On Gain Design in Virtual Output Feedback for Model Updating

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Abstract. The set of equations to be solved for parameter estimation in model updating has no unique solution when, as will often be the case in structural applications, the dimensionality of the model exceeds the number of target parameters estimated from experiments. One approach for enlarging the target space is to create closed-loop systems that, in addition, can be designed with pole sensitivities favorable for updating the model. The present paper will focus on designing gains for model updating using a recently proposed virtual implementation of output feedback, which allows computation of several closed-loop systems from a single open-loop realization and removes the constraint of closed-loop stability. The gains are designed through an eigenstructure assignment procedure, in which the model parameters of interest in the updating are divided into two different classes; one where the pole sensitivities with respect to the parameters are to be enhanced and one where they are to be reduced. A numerical example with a structural system is presented that demonstrates the merit of the proposed gain design procedure.

Keywords: model updating, output feedback, virtual implementation, gain design

1 Introduction

Parameter estimation for updating numerical models of structural systems is often resolved in an optimization setting, where some cost function expressing the discrepancy between experimental target poles and model-predicted ones is minimized [1]. Intrinsic deficiencies in this approach are that the structural system is never within the model space manifold and that the experimentally identified poles constituting the target vector are subject to estimation uncertainties [2]. What also hinder the applicability when strictly using poles to form the target vector are that the number of poles that can be identified is typically low and, in addition, that these have limited sensitivity to the parameters of interest.

A recognized procedure for enlargement of the target space is to test the structure in question under known perturbations and/or changed boundary conditions [3–5]. Another approach, which avoids testing under modified structural conditions, is to operate in closed loop and increase the target space by designing multiple systems using different gains [6–8]. However, despite the noticeable merits of the approach, which also include allowing for eigenstructure assignment to tailor pole sensitivities, the applicability has thus far been hampered by the practical overhead associated with real-time testing in closed loop.

The applicability of closed-loop model updating has, however, recently been promoted by a proposed virtual implementation, where closed-loop eigencharacteristics are computed directly from processing of open-loop input-output data [9, 10]. Besides eliminating the practical overhead associated with real-time operation, the virtual implementation also removes the constraint of closed-loop stability and allows computation of several closed-loop eigenstructures based on a single open-loop realization. The latter obviously implies that the target space can be readily increased by use of different gains, thus the task becomes the design and selection of these. Ultimately, the goal is to increase the Fisher information on the parameters to be updated, which can be achieved qualitatively by designing gains through optimizing some cost function promoting pole sensitivity or, as is currently being explored [11–14], quantitatively by simply generating an amount of random gains that highly overdetermine the set of equations to be solved in the updating.

In the present paper, we will follow the qualitative path of gain design for model updating, and we choose to divide the system parameters into three classes; 1) those with large uncertainties, 2) those with less but still notable uncertainties, and 3) those which are (almost) known exactly. While it is obvious that the parameters in the first class and the third class are, respectively, included and discarded in the model updating, a question opens up regarding the parameters in the second class. Whether or not to include these in the updating boils down to a tradeoff between the error introduced by discarding them (and hence treating them at their nominal values) and, if included, the increased size of the free parameter space to be handled in the optimization. In this study, we opt for the former and, in the gain design, minimize the sensitivity of the poles with respect to these parameters.

The paper is organized as follows: in section 2, the basic principles of static output feedback, including the virtual implementation and eigenstructure assignment, are briefly presented. Section 3 discusses the design of gains and section 4 outlines the model updating formulated as an optimization problem. The points made in the theoretical part of the paper will be demonstrated in a numerical example in section 5, while some concluding remarks are provided in section 6.

2 Output feedback

We consider a linear and time-invariant structural system, $\mathcal{P}$, which is described in discrete time by the state-space formulation

$$x(k+1) = A_d x(k) + B_d u(k) \tag{1a}$$

$$y(k) = Cx(k), \tag{1b}$$

where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^r$, and $y(k) \in \mathbb{R}^m$ are the state, input, and output vectors, while $A_d \in \mathbb{R}^{n \times n}$, $B_d \in \mathbb{R}^{n \times r}$, and $C \in \mathbb{R}^{m \times n}$ are the system matrices for which it is assumed that $\{A_d, B_d\}$ is controllable and $\{A_d, C\}$ observable. Also worth of explicit note is that eq. (1b) holds directly when measurements are displacements, velocities, or non-collocated accelerations, and it can also be used in the case of collocated acceleration measurements if the direct transmission term is subtracted from the measurements. One of the mentioned conditions will be assumed to hold in this paper.

Let $\mathcal{P}$ operate under the influence of static output feedback of the form

$$u(k) = -Gy(k) + v(k) \tag{2}$$

with some excitation $v(k) \in \mathbb{R}^r$ and gain $G \in \mathbb{R}^{r \times m}$, which we, for simplicity, restrict to be real from the outset. Substituting eq. (2) into eq. (1a) yields the closed-loop formulation

$$x(k+1) = (A_d - B_d GC)\, x(k) + B_d v(k) \tag{3}$$

from which the closed-loop state matrix, $\bar{A} = A_d - B_d GC$, is defined.

2.1 Virtual implementation

The transfer matrix of $\mathcal{P}$ in open loop, $H(z) \in \mathbb{C}^{m \times r}$, is defined as

$$H(z) = C(sI - A_d)^{-1} B_d, \tag{4}$$

so with the feedback law specified in eq. (2), we establish

$$y(z) = H(z)\left(-Gy(z) + v(z)\right), \tag{5}$$

which results in the closed-loop transfer matrix

$$\bar{H}(z) = \left(I + H(z)G\right)^{-1} H(z). \tag{6}$$

It is appreciated that closed-loop eigenstructures can be identified directly from an open-loop realization. In fact, by using eq. (6) with different gains, one can, in principle, generate as many closed-loop systems as required from just a single open-loop realization.

We close this part by noting that the virtual implementation follows directly when the identification of the open-loop system is conducted in frequency domain. If, however, a time-domain identification scheme is used, one must transform to z-domain to compute eq. (6) and then return to time-domain to finish the identification. An approach for this, which is based on mapping observer Markov parameters to $H(z)$, is provided in [10]. Here, it is also brought to attention that the closed-loop system can have unstable poles, because these will be filtered when doing the inverse z-transformation to return to time-domain.

2.2 Pole and eigenvector placement

Let $\Lambda_{\mathcal{M}} = \{\lambda_1, \lambda_2, \ldots, \lambda_p\}$ denote a subset of $p \leq n$ poles that are to be placed, then, for $\lambda_j \in \Lambda_{\mathcal{M}}$, it follows from eq. (3) that

$$(A_d - B_d G C - \lambda_j I)\, \psi_j = 0, \tag{7}$$

or, in partitioned form,

$$\begin{bmatrix} A_d - \lambda_j I & -B_d \end{bmatrix} \begin{bmatrix} \psi_j \\ b_j \end{bmatrix} = 0 \tag{8}$$

with $b_j = GC\psi_j$. The number of vectors that satisfy eq. (8) equals the nullity of $\begin{bmatrix} A_d - \lambda_j I & -B_d \end{bmatrix}$ and is no less than the number of inputs, r.

Defining $\Psi = \begin{bmatrix} \psi_1 \; \psi_2 \; \ldots \; \psi_p \end{bmatrix} \in \mathbb{C}^{n\times p}$, $\Phi = C\Psi \in \mathbb{C}^{m\times p}$, and $Z = [b_1 \; \ldots \; b_p] \in \mathbb{C}^{r\times p}$, it follows that

$$G\Phi = Z, \tag{9}$$

hence showing that the gain, G, is found by inversion when Φ has full rank and is square; with the latter obviously requiring that the number of placed poles, p, equals the number of outputs, m. Since transposition of a square matrix does not change its eigenvalues, operating with left-side eigenvectors and the appropriate transposes is also valid and allows placement of r poles.

3 Gain design

Since $\bar{A} \in \mathbb{R}^{n\times n}$, the poles to be placed for the closed-loop system with gain G_i come in the self-conjugate subset

$$\Lambda_{\mathcal{M}}(G_i) = \{\lambda_1, \ldots, \lambda_l, \lambda_1^*, \ldots, \lambda_l^*\} \tag{10}$$

where superscript $*$ denotes complex conjugate and $p = 2l$. The number of poles selected for each gain can differ, but here we simplify the notation by taking l to be the same for all the gains.

The system parameters are gathered in $\theta = \{\theta_\alpha, \theta_\beta, \theta_\gamma\}$, where we, as described in section 1, have defined three groups of which two of them, namely, $\theta_\alpha \in \mathbb{R}^{s_\alpha}$ and $\theta_\beta \in \mathbb{R}^{s_\beta}$, are included in the design of the gains. θ_α contains the parameters associated with large uncertainties and θ_β those with less but still notable uncertainties, thus the model updating is carried out in a setting where only the n_α parameters collected in θ_α are estimated.

Assume that q gains are gathered in the compound matrix

$$\mathcal{G} = \begin{bmatrix} G_1^T \ldots \; G_q^T \end{bmatrix}^T \in \mathbb{R}^{qr\times m}, \tag{11}$$

which is designed to maximize the sensitivity of

$$\Lambda_{\mathcal{M}} = \{\Lambda_{\mathcal{M}}(G_1), \ldots, \Lambda_{\mathcal{M}}(G_q)\} \in \mathbb{C}^{ql} \tag{12}$$

with respect to θ_α and minimize the sensitivity of $\Lambda_{\mathcal{M}}$ with respect to θ_β. Let

$$J = \begin{bmatrix} J_\alpha\left(G_1\right) & J_\beta\left(G_1\right) \\ \vdots & \vdots \\ J_\alpha\left(G_q\right) & J_\beta\left(G_q\right) \end{bmatrix} = \begin{bmatrix} \frac{\partial \Lambda_{\mathcal{M}}(G_1)}{\partial \theta_\alpha} & \frac{\partial \Lambda_{\mathcal{M}}(G_1)}{\partial \theta_\beta} \\ \vdots & \vdots \\ \frac{\partial \Lambda_{\mathcal{M}}(G_q)}{\partial \theta_\alpha} & \frac{\partial \Lambda_{\mathcal{M}}(G_q)}{\partial \theta_\beta} \end{bmatrix} \in \mathbb{C}^{ql \times (s_\alpha + s_\beta)} \tag{13}$$

be an extended Jacobian, which is partitioned into the sensitivities with respect to θ_α, gathered in $J_\alpha \in \mathbb{C}^{ql \times s_\alpha}$, and the sensitivities with respect to θ_β, gathered in $J_\beta \in \mathbb{C}^{ql \times s_\beta}$. For details on the computation of the sensitivities, the reader is referred to [8].

The Fisher information on, respectively, θ_α and θ_β is

$$\mathcal{F}_\alpha = J_\alpha^H \Sigma^{-1} J_\alpha \tag{14a}$$
$$\mathcal{F}_\beta = J_\beta^H \Sigma^{-1} J_\beta, \tag{14b}$$

where superscript H denotes conjugate transpose and where we have assumed normality and that the covariance on the poles, $\Sigma \in \mathbb{C}^{ql \times ql}$, is independent of the parameters. If Σ is taken as the identity, we see that the Fisher information is simply the dot product of the Jacobian by itself, which suggests that the gains can be designed from

$$\arg\max_{\mathcal{G}} \; ||J_\alpha||_* \tag{15a}$$
$$\arg\min_{\mathcal{G}} \; ||J_\beta||_*, \tag{15b}$$

as $||(\bullet)||_* = \text{trace}\left(\sqrt{(\bullet)^H(\bullet)}\right)$ is the nuclear norm of $(\bullet)$. A scalar cost function is conveniently formulated as

$$\begin{aligned} &\arg\min_{\mathcal{G}} && ||J_\alpha||_*^{-1} + ||J_\beta||_* \\ &\text{subject to} && \forall i \in [1, q] : \|G_i\| \leq \xi \end{aligned} \tag{16}$$

from which the compound gain, $\mathcal{G}$, is found under the noted constraint on the gains' norm. ξ must be selected such no undue error arises in the closed-loop pole estimates, which, according to eq. (6), implies that $I + H(z)G_i$ must be well-conditioned. In the numerical example in section 5, ξ is selected heuristically.

We note that the gain design, for necessary practicality, is carried out by use of a model of a structural reference state, which does not take into account the current realization of the system parameters. It is also worth mentioning that the outlined eigenstructure assignment procedure will, as elaborated in [8], typically yield a subset of unstable poles when one operates with homogeneous measurands. While this is obviously an intractable condition for physical real-time testing, eigenstructures with unstable poles are fully acceptable in the virtual implementation.

4 Model updating formulation

The parameters to be updated are $\theta_\alpha \in \mathbb{R}^{s_\alpha}$, and the updating is formulated as the following constrained optimization problem based on the use of q gains:

$$\begin{aligned} &\underset{\theta_\alpha \in \mathbb{R}^{s_\alpha}}{\arg\min} && \left\| \hat{\Lambda}_{\mathcal{M}} - \tilde{\Lambda}_{\mathcal{M}} \left(\theta_\alpha\right) \right\| \\ &\text{subject to} && \forall i \in [1,\, s_\alpha] : \tau_i \leq \theta^0_{\alpha,i} \leq \upsilon_i, \end{aligned} \tag{17}$$

where $\tau_i, \upsilon_i \in \mathbb{R}$ are lower and upper bounds on the ith nominal parameter, $\theta^0_{\alpha,i}$, while $\hat{\Lambda}_{\mathcal{M}} \in \mathbb{C}^{ql}$ and $\tilde{\Lambda}_{\mathcal{M}} \left(\theta_\alpha\right) \in \mathbb{C}^{ql}$ are, respectively, the estimated target poles and the corresponding model-predicted ones. Needless to say, the premise is to enforce $ql \geq s_\alpha$ such the system of equations to be solved in the optimization scheme is not underdetermined.

5 Numerical examination

We consider the shear building model depicted in fig. 1 and use the terms *nominal model* and *simulation model* to refer to, respectively, the model to be updated and the model used to simulate experiments. In the nominal model, all inter-story stiffnesses and floor masses are, respectively, 500 and 1 in some consistent set of units, and classical damping is assumed such each mode has a damping ratio of 2 % in open loop. In the simulation model, perturbations are introduced such the floor masses are {1.02, 0.96, 1, 0.98, 1.04} and the inter-story stiffnesses are {505, 503, 493, 400, 501}, where we note that the low value of the fourth inter-story stiffness could be due to, for example, structural damage.

In the simulations, noise excitation with unit standard deviation—low-pass-filtered such that only the first three modes are consistently excited—is applied as shown in fig. 1, and the output is taken, with a sampling frequency of 100 Hz,

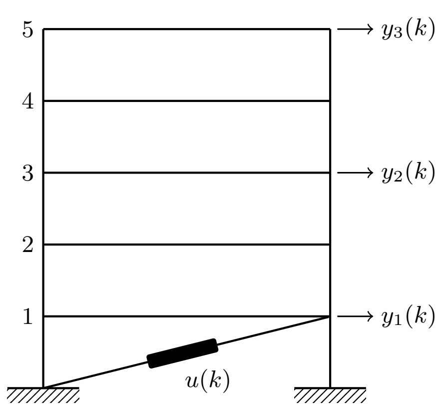

Fig. 1. Shear building with one input, $u(k)$, and three displacement outputs, $y_i(k)$.

as displacement response measured at floors 1, 3, and 5. The response is contaminated with 2 % additive white noise, and the open-loop system identification is carried out using the Eigensystem Realization Algorithm [15].

In the gain design, the parameters θ_α and θ_β are composed of, respectively, the inter-story stiffnesses and the floor masses. We design two gains and select three poles from the identification of each of the resulting closed-loop systems, which provide the target vector $\hat{\Lambda}_{\mathcal{M}} \in \mathbb{C}^6$. The corresponding poles of the model, $\tilde{\Lambda}_{\mathcal{M}}(\theta_\alpha) \in \mathbb{C}^6$, are taken as those with the smallest discrepancy to the target poles. From this outset, the cost function defined in eq. (17) is minimized with the constraints set to lower and upper bounds of 70 % and 130 % on the nominal inter-story stiffness parameters, θ_α^0. The minimization is conducted using the "fmincon" algorithm in MATLAB®, and it converges to the results presented in fig. 2. Evidently, we estimate the inter-story stiffness parameters with a maximum absolute error of 3 %.

6 Conclusion

This paper explores the design of gains through eigenstructure assignment in a recently proposed virtual implementation of static output feedback for parameter estimation. The gains are designed in an optimization setting, where pole sensitivities with respect to highly uncertain parameters are maximized and pole sensitivities with respect to parameters with small uncertainties are minimized. In this way, only the parameters associated with large uncertainties are included in the model updating while the rest are assigned their nominal values.

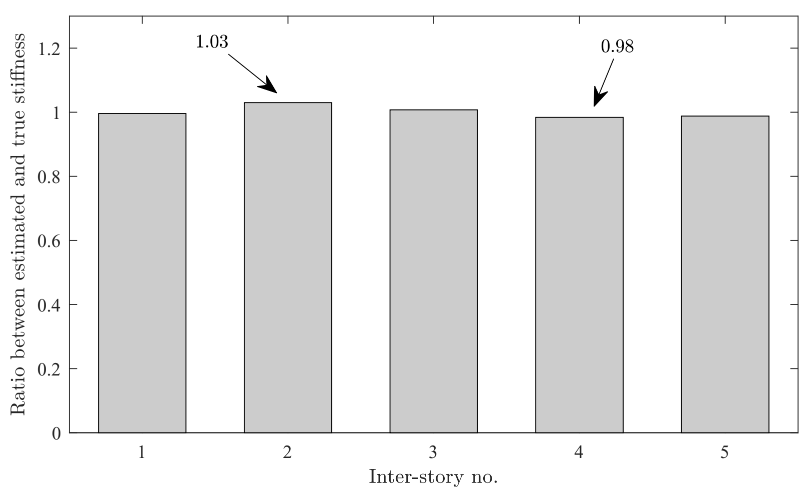

Fig. 2. Updating results for the inter-story stiffness in the shear building model.

A numerical examination of a shear building model is conducted to demonstrate the governing concept. Here, the inter-story stiffness parameters are assumed to have large uncertainties (due to, for example, damage) and the floor masses small uncertainties. It is shown how the proposed gain design procedure allows for parameter estimation with a maximum error of 3 % in a setting with open-loop output signals corrupted with 2 % additive noise.

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