A Multi-Leader Stackelberg Game for Two-Hop Systems with Wireless Energy Transfer

Shiyang Leng and Aylin Yener
Wireless Communications and Networking Laboratory (WCAN)
School of Electrical Engineering and Computer Science
The Pennsylvania State University, University Park, PA 16802.
sfl5154@psu.edu *yener@engr.psu.edu*

***Abstract*—We study a two-hop network with wireless energy transfer (WET) from the source to multiple energy harvesting relays. Both the source and relays intend to transmit dedicated information to the destination. The source, without direct reliable channels to the destination, needs the relays to forward signals, while the relays are short of energy and have to harvest energy from the source to transmit their own data and relaying the source's data. Relays use time division to harvest then transmit. For the multiple access channel (MAC) from the relays to the destination, we consider both time division multiple access (TDMA) between the relays and simultaneous transmission (ST) by all relays. The source and the relays are all selfish and aim to maximize their own utility. We take a game theoretic viewpoint to model the hierarchical competition between the source and the relays. In particular, multi-leader Stackelberg games are formulated where the relays play as the leaders and the source plays as the follower. The existence and the uniqueness of Stackelberg equilibrium (SE) are analyzed, based on which algorithms are proposed to achieve the SE. The numerical results verify that the proposed algorithms improve the system performance comparing to the baseline scheme.**

***Index Terms*—Wireless energy transfer, two-hop system, multi-leader Stackelberg game, Stackelberg equilibrium.**

I. Introduction

Energy harvesting has been considered a promising solution to enable perpetual operation of wireless networks. Energy harvesting sources for wireless nodes include solar radiation, body heat and motion, and electromagnetic waves. Recently, energy harvesting from radio frequency (RF) signals has been considered in energy and data cooperation networks [1]. In addition to data cooperation in conventional wireless systems, energy cooperation, as a viable design to improve system performance [2], can be accomplished by transferring energy as RF signals from energy-abundant nodes to energy-deprived nodes. Wireless energy transfer (WET) has been studied in cellular systems, and cognitive radio networks among others [1], [3].

A relay network with WET has first been proposed in [4], where the relay is energy-constrained and harvests energy from the received RF signals to forward the source's information. Both power splitting and time switching energy harvesting protocols are investigated. Following this two-hop system model, growing research has focused on wireless information and power transfer (WIPT) in relay networks from different perspectives. In particular, several works have studied WIPT relaying systems from the perspective of game theory [5]–[8]. Reference [5] has considered multiple source-destination pairs communicate via an energy harvesting relay. An auction based harvested energy allocation strategy is proposed and the properties of the equilibrium are discussed. In [6], a WIPT system with multiple source-destination pairs communicating through their dedicated relays is considered. A non-cooperative game is formulated for the competition among relays whose strategies are the power splitting ratios. In [7], [8], an energy harvesting relay with its own objective is considered. In [7], the relay has its own objective of harvesting energy and a Nash bargaining game is proposed to balance the information transmission efficiency of multiple source-destination pairs and the harvested energy of the relay. Reference [8] considers relays harvesting energy from the source's signal not only for forwarding source's information but also for transmitting their own data. Single-leader Stackelberg games are formulated respectively considering the source as leader and the relay as leader. For the source as leader, the delivery of source's information is given priority and the multiple relays play as followers, among which one relay with the best channel state is selected for transmission by Vickery auction.

In this paper, we consider a two-hop system with both data and energy cooperation based on the model studied in [8]. In particular, multiple relays are allowed to transmit to the destination with the primary objective of their own data delivery in contrast to [8]. Data cooperation is performed for the information of the source, which is delivered to the destination via decode-and-forward (DF) relaying at multiple relays. On the other hand, WET is adopted from the source to the relays to supply energy for data transmission from the relays to the destination. In the second hop, for the multiple access channel (MAC) from the relays to the destination, both simultaneous transmission (MAC-ST) and time division multiple access (MAC-TDMA) are investigated. We assume the source and the relays are selfish in the sense that each node aims to maximize its own utility. We adopt a multi-leader Stackelberg game framework. Multi-leader Stackelberg games have been utilized in wireless games to capture the competition between hierarchical agents [9], [10], but, to date, not in wireless energy transfer. Our model is that each relay plays as a leader in the game and determines its strategy by anticipating the strategy of the follower and competing non-cooperatively with other leaders. The source plays as the follower and chooses its strategy in response to the

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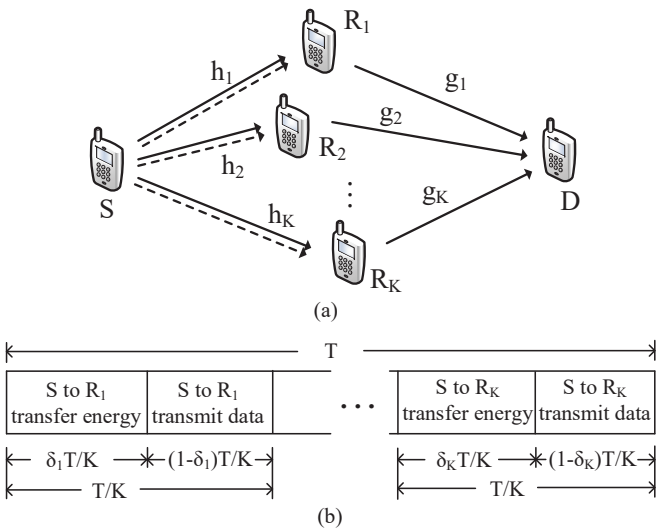

Fig. 1. (a) A two-hop network with K relays. Solid lines and dash lines indicate information transmission and energy transfer, respectively. (b) First hop transmission.

leaders' strategies. For MAC-ST, we provide an algorithm to obtain the solution of the game and prove a sufficient condition for the uniqueness of SE by the theory of variational inequality (VI). For MAC-TDMA, we solve a multi-leader Stackelberg game with a shared constraint among leaders. An algorithm is proposed to achieve the SE, which is also Pareto optimal. Numerical results demonstrate system performance is improved by both transmission protocols as compared to the single-leader Stackelberg game in [8], and MAC-TDMA provides a more significant outperformance.

II. System Model

We consider a two-hop Gaussian channel with one source, S, K relays, R_k, $k \in \mathcal{K} = \{1, 2, \ldots, K\}$, and one destination, D, shown in Fig. 1(a). Both the source and the relays intend to transmit information to the destination. Assume there is no direct channel available for reliable communications as the source and the destination are located far apart from each other. This necessitates the cooperation of relays to forward the messages of the source. Decode-and-forward relaying is adopted. The relays have no access to any other energy sources except harvesting energy from RF signals. Each relay harvests energy from the received signals of the source to sustain transmission of both its own and the source's data to the destination. The source provides WET to relays as a trade for channel access and signal relaying to the destination.

Assume transmission duration T in each hop. We consider an additive white Gaussian noise channel with quasi-static channel gains that are known at the associated transmitters and receivers. Let h_k and g_k denote the channel gains from S to R_k and from R_k to D, respectively. In the first hop, the transmission time T is divided into K slots with equal length $\frac{T}{K}$, shown in Fig. 1(b). In particular, S transmits dedicated signals to R_k in the kth slot. R_k adopts harvest-then-transmit protocol, where δ_k fraction of its slot is reserved for energy harvesting and the remaining $1-\delta_k$ fraction for information receiving. S agrees on $\delta_k \in [0,1]$ chosen by R_k and transmits with power $\frac{p_k}{\delta_k \frac{1}{K}}$ and $\frac{p_k}{(1-\delta_k)\frac{1}{K}}$ for energy transfer and data transmission, respectively. The average transmit power for R_k over T is $2p_k$, where p_k is restricted to be no larger than the maximum value P_k. Over the two-hop transmission time $2T$, the rate of S via R_k is given by

$$R_{\mathrm{S}_k} = \frac{1-\delta_k}{2K} \log\left(1 + \frac{h_k}{\sigma_k^2} \frac{p_k}{(1-\delta_k)\frac{1}{K}}\right), \tag{1}$$

where σ_k^2 is the received noise power at R_k. The harvested energy at R_k is $\eta_k h_k p_k T$, where $\eta_k \in (0,1)$ is the energy harvesting efficiency incorporating energy conversion loss and processing cost [8].

In the second hop, each relay transmits its own and the source's data to the destination which is equipped with a single-user decoder. Assume signal receiving, decoding, and encoding at the relays incur negligible energy cost and the consumed energy is for data transmission only. We consider two transmission protocols, MAC-ST and MAC-TDMA. For MAC-ST, all relays transmit to D simultaneously over T. R_k transmits with power $\eta_k h_k p_k$. The destination decodes the received signal of R_k treating the signals from other relays as noise. The rate of R_k is given by

$$R_{\mathrm{R}_k}{}^{\mathrm{ST}} = \frac{1}{2} \log\left(1 + \frac{g_k \eta_k h_k p_k}{\sum_{j \neq k}^{K} g_j \eta_j h_j p_j + \sigma_{\mathrm{D}_k}^2}\right), \tag{2}$$

where $\sigma_{\mathrm{D}_k}^2$ is the received noise power at D for signal from R_k. For MAC-TDMA protocol, each relay transmits to D in its assigned slot of length $\gamma_k T$, where $\gamma_k \in [0,1]$, $\forall k$ and $\sum_{k=1}^{K} \gamma_k \leq 1$. The transmit power of R_k is $\frac{\eta_k h_k p_k}{\gamma_k}$, $\forall k$. Then, the rate of R_k is given by

$$R_{\mathrm{R}_k}{}^{\mathrm{TDMA}} = \frac{\gamma_k}{2} \log\left(1 + \frac{g_k}{\sigma_{\mathrm{D}_k}^2} \frac{\eta_k h_k p_k}{\gamma_k}\right). \tag{3}$$

We define the utility of S as the average payoff over $2T$ by transmitting data to all relays. The cost of transmission to R_k is given by $2\mu_k p_k T$, where $2p_k T$ is the energy consumption and μ_k denotes the predetermined cost per energy unit. Then, the utility U_{S} is expressed as

$$U_{\mathrm{S}} = \sum_{k=1}^{K} R_{\mathrm{S}_k} - \mu_k p_k. \tag{4}$$

The utility of R_k is considered as the rate of delivering its own data to D, which is given by

$$U_{\mathrm{R}_k} = R_{\mathrm{R}_k}{}^{m} - R_{\mathrm{S}_k}, \quad m \in \{\mathrm{ST}, \mathrm{TDMA}\}. \tag{5}$$

In the sequel, we study multi-leader Stackelberg games for MAC-ST and MAC-TDMA.

III. Multi-Leader Stackelberg Games

The multi-leader Stackelberg game, a generalization of the single-leader Stackelberg game, consists of multiple upper-level players, the leaders, and a group of lower-level players, the followers [11]. Each leader has the privilege of choosing its strategy by anticipating the response of

the followers and competing with the other leaders. Each follower optimizes its strategy for the followers' game reacting to leaders' strategies while competing with other followers non-cooperatively.

We consider static games with a group of rational players. Every player knows the complete and perfect information of the set of players, the set of strategies and the utility functions of all players, and does best response [12]. Here, each relay, as a leader, decides its strategy by considering the strategy of the follower, the source, and competing with other relays. Denote the strategy of leader R_k by $\mathbf{q}_k$. Let $\mathbf{q} \triangleq (\mathbf{q}_k)_{k=1}^K$ be the strategies of all relays, $\mathbf{q}_{-k} \triangleq (\mathbf{q}_1, \ldots, \mathbf{q}_{k-1}, \mathbf{q}_{k+1}, \ldots, \mathbf{q}_K)$ denote the strategies of other relays except R_k, and $\mathbf{p} \triangleq (p_k)_{k=1}^K$ denote the strategy of the source. The strategy sets of relays for MAC-ST and MAC-TDMA are defined respectively as, $\forall k$,

$$Q_k = \{\mathbf{q}_k \triangleq \delta_k \in \mathbb{R}_+ : \delta_k \leq 1\}, \tag{6}$$

$$Q_k = \{\mathbf{q}_k \triangleq (\delta_k, \gamma_k) \in \mathbb{R}_+^2 : \delta_k \leq 1, \gamma_k \leq 1, \sum_{k=1}^K \gamma_k \leq 1\}. \tag{7}$$

Let $\mathcal{Q} \triangleq \prod_{k=1}^K Q_k$ and $\mathbf{f} \triangleq (U_{\mathrm{R}_k}(\mathbf{q}))_{k=1}^K$. The leaders' game $\mathcal{G} = (\mathcal{K}, \mathcal{Q}, \mathbf{f})$ is a non-cooperative game, where each leader R_k solves the optimization problem

$$\max_{\mathbf{q}_k \in Q_k} \; U_{\mathrm{R}_k}(\mathbf{q}_k, \mathbf{q}_{-k}, \mathbf{p}). \tag{8}$$

Reacting to leaders' strategies $\mathbf{q}$, the follower, S, maximizes its utility by choosing strategy $\mathbf{p}$ in the strategy set $\mathcal{P} = \{\mathbf{p} \in \mathbb{R}_+^K : p_k \leq P_k, \forall k \in \mathcal{K}\}$. S solves the optimization problem

$$\max_{\mathbf{p} \in \mathcal{P}} \; U_{\mathrm{S}}(\mathbf{p}, \mathbf{q}). \tag{9}$$

The Stackelberg equilibrium (SE) of the multi-leader Stackelberg game is defined as follows [11].

Definition 1: Let $\mathbf{q}_k^*$ be the optimal solution for problem (8) and $\mathbf{p}^*$ be the optimal solution for problem (9). Then, $(\mathbf{q}^*, \mathbf{p}^*)$ is an SE for the proposed multi-leader Stackelberg game if for any $(\mathbf{q}, \mathbf{p}) \in \mathcal{Q} \times \mathcal{P}$

$$U_{\mathrm{R}_k}(\mathbf{q}_k^*, \mathbf{q}_{-k}^*, \mathbf{p}^*) \geq U_{\mathrm{R}_k}(\mathbf{q}_k, \mathbf{q}_{-k}^*, \mathbf{p}^*), \; \forall k \in \mathcal{K}, \tag{10}$$

$$U_{\mathrm{S}}(\mathbf{p}^*, \mathbf{q}^*) \geq U_{\mathrm{S}}(\mathbf{p}, \mathbf{q}^*). \tag{11}$$

Next, we solve the SE of the proposed game by finding the Nash equilibrium (NE) of the leader's non-cooperative game $\mathcal{G}$. By backward induction, the source's strategy is analytically solved and substituted into each leader's optimization problem, then the NE can be found for the game $\mathcal{G}$.

A. MAC-ST

The source's optimization problem (9) is a convex problem for any given $\mathbf{q}$. The unconstrained solution maximizing the objective function in (9) is obtained by taking the gradient with respective to $\mathbf{p}$ and equating to zero, i.e., $\nabla_{\mathbf{p}} U_{\mathrm{S}}(\mathbf{p}, \mathbf{q}) = \mathbf{0}$. Then, projecting to the strategy set $\mathcal{P}$ gives

$$p_k = \min\left\{P_k, \max\left\{0, \frac{1-\delta_k}{K}\phi_k\right\}\right\}, \; \forall k \in \mathcal{K}, \tag{12}$$

where $\phi_k \triangleq \frac{1}{2\mu_k} - \frac{\sigma_k^2}{h_k}$.

It can be observed that p_k is nonincreasing in δ_k. As δ_k increasing, rate R_{S_k} decreases due to a shorter data transmission phase. As a response, S has to lower its transmit power to reduce the energy cost so as to maintain a positive utility. Furthermore, p_k is also nonincreasing in μ_k and nondecreasing in h_k, which implies p_k equals zero if $\phi_k = 0$ due to a large μ_k and/or a small h_k. In this case, the utility of S via R_k and the utility of R_k both end up with zero regardless of the choice of δ_k since R_k is unable to transmit without harvested energy. Hence, we only consider $\phi_k > 0$ in the following. The transmit power of S to R_k can be rewritten as

$$p_k = \begin{cases} P_k, & \text{if } \delta_k \in [0, \bar{\delta}_k), \\ \frac{1-\delta_k}{K}\phi_k, & \text{if } \delta_k \in [\bar{\delta}_k, 1]. \end{cases} \tag{13}$$

where $\bar{\delta}_k \triangleq 1 - \min\{\frac{KP_k}{\phi_k}, 1\}$.

With the knowledge of the source's strategy, each relay makes its decision by solving optimization problem (8). For $\delta_k \in [0, \bar{\delta}_k)$, we have

$$U_{\mathrm{R}_k}(\delta_k, \boldsymbol{\delta}_{-k}) = \frac{1}{2}\log\left(1 + \frac{g_k\eta_k h_k P_k}{\sum_{j\neq k}^K g_j\eta_j h_j p_j + \sigma_{\mathrm{D}_k}^2}\right) - \frac{1-\delta_k}{2K}\log\left(1 + \frac{h_k}{\sigma_k^2}\frac{P_k}{(1-\delta_k)\frac{1}{K}}\right). \tag{14}$$

It can be easily verify that $U_{\mathrm{R}_k}(\delta_k, \boldsymbol{\delta}_{-k})$ increases on δ_k, which implies that the optimal δ_k lies in $[\bar{\delta}_k, 1]$. Thus, in the sequel we focus on $\delta_k \in [\bar{\delta}_k, 1]$. The strategy set can be redefined as $Q_k = \{\delta_k \in \mathbb{R}_+ : \bar{\delta}_k \leq \delta_k \leq 1\}$, and (8) is rewritten as a convex optimization problem in terms of δ_k, that is

$$\max_{\delta_k \in Q_k} \; U_{\mathrm{R}_k}(\delta_k, \boldsymbol{\delta}_{-k}) = \frac{1}{2}\log\left(1 + \frac{\alpha_k(1-\delta_k)}{\sum_{j\neq k}^K \alpha_j(1-\delta_j) + \sigma_{\mathrm{D}_k}^2}\right) - (1-\delta_k)\beta_k, \tag{15}$$

where $\alpha_k \triangleq g_k\eta_k h_k\phi_k/K$ and $\beta_k \triangleq \frac{1}{2K}\log(1 + \frac{h_k\phi_k}{\sigma_k^2})$. This game $\mathcal{G}$ is a Nash equilibrium problem (NEP), which is closely related to the variational inequality (VI) problem. Specifically, the equivalency between the NEP and a properly defined VI problem is given as follows [13].

Theorem 1: Given the game $\mathcal{G} = (\mathcal{K}, \mathcal{Q}, \mathbf{f})$ in (15), where for each leader k, the strategy set Q_k is closed and convex, the utility function $U_{\mathrm{R}_k}(\mathbf{q}_k, \mathbf{q}_{-k})$ is continuously differentiable in $\mathbf{q}$ and concave in $\mathbf{q}_k$ for every fixed $\mathbf{q}_{-k}$, then, the NEP is equivalent to the VI($\mathcal{Q}, \mathbf{F}$), where $\mathbf{F}(\mathbf{q}) = (-\nabla_{\mathbf{q}_k} U_{\mathrm{R}_k}(\mathbf{q}))_{k=1}^K$.

Moreover, the existence of an NE of the game $\mathcal{G}$ is given by the following theorem.

Theorem 2: There is at least one NE for the game $\mathcal{G}$.

Proof: By Theorem 1, NEs of $\mathcal{G}$ exist if the equivalent VI($\mathcal{Q}, \mathbf{F}$) problem has a nonempty solution set. Based on the properties of VI, given VI($\mathcal{Q}, \mathbf{F}$) defined in Theorem 1, the solution set of VI($\mathcal{Q}, \mathbf{F}$) is nonempty, closed, and convex if $\mathcal{Q}$ is convex and compact, and $\mathbf{F}$ is monotone on $\mathcal{Q}$ [13]. It can be easily verify that $\mathcal{Q}$ is convex and compact. And $\mathbf{F}$ is monotone if and only if $U_{\mathrm{R}_k}(\mathbf{q}_k, \mathbf{q}_{-k})$ is concave in $\mathbf{q}_k$ for

Algorithm 1 Distributed iterative algorithm

1: Choose an initial strategy $\boldsymbol{\delta}^{(0)} = (\delta_k^{(0)})_{k=1}^K \in \mathcal{Q}$, set $n = 0$.
2: **repeat**
3: **for** $k = 1, \ldots, K$ **do**
4: compute $\delta_k^{(n+1)}$ as in (16) for given $\boldsymbol{\delta}_{-k}^{(n)}$.
5: **end for**
6: set $\boldsymbol{\delta}^{(n+1)} = (\delta_k^{(n+1)})_{k=1}^K$ and $n \leftarrow n+1$.
7: **until** $\boldsymbol{\delta}^{(n+1)}$ satisfies a suitable termination criterion.
8: Compute the strategy of the source $\mathbf{p}$ as in (12).

given $\mathbf{q}_{-k}$, $\forall k \in \mathcal{K}$, which is true. Therefore, there is at least one NE of $\mathcal{G}$. ■

Due to the selfish nature of players, each leader dedicates to maximize its own objective function. This naturally gives rise to a distributed iterative algorithm, where each leader, given the strategies of other leaders, updates its strategy by solving (15) at each iteration [13]. (15) can be solved by the first-order optimality condition, i.e., $\frac{\partial U_{\mathrm{R}_k}(\delta_k, \boldsymbol{\delta}_{-k})}{\partial \delta_k} = 0$, and mapping to the feasible set. We obtain

$$\delta_k = \min\left\{1, \max\left\{\bar{\delta}_k, 1 - \frac{1}{2\beta_k} + \frac{\sum_{j\neq k}^{K} \alpha_j(1-\delta_j) + \sigma_{\mathrm{D}_k}^2}{\alpha_k}\right\}\right\}. \quad (16)$$

In Algorithm 1, we summarize the distributed iterative algorithm based on the above discussion. Each relay has to know the local CSI and its signal-to-interference-plus-noise ratio (SINR) measured at the destination in every iteration. A sufficient condition of the uniqueness of the NE of the game is provided next, under which the global convergency to the NE can be guaranteed.

Theorem 3: The game $\mathcal{G}$ in (15) has an unique NE if matrix $\mathbf{M} \in \mathbb{R}^{K\times K}$ is positive definite, where each entry of $\mathbf{M}$ is

$$[\mathbf{M}]_{kj} \triangleq \begin{cases} 1, & \text{if } k = j \\ -\frac{\sum_{i=1}^{K} \alpha_i(1-\bar{\delta}_i) + \sigma_{\mathrm{D}_j}^2}{\sigma_{\mathrm{D}_k}^2}, & \text{if } k \neq j \end{cases}, \forall k, j \in \mathcal{K}. \quad (17)$$

Proof: By Theorem 1, the game $\mathcal{G}$ has an unique NE if and only if the solution of VI$(\mathcal{Q}, \mathbf{F})$ is unique. Based on the properties of VI, the VI$(\mathcal{Q}, \mathbf{F})$ admits a unique solution if $\mathbf{F}$ is strongly monotone [13]. In the Appendix, we give a proof of the strong monotonicity of $\mathbf{F}$ under the condition of positive definiteness of $\mathbf{M}$. ■

B. MAC-TDMA

Now, we investigate MAC-TDMA transmission in the second hop. The source's game is solved as in (12) and (13). Each relay is aware of the source's response and determines its strategy $\mathbf{q}_k = (\delta_k, \gamma_k)$ based on that. Considering the strategy set Q_k in (7), the utility for $\delta_k \in [0, \bar{\delta}_k)$ is given by

$$U_{\mathrm{R}_k}(\mathbf{q}_k, \mathbf{q}_{-k}) = \frac{\gamma_k}{2} \log\left(1 + \frac{g_k}{\sigma_{\mathrm{D}_k}^2} \frac{\eta_k h_k P_k}{\gamma_k}\right) - \frac{1-\delta_k}{2K} \log\left(1 + \frac{h_k}{\sigma_k^2} \frac{P_k}{(1-\delta_k)\frac{1}{K}}\right). \quad (18)$$

We see $U_{\mathrm{R}_k}(\mathbf{q}_k, \mathbf{q}_{-k})$ increases on δ_k for any $\gamma_k \in Q_k$, thus, we can focus on $\delta_k \in [\bar{\delta}_k, 1]$ for optimal solution.

The optimization problem of relay k is

$$\max_{\mathbf{q}_k \in Q_k} U_{\mathrm{R}_k}(\mathbf{q}_k, \mathbf{q}_{-k}) = \frac{\gamma_k}{2} \log\left(1 + \frac{\alpha_k(1-\delta_k)}{\gamma_k \sigma_{\mathrm{D}_k}^2}\right) - (1-\delta_k)\beta_k, \quad (19)$$

where Q_k is redefine as $Q_k = \{\mathbf{q}_k = (\delta_k, \gamma_k) \in \mathbb{R}_+^2 : \bar{\delta}_k \leq \delta_k \leq 1, \gamma_k \leq 1, \sum_{k=1}^K \gamma_k \leq 1\}$, and α_k and β_k are defined as in (15). We observe that (19) is a convex optimization problem, since the objective function is a perspective function concave on (δ_k, γ_k), and the feasible set $\mathcal{Q}_k$ is closed and convex. With the shared constraint $\sum_{k=1}^K \gamma_k \leq 1$ in the strategy set, the game $\mathcal{G}$ becomes a generalized Nash equilibrium problem (GNEP), where each player's strategy set depends on the rival players' strategies. Different from the equivalency between the VI and NEP in Sec. III-A, the relationship between the solution of VI and that of GNEP is stated in the following theorem [13].

Theorem 4: Given the game $\mathcal{G} = (\mathcal{K}, \mathcal{Q}, \mathbf{f})$ in (19), consider the VI$(\mathcal{Q}, \mathbf{F})$, where $\mathbf{F}(\mathbf{q}) = (-\nabla_{\mathbf{q}_k} U_{\mathrm{R}_k}(\mathbf{q}))_{k=1}^K$. Every solution of VI$(\mathcal{Q}, \mathbf{F})$ is a solution of the GNEP with the shared constraint, which is called a variational solution.

In particular, notice that $\mathbf{F}(\mathbf{q}) = (-\nabla_{\mathbf{q}_k} U_{\mathrm{R}_k}(\mathbf{q}))_{k=1}^K$ is the negative gradient of the concave function $\bar{f}(\mathbf{q}) = \sum_{k=1}^K U_{\mathrm{R}_k}(\mathbf{q})$, i.e., $\mathbf{F}(\mathbf{q}) = -\nabla_{\mathbf{q}} \bar{f}(\mathbf{q})$ [14]. Thus, finding the solution of VI$(\mathcal{Q}, \mathbf{F})$ coincides with solving the convex optimization problem given by

$$\max_{\mathbf{q}\in\mathcal{Q}} \bar{f}(\mathbf{q}) = \sum_{k=1}^{K} \frac{\gamma_k}{2} \log\left(1 + \frac{\alpha_k(1-\delta_k)}{\gamma_k \sigma_{\mathrm{D}_k}^2}\right) - (1-\delta_k)\beta_k. \quad (20)$$

Due to the convexity of the problem (20) and the independent constraints on $\boldsymbol{\delta} = (\delta_k)_{k=1}^K$ and $\boldsymbol{\gamma} = (\gamma_k)_{k=1}^K$, alternating optimization on $\boldsymbol{\delta}$ and $\boldsymbol{\gamma}$ converges to the global optimum. By the concavity of $\log(\cdot)$, we have for any given $\boldsymbol{\delta} \in \mathcal{Q}$,

$$\sum_{k=1}^{K} \frac{\gamma_k}{2} \log\left(1 + \frac{\alpha_k(1-\delta_k)}{\gamma_k \sigma_{\mathrm{D}_k}^2}\right) \leq \frac{1}{2} \log\left(\sum_{k=1}^{K} \gamma_k \left(1 + \frac{\alpha_k(1-\delta_k)}{\gamma_k \sigma_{\mathrm{D}_k}^2}\right)\right) \leq \frac{1}{2} \log\left(1 + \sum_{k=1}^{K} \frac{\alpha_k(1-\delta_k)}{\sigma_{\mathrm{D}_k}^2}\right), \quad (21)$$

where the equalities hold when $\sum_{k=1}^K \gamma_k = 1$ and

$$\gamma_k = \frac{\alpha_k(1-\delta_k)}{\sum_{k=1}^K \alpha_k(1-\delta_k)}, \ \forall k \in \mathcal{K}. \quad (22)$$

It implies that by allocating transmission time to each relay proportional to the power in its channel, we achieve the optimal system throughput $\sum_{k\in\mathcal{K}} {R_{\mathrm{R}_k}}^{\mathrm{TDMA}}$ for given $\boldsymbol{\delta}$. On the other hand, given $\boldsymbol{\gamma} \in \mathcal{Q}$, the objective function in terms of $\boldsymbol{\delta}$ is maximized by the first-order optimality condition $\frac{\partial \bar{f}(\boldsymbol{\delta})}{\partial \delta_k} = 0$ and projecting to interval $[\bar{\delta}_k, 1]$, $\forall k$, which gives

$$\delta_k = \min\left\{1, \max\left\{\bar{\delta}_k, 1 + \gamma_k\left(\frac{\sigma_{\mathrm{D}_k}^2}{\alpha_k} - \frac{1}{2\beta_k}\right)\right\}\right\}. \quad (23)$$

We summarize the distributed alternating optimization algorithm in Algorithm 2. In every iteration, $\boldsymbol{\gamma}$ is calculated as in (22) centrally and feedbacked to relays, which can be done at the destination, then each relay updates δ_k as in (23)

Algorithm 2 Alternating optimization algorithm

1: Choose an initial strategy $\mathbf{q}^{(0)} = (\delta_k^{(0)}, \gamma_k^{(0)})_{k=1}^K \in \mathcal{Q}$, set $n = 0$.
2: **repeat**
3: calculate $\boldsymbol{\gamma}^{(n+1)}$ as in (22) for given $\boldsymbol{\delta}^{(n)}$.
4: **for** $k = 1, \ldots, K$ **do**
5: compute $\delta_k^{(n+1)}$ as in (23) for given $\gamma_k^{(n+1)}$.
6: **end for**
7: set $\mathbf{q}^{(n+1)} = (\delta_k^{(n+1)}, \gamma_k^{(n+1)})_{k=1}^K$ and $n \leftarrow n+1$.
8: **until** $\mathbf{q}^{(n+1)}$ satisfies a suitable termination criterion.
9: Compute the strategy of the source $\mathbf{p}$ as in (12).

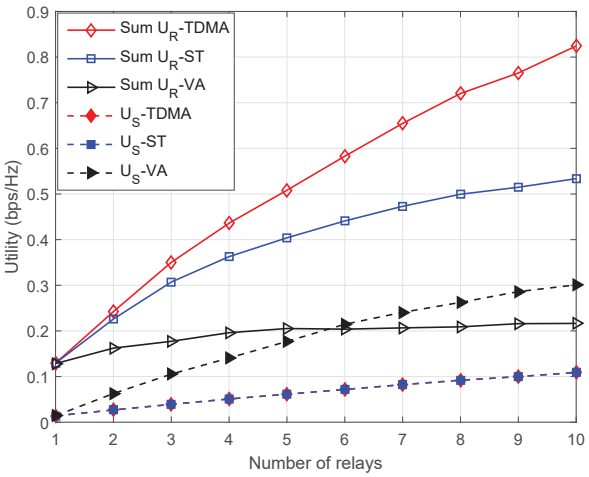

Fig. 2. Utilty versus number of relays for $\mu = 0.01$ bits/J/Hz, $P = 0.05$ W.

based on local CSI. Notably, in this way, we achieve an NE that maximizes the sum utility of the game $\mathcal{G}$, thus, the NE is most socially stable and also Pareto optimal [12].

Notice that given γ_k in (22), the problem (20) is equivalent to maximize the sum utility of relays by optimizing δ_k in a centralized way, meaning maximizing the objective function $\frac{1}{2}\log(1+\sum_{k=1}^K \frac{\alpha_k(1-\delta_k)}{\sigma^2_{\mathrm{D}_k}}) - \sum_{k=1}^K (1-\delta_k)\beta_k$. And it can be verified that the optimal δ_k solved by this means is identical as (16) for MAC-ST. This implies that the transmit power p_k and the rate of source R_{S_k}, thus, the source utility U_{S} for MAC-TDMA are the same as those for MAC-ST, but a higher system throughput $\sum_{k\in\mathcal{K}} R_{\mathrm{R}_k}{}^{\mathrm{TDMA}}$ can be achieved since for each given $\boldsymbol{\delta}$, $\boldsymbol{\gamma}$ is optimized as in (21) and (22) that maximizes $\sum_{k\in\mathcal{K}} R_{\mathrm{R}_k}{}^{\mathrm{TDMA}}$.

IV. Numerical Results

In this section, we present numerical results of the proposed algorithms. We consider carrier frequency 900 MHz with bandwidth 1 MHz. The noise power spectrum density is 10^{-19} W/Hz. Rayleigh fading with average power -3 dB is used to model the small-scale multipath fading. For the large-scale fading, we adopt the free space path loss model with path loss exponent 2 and reference distance 1 meter. The distance between the source and the destination is 100 meters and the relays are located uniformly in between. The antenna gain at all receivers is set to be 6 dBi. We set

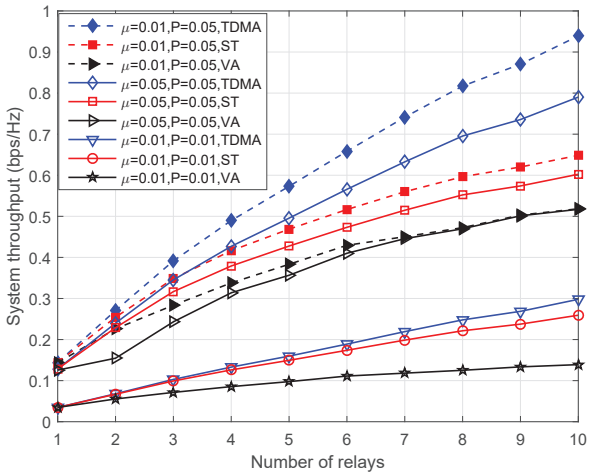

Fig. 3. System throughput versus number of relays.

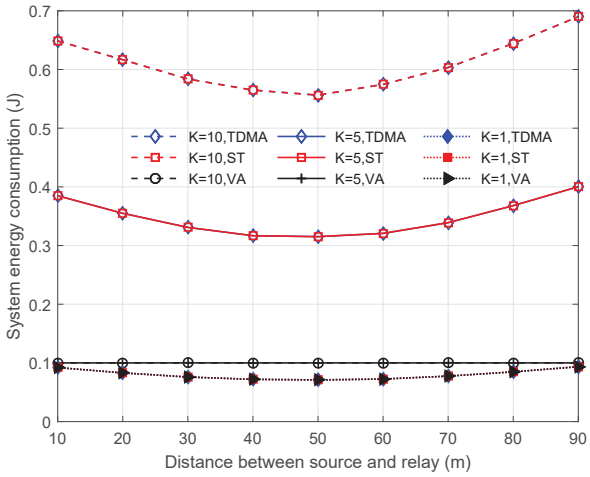

Fig. 4. System energy consumption versus distance between the source and the relay for $\mu = 0.01$ bits/J/Hz, $P = 0.05$ W.

$T = 1$ and $\eta_k = 0.8$, $\mu_k = \mu$ (bit/Hz/J), and $P_k = P$ (W) for all k. The numerical results are averaged over thousands of channel fading realizations. In Figs. 2-4, system performance metrics in terms of sum utility of relays, utility of source, system throughput $\sum_{k\in\mathcal{K}} R_{\mathrm{R}_k}$, and system energy consumption are evaluated by varying the number of relays and system parameters μ and P. Our proposed algorithms for MAC-ST and MAC-TDMA are compared with the baseline in [8], where the relay with best channel state is chosen for transmission via Vickery auction (VA) and a single-leader Stackelberg game is solved with the source as the leader and the selected relay as the follower. Observe that the utilities and system throughput increase on the number of relays.

In Fig. 2, the proposed algorithms outperform the baseline on the sum utility of relays, but are worse on the source's utility. This is due to the fact that the leaders in Stackelberg game have the priority to choose the most beneficial strategy and the followers make a following movement. Thus, the relays as the leaders in the proposed algorithms achieve higher utilities. While, it is the source at an advantage as the leader in the baseline. Furthermore, the relays' sum utility of MAC-TDMA is higher than that of MAC-ST due

to interference free, and the source's utilities of both are identical confirming the conclusion in Sec. III-B. In terms of the system throughput in Fig. 3, the proposed algorithms perform better and MAC-TDMA provides highest throughput. In particular, we show the results for $P = 0.01, 0.05$ W and $\mu = 0.01, 0.05$ bits/J/Hz. The throughput of all algorithms grows when P increases. But the superiority of MAC-ST to VA is less obvious for large P, since MAC-ST has a superior performance for small SINR. We also see the impact of μ that system throughput turns down for a higher μ. Fig. 4 shows the system energy consumption. The MAC-TDMA and MAC-ST coincide as discussed in Sec. III-B, and consume more energy as more relays are occupied. However, since only one relay is transmitting in baseline, the consumed energy is no larger than $2P$. When $K = 1$, all algorithms result in the same energy consumption as the only relay transmits. Moreover, when relays are close to the destination, more energy is consumed than the case of relays close to the source since energy transfer for larger distance becomes less efficient.

V. Conclusion

In this paper, we have studied a multi-leader Stackelberg game in a two-hop system with wireless energy transfer from the source to multiple energy harvesting relays. Considering each relay's objective of transmitting its own data to the destination, we have formulated a game with the relays as leaders and the source as the follower. Two transmission protocols, MAC-ST and MAC-TDMA, for the second hop have been investigated, where the NEP and the GNEP have been solved, respectively. Numerical results have verified that the proposed algorithms, allowing multiple relays transmitting, achieve better system performance than the baseline. Future works include the impact of incorrect incomplete system information in this game theoretic framework, and dynamic multi-shot games for time-varying channels.

Appendix
Proof of Theorem 3

The proof follows Appendix B in [15]. Denote $F_k(\boldsymbol{\delta}) = -\nabla_{\delta_k} U_{\mathrm{R}_k}(\boldsymbol{\delta}) = \frac{1}{2}\left(\sum_{j=1}^{K} \hat{\alpha}_{jk}(1-\delta_j) + \hat{\sigma}_k^2\right)^{-1} - \beta_k$, where $\hat{\alpha}_{jk} \triangleq \frac{\alpha_j}{\alpha_k}$ and $\hat{\sigma}_k^2 \triangleq \frac{\sigma_{\mathrm{D}_k}^2}{\alpha_k}$ for all k. Note that $\hat{\alpha}_{kk} = 1$. We show that $\mathbf{F}(\boldsymbol{\delta}) = \big(F_k(\boldsymbol{\delta})\big)_{k=1}^{K}$ is strongly monotone when $\mathbf{M}$ defined in Theorem 3 is positive definite. That is, given two solutions $\boldsymbol{\delta}, \boldsymbol{\delta}' \in \mathcal{Q}$, there exists a constant $c > 0$ such that $(\boldsymbol{\delta} - \boldsymbol{\delta}')(\mathbf{F}(\boldsymbol{\delta}) - \mathbf{F}(\boldsymbol{\delta}'))^T \geq c \left\|\boldsymbol{\delta} - \boldsymbol{\delta}'\right\|^2$ for $\mathbf{M}$ positive definite. For the ease of notation, we define for all $k \in \mathcal{K}$,

$$\omega_k \triangleq \sqrt{\textstyle\sum_{j=1}^{K} \hat{\alpha}_{jk}(1-\delta_j) + \hat{\sigma}_k^2} \cdot \sqrt{\textstyle\sum_{j=1}^{K} \hat{\alpha}_{jk}(1-\delta'_j) + \hat{\sigma}_k^2}$$

and $e_k \triangleq \frac{(1-\delta_k)-(1-\delta'_k)}{\sqrt{2}\omega_k}$. Then, for any k, we have

$$(\delta_k - \delta'_k)(F_k(\boldsymbol{\delta}) - F_k(\boldsymbol{\delta'})) = e_k^2 + \textstyle\sum_{j=1, j\neq k}^{K} e_k \frac{\hat{\alpha}_{jk}\omega_j}{\omega_k} e_j$$

$$\geq e_k^2 - \textstyle\sum_{j=1, j\neq k}^{K} \left| e_k \frac{\hat{\alpha}_{jk}\omega_j}{\omega_k} e_j \right| \overset{(a)}{\geq} e_k^2 - \textstyle\sum_{j=1, j\neq k}^{K} |e_k| \frac{\hat{\alpha}_{jk}\omega_j}{\omega_k} |e_j|$$

$$\overset{(b)}{\geq} e_k^2 + \textstyle\sum_{j=1, j\neq k}^{K} |e_k| [\mathbf{M}]_{kj} |e_j| = |e_k| \textstyle\sum_{j=1}^{K} [\mathbf{M}]_{kj} |e_j|,$$

where (a) is by Cauchy-Schwarz inequality, and (b) is because $\hat{\sigma}_k^2 \leq \omega_k \leq \omega_k^{\max} \triangleq \sum_{i=1}^{K} \hat{\alpha}_{ik}(1-\bar{\delta}_i) + \hat{\sigma}_k^2$. Then, since $|e_k| \geq \frac{|(1-\delta_k)-(1-\delta'_k)|}{\sqrt{2}\omega_k^{\max}} \geq \frac{|(1-\delta_k)-(1-\delta'_k)|}{\max\limits_{k\in\mathcal{K}} \sqrt{2}\omega_k^{\max}}$, we obtain

$$(\boldsymbol{\delta}-\boldsymbol{\delta}')\big(\mathbf{F}(\boldsymbol{\delta})-\mathbf{F}(\boldsymbol{\delta}')\big)^T \geq |\mathbf{e}|\,\mathbf{M}\,|\mathbf{e}|^T \geq \frac{\lambda_{\min}(\mathbf{M})}{\max\limits_{k\in\mathcal{K}} 2(\omega_k^{\max})^2} \left\|\boldsymbol{\delta}-\boldsymbol{\delta}'\right\|^2,$$

where $\lambda_{\min}(\mathbf{M})$ is the minimum eigenvalue of $\mathbf{M}$. Since $\mathbf{M}$ is positive definite, $\lambda_{\min}(\mathbf{M}) > 0$. Thus, we find the constant $c = \frac{\lambda_{\min}(\mathbf{M})}{\max\limits_{k\in\mathcal{K}} 2(\omega_k^{\max})^2} > 0$ such that $\mathbf{F}$ is strongly monotone.

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