ &\quad - \mathbb{E}_\epsilon\big[(q_{iL}^N(z_r) - \bar{D}_i(q_e^N(z_r)))^+ + (q_e^N(z_r) \\ &\quad - \bar{D}_e(q_{iL}^N(z_r)))^+ \,|t_i = L\big] \\ &= \mathbb{E}_\epsilon\big[(q_{iL}^N(z_r) - D_i)^+ + (q_e^A(q_{iL}^N(z_r);z_r) \\ &\quad - \bar{D}_e(q_{iL}^N(z_r)))^+ \,|t_i = L\big] \\ &\quad - \mathbb{E}_\epsilon[(q_{iL}^N(z_r) - D_i)^+ + (q_e^N(z_r) - \bar{D}_e(q_{iL}^N(z_r)))^+ \,|t_i = L] \\ &= \mathbb{E}_\epsilon[(q_e^A(q_{iL}^N(z_r);z_r) - \bar{D}_e(q_{iL}^N(z_r)))^+ \,|t_i = L] - \mathbb{E}_\epsilon\big[(q_e^N(z_r) \\ &\quad - \bar{D}_e(q_{iL}^N(z_r)))^+ \,|t_i = L\big] \\ &= -\int_{q_e = q_{eL}^F(z_r)}^{q_e^N(z_r)} \mathbb{P}\big(q_e \ge \bar{D}_e(q_{iL}^N(z_r))\,|t_i = L\big)dq_e < 0,\end{aligned}$$

where the second equality follows Theorem 1 part 2 that $q_e^N(z_r) > q_{eL}^F(z_r)$ and the property that $(q_i - \bar{D}_i(q_e))^+ = (q_i - D_i)^+$ when $q_e \ge q_{eL}^F(z_r)$, the inequality follows Theorem 1 part 2 that $q_e^N(z_r) > q_{eL}^F(z_r)$, the property that $\mathbb{P}(q_{eL}^F(z_r) \ge \bar{D}_e(q_{iL}^N(z_r))\,|t_i = L) = z_r > 0$, and the property that $\mathbb{P}(q_e \ge \bar{D}_e(q_{iL}^N(z_r))\,|t_i = L)$ is continuous in q_e.

We have

$$\begin{aligned}\Delta U_L(z_r) &= \mathbb{E}_\epsilon\big[(\bar{D}_i(q_e^A(q_{iL}^N(z_r);z_r)) - q_{iL}^N(z_r))^+ + (\bar{D}_e(q_{iL}^N(z_r)) \\ &\quad - q_e^A(q_{iL}^N(z_r);z_r))^+ \,|t_i = L\big] \\ &\quad - \mathbb{E}_\epsilon\big[(\bar{D}_i(q_e^N(z_r)) - q_{iL}^N(z_r))^+ + (\bar{D}_e(q_{iL}^N(z_r)) \\ &\quad - q_e^N(z_r))^+ \,|t_i = L\big] \\ &\ge \mathbb{E}_\epsilon[(\bar{D}_e(q_{iL}^N(z_r)) - q_e^A(q_{iL}^N(z_r);z_r))^+ \,|t_i = L] \\ &\quad - \mathbb{E}_\epsilon[\bar{D}_e(q_{iL}^N(z_r)) - (q_e^N(z_r))^+ \,|t_i = L] \\ &= \int_{q_e = q_{eL}^F(z_r)}^{q_e^N(z_r)} \mathbb{P}\big(q_e \le \bar{D}_e(q_{iL}^N(z_r))\,|t_i = L\big)dq_e \\ &> 0,\end{aligned}$$

where the first inequality follows Theorem 1 part 2 that $q_e^N(z_r) > q_{eL}^F(z_r)$ and the property that $(\bar{D}_i(q_e) - q_i)^+$ is increasing in q_e, the second inequality follows Theorem 1 part 2 that $q_e^N(z_r) > q_{eL}^F(z_r)$, the property that $\mathbb{P}(q_{eL}^F(z_r) \le \bar{D}_e(q_{iL}^N(z_r))\,|t_i = L) = 1 - z_r > 0$, and the property that $\mathbb{P}(q_e \le \bar{D}_e(q_{iL}^N(z_r))\,|t_i = L) = 1 - z_r > 0$ is continuous in q_e. □

Proof of Theorem 6. When $t_i \in \{H, L\}$, inequality (5) holds if and only if $c_o^s \Delta O_{t_i}(z_r) + c_u^s \Delta U_{t_i}(z_r) \ge 0$. Following Lemma 2 of the signs of $\Delta O_{t_i}(z_r)$ and $\Delta U_{t_i}(z_r)$ for $t_i \in \{H, L\}$, these conditions are equivalent to the conditions $c_o^s \gamma_H(z_r) \ge c_u^s$ and $c_o^s \gamma_L(z_r) \le c_u^s$.

For downside-protection contracts, we now show that conditions $c_o^s \gamma_H(z_r) \ge c_u^s$ and $c_o^s \gamma_L(z_r) \le c_u^s$ are equivalent to conditions $\frac{c_u^s}{c_o^s} \le \gamma_H(z_r)$ and $\frac{c_u^s}{c_o^s} \ge \gamma_L(z_r)$. First, suppose conditions $c_o^s \gamma_H(z_r) \ge c_u^s$ and $c_o^s \gamma_L(z_r) \le c_u^s$ hold. Because $c_u^s > 0$ and $\gamma_H(z_r) > 0$, the condition $c_o^s \gamma_H(z_r) \ge c_u^s$ implies $c_o^s > 0$. Thus, we have $\frac{c_u^s}{c_o^s} \le \gamma_H(z_r)$ and $\frac{c_u^s}{c_o^s} \ge \gamma_L(z_r)$. Second, suppose conditions $\frac{c_u^s}{c_o^s} \le \gamma_H(z_r)$ and $\frac{c_u^s}{c_o^s} \ge \gamma_L(z_r)$ hold. Because $c_u^s > 0$ and $\gamma_L(z_r) > 0$, the condition $\frac{c_u^s}{c_o^s} \ge \gamma_L(z_r)$ implies $c_o^s > 0$. Thus, we have $c_o^s \gamma_H(z_r) \ge c_u^s$ and $c_o^s \gamma_L(z_r) \le c_u^s$. Therefore, conditions $c_o^s \gamma_H(z_r) \ge c_u^s$ and $c_o^s \gamma_L(z_r) \le c_u^s$ hold if and only if conditions $\frac{c_u^s}{c_o^s} \le \gamma_H(z_r)$ and $\frac{c_u^s}{c_o^s} \ge \gamma_L(z_r)$ hold.

For upside-protection contracts, because $c_o^s < 0$, conditions $c_o^s \gamma_H(z_r) \ge c_u^s$ and $c_o^s \gamma_L(z_r) \le c_u^s$ are equivalent to the conditions $\frac{c_u^s}{c_o^s} \ge \gamma_H(z_r)$ and $\frac{c_u^s}{c_o^s} \le \gamma_L(z_r)$.

Now, we prove part 3. If $a_{ie} = a_{ei} = 1$, then the change in the supply chain's excess inventory and unmet demand, $\Delta O_{t_i}(z_r)$ and $\Delta U_{t_i}(z_r)$, respectively, can be written in the following compact form:

$$\Delta O_{t_i}(z_r) \triangleq \mathbb{E}_\epsilon[(q_{it_i}^N(z_r) + q_e^A(q_{it_i}^N(z_r); z_r) - D_i - D_e)^+|t_i] - \mathbb{E}_\epsilon[(q_{it_i}^N(z_r) + q_e^N(z_r) - D_i - D_e)^+|t_i]$$

$$\Delta U_{t_i}(z_r) \triangleq \mathbb{E}_\epsilon[(D_i + D_e - q_{it}^N(z_r) - q_e^A(q_{it}^N(z_r); z_r))^+|t_i] - \mathbb{E}_\epsilon[(D_i + D_e - q_{it_i}^N(z_r) - q_e^N(z_r))^+|t_i]$$

Suppose a downside-protection contract with $z_r \geq z_I$ prevents information leakage. On the one hand, note that Theorem 5 implies $c_o^s > 0$. Thus,

$$\frac{c_u^s}{c_o^s} = \frac{c_u^I - c_u^r}{c_o^s} = \frac{\frac{z_I}{1-z_I}c_o^I - \frac{z_r}{1-z_r}c_o^r}{c_o^s} \leq \frac{\frac{z_r}{1-z_r}c_o^I - \frac{z_r}{1-z_r}c_o^r}{c_o^s} = \frac{z_r}{1-z_r},$$

where the inequality follows the condition $z_r \geq z_I$ and $c_o^s > 0$.

On the other hand, when $t_i = L$, we have $q_e^A(q_{iL}^N(z_r); z_r) = q_{eL}^F(z_r) < q_e^N(z_r)$, $F(q_{iL}^N(z_r) + q_e^A(q_{iL}^N(z_r); z_r) - \mu_L^i - \mu_L^e) = z_r \in (0,1)$. Because $F(\epsilon') > F(\epsilon)$ if $F(\epsilon) \in (0,1)$ and $\epsilon' > \epsilon$, $F(q_{iL}^N(z_r) + q_e^N(z_r) - \mu_L^i - \mu_L^e) \in (z_r, 1]$. Hence, the intermediate value theorem implies that there exists $v_L \in (z_r, 1)$, such that

$$\Delta O_L(z_r) = \int_{q=q_{iL}^N(z_r)+q_e^N(z_r)}^{q_{iL}^N(z_r)+q_e^A(q_{iL}^N(z_r);z_r)} F(q - \mu_L^i - \mu_L^e)dq = -\int_{q=q_{iL}^N(z_r)+q_e^A(q_{iL}^N(z_r);z_r)}^{q_{iL}^N(z_r)+q_e^N(z_r)} F(q - \mu_L^i - \mu_L^e)dq = -v_L(q_e^N(z_r) - q_{eL}^F(z_r)),$$

and

$$\Delta U_L(z_r) = \int_{q=q_{iL}^N(z_r)+q_e^N(z_r)}^{q_{iL}^N(z_r)+q_e^A(q_{iL}^N(z_r);z_r)} (1 - F(q - \mu_L^i - \mu_L^e))dq = \int_{q=q_{iL}^N(z_r)+q_e^A(q_{iL}^N(z_r);z_r)}^{q_{iL}^N(z_r)+q_e^N(z_r)} (1 - F(q - \mu_L^i - \mu_L^e))dq = (1 - v_L)(q_e^N(z_r) - q_{eL}^F(z_r)).$$

Hence,

$$\gamma_L(z_r) = \frac{-\Delta O_L(z_r)}{\Delta U_L(z_r)} = \frac{v_L}{1 - v_L} > \frac{z_r}{1 - z_r} \geq \frac{c_u^s}{c_o^s},$$

which violates the nonleakage conditions given by Theorem 6 part 1.

Therefore, the condition $z_r \in (0, z_I)$ is necessary for downside-protection contracts to prevent information leakage.

Suppose an upside-protection contract with $z_r \leq z_I$ prevents information leakage. On the one hand, recall that an upside-protection contract is defined to satisfy the condition implies $c_o^s < 0$. Thus,

$$\frac{c_u^s}{c_o^s} = \frac{c_u^I - c_u^r}{c_o^s} = \frac{\frac{z_I}{1-z_I}c_o^I - \frac{z_r}{1-z_r}c_o^r}{c_o^s} \leq \frac{\frac{z_r}{1-z_r}c_o^I - \frac{z_r}{1-z_r}c_o^r}{c_o^s} = \frac{z_r}{1-z_r},$$

where the inequality follows the condition $z_r \leq z_I$ and $c_o^s < 0$. On the other hand, when $t_i = H$, we have $q_e^A(q_{iH}^N(z_r); z_r) = q_{eH}^F(z_r) > q_e^N(z_r)$, $F(q_{iH}^N(z_r) + q_e^N(z_r) - \mu_H^i - \mu_H^e) = z_r \in (0,1)$. Because $F(\epsilon') > F(\epsilon)$ if $F(\epsilon) \in (0,1)$ and $\epsilon' > \epsilon$, $F(q_{iH}^N(z_r) + q_e^A(q_{iH}^N(z_r); z_r) - \mu_H^i - \mu_H^e) \in (z_r, 1]$. Hence, intermediate value theorem implies that there exists $v_H \in (z_r, 1)$, such that

$$\Delta O_H(z_r) = \int_{q=q_{iH}^N(z_r)+q_e^N(z_r)}^{q_{iH}^N(z_r)+q_e^A(q_{iH}^N(z_r);z_r)} F(q - \mu_H^i - \mu_H^e)dq = v_H(q_{eH}^F(z_r) - q_e^N(z_r)),$$

$$\Delta U_H(z_r) = -\int_{q=q_{iH}^N(z_r)+q_e^N(z_r)}^{q_{iH}^N(z_r)+q_e^A(q_{iH}^N(z_r);z_r)} (1 - F(q - \mu_H^i - \mu_H^e))dq = -(1 - v_H)(q_{eH}^F(z_r) - q_e^N(z_r)).$$

Hence,

$$\gamma_H(z_r) = \frac{\Delta O_H(z_r)}{-\Delta U_H(z_r)} = \frac{v_H}{1 - v_H} > \frac{z_r}{1 - z_r} \geq \frac{c_u^s}{c_o^s},$$

which violates nonleakage conditions given by Theorem 6 part 2.

Therefore, the condition $z_r \in (z_I, 1)$ is necessary for upside-protection contracts to prevent information leakage. □

Appendix C. Additional Results on Nonleakage Conditions and Profits

We study two market environments that differ from the market environment studied in Section 6 only in λ', with $\lambda' = 0.5$ and $\lambda' = 0.1$, respectively. All other parameters remain the same, that is, $\mu_H^i = 12$, $\mu_L^i = 10$, $\mu_H^e = 8$, $\mu_L^e = 6$, $\lambda = 0.5$, $\eta = 0.6$, $p = 1$, $c = 0.9$, $a_{ie} = 0.9$, $a_{ei} = 0.9$, and ϵ is uniformly distributed on $[-3,3]$. The shaded areas in Figure C.1 represent all buy-back contracts that prevent information leakage.

Next, we analyze the impact of buy-back contracts on each firm's expected profit. Among those buy-back contracts that fall in the shaded area of Figure C.1, we randomly pick five of them (marked in circles in the figure) and report the resulting profits in Tables C.1 and C.2. We report additional results in Table C.3 on each firm's expected profit in equilibrium under a series of buy-back contracts with different parameters. For each selected wholesale price w, we consider three different values of the buy-back price b. We make the following observations. By fixing the wholesale price w, the integrated firm's total expected profit under the buy-back price b that prevents information is very close to the ones under other buy-back prices that lead to the leakage equilibrium. In addition, these leakage-proof contracts only result in a small loss from the integrated firm's optimal expected profit.

Appendix D. Intuitive Criterion

We remark that all three types of equilibriums characterized in Theorem 2, the separating equilibrium, the pooling equilibrium, and the semiseparating equilibrium, survive the intuitive criterion. It is worth noting that the pooling and the semiseparating equilibriums do not survive the intuitive criterion in many other signaling games (see, e.g., Cho and Kreps 1987). In contrast, these two types of equilibriums survive the intuitive criterion in our newsvendor competition game. First, in our model, the entrant orders at least the quantity that maximizes her expected profit derived from her own market, $q_{eL}^F(z_r)$, because each retailer satisfies her own demand before leftover demand spills over to the other retailer, irrespective of the incumbent's order quantity and the entrant's belief on the incumbent's demand state. Second, when the incumbent's demand state is low, the entrant's demand state is also low with certainty and the market uncertainty ϵ is split between the incumbent and the entrant with fixed proportions η and $1 - \eta$, respectively.

Therefore, when the incumbent distorts her order quantity to be less than her equilibrium order quantity in her low demand

Figure C.1. Buy-Back Contracts That Prevent Information Leakage

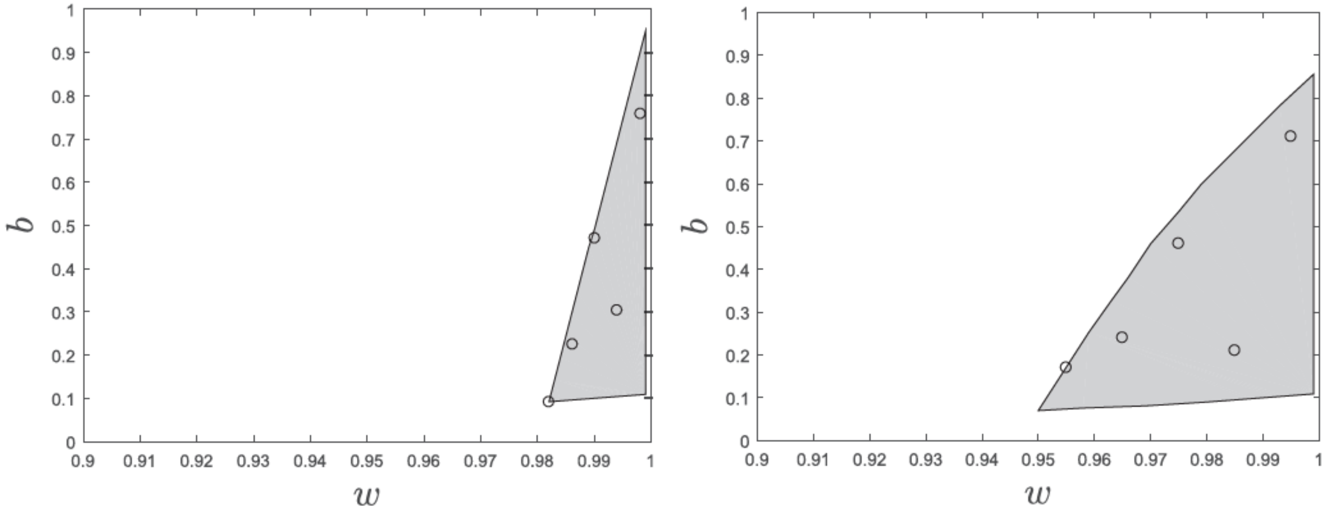

Table C.1 Expected Profits Under Buy-Back Contracts That Prevent Information Leakage

$T=(w,b)$	t_i	$\Pi^i_{t_i l}(T)$	$\Pi^e_{t_i l}(T)$	$\Pi^s_{t_i l}(T)$	$\Pi^I_{t_i l}(T)$	$\Pi^I_{t_i l}(T)/\Pi^{I*}_{t_i}$
(0.98,0.09)	H	0.19	0.09	1.25	1.53	99.08%
	L	0.15	0.09	1.08	1.31	99.07%
	$\mathbb{E}_{t_i}[\cdot]$	0.17	0.09	1.17	1.42	99.07%
(0.98,0.23)	H	0.14	0.07	1.31	1.53	99.08%
	L	0.12	0.07	1.13	1.31	99.07%
	$\mathbb{E}_{t_i}[\cdot]$	0.13	0.07	1.22	1.42	99.07%
(0.99,0.47)	H	0.10	0.05	1.38	1.53	99.08%
	L	0.08	0.05	1.18	1.31	99.06%
	$\mathbb{E}_{t_i}[\cdot]$	0.09	0.05	1.28	1.42	99.07%
(0.99,0.30)	H	0.06	0.03	1.42	1.51	97.86%
	L	0.05	0.03	1.23	1.31	98.48%
	$\mathbb{E}_{t_i}[\cdot]$	0.06	0.03	1.32	1.41	98.15%
(1.00,0.76)	H	0.02	0.01	1.48	1.51	97.85%
	L	0.02	0.01	1.28	1.31	98.48%
	$\mathbb{E}_{t_i}[\cdot]$	0.02	0.01	1.38	1.41	98.14%

Table C.2 Expected Profits Under Buy-Back Contracts That Prevent Information Leakage

$T=(w,b)$	t_i	$\Pi^i_{t_i l}(T)$	$\Pi^e_{t_i l}(T)$	$\Pi^s_{t_i l}(T)$	$\Pi^I_{t_i l}(T)$	$\Pi^I_{t_i l}(T)/\Pi^{I*}_{t_i}$
(0.96,0.17)	H	0.46	0.22	0.84	1.52	99.45%
	L	0.37	0.22	0.73	1.32	99.55%
	$\mathbb{E}_{t_i}[\cdot]$	0.42	0.22	0.78	1.42	99.50%
(0.97,0.24)	H	0.36	0.17	0.99	1.52	99.35%
	L	0.29	0.17	0.86	1.32	99.43%
	$\mathbb{E}_{t_i}[\cdot]$	0.32	0.17	0.92	1.42	99.38%
(0.98,0.46)	H	0.26	0.12	1.14	1.52	99.35%
	L	0.21	0.12	0.99	1.32	99.43%
	$\mathbb{E}_{t_i}[\cdot]$	0.23	0.12	1.07	1.42	99.39%
(0.99,0.21)	H	0.15	0.07	1.28	1.51	98.69%
	L	0.12	0.07	1.11	1.31	98.63%
	$\mathbb{E}_{t_i}[\cdot]$	0.14	0.07	1.20	1.41	98.66%
(1.00,0.71)	H	0.05	0.02	1.43	1.51	98.52%
	L	0.04	0.02	1.24	1.31	98.53%
	$\mathbb{E}_{t_i}[\cdot]$	0.05	0.02	1.34	1.41	98.52%

state $q^F_{iL}(z_r)$, she would never receive any spillover from the entrant. Note that the incumbent's equilibrium order quantity in her low demand state, $q^F_{iL}(z_r)$, enables her to capture the maximum expected profit derived from her own market. Therefore, the incumbent would be worse off if she distorted her order quantity in the downward direction.

Table C.3 Expected Profits Under Buy-Back Contracts

$T = (w, b)$	Supplier's decision	$\mathbb{E}_{t_i}[\Pi^i_{t,l}(T)]$	$\mathbb{E}_{t_i}[\Pi^e_{t,l}(T)]$	$\mathbb{E}_{t_i}[\Pi^s_{t,l}(T)]$	$\mathbb{E}_{t_i}[\Pi^I_{t,l}(T)]$	$\mathbb{E}_{t_i}[\Pi^I_{t,l}(T)]/\mathbb{E}_{t_i}[\Pi^{I,*}_{t_i}]$
(0.94,0.10)	Always leak	0.56	0.33	0.61	1.50	99.24%
(0.94,0.30)	Never leak	0.59	0.30	0.60	1.50	98.91%
(0.94,0.50)	Always leak	0.57	0.34	0.59	1.50	98.99%
(0.96,0.20)	Always leak	0.37	0.22	0.92	1.51	99.43%
(0.96,0.40)	Never leak	0.39	0.20	0.91	1.50	99.35%
(0.96,0.60)	Always leak	0.38	0.23	0.91	1.51	99.66%

Endnotes

[1] If $c^r_o < 0$ or if $c^r_o = 0$ and $c^r_u > 0$, then the incumbent's optimal decision would be to order an infinite quantity; if $c^r_o \geq 0$ and $c^r_u \leq 0$, then the incumbent's optimal decision would be to order nothing.

[2] Contracts that do not satisfy this property, such as the quantity flexibility contracts, are beyond the scope of the family of contracts considered in this paper.

[3] Otherwise, the supplier's profit is independent of retailers' order quantities. In that case, the supplier has no incentive to participate in the business.

[4] We consider a fixed-fee rebate contract that combines the two-part tariff and linear rebate by introducing a rebate term into a two-part tariff contract (see Cachon and Lariviere 2005). This contract prevents the supplier's expected profit from being negative. The linear-rebate contract alone is not implementable because the supplier's expected profit is negative (see Taylor 2002 who, as a remedy, proposes a target-rebate contract under which rebates are implemented if sales quantity exceeds a target level).

[5] For a revenue-sharing contract (w, f), we have $c^r_u = fp - w$ and $c^r_o = w$. If we compare this with the wholesale-price contract $(w + (1-f)p)$, which has $c^r_u = fp - w$ and $c^r_o = w + (1-f)p$, then this revenue-sharing contract has a smaller unit overage cost and the same unit underage cost. Therefore, a revenue-sharing contract is a downside-protection contract.

[6] We remark that the information exchange between the supplier and the entrant (i.e., information leakage) is not a cheap-talk. In other words, the supplier can credibility leak the incumbent's order quantity information to the entrant by showing, for example, the paid invoice because the supplier receives a binding order (not a forecast) and a corresponding payment from the incumbent.

[7] Throughout the paper, we use "decreasing," "increasing," "greater than," and "smaller than" in the weak sense.

[8] This property holds because (1) two retailers' demands are perfectly correlated when the incumbent's demand state is low; that is, the entrant's demand state is low with probability 1 and the market uncertainty ϵ is split between the incumbent and the entrant with fixed proportions η and $1-\eta$, respectively; (2) each retailer has the priority to satisfy her own demand before it spills to the other retailer.

[9] The thresholds on the market state and its likelihood, that is, $\underline{\mu}^i_H$ and $\underline{\Lambda}(\mu^i_H)$, are defined in the proof.

[10] Recall that for downside-protection or upside-protection contracts, if the supplier's unit underage cost c^s_u or the supplier's unit overage cost c^s_o is zero, then the contract cannot prevent the supplier from inducing retailers to order as much as possible. By contrast, a two-sided protection contract with either the supplier's unit underage cost c^s_u or the supplier's unit overage cost c^s_o that is equal to zero may prevent the supplier from doing so. Consider a two-sided contract that has zero unit underage cost $c^s_u = 0$ and strictly positive unit overage cost $c^s_o > 0$. Under this contract, the supplier is neither rewarded nor punished from the supply chain's excess inventory. However, he is penalized from the supply chain's inventory shortage. Therefore, the supplier suffers from inducing retailers to order too much. Following a similar argument, consider a two-sided contract that has zero unit overage cost $c^s_o = 0$ and strictly negative unit underage cost $c^s_u < 0$. Under this contract, the supplier is neither rewarded nor punished from the supply chain's excess inventory. However, he is rewarded from the supply chain's inventory shortage. Therefore, the supplier has incentive to motivate retailers not to order too much.

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