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A Power Market Model in Presence of Strategic Prosumers

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***Abstract*—Prosumers, with ability to act both as a supplier and a consumer in a power market, have received considerable attention recently. Possessing with distributed energy resources, their capability to operate in an isolated mode, shielding from the main grid, has also been promoted as a vital option to enhance the power system's resilience. One emerging concern is the prosumer's ability to manipulate the power market as a buyer or as a seller. This study vets the outcomes of a power market in presence of strategic prosumers. We formulate the optimization problem faced by the prosumers in a complementarity problem. We posit a situation in which a strategic prosumer owns a renewable unit with variant output and a dispatchable backup unit, and participates in a competitive market. The prosumer is assumed to maximize its benefit by deciding amount of power to buy from or sell into the main grid, amount of renewable power to forego consumption, and amount of power to produce from backup unit. The model is applied to a case study of the IEEE 24-bus RTS as an illustrative example. We summarize the model properties and findings in three propositions and report distributional impacts of economic rent among conventional suppliers and consumers.**

***Index Terms*—Prosumers, Renewables, Complementarity Problem, Market Power**

Notations

1) Sets and Indices

F, I, R	The total number of generation firms, nodes, and transmission lines.
$H_{fi} \in H$	Set of generating units at i owned by f.

2) Parameters

$PTDF_{ki}$	Power transmission distribution factor for a unit of power. injected at hub and withdrawn at node i through branch k.
$P_i^0(Q_i^0)$	Vertical (Horizontal) intercept of the demand at i (\$/MWh).
T_k	Thermal limit for line k (MW).
X_{fih}	Production capacity for generation unit h belonging to firm f at node i.
$K_i(G_i)$	Exogenous renewable output (backup capacity) owned by the prosumer at node i (MW).
$A_i^0(B_i^0)$	Vertical (Horizontal) intercept of prosumer's marginal benefit for consumption at node i (\$/MWh).

3) Primal Variables

p_i	Wholesale power price at i (\$/MW).
z_{fi}	Sales (+) or purchases (-) by prosumer at i to/from firm f (MW)
l_i	Prosumer's demand at i (MW)
g_i	Power produced by prosumer's unit at i (MW)
x_{fih}	Power generated by unit fih (MW).
s_{fi}	Sales by firm f at node i (MW).
w_i	Transmission price charged by grid owner to move power from hub to i (\$/MW).
a_i	Power sell/buy (+/-) by arbitrager at i (MW).
d_i	Consumer's demand at node i (MW).
y_i	Power injected (+)/withdrew at node i (MW)

4) Dual Variables

λ_k	Dual variable of to branch k limits (\$/MW).
ρ_{fih}	Dual variable of capacity constraint of unit h of firm f at node i (\$/MW).
θ_f	Dual variable of firm f's supply and demand balance condition (\$/MW).
δ_i	Dual variable of prosumer's power balance constraint (\$/MW).
κ_i	Dual variable of prosumer's dispatchable generation capacity (\$/MW).
μ_i	Dual variable of prosumer's sales limit constraint (\$/MW).

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I. INTRODUCTION

THE electric power markets are undergoing rapid and fundamental transformations. The urge for an increase in renewable capacity and generation, in part owing to the effort of mitigating climate change and pursuing sustainability, has led to significant changes in the design and operation of modern power grids. With the availability of smart meters together with advances in IT-related technologies, a growing body of customers with renewable power generation capabilities combined with emerging distributed technologies, such as electric vehicles and storage, has altered the conventional demand-side paradigm in electricity markets.

This major shift in power markets towards a more engaged and flexible demand-side involvement has direct impacts on the behavior and participation of various agents in the market. Specifically, we see the advent of *prosumers*, i.e., agents who are capable of concurrent generation and consumption of power as opposed to the conventional consumers or suppliers who only participate in one side of the market. Given an increasing proportion of customers in the power market transforming into this emerging entity, with the duality of consumption and generation, it is expected to have signifi-

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cant implications on the design and operation of the future competitive power market [1].[1]

The interactions between prosumers and the wholesale power market are facilitated by aggregators who collect and integrate demand response (DR) and distributed energy resources (DER) at the distribution level and offer the *aggregated* energy bundle as a product to the wholesale market. Aggregators install and operate renewable facilities, such as solar panels, energy storage, electric vehicle charging, and smart energy management systems and are responsible for operation of generating fleet over a wide and diverse set of households and geographical areas constituting a substantial distributed generation and energy management capability [2] [3]. This provides an economic leverage for prosumers participating in the wholesale power markets far beyond ordinary customers as they are capable of integrating considerable resources over space and time, and at the same time, also fundamentally changes the business models of the electricity markets.

However, the introduction of prosumers into wholesale power markets poses several challenges in terms of operation and planning of a power system, in part driven by economic incentives offered to new participants in the power market. The operation and planning of a power system with increased participation of prosumers would shift the market's focus towards a more distribution level paradigm. This would consequently affect decisions on transmission grid expansion and investments on distribution and power generation facilities [4]. Furthermore, the economic incentives for market participants and the resulting market outcomes might change considerably as more participants are capable of concurrent generation and consumption. In other words, the market outcomes in presence of a prosumer would likely be different from those of a market with conventional consumers, especially when prosumers are allowed to deviate from price-taking assumptions. As these entities are relatively new to the market, they might be subject to relatively less oversight, partly as the result of underdeveloped regulatory framework to address their behavior.

Given the recent paradigm shift in the power markets towards an architecture with an increased presence of prosumers, an interesting question is how this emerging entity, i.e., prosumers with the ability of acting as both a producer and a consumer, might interact with the wholesale market and affect market outcomes as well as other entities in the market. The impact of strategic prosumers has received some attention in the literature. For example, [5] examines how a demand aggregator, operating a conventional generator and a green energy management system, affects the wholesale market by considering the aggregator exercise a quantity-based or Cournot strategy. The paper, however, does not account for the capability of concurrent generation and consumption by the prosumers, thereby underestimating the ability of the prosumers affecting the market. A more recent paper [6] considers a problem with a different information structure by postulating a load aggregator as a leader while other entities, i.e., producers, consumers and the grid operator, are followers in a Stackelberg setting. The load aggregator operates renewables, a wind source for example, and contemplates to "spillover" and "curtail" its wind power to reduce energy offering into the wholesale market in order to push up the wholesale power prices. Resemble to reference [5], buyer's market power is not considered in the analysis. As prosumers are expected to play a crucial role in the future power market, especially with their continuous growth in the market, models that explicitly formulate prosumer's behavior and endogenize power price formation will prove to be an important tool to assess its impact on market outcomes.

Other papers also contributed to modeling prosumers' behavior. Authors in [7] implement a two-stage stochastic programming approach to optimize a prosumer's bidding (first stage) and scheduling decisions (second stage) with the objective of minimizing the prosumer's expected cost. However, the power prices are assumed to be exogenous, and the paper fails to reflect the interplay between prosumer's decisions and price formation at the wholesale market. Reference [8] investigates demand response participation in the wholesale power market in which a DR aggregator offers contracts to customers based on physical constraints and capabilities, such as storage, on-site generation, load shifting, and load shedding and maximizes its expected payoff. While power price paths are simulated based on time series and artificial neural network techniques, it is subject to the same limitation as [7]. Another study examines optimal contract design between a retailer and an end-user when facing uncertain power prices [9]. A power retailer, to some extent, is similar to a prosumer as it is capable of both purchasing and selling electricity. The authors in [9], however, treat the wholesale prices as exogenous (similar to [7]– [8]) and take the contract price as the decision variable. Therefore, a common thread of the existing papers is to treat the wholesale power prices as given, and focus their attention on finding optimal contracts with customers or schedules while maximizing expected payoff. In other words, the interplay between the prosumers' strategic actions on the wholesale power prices is commonly not considered. Prosumers' strategic actions could play an important role in the future because the number of prosumers is expected to grow significantly. This is in part facilitated by the emerging decentralized and layered marketplace, such as DSOs to govern and facilitate energy transactions, together with prominent incentive-compatible business models to minimize transaction cost and maximize business opportunity [6], [10].

Our paper extends the existing work by Hobbs [11] with an explicit formulation of the prosumer's problem in a bottom-up complementarity framework, which allows interactions of the prosumers with other entities in the market, e.g., conventional generators, consumers and the grid operator, to be investigated. The prosumers can be either a seller or a buyer, acting strategically or competitively, as oppose to merely sellers as in [5]– [7]. Power prices and transmission charges, and decisions

[1]For example, recent focus of the power engineering community has been on developing a platform that allows a distribution system operator (DSO) to coordinate and to align with prosumers and an independent system operator (ISO) at the transmission level to facilitate energy transactions. In particular, the final rule of the FERC Order 745 stipulates that demand response resources participating in an organized wholesale energy market must be compensated for the service they provide to the energy market at the market price for energy, namely the locational marginal price (LMP). Moreover, issues related to the DER aggregation reforms have been discussed by the FERC.

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of all the entities in the market are endogenously determined, rather than exogenously given as in [7]– [9].[2] (Therefore, our model considers only the high-voltage transmission network, abstracting from representing low-voltage distribution network.) In particular, our formulation does not *ex ante* fixate the prosumer's role, either as a producer or a consumer, in the market, but, instead, allows solutions of the model to decide which one of the two roles that prosumer should be when maximizing its benefit. That is, whether the prosumer sells power into or buys power from the wholesale market in equilibrium is not known before solving the model. Moreover, our analysis, which explicitly decouples the prosumer's marginal benefit and the bulk energy consumers' willingness-to-pay without *a priori* fixation of prosumer's role, a producer or a consumer, also advances bottom-up modeling of prosumers' behavior. In fact, how to treat prosumers' demand in the model when their role in the equilibrium in the bulk energy market is unknown a priori is actually not trivial. Finally, we provide rigorous proof of the existence and discuss uniqueness of the solutions to enhance our understanding the properties of the models. Thus, the paper advances current knowledge of studying prosumers' behavior by allowing an endogenous treatment of power price formation process and simultaneously modeling the prosumers as both a buyer and a seller.

The paper is organized as follows. Section II gives a detailed formulation of optimization problems faced by each entity in the market, their first-order conditions as well as market clearing conditions that define equilibrium. The models developed in Section II is then applied to a case study of IEEE 24-bus Test System in Section III. We report main results, provide proofs of solution properties, and generalize our findings in two propositions that emerge from our analyses. Additional numerical simulations are conducted to illustrate our findings. Concluding remarks are given in Section IV. We document our proofs of the three propositions in online Appendices.

II. Model

Our work is based on work by Hobbs [11] and extends his work by introducing prosumers in the model. We use capital letters to indicate parameters and sets. Lowercase letters refer to variables and indices. Dual variables are designated with greek lower-case letters. In the following presentation, "$x \perp y$" implies $x^T y = 0$.)

A. Individual Optimization Problems

This section proceeds as follows. First, we introduce the optimization problem faced by each entity in the market, including prosumers, producers, the grid operator, and an arbitrageur. Second, we derive the Karush-Kuhn-Tucker (KKT) conditions associated with each variable in the optimization problem. Third, the collection of KKT conditions together with market clearing conditions will define a market equilibrium problem in form of a linear complementarity problem, which can then be solved using complementarity solvers, e.g., PATH [14].

1) Consumers: Consumers are assumed to be price-taking agents, and their willingness-to-pay for power is represented by the inverse function in the complementarity form:

$$0 \leq d_i \perp p_i - \left(P_i^0 - (P_i^0/Q_i^0)d_i\right) \geq 0, \qquad \forall i = 1, \dots, I \tag{1}$$

where P_i^0 and Q_i^0 represent the vertical and horizontal intercepts of the inverse demand function, respectively, at demand node i. The vertical intercept, also referred to as choke price, indicates that consumption drops to zero when price exceeds P_i^0. The function is positive but decreasing in d_i. When there is no regular consumer in node i, we then model that location with a sufficiently small P_i^0 so that the quantity demanded, d_{it}, is equal to zero in equilibrium.

2) Prosumers: The prosumer at node i possesses renewable energy with a negligible short-run marginal cost.[3] We assume that prosumers only engage in power sales or purchase at their local node. [4] (The assumption is consistent with layered grid structure envisioned in [15].) Thus, the wheeling cost will cancel out in this. That is, the prosumer gets paid by ω_i when moving power to the hub and pays ω_i when selling from the hub to the node i. The output from renewable is denoted by K_i, which is uncertain because it is limited by available natural resources, e.g., solar and wind. Meanwhile, it also owns a dispatchable or backup resource with a capacity of G_i in order to hedge against uncertain output K_i. For our purposes, the prosumer's benefit function of consuming electricity around level K_i is given by $B_i(l_i)$, where l_i corresponds to the quantity consumed by prosumer when renewable output equals K_i (Fig.1).[5] The benefit function

[2] Bottom-up complementarity models formulated based on game-theoretical framework and built upon individual entities' optimization problems have emerged as a popular tool to assess the impact of newly enacted regulations, proposed market designs, emerging technologies, and other considerations in the energy sector. The strengths of this model lies in its ability to incorporate heterogeneity in generating technologies, physical systems, e.g., transmission, various institutions and emerging entities in analyses. Examples include [12], [13].

[3] Individual behind-the-meter prosumers, e.g., owner of roof-top solar, might have limited access to the wholesale or bulk market and be subject to a tariff that does not reflect value of their surplus energy. We assume that our prosumer problem is the result of the aggregation of a large number of end-prosumers, thereby allowing them interact with the bulk market directly. Thus, in a way, we model end-prosumers and the aggregator as a joint entity. One can think about that end-prosumers, who are subject to uncertain level of renewable output, enter bilateral agreement, a contract, with an aggregator while allowing the aggregator to operate their aggregated dispatchable capacity. In this case, the premium associated with the bilateral contracts will be an internal wealth transfer between end-prosumers and the aggregator. Please see the Appendix for the formal proof of their equivalence.

[4] Allowing prosumers to sell surplus power from its local node i to other locations is expected to produce the same market outcomes. This is because the price difference between two nodes, e.g., i and j, is equal to the transmission cost of moving power from i to j. Thus, while selling power to node j might earn extra revenue (i.e., $p_j - p_i$), it will be exactly offset by the transmission cost; see, for example, [11] for the equivalence between Poolco and bilateral markets.

[5] B_i is entirely separate and different from $p_i(d_i)$, which represents willingness-to-pay or benefit of consumers in the wholesale market. Its interaction with the main grid is through shifting the wholesale's supply to left (right) when purchasing from (selling to) the wholesale market. It represents a local benefit function centered around consumption level at K_i. As a prosumer engages in the market, directly through bilateral trading with firms, there is limited opportunity for the market to solicit prosumers' preferences through market settlements, i.e., a preference revelation process.

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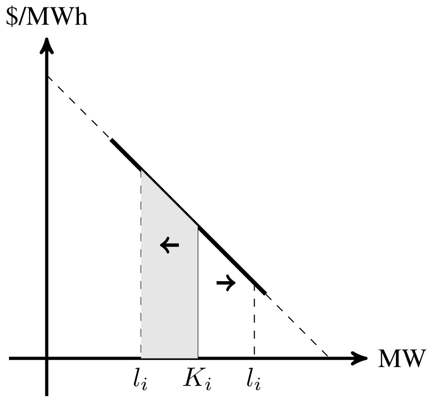

Fig. 1. An illustration of prosumer's marginal benefit function

$B(\cdot)$ is assumed to be increasing and strictly concave. The monotonicity of $B(\cdot)$ indicates that the prosumer's objective function is increasing in the level of consumption. We posit that a prosumer maximizes its profit by deciding i) amount of power to buy from ($z_{fi} < 0$) or sell to ($z_{fi} > 0$) firm f in node i through bilateral contracts[6], ii) amount of forgone consumption, $K_i - l_i$, and iii) amount of power to be generated from the backup dispatchable technology, g_i. The prosumer faces a price-responsive demand characterized by its marginal benefit function. Its maximal consumption is capped by the horizontal intercept of its marginal benefit function.

The optimization problem faced by the prosumer at node i is displayed as follows. (The greek variables within the parenthesis to the right of an equation render the corresponding dual variable.)

$$\underset{z_{fi}, l_i \geq 0, g_i \geq 0}{\text{maximize}} \quad p_i \left(\sum_f z_{fi} \right) - \int_{l_i}^{K_i} B_i'(x)dx - C_i^g(g_i) \tag{2a}$$

subject to

$$\sum_f (z_{fi}) + l_i - K_i - g_i = 0 \qquad (\delta_i), \tag{2b}$$

$$g_i \leq G_i. \qquad (\kappa_i) \tag{2c}$$

The three terms in the objective function of (22), in order, correspond to revenue (+) or cost (-) from transactions with the wholesale market, foregone benefit (if $K_i > l_i$) or incremental benefit (if $l_i > K_i$) of consuming power, and generation costs incurred from backup generation, respectively. Two constraints are associated with the prosumers' problem. (2b) states that the sum of renewable output K_i and self generation g_i net of sales to the wholesale market or $\sum_f z_{fi}$ equals the quantity consumed l_i. (2c) limits the output g_i by its capacity G_i.Note that the transactions of prosumer with the wholesale market does not involve the wheeling charge w_i since it only sells or buys from the node where it produces its power. That is, the prosumer gets paid by w_i when moving power to the hub and pay w_i when selling from the hub to the node i. This way of modeling prosumers is consistent with the layer structure of future power market discussed in paper [15]

When a prosumer is modeled in our analysis as a price-taker, it takes the price p_i as given and decides on (z_{fi}, l_i, g_i) accordingly. However, when a prosumer in our model is designated as a strategic entity, it realizes that by "contracting" some of its procurement of power, it could lower the power price, thereby exercising buyer's market power. On the contrary, it is also aware that if it reduces power sales slightly, it might be able to push up power prices, thereby exercising seller's market power. This highlights the capability of the model to capture the duality of a prosumer in a unified framework. As we demonstrate later, which of the two strategies should be implemented depends on the prosumer's net position in the energy market, which is affected by the zero marginal cost renewable output K_i. While a prosumer only participates in the wholesale market indirectly through bilateral contracts rather than, say directly submitting bids into the market, one can assume that it acquires "strategic" knowledge through its repeated observations of power price clearance processes of the wholesale market.

Knowing that a prosumer can manipulate the power market through changes in procurement or purchase quantities, we then re-write (22) as

$$\begin{aligned} \underset{z_{fi}, l_i \geq 0, g_i \geq 0}{\text{maximize}} \quad & p_i(z_{1i}, z_{2i}, \cdots, z_{Fi}) \left(\sum_f z_{fi} \right) \\ & - \int_{l_i}^{K_i} B_i'(x)dx - C_i^g(g_i), \end{aligned} \tag{2a*}$$

by representing p_i as a function of z_{fi}. One way of representing prosumer's ability to manipulate the wholesale power market in the model is by treating its belief as a parameter based on conjecture variation approach. One benefit of using this approach is that the parameter can be altered in order to explore the impact of a prosumer's belief of its "manipulating" strength on market outcomes. However, the approach is mainly useful in a situation when the demand function of underlying commodity is unobservable. An example of this is modeling market power of tradable pollution permit market where the demand for tradable permits is actually implied from output decisions of generators in the power market [13]. Because 1) our interests lie in understanding market outcomes when a prosumer behaves compatibly with a producer, and 2) classic results indicating that quantity pre-commitment and Bertrand competition yield Cournot outcomes [16], we believe a Cournot or quantity-based formulation is more apt for our analysis. Therefore, the first-order conditions associated with prosumers then can then be displayed as follows.

[6]Because the equivalence between a power market based on pool-type transactions and on bi-lateral contracts have been alluded to in [11], we believe that our assumption herein is reasonable and can be seen as an extension.

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$$\text{For } z_{fi}: p_i - \delta_i = 0, \forall f, i \tag{3a}$$

$$\text{For } z_{fi}: p_i - (P_i^0/Q_i^0)\sum_f z_{fi} - \delta_i = 0, \forall f, i \tag{3a*}$$

$$0 \le l_i \perp A_i^0 - B_i^0 l_i - \delta_i \le 0, \forall i \tag{3b}$$

$$0 \le g_i \perp -C_i^{g\prime} - \kappa_i + \delta_i \le 0, \forall i \tag{3c}$$

$$\text{For } \delta_i: l_i - K_i - g_i + \sum_f z_{fi} = 0, \forall i \tag{3d}$$

$$0 \le \kappa_i \perp g_i - G_i \le 0, \forall i \tag{3e}$$

A total of five conditions are associated with the prosumers' problem. In cases when the prosumer exercises market power, the first-order conditions of variables z_{fi} are given in (3a*). Comparing (3a) to (3a*), the difference is $(P_i^0/Q_i^0)\sum_f z_{fi}$. This term acts similarly to those under the standard Cournot formulation, allowing market equilibrium to be different from the competitive outcomes. The sign defined by $\sum_f z_{fi}$ leads the prosumers to either exercising seller's or buyer's market power. Note that when prosumers are price-takers, their consumption l_i is related to the its marginal benefit $A_i^0 - B_i^0 l_i$. When $l_i > 0$, (3b) indicates that $A_i^0 - B_i^0 l_i = \delta_i$, which is equal to p_i from (3a). That is, l_i is implicitly capped by $A_i^0 - B_i^0 l_i = p_i$ or the marginal benefit equals the equilibrium price.

3) Producers: As alluded to earlier, we assume suppliers or firms are price-takers in the wholesale power market as they are constantly subject to rigorous regulatory oversight.[7] We assume that firm f maximizes its profit by deciding the output x_{fih} and sales s_{fi}. A supplier f's problem is given as follows:

$$\underset{s_{fi}\ge 0, x_{fih}\ge 0}{\text{maximize}} \quad \sum_i (p_i - w_i)(s_{fi} - z_{fi}) - \sum_{fih}(C_{fih}(x_{fih}) - w_i x_{fih}) \tag{4a}$$

subject to

$$x_{fih} \le X_{fih}, \forall i, h \in H_{fi} \qquad (\rho_{fih}), \tag{4b}$$

$$\sum_i (s_{fi} - z_{fi}) - \sum_{i,h\in H_{fi}} x_{fih} = 0 \qquad (\theta_f) \tag{4c}$$

The first term in the objective function (4) is the revenue received from power sales $s_{fi} - z_{fi}$ while paying for the wheeling charge w_i. The second term gives generation cost, minus transmission charge $-w_i$, effectively representing a payment received by the generator from the grid operator for its service of providing counterflow to de-congest the line from i to hub. The cost function C_{fih} is convex and marginally increasing as in the literature [19].

Turning to the constraints, (4b) limits the output x_{fih} to be less than its capacity X_{fih}. (4c) assures that total power sales equal its supply while accounting for its bilateral transactions with the prosumers. More specifically, when z_{fi} is negative, (4c) suggests that additional x_{fih} needs to be produced by the generator to satisfy demand other than s_{fi}. This effectively reduces the amount of power available to the power pool, thereby, expectedly, driving up the wholesale prices. Similarly, when z_{fi} is positive, output from firm f is reduced as a portion of the wholesale demand is met by the prosumers. The reverse analogue is applied so the power prices are expected to lower in this case. By formulating this way, it allows the model decouple the bulk energy demand, defining p_i in (1), from the prosumers' marginal benefit function B_i' in (2a).

[7]For example, the PJM market is reported to be competitive, i.e., prices set by marginal offering units close to their marginal costs [17]. Likewise, the day-ahead market in California is generally competitive [18]. However, regulator and market monitor are always concerned about the exercise of market power in local load pocket due to congestion is always a concern, see [17] and [18].

The KKT conditions of the producer f in the wholesale market are summarized as follows:

$$0 \le s_{fi} \perp p_i - w_i - \theta_f \le 0, \forall i \tag{5a}$$

$$0 \le x_{fih} \perp -C'(x_{fih}) + w_i - \rho_{fih} + \theta_f \le 0, \forall i, h \in H_{fi} \tag{5b}$$

$$\text{For } \theta_f: \sum_i (s_{fi} - z_{fi}) - \sum_{i,h\in H_{fi}} x_{fih} = 0, \ \forall f \tag{5c}$$

$$0 \le \rho_{fih} \perp x_{fih} - X_{fih} \le 0, \forall i, h \in H_{fi} \tag{5d}$$

4) Grid Operator: The grid owner operates the power network and decides on the allocation of transmission resources while charging producers w_i to move power from hub to node i. The optimization problem faced by the grid operator is given in (6).

$$\underset{y_i}{\text{maximize}} \quad \sum_i w_i y_i \tag{6a}$$

subject to

$$-T_k \le \sum_i PTDF_{ki} y_i \le T_k \qquad (\lambda_k). \tag{6b}$$

The grid operator is a price-taker with respect to w_i and aims to maximize its revenue by deciding y_i given the power flow in each line k is within its thermal limit T_k. Similar to [19], power flows in the network are governed by the power distribution transfer factor (PTDF) based on linearized Directed-Current principle [20]. In this context, the grid operator maximizes the value obtained from the sales of nodal transmission rights based on the topology of the network [21]. The grid operator represents the behavior of the transmission operator or *line owner* that seeks to maximize the value of its network given the set of prices w_i [22]. The grid operator's KKT conditions then are given as follows:

$$w_i - \sum_k PTDF_{ki}(\lambda_k^+ - \lambda_k^-) = 0 \qquad \forall i \tag{7a}$$

$$0 \le \lambda_k^+ \perp \sum_i PTDF_{ki} y_i - T_k \le 0 \qquad \forall k \tag{7b}$$

$$0 \le \lambda_k^- \perp -\sum_i PTDF_{ki} y_i - T_k \le 0 \qquad \forall k \tag{7c}$$

5) Arbitrager: We include an arbitrager in our model, as it has been shown that solutions from models with an arbitrager are equivalent to that of a POOL-type power market when the market is imperfectly competitive [11]. Moreover, [23] proved that when considering an arbitrager, the cost of moving power from node i to j will equal the price difference between nodes or $p_j - p_i$. The implicit assumption here is that the arbitrager has full knowledge of power prices at each node. An arbitrager moves power from a bus where the market price is lower to one with a higher price. The arbitrager's optimization problem is as follows:

$$\underset{a_i}{\text{maximize}} \quad \sum_i (p_i - w_i) a_i \tag{8a}$$

$$\text{subject to}$$

$$\sum_i a_i = 0. \qquad (p^{hub}) \tag{8b}$$

One constraint, (8b), is associated with this problem, guaranteeing total sales = total purchases, with its dual variable denoting the market price at hub or p^{hub}. The arbitrager's KKT conditions are given in (9):

$$p_i - w_i - p^{Hub} = 0 \qquad \forall i \tag{9a}$$

$$\sum_i a_i = 0. \tag{9b}$$

6) Market Clearing Conditions: While each market participant's optimization problem represents its behavior in the wholesale market, the market clearing conditions tie them all together and ensure the demand and supply balance. This is shown in (10).

$$\sum_f s_{fi} + a_i - \sum_{f,h \in H_{fi}} x_{fih} - \sum_f z_{fi} = y_i, (\omega_i), \forall i \tag{10}$$

Note that the first two terms together, $\sum_f s_{fi} + a_i$, equals the demand at node i: d_i, as in (1) to determine the whole price at node i. The collection of all KKT conditions for each market participant (3)–(9) in addition to the market clearing condition (10) forms the set of equalities and inequalities, termed as a mixed complementarity problem (MiCP), which defines a market equilibrium [24], [25]. The MiCP is formulated in AMPL and solved using the PATH solver [14]. In the following we state the existence of a market equilibrium. The proof is provided in Appendix A-A.

Proposition 1. *(Existence) Assume that the prosumer's marginal benefit function $B_i'(\cdot) : \Re^1 \to \Re^1$ is continuous and monotonically decreasing. Further assume that the prosumer's generation cost function $C_i^g(\cdot)$ and the supplier's cost function $C_{fih}(\cdot)$ are continuously differentiable, for all $i = 1, \ldots, I$, $f = 1, \ldots, F$, and $h \in H_{fi}$. Then a market equilibrium exists, which is defined as the collection of primal variables $(z, l, g, s, x, y, a, w, p)$ and the dual variables $(\delta, \kappa, \theta, \rho, \lambda, p^{Hub})$ that simultaneously satisfy the optimality conditions* (3)*,* (5)*,* (7)*,* (9)*, together with* (10)*.*

III. Numerical Case Studies

A. Data, Assumptions and Scenarios

To analyze the power market outcomes in presence of strategic prosumers, the IEEE Reliability Test System (RTS 24-Bus) [26] is used. The topology of the system consists of 24 buses, 38 transmission lines, and 17 constant-power loads with a total of 2,850 MW. We aggregate 32 generators into 13 generators by combining those with the same marginal cost and located at the same node. Six generation units, however, are excluded from the dataset since they are hydro power units operating at their maximum output of 50 MW [27]. Because the wholesale market is assumed to be perfectly competitive, we assume that all the generators are owned by a single firm. In order to be able to analyze the impact of transmission congestion, the capacity of line 7 in the test case is reduced to 150 MW. The marginal cost of generation is represented by a quadratic function parameterized by intercept C_0 and slope C_1.

Furthermore, a prosumer is assumed to be located at node 1 with the same preferences and quantity demanded of power consumption as consumers located in that node. That is, both prosumer and consumers in node 1 are assumed to have the same demand function. The prosumer owns a renewable generating unit that produces a varying amount of power (contingent on available natural resources) and a dispatchable unit as a backup option.

The RTS 24-Bus case is first formulated as a least-cost minimization problem and solved with fixed nodal load in order to get dual variables associated with load constraints. The dual variables together with an assumed price elasticity of -0.2 is then used to calculate P_i^0 and Q_i^0. The magnitude of price elasticity of demand is comparable with what has been reported in [28].

We examine a total of six scenarios, varied by the level of renewable output owned by the prosumers as well as strategic assumption of the prosumers. More specifically, renewable output K_1 is assumed to have three levels: 25, 50, and 120 MW. The prosumer is designated as either a price-taker or as a strategic entity while other entities in the market are assumed to be price-takers.[8] Additional sensitivity analyses are conducted in order to numerically illustrate two propositions.

B. Main Results

Tables I and II report the main results of our analysis, involving perfect competition and strategic prosumers, respectively. We organize each of the two tables into three parts, corresponding to outcomes associated with the prosumers, wholesale power market, and economic rent distribution. Each table also contains three columns (a)–(c), from left to right, respectively, for cases with 25, 50 and 120 MW of renewable output. To facilitate our expositions, we define a prosumer's net position as follows. A *short* position if the prosumer engages in the market to purchase power, i.e., $\sum_f z_{fi} < 0$. A *long* position is when the prosumer engages in the market to sell power, i.e., $\sum_f z_{fi} > 0$. With this definition in mind, we report prosumer's net sale (+) or purchase (–) to/from the power pool, consumed energy or load, the self generation from

[8] These three levels of renewable outputs are carefully selected so that the prosumers will be in a long as well as short position in the resulting equilibria. Moreover, K_i is capped above in order to prevent the prosumer's marginal benefit from becoming negative. One explicable justification of this assumption is that a prosumer, with the goal of energy self-reliance, is less likely to install excessive renewable capacity with an effective output much greater than its expected demand. If possessing a considerable amount of renewables, the prosumer, mostly likely, will always be in a long position and act as a producer. That case would be less interesting.

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TABLE I
RESULTS UNDER PERFECT COMPETITION CASES

Variables \Scenarios Renewable output [MW]	(a) 25	(b) 50	(c) 120
Prosumer's sale(+)/purchase(-) [MWh]	-66.27	-44.11	19.96
Prosumer's load [MWh]	102.02	103.09	105.32
Prosumer's generation [MWh]	10.75	8.98	5.25
Marginal cost of backup [$/MWh]	45.75	43.98	40.28
Prosumer's surplus [$K]	9.89	11.05	14.05
Total power demand [MWh]	2,847.32	2,851.35	2,858.81
Total power production [MWh]	2,913.59	2,895.45	2,838.85
Power price in node 1 [$/MWh]	45.75	43.98	40.28
Sale-weighted power price [$/MWh]	35.52	35.41	35.17
Producers' surplus [$K]	39.23	41.12	45.72
Consumers' surplus [$K]	255.74	256.26	257.27
Grid operator's revenue [$K]	10.18	8.50	5.08
Social Surplus [$K]	305.16	305.87	308.07

TABLE II
RESULTS UNDER STRATEGIC PROSUMER CASES

Variables \Scenarios Renewable output [MWh]	(a) 25	(b) 50	(c) 120
Prosumer's sale(+)/purchase(-) [MWh]	-19.65	-12.97	7.52
Prosumer's load [MWh]	84.49	91.38	112.48
Prosumer's generation [MWh]	39.84	28.41	0.00
Marginal cost of backup [$/MWh]	74.84	63.41	35.00
Prosumer surplus [$K]	9.23	10.78	13.99
Total power demand [MWh]	2,855.25	2,855.85	2,857.69
Total power production [MWh]	2,874.91	2,868.82	2,850.17
Power price in node 1 [$/MWh]	42.21	41.88	40.88
Sale-weighted power price [$/MWh]	42.21	35.26	35.21
Producers' surplus [$K]	43.08	43.52	44.89
Consumers' surplus [$K]	256.78	256.87	257.11
Grid operator's revenue [$K]	6.82	6.52	5.62
Social Surplus [$K]	306.68	306.91	307.62

backup generation, and its surplus. We also report a number of variables associated with the power market, including total generation, total demand, power price in node 1, and sale-weighted power price, which is computed as $\sum_i p_i d_i / \sum_i d_i$. Note that the difference between total power generation and total power demand is equal to the power purchase by the prosumer. The last section summarizes the economic rent distribution in the power pool, including that of consumers, producers and grid operator.

A number of observations emerge from these two tables. When the renewable output is equal to 25 and 50 MW, either as a price-taker or a strategic entity, the prosumer's load is met by self generation plus purchase from the power pool. Thus, in both cases, the prosumer is in a short position. Epitomized by column (a) in Table I, the prosumer's load, 102.02MW, is met by 25MW from renewables, a power purchase of 66.27 MW from the power pool, and self generation of 10.75 MW from the backup unit. Intuitively, other than renewables, there are two competing power sources available to the prosumer, one is by self generation, and the other is by power purchases from the pool. These two options are perfect substitutes for each other, and profit-maximization principle requires the prosumer to use the option, among the two, that has a lower cost or utilize them both insofar that the marginal cost of two options become equal when market is perfectly competitive. Indeed, Table I indicates that the prosumer decides to produce the backup option to the level such that its marginal cost is equal to the pool power price in all scenarios.

However, it is not the case when the prosumer is designated as a strategic agent. In particular, while the prosumer remains in a short position with renewable output equal to 25 MW and 50 MW in columns (a) and (b) in Table II, its power consumption of 84.49 MW in (a) is supplied by 25 MW from renewables, 19.65 MW from power purchase, and 39.84 MW from self generation, respectively. The marginal cost of the backup generation, in this scenario, is actually significantly higher than the power price in node 1 by a margin of $32.63/MWh (=$74.84 - 42.21$) or 77%. It is this self "over generation" that allows the prosumer to lower its power procurement, suppressing power demand in the power pool, which leads to a lower power price in node 1 compared to its correspondent in Table I, i.e., 42.21 v.s. $47.74/MWh. A similar tactic is applied by the prosumer in column (b) of Table II to reduce the power price from $43.98/MWh in Table I to $41.88/MWh.

Furthermore, the power price in node 1 as well as the sale-weighted average power price of the market decline in accordance with the increases in renewable output. Because renewable energy represents a zero marginal cost resource, its abundance suppresses power prices in the market. For instance, the power price in node 1 (the sale-weighted average power price) in Table I decreases from $45.75/MWh ($35.52/MWh) in (a) to $40.28/MWh ($35.17/MWh); a similar trend can also be observed in Table II. Given the current parameter setting, the prosumer's foregone benefit of not consuming renewable is equal to zero as the prosumer's load in equilibrium is greater than renewable output (see Fig.1).[9] The prosumer, as expected, benefits from zero marginal cost renewables as its surplus increases are commensurate with incremental output from renewables.

With the prosumers participating in the market, the total generation and total consumption (excluding the prosumers) in the power market are not equivalent, depending on the prosumer's net position in the market. When the prosumers are in a short position, e.g., scenarios 25 MW and 50 MW in columns (a) and (b), the total generation from producers is greater than the total consumption by consumers, with excessive generation purchased by the prosumer. For instance, as indicated in scenario (a) in Table I, a total of 66.27 MW (= $2,913.59 - 2,847.32$) is procured by the prosumers.

Now we turn to the economic rent analysis. Interestingly, the prosumer is worse off in Table II compared to Table I. This is in part because producers and consumers in the power pool are designated as price takers. Generally, there are two counteracting forces that jointly determine the market equilibrium. When the prosumer exercises buyer's market power to lower the cost of its procurement, with an attempt to lower power prices, consumers will increase their quantity demanded when

[9] Of course, had evaluation of the power by the consumers in the power pool been significantly higher than that of the prosumers, it is possible to observe the prosumer forego some consumption in order to profit from the power pool.

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seeing lower prices, earning additional economic rent, thereby working against the prosumer. (Had the consumers been with a fixed demand or less price responsive, the prosumer will more likely succeed in the attempt to manipulate the power prices in its favor.) On the contrary, when the prosumer is in a long position, its endeavor to exercise seller's market power in order to push up the power prices is also likely to be thwarted by the increases in power sales from the price-taking producers. For instance, when the prosumer is in a relatively "longer" position, the producers' ability to negating the impacts from the prosumer is more than offset by the ability of the prosumer to exercise buyer's market power to lower the power prices, leading to a lower surplus \$44.89K in Table II compared to \$45.72K in Table I. Moving from columns (a) to (b), when the prosumer is in a relatively "weaker" short position, the producers in the wholesale market would then benefit from the prosumers' strategy, leading to a higher surplus in Table II than that of Table I. Overall, considering columns from (a) to (c) with increasing more renewables, elevation in the surplus by the consumers and producers leads to increases in the social surplus.

C. Sensitivity Analyses

Section III-B summarizes the main results of the sensitivity analyses. The focus was placed on learning the underlying strategies used by the prosumer and the consequential impacts on the market equilibria. This section reports the outcomes from additional sensitivity analyses by altering the renewable output K_i from 25 to approximately 120 MW to generalize our findings.[10] The sensitivity analyses in this section serve for two purposes. One is to explore the impact of the prosumers on the market when they own either *relatively* small or large size of renewable asset. The second one is to understand the impact of renewable stochasiticity on the market outcomes. In particular, we assume that there is an expectation on the renewable output by the prosumers. Thus, a high value of K_i corresponds to the situation where output from the renewables is greater than the expectation while small value of K_i is for the situation that output from the renewables is less than the expectation. We summarize the findings formally in two propositions while illustrating the results numerically in Figs. 2–3. The proofs of these two propositions can be found in the online appendices.

Proposition 2. *If a prosumer is in a short (long) position as a price-taker, he/she will also be in a short (long) position as a strategic entity, and vice versa.*

As alluded to earlier, the prosumers' net position cannot be determined *a priori*, but is a consequence of market interactions. This proposition suggests that the strategy executed by the prosumers would only impact the quantity of their decision variables $\sum_f z_{fi}$, the MW of power to buy from or sell to the main grid, but not its net position, i.e., buy (-) or purchase (+). Fig.2 illustrates Proposition 2 numerically by graphing the prosumers' net position against the renewable output K_i,

[10] We limit our attention to those cases with a positive marginal benefit possessed by the prosumers. A larger K_i beyond 120 MW or so will lead the prosumers' marginal benefit to be negative. We therefore rule those cases out.

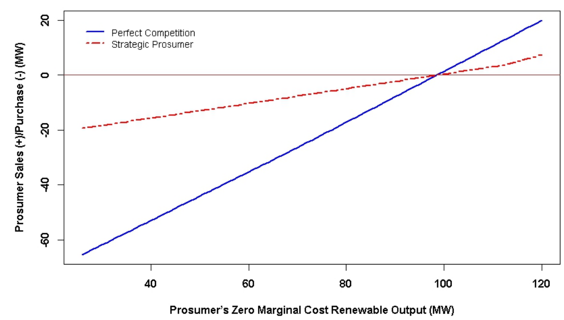

Fig. 2. Plot of prosumers' net position t_i against renewables output K_i under the price-taker and strategic entity scenarios

where solid and dashed lines represent the scenarios with the prosumers as a price-taker and a strategic entity, respectively. We also plot a horizontal line linking K_i to the case when the prosumers behave in an isolated or island mode without any interaction with the power pool or $\sum_f z_{fi} = 0$. Proposition 2 states that regardless of strategy assumption, if a prosumer is in a short position as a price-taker, it will also be in a short position as a strategic entity. That is, for a given K_i, both lines will stay at the same side (below or above) separated by the horizontal line.

Proposition 3. *A prosumer is better off by participating in the market as a price-taker rather than a strategic entity exercising Cournot strategy.*

Fig. 3 illustrates Proposition 3 numerically by graphing the prosumers' surplus against renewable output K_i under both price taking and strategic scenarios. Fig 3 reminisces the observations that we had pointed out previously in Section III-B: when the prosumer is in a short position, contracting the power procurement in order to lower the power price is not an economically viable strategy when the other participants in the market act as price takers. The initial gap of the prosumer's surplus between the two scenarios begins with around \$5K when $K_i = 25$MW. The gap then shrinks with an increase of renewables output K_i, and eventually asymptotically verges to null when $K_i = 85$ MW. The line when a prosumers is a price-taker either overlaps with or lies above that of a strategic entity.

IV. Conclusions

Prosumers' ability to act as a producer and a consumer, a duality that is not commonly seen if not unprecedented in the sector, also creates new opportunity or challenge to the energy sector. This paper extends the existing work by explicitly formulating the optimization problem faced by a prosumer in a complementarity problem. We conclude that exercising market power will not alter a prosumer's net position in equilibrium. That is, if the prosumer is in a short position as a price-taker, i.e., buy power from the main grid, it will also be in a short position had it been a strategic entity. The paper also discovers that while the prosumer is capable of manipulating the power prices by either exercising buyer's (seller's) power to lower

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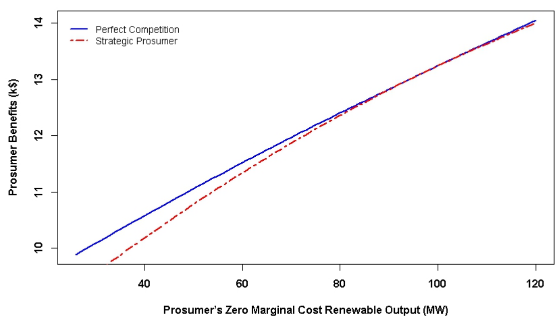

Fig. 3. Impact of prosumer strategy on prosumer surplus under different levels of renewable outputs

(increase) power prices, it actually is better off if acting as a price-taker while other entities in the market behave competitively. Finally, our analysis concludes that when the renewable output is low (so that the prosumer needs to purchase power from the main grid in equilibrium), consumers could benefit from lower power prices at the expense of producers as the prosumer decreases its power procurement from the power market in order to lower the power prices. On the other hand, as renewable output increases, thus the prosumer becomes a net seller to the main grid; its economic incentive is then more aligned with other conventional suppliers.

Our analysis is subject to a number of worth-noting limitations. First, we did not consider the possibility that the prosumer operates an energy storage system. In reality, many prosumers might own and operate energy storages, e.g., electrical vehicles, in order to take advantage of lower power prices during off-peak periods. Accounting for this will call for a multiple-period model with a consideration of cross-elasticity of energy demand among time periods in order to examine the effect of power price in one time period on the demand other time periods. Second, we maintain the assumption that market participants, other than the prosumer, are price takers. While the model in Section II is readily modified to simulate strategic behavior of conventional producers, allowing other producers behave strategically might complicate the analysis so that we might find it difficult to isolate the impact induced by the prosumers. Third, while we simulate different levels of renewable outputs, our analysis is essentially deterministic. Moving to a stochastic modeling framework, for example, by using scenario paths of renewable outputs and correlated demand, will, undoubtedly, be more realistically to represent the reality faced by the power market. Fourth, an aggregator, who acquires adequate trading experience, might be able to engage in spatial arbitraging to explore price difference in different location. We leave the aforementioned considerations to our future work.

Appendix A
Proofs of the Propositions

A. Proof of Proposition 1

Proof: We prove existence by showing that all the primal variables without explicit bounds, s, z, y, w and a, are all implicitly bounded. With the continuity assumptions of all the objective functions, we know that all the optimization problems involved, (22), (4), (6), and (8), have an optimal solution by the well-known Weierstrass' extreme value theorem, so long as the problems are feasible. Then since the constraints in all the optimization problems are linear, the linear constraint qualification holds, which guarantees the existence of dual variables satisfying the complementarity conditions in (3), (5), (7), and (9).

Note that feasibility here is not an issue since by letting $l_i = K_i$, $i = 1, \ldots, I$ and all other variables (g, z, s, x, y and a) to be 0, all the optimization problems' constraints, together with the flow balance constraint (10), can be satisfied. Hence, the set of joint feasibility of the optimization problems (22), (4), (6), (8), and (10) is not empty.

Next, we start with the arbitrager's problem (8) first. It has been shown in [29] (Equation (6) in [29]) that the two sets of equations in (9), together with the expression of the market price p_i as in (1), can uniquely determine the value for p^h and a_i, $i = 1, \ldots, I$, with a given set of values of w_i and s_{fi}, $i = 1, \ldots, I$, $f = 1, \ldots, F$. For simplicity, we denote a_i as $a_i(w, s)$, indicating that a_i is a linear function of w and s.

We next consider the grid operator's problem (6). It has been shown in [30] that the Mangasarian-Fromovitz Constraint Qualification (MFCQ) holds for the grid operator's problem at any feasible point. It is well-known that MFCQ is equivalent to the set of multipliers to be bounded [31]. Hence, $\lambda_k^{+,-}$ are bounded for $k = 1, \ldots, R$ in (7), which implies that w_i is bounded for $i = 1, \ldots, I$.

Now consider the inverse demand function at $i = 1, \ldots, I$:

$$p_i = P_i^0 - \frac{P_i^0}{Q_i^0} d_i = P_i^0 - \frac{P_i^0}{Q_i^0} \left(\sum_f s_{fi} + a_i(w, s) \right),$$

which determines the wholesale electricity price at each node i. In an equilibrium, we must have $0 \leq p_i \leq P_i^0$. If $p_i > P_i^0$ for some i, then the demand $d_i < 0$, which is impossible; on the other hand, if $p_i < 0$, then suppliers can simply choose to produce nothing (i.e., $x = s = 0$) to avoid a net loss in profit. Hence, we can only consider the set of s and w within which that yield a p_i within $[0, P_i^0]$ for each i. Based on the expression of $a_i(w, s)$ with respect to s (as in (6) in [29]), p_i can be re-written as a linear function of s and w, with coefficient of $s_f i$ all non-positive. Then the boundness of w_i and p_i, together with $s_{fi} \geq 0$, implies the boundedness of $s_f i$, for each f and i. Hence, the supplier's problem (4) assumes a finite optimal solution.

For the prosumer's problem (22), even though the z variables are not explicitly bounded (and hence the l variables are not bounded above), the level set of the problem must be bounded. This is so since (22) is a maximization problem; if $\sum_f z_{fi}$ goes to $-\infty$, the corresponding l_i will become $+\infty$, which will make the first term $p_i(\sum_f z_{fi})$ and the second term $-\int_{l_i}^{K_i} B_i'(x)dx$ in the objective function become negative infinity (since $B_i'(x)$ is a decreasing function by the assumption). This cannot happen to a feasible maximization problem. Hence (22) also assumes a finite optimal solution.

The finite optimal solutions of all optimization problems with linear constraints, together with the fact of a non-empty joint feasible set, yields the existence of a market equilibrium. ■

Note that by further assuming that the benefit function $B_i(\cdot)$ and the production cost functions $C_i^g(\cdot)$ and $C_{fih}(\cdot)$ are all convex quadratic, for all i, f and $h \in H_{f,i}$, we can also prove the uniqueness of the following quantities in a market equilibrium: l_i, g_i, $\sum_f z_{fi}$, $\sum_f s_{fi}$, and x_{fih}, for all i, f and $h \in H_{f,i}$. The proof is very similar to that in the e-companion of [32]. However, the detailed proof is lengthy, as we would need to stack all the optimization problems' KKT conditions as a single MiCP and define various matrix and vector notations. We do not feel that the uniqueness result adds any particular insights to the main conclusion of this paper, and hence do not show the details here.

B. Proof of Proposition 2

Proof: For easy exposition, we assume there are consumers, a producer and a prosumer in the market. We then drop the subscripts accordingly. Recall the prosumer's optimization problem in (22) and definition of its z ($z > 0$: long and $z < 0$: short), assuming that (22b) is binding, we can rewrite (2a) as follows:

$$\underset{z}{\text{maximize}} \quad pz + B(K - z) \tag{11a}$$

We further assume that the prosumer only owns zero marginal cost renewables, i.e., $G = 0$ so that constraint (22c) is omitted. Similarly, the producer's optimization problem can be simplified and rewritten as:

$$\underset{s}{\text{maximize}} \quad ps - C(s) \tag{12a}$$

Here, s represents the producer's sales. For simplicity, we assume that $C_0 = 0$ so that cost takes a form of $C(s) = \frac{1}{2}cs^2$. Furthermore, given (1) and $G = 0$, consumers quantity demanded equals $d = s + z$.

Perfect Competition. Taking the first order condition of (11) and (12), together with $d = s + z$, we have three conditions and three unknown (z, s, d):

$$p = A^0 - B^0(K - z) \tag{13a}$$

$$p = cs \tag{13b}$$

$$p = P^0 - (P^0/Q^0)d \tag{13c}$$

Solving the system of equations in (13) for z, we have:

$$z = \frac{P^0 c - (\frac{P^0}{Q^0} + c)(A^0 - B^0 K)}{B^0(\frac{P^0}{Q^0} + c) + \frac{P^0}{Q^0}c} \tag{14}$$

Market Power. Accounting for the prosumers' market power, we re-write (11) as follows:

$$\underset{z}{\text{maximize}} \quad [P^0 - (\frac{P^0}{Q^0})(s + z)]z + B(K - z) \tag{15a}$$

The first order conditions for the three agents under market power would then be:

$$P^0 - \frac{P^0}{Q^0}(s + z) - \frac{P^0}{Q^0}t = A^0 - B^0(K - z) \tag{16a}$$

$$(13b) \quad \& \quad (13c)$$

Solving the system of equations again for z, we have:

$$z = \frac{P^0 c - (\frac{P^0}{Q^0} + c)(A^0 - B^0 K)}{B^0(\frac{P^0}{Q^0} + c) + \frac{P^0}{Q^0}(\frac{P^0}{Q^0} + 2c)} \tag{17}$$

The denominator of (14) and (17) are positive given that all the parameters are positive. Thus, given that the numerators in (14) and (17) are equivalent, this suggests that sign of z, the prosumer's net sale, under both cases will be the same. Namely, If a prosumer is in a short (long) position as a price-taker, he/she will also be in a short (long) as a strategic entity, and vice versa.

When the prosumer also owns a dispatchable with a marginal cost of c', z under perfect competition and market power cases can be expressed as (18) and (19), respectively.

$$z = \frac{P^0(c + \frac{cB^0}{c'}) - (\frac{P^0}{Q^0} + c)(A^0 - B^0 K)}{B^0(\frac{P^0}{Q^0}\frac{c}{c'} + \frac{P^0}{Q^0} + c) + \frac{P^0}{Q^0}c} \tag{18}$$

$$z = \frac{P^0(c + \frac{cB^0}{c'}) - (\frac{P^0}{Q^0} + c)(A^0 - B^0 K)}{B^0(\frac{P^0}{Q^0} + c) + \frac{P^0}{Q^0}(\frac{P^0}{Q^0} + 2c) + \frac{\frac{P^0}{Q^0}B^0 c}{c'}} \tag{19}$$

A similar conclusion can be drawn with regard to the Proposition 1. ■

C. Proof of Proposition 3

Proof: By the same token as the proof of Proposition 2, we denote the power price and the prosumer's net position z for the perfect competition and market power cases by subscripts c and m, respectively.

Perfect Competition. the prosumer surplus defined in (11a) is equal to $p_c z_c + B^0(K - z_c)$. By substituting p_c and z_c from the solution to (13), we get $p_c z_c + B^0(K - z_c) = \frac{B^0}{2}z_c^2$.

Market Power. Similarly, substituting p_m and z_m from the solution to (16), we have $p_m z_m + B^0(K - z_m) = \frac{B^0}{2}z_m^2$.

Finally, substitute (14) and (17), respectively, for z_c and z_m, it implies that $\frac{B^0}{2}z_m^2 < \frac{B^0}{2}z_c^2$. Accordingly, we conclude that $p_c z_c + B^0(K - z_c) > p_m z_m + B^0(K - z_m)$. That is, a prosumer is better off with a higher profit when behaving as a price taker regardless of its position in the market. ■

D. Model Equivalence

We consider a situation where prosumer in node i enters a bilateral contract with the aggregator in i to purchase firm energy (l_i^c) at a contract price (p_i^c). By doing so, the prosumer relinquishes its control over dispatch unit, g_i, to the aggregator. The optimization problem faced by the prosumer i is as follows:

$$\underset{l_i \geq 0, l_i^p \geq 0}{\text{maximize}} \quad -p_i^c l_i^p - \int_{l_i}^{K_i} B_i'(x)dx \tag{20a}$$

$$\text{subject to}$$

$$l_i - l_i^p \leq 0 \qquad (\epsilon_i). \tag{20b}$$

The first-order conditions for l_i and l_i^p are, respectively, displayed as follows:

$$0 \le l_i \perp A_i^0 - B_i^0 l_i - \epsilon_i \le 0 \quad (21a)$$
$$0 \le l_i^p \perp -p_i^c + \epsilon_i \le 0 \quad (21b)$$

The aggregator i decides 1) amount of energy l_i^a to contract with prosumers, 2) amount of energy to purchase from ($z_{fi} < 0$) or sell to ($z_{fi} > 0$) the wholesale market, and 3) amount of g_i to generate while subject to sales and output constraints.

$$\underset{z_{fi}, l_i^a \ge 0, g_i \ge 0}{\text{maximize}} \quad p_i^c l_i^a + p_i \sum_f z_{fi} - \sum_i C_i^g(g_i) \quad (22a)$$

$$\text{subject to}$$

$$\sum_f z_{fi} + l_i^a - K_i - g_i = 0 \qquad (\delta_i), \quad (22b)$$

$$g_i \le G_i \qquad (\kappa_i) \quad (22c)$$

The first-order conditions of the aggregator i's problem for variables z_{fi}, l_i^a, g_i, and κ_i are summarized as follows.

$$p_i - \delta_i = 0 \quad (23a)$$
$$0 \le l_i^a \perp p_i^c - \delta_i \le 0 \quad (23b)$$
$$0 \le g_i \perp -C_i^{g\prime} + \delta_i - \kappa_i \le 0 \quad (23c)$$
$$0 \le \kappa_i \perp g_i - G_i \le 0 \quad (23d)$$

At equilibrium, $l_i^a = l_i^p$ with p_i^c defining the contract premium:

$$l_i^a = l_i^c, \qquad (p_i^c) \quad (24)$$

Assuming that $l_i^a = l_i^p > 0$, we have i) $p_i^c - \delta_i = 0$ from (23b), which is equivalent to (3a) and ii) $p_i^c = \epsilon_i$ from (21b). Thus, we can conclude that $\epsilon_i = \delta_i$. (21a) can then be written as $0 \le l_i \perp A_i^0 - B_i^0 l_i - \delta_i \le 0$, which is equivalent to (3b). Moreover, (23a)=(3a), (23c)=(3c), and (23d)=(3e). This establishes the equivalence of the model in Section (II-A2) and the model here.

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