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Dynamic Pricing for Autonomous Vehicle E-hailing Services Reliability and Performance Improvement

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Abstract— **As Autonomous Vehicles (AVs) become possible for E-hailing services operate, especially when telecom companies start deploying next-generation wireless networks (known as 5G) , many new technologies may be applied in these vehicles. Dynamic-route-switching is one of these technologies, which could help vehicles find the best possible route based on real-time traffic information. However, allowing all AVs to choose their own optimal routes is not the best solution for a complex city network, since each vehicle ignores its negative effect on the road system due to the additional congestion it creates. As a result, with this system, some of the links may become over-congested, causing the whole road network system performance to degrade. Meanwhile, the travel time reliability, especially during the peak hours, is an essential factor to improve the customers' ride experience. Unfortunately, these two issues have received relatively less attention. In this paper, we design a link-based dynamic pricing model to improve the road network system and travel time reliability at the same time. In this approach, we assume that all links are eligible with the dynamic pricing, and AVs will be perfect informed with update traffic condition and follow the dynamic road pricing. A heuristic approach is developed to address this computationally difficult problem. The output includes link-based surcharge, new travel demand and traffic condition which would improve the system performance close to the System Optimal (SO) solution and maintain the travel time reliability. Finally, we evaluate the effectiveness and efficiency of the proposed model to the well-known test Sioux Falls network.**

I. INTRODUCTION

In the past few years, autonomous vehicles (AVs) equipped with advanced sensor technologies and able to drive themselves without any human intervention have been developed [1]. They can provide many advantages when compared with human's driving. For example, Avs have the potential to reduce crashes, smoothing traffic, and reducing the congestion time [2]. However, due to the new features associated with of AVs, more advanced research is needed to learn about their travel behavior and system performance, especially regarding to city congestion issues.

Road pricing, especially the congestion pricing, is not new in the transportation industry. Singapore began implementing congestion pricing in 1975, which set up a restricted driving area and levied extra toll charges during the peak traffic hours [3]. Today, countries like the UK, Australia, Sweden, and Finland are all have some version of a congestion road pricing scheme. Results indicate that implementing the proper level of toll/surcharge is one of the best viable solutions to reduce congestion, along with other social and environmental negative externalities like air pollution, greenhouse gas emissions, visual intrusion, noise, and road accidents [4].

In this paper, we will apply the theoretical study of using road pricing to control congested traffic and improve the network system performance. Pigou [5] first proposed congestion pricing theory by using externalities to measure optimal congestion charges, which originated from the concept of the economics of welfare. In line with this theory, those who use congested roads should pay a toll equal to the difference between the marginal social cost and the marginal private cost [6,7]. However, the assumption of first-best pricing, which implies road congestion is caused only by underpricing road users' travel cost, is not perfect in practice. It sometimes overlooks other factors like supply and demand changes after surcharges have been applied. If they are simply applied without considering the consequences of their application, it may distort the allocation of traffic assignment over the entire traffic network [8].

Due to the imperfections of first-best pricing, second-best pricing, which incorporates feedback from the system, was first explored by Lévy-Lambert [9] and Marchand [10]. They focused on the simplest version of a "classic two-route problem" in which a non-tolled alternative road runs parallel to a toll road in order to determine toll levels. More recently, Wang and Lownes [11] applied a link-based surcharge mechanism to adapt a full network to E-hailing service use. In general, the problem with second-best road pricing is finding a set of optimal values for toll charges in order to minimize total travel time, maximize toll revenue, or accomplish both, while also considering the choice behavior of network users. Regardless of the pricing mechanisms set around different modes of transportation or around new innovative technology like autonomous vehicles, the second-best pricing model can incorporate further consideration of factors in the model in order to achieve multiple goals at the same time.

In this paper, we design a link-based dynamic pricing

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TABLE I. NOTATION INDEX

$a \in A$	Link a belonging to set of links A.	d_w	Traffic Demand of O-D pair w
$i \in N$	Node i belonging to set of nodes N.	D_w	Travel Demand function for O-D pair w
$w \in W$	O-D pair w belonging to set of O-D pairs W	f_k^w	Flow on path k connecting O-D pair w
$k \in K$	Path in set of paths K	x_a	Traffic volume of link a
α	Travel time variation allowance rate	$t_a(x)$	Travel time function for link a with volume x
β	Travel cost variation allowance rate	M_a	Marginal cost for link a
μ	Total travel cost	S_a^n	Link surcharge rate in iteration n
μ_w^0	Initial general travel cost for O-D pair w before the surcharge	$\delta_a^{w,k}$	Equal to 1 if link a on the path k between the O-D pair w, otherwise equal to 0

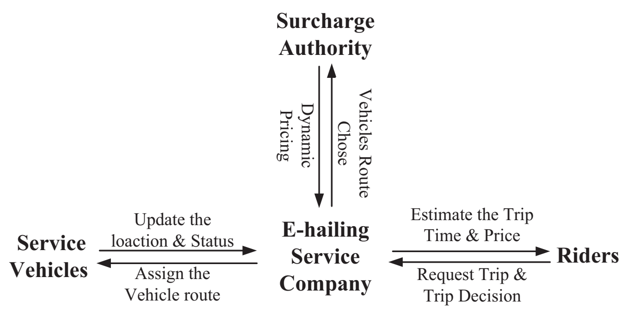
model to improve the road network system and travel time reliability simultaneously. Different than the previous dynamic pricing studies focus on toll pricing and traffic assignment e.g., Nikolic et al.[15], Lu and Mahmassani [16] and Sharron et al. [17], this study more focus on the AVs transportation network system performance improvement and service reliability control. As the result, the prices on congested routes are higher than uncongested routes, which lead to AVs to switching to alternative uncongested routes and thus results in fewer AVs selecting those congested routes. Additionally, with service reliability control, the increasing of the dynamic pricing would lead some customers shift their ride to a less congested time period or shift their e-hailing ride to more economic mode e.g. transit, bicycle, etc. Overall, the iterative process of pricing and predicting impact leads to a stable solution that should improve system performance, maintain the traffic level, and ensure the customers' travel reliability at the same time. The contributions of this work are:

- An efficient algorithm that will update the travel demand while ensuring each link will maintain a non-congestion condition. Previous work either focused on one travel demand estimation only or limited the travel reliability in a regional network [18].
- A bi-level mathematic model with consideration of customers choose to give up their e-hailing ride or shift their ride to a less congested time period.
- A platform output that would not only include optimal pricing, but also the surcharge revenue, the new travel demand under this pricing and the associated traffic condition.

II. NETWORK AND MODELS

As illustrated in Fig. 1. there are four main components in this model: the Surcharge Authority, the E-hailing service company, the E-hailing customers, and the Autonomous Service vehicles (AVs). With the customers' trip requests, E-hailing service companies calculate the riders' estimated trip time and pricing based on current autonomous vehicle locations, status, and surcharge authority based dynamic pricing. If the riders accept the trips' rates and times, the E-hailing company will assign the AVs to the customers and update each vehicle's route. Each vehicle's route information will be updated and send to the surcharge authority at each time segment, and the surcharge authority will use this information to calculate dynamic pricing for next time interval.

Figure 1. Role Components of the Model



In this paper, we propose a link based dynamic pricing system for Autonomous Vehicle (AV) Ride-sharing E-hailing Services. The goals include optimizing traffic system performance while maintaining each rider's travel time reliability. By introducing link based dynamic pricing: a) road network usage will change from depending on each vehicle's shortest path to employing the path which will minimize total system travel time; b) trip travel time will be guaranteed in a certain range through the demand changes instigated by pricing. Previous work either assumes that the demand is fixed or only targets optimal road network traffic conditions [19]. The generalization of the dynamic pricing problem formulation is explained in the following subsection, and a list of parameters and variables are demonstrated in following table I.

A bi-level programming model is proposed to solve this dynamic pricing problem. The outer layer of this model represents the surcharge authority, which calculates the most effective pricing to achieve System Optimal (SO) traffic conditions and guarantee the trip travel time in an acceptable range. The objective functions and subjective constraints are defined as follows:

$$Min \sum_a x_a \, t_a(x_a) \tag{1}$$

Subject to:

$$t_a^n(x_a) \leq (1+\alpha) t_{a,0} \tag{2}$$

$$\sum_k f_k^{w,n} \leq d_{w,0} \quad \forall\, k, w \tag{3}$$

$$f_k^{w,n} \geq 0 \quad \forall\, k, w \tag{4}$$

Definitional constraints:

$$x_a^n = \sum_k \sum_w \delta_a^{w,k,n} \cdot f_k^{w,n} \tag{5}$$

$$M_a^n = x_a^n \frac{dt_a^n(x_a^n)}{dx_a^n} \tag{6}$$

$$S_a^n = \left(\frac{1}{n}\right)(1+)M_a^n + \left(1-\frac{1}{n}\right)S_a^{n-1} \tag{7}$$

Objective function (1), which represents the surcharge authority objective, is the standard system optimization formula. Equation (2) is the link travel time constraint, which ensures each link travel time does not exceed the given threshold. Equations (3) and (4) are the constraints which define all path volumes that fall between 0 and the original total demand. Equation (1) and constraints (3) and (4) are similar to the User Equilibrium assignment [12]. Equation (5) is the definitional constraint from path to link formulation transformation. Equations (6) and (7) are link pricing calculation mechanisms based on the marginal externality.

The inner layer, which includes the AVs, E-hailing service companies, and the AVs' three components, and aims to serve the maximum original demand, is formulated as follows:

$$Max \sum_W \mathrm{D_w}(\mathrm{\mu_w}) \tag{8}$$

Such that:

For each OD pair:

$$\sum_a \delta_a^{w,k} \cdot S_a^n + \sum_a \delta_a^{w,k} \cdot t_a(x_a^n) = \mu_w^n \quad \forall\, a, w \tag{9}$$

$$\mu_w^n \leq (1+\beta)\, \mu_w^0 \quad \forall\, w \tag{10}$$

$$d_w^n \leq d_{w,0} \quad \forall\, w \tag{11}$$

And for each link, it follows:

$$Min \sum_a \int_0^{x_a} t_a^n(\omega) + S_a^n d\omega \tag{12}$$

Subject to:

$$\sum_k f_k^{w,n} = d_w^n \quad \forall\, k, w \tag{13}$$

$$f_k^{w,n} \geq 0 \quad \forall\, k, w \tag{14}$$

The object function (8) is defined as the maximum number of trips based on original demand. The general travel cost function is exhibited in equation (9), demonstrating the relationship between each OD trip travel cost associated with the correlated link's travel time and link pricing. Equations (10) and (11) are the cost constraints for the OD trip after link pricing is applied. At the trip level, equations (9) ~ (11) determine the OD trip service demand for each iteration. At the link-based level, Equations (12) ~ (14) assume each vehicle has chosen its least costly route, which means these equations can estimate traffic condition based on the new link pricing and service demands through this process.

III. Solution Algorithm

A heuristic solution algorithm is developed to solve this E-hailing autonomous vehicle pricing and travel time reliability problems. Four major phases are involved in the process to improve system performance: in the first phase, the algorithm aims to minimize total system travel time, while the second phase aims to serve as much original demand as possible. In the third phase, the AVs obtain all previously-calculated information with the assumption that it is perfect and use it to make their route choices. In the last phase, each link travel time will be ensured in a certain range. If the volume increases over capacity, the incremental process will end. The aggregation of each rider's travel decisions characterizes the traffic conditions of each specific time period that follows, and the surcharge authority accurately obtains this information. The link pricing rate is then recalibrated and updated. This loop continues until the difference between iterations of link pricing rates in the objective function falls below a critical threshold. The essential steps in this process are shown below:

Algorithm 1 Algorithm for optimal pricing and traffic estimation

Input: road network and original travel demand

Output: optimal price, demand, and traffic conditions under the pricing.

1. Initiate the network by assigning the traffic based on original demand and each trip travel cost μ_w^0.
2. Based on the link volume x_a from previous step, calculate the marginal price M_a^n and associate MSA surcharge rate S_a^n.
3. Estimate each OD pair w travel demand d_w based on the updated link surcharge rate while ensuring the link travel time $t_a(x)$.
4. Re-estimate the traffic assignment with the update demand and surcharge rate.
5. Convergence check: if the link Surcharge rate difference is smaller than the criteria $\sum_a |S_a^n - S_a^{n-1}| < \varepsilon$, otherwise return to step 2.

A. Initialize the network

In the initial stage, the time period is divided into $t \in T$ time segments to coordinate traffic information update frequency. Within the initial time segment t_0, the network is set to the no-surcharge scenario by carrying the full load of demand and distributing traffic with the UE assignment. The original O-D trip travel cost is calculated using the total link cost of its shortest path with the estimated traffic condition:

$$\mu_w^0 = \sum_a \delta_a^{w,k} \cdot t_a\left(x_a^0\right)$$
$$\delta_a^{w,k} = \begin{cases} 1, & \text{link on shortest path} \\ 0, & \text{otherwise} \end{cases} \quad (15)$$

As shown, $\delta_a^{k,w}$ is an indicator variable for each link: it is equal to 1 if link a is on the shortest path k between the O-D pair w; it is equal to 0 if link a is not on that path. Through this step, the complex network path-based cost problem is transferred to the link-based calculation.

B. Calculate the link dynamic pricing

The E-hailing AVs' dynamic pricing is calculated using each time segment to coordinate the traffic information update. The two process are: using marginal pricing computing to optimize the network balance; and using an iterative heuristic approach to capture the travelers' reactions.

Link-based marginal cost computing implies the negative externality of adding an additional car, which imposes costs on all the other existing cars on the road. The value of this negative externality is equal to the arc elasticity of the link cost (subject to current traffic volume), which can be written as:

$$M_a^n = x_a^n \frac{dt_a^n(x_a^n)}{dx_a^n} \quad (17)$$

The Appendix shows the difference between the non-surcharge scenario and the System Optimal (SO) scenario is the link-based marginal cost. Which indicate that, if all AVs choose their routes based not only on their least costly paths, but also considering their negative externality on all other existing vehicles, the final traffic condition is the System Optimal (SO) solution.

Meanwhile, the assumption of perfect information allows the AVs to receive the dynamic link pricing for the network during each time interval as the surcharge authorities obtain the traffic conditions at the same time. In order to reduce this back-and-forth process, the Measure of Success Average method (MSA) is introduced here. The link iteration pricing is updated with equation (18), where M_a^n is the current marginal cost of link a, and S_a^{n-1} is the link pricing rate of the previous iteration.

$$\begin{aligned} S_a^n &= \left(\frac{1}{n}\right) M_a^n + \left(1 - \frac{1}{n}\right) S_a^{n-1} \\ &= \left(\frac{1}{n}\right)\left(x_a^n \frac{dt_a^n}{dx_a^n}\right) + \left(1 - \frac{1}{n}\right) S_a^{n-1} \end{aligned} \quad (18)$$

C. Update travel demand

In this step, with the least costly path already having been identified for each O-D pair, the inverse travel cost function is applied to estimate the new demand [11]. In the economic inverse demand function "the price of a good represents the marginal willingness to pay for an extra unit of the good by anyone who is demanding that good" [13]. The inverse travel cost function at any given trip's cost measures how many AVs would be willing to take the ride which they intended in the non-surcharge scenario. The inverse cost function of travel demand, which is equal to the travelers' willingness to pay for their desired trip, estimates the demand.

$$\mathrm{d}_w^n = \mathrm{D_w}(\mu_w^n) \quad (19)$$

With the cost of μ_w^n, traveling on path $\mathrm{w} \in W$, the inverse cost function is $\mu_w^n = \mathrm{D_w^{-1}}(\mathrm{d}_w^n)$. t_w^n and S_w^n represent the route total travel time cost and the total surcharge rate in iteration n, which are two major components of total travel cost:

$$\mu_w^n = \mathrm{t}_w^n + S_w^n \quad (20)$$

In this heuristic approach, in order to coordinate the travel time reliably, which constrains equation (2), and serves as much of the original travel demand as possible, which is the solution to equation (8) ~ (14). However, the mathematic solution to this problem is not straightforward since each link in the network can be used by multiple routes. This means if one AV switches its route, it could affect multiple AVs, causing them to switch their routes or even cancel their trips. And this problem becomes more complicated considering reliable the travel time simultaneously. Therefore, we propose the following incremental algorithm to solve this complex problem:

Algorithm 2 Algorithm for travel demand estimation while ensuring travel time reliability

```
Begin
1:   Set ∀x*_a = 0, incremental iteration i = 1, incremental rate p;
     While
2:   The initial demand has not been fully served: i <= 1/p;
         Do {
3:           Perform all-pair-shortest path assignment for all demand
         w ∈ W based on μ^n_a
4:           Set each link on the shortest path w ∈ W:  δ^{w,n}_a = 1,
         Otherwise: δ^{w,n}_a = 0
             While
5:               OD pair w ∈ W and
             Σ_a δ^{w,n}_a · [ t_a( 1 + 0.15 · (x*_a / C_a)^4 ) + S^n_a ] ≤ μ^0_w;
             Do {
6:               Update link volume: x*_a += δ^w_a · D^0_w · p;
7:               Update OD pair demand: D^n_w += D^0_w · p
8:               Move to next OD pair w ∈ W;
             }
             Check each link:
9:               If: t^n_a(ω) ≥ (1 + α) t_{a,0}
10:                  Increase the surcharge rate by:
                     S^n_a = S^n_a · (1 + β%)
         }
       i++;
     }
End;
```

The pseudo-code of algorithm 2 demonstrates the incremental process of the new travel demand estimation while ensuring travel time reliability. Lines 1 and 2 are the outer layer loop and searching step length settings for this algorithm. Lines 3 and 4 identify the all-pair-shortest-path based on current traffic conditions and price. Similar to [14], performing all-pair-shortest-path process in the beginning of the incremental loading loop will reduce the computation complexity. Since this program will run multiple iterations, the difference between conducting it in the beginning and during incremental loading will be eliminated. Links 5 ~8 are the first criteria to check in order to see if p percent of original demand will add to the network based on the travel cost; while lines 9 and 10 are the second criteria to check in order to see if p percent demand added to the network will cause link congestion. The program will move to the next iteration after these two criterial checks.

D. Convergence Check

The heuristic approach ends when the convergence criteria meet, which is when the sum of all links' absolute link pricing differences between the last two iterations are smaller than ε:

$$\sum_a |S_a^n - S_a^{n-1}| < \varepsilon \tag{21}$$

Overall, this heuristic approach simulates the back-and-forth negotiation process among the surcharge authority, the E-hailing company, the AVs and the customers. Since this process is based on each component's reaction, it will not encounter feasible solution issues. That means at the worst-case scenarios, the dynamic pricing of the link surcharge will become extremely high, which will exclude some customers to ensure that the link maintains its non-congested condition. Leftover customers may shift their ride to a less congested time period or switch to another mode of transportation (like public transit).

IV. Evaluation Results

The method described above will be demonstrated in the well-known and well-studied 24 nodes, 76 links, and 360,600 total OD trips comprising the Sioux Fall network (assume all demands are running for the e-hailing services). While it has been noted that this network bears little physical sameness to Sioux Falls, South Dakota today, the network and its associated data have been widely used in variety of transportation network analysis studies.[20] The different lines (solid/dot/dash) indicate the link volume-to-capacity (v/c) ratio based on User Equilibrium Assignment (as shown in Fig. 2). Due to the large travel demand, 37 out of 76 links are over capacity, and eight links have a v/c ratio over 1.5. Many of the high-volume links are clustered around the network center at nodes 11 and 6, as they are where the travel is concentrated.

Fig. 3 illustrates the link surcharge rates and v/c ratios after the e-hailing surcharge has been applied. With a 44.75% decrease in demand, most of the links' congestion conditions have been reduced. An important note here is that links between 13 and 24, and 10 and 16, have been charged the highest rate even though they are not seriously congested in the non-surcharge scenario. The reason for this is that even with the surcharge, the demands for using nearby alternative links are still high, and a high surcharge rate prevents a large amount of traffic from switching to these non-congested roads.

Table II exhibits different alternative routes in three traffic conditions. In the most congested route, the traffic condition improved after the surcharge was applied. However, due to the longer distances in the alternative routes, the traveler would expect a much longer travel time even when the surcharge rate is still high. Similarly, in the median congested route category, since many of these routes are also located in the center of the city, the alternative routes still face a considerable amount of surcharge and extra travel time. This is the opposite case in the un-congested category, since travelers expect a higher surcharge rate and longer travel time because their alternative route will go through some congested links.

Figure 2. Sioux Falls Network under UE Assignment

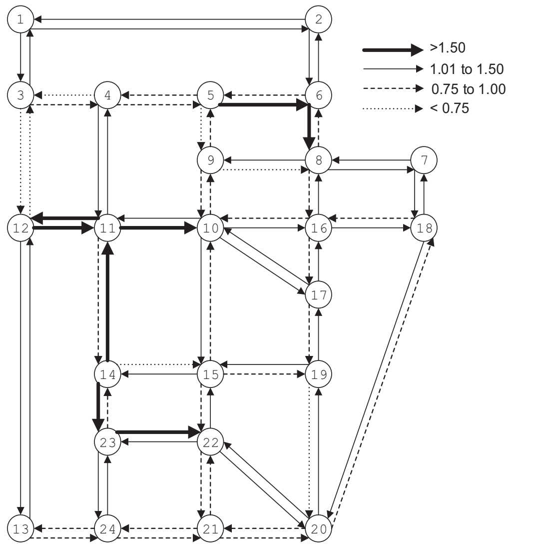


Figure 3. Sioux Fall Network with Dynamic Pricing

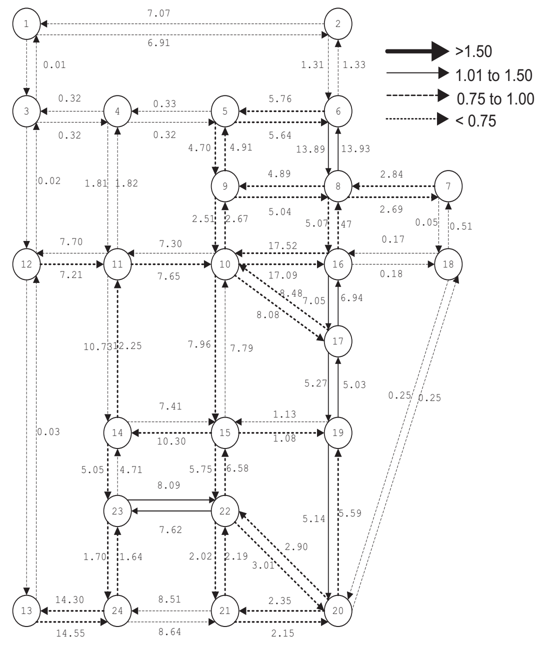


TABLE II. PATH ANALYSIS

Sioux Falls Network Analysis				
Most Congested	From 12 to 10			
		Links	Time	Surcharge
	Path 1	12-11-10	13.21	15.00
	Path 2	12-3-4-11-10	20.99	9.98
	Path 3	12-3-4-5-9-10	21.01	8.05
Median Congested	From 10 to 18			
		Links	Time	Surcharge
	Path 1	10-16-18	7.83	17.27
	Path 2	10-17-16-18	16.58	15.21
	Path 3	10-9-8-7-18	23.54	10.45
Un-Congested	From 5 to 3			
		Links	Time	Surcharge
	Path 1	5-4-3	6.19	0.65
	Path 2	5-6-2-1-3	19.3	14.05
	Path 3	5-9-10-11-12-3	28.22	21.46

TABLE III. SUMMARY OF CHANGE

	Before Surcharge	After Surcharge	Percentage Change
Total system travel time	7,476,972	3,705,927	-50.44%
Travel Demand	360,600	199,240	-44.75%
Average link travel time	8.81	7.87	-10.65%
Average link V/C ratio	1.09	0.75	-32.86%
Average Surcharge rate	0.00	3.09	N/A
Average link travel cost	8.81	11.25	27.74%

Overall, as table III shows, with a 44.75% demand change, the total system travel time was reduced by 50.44% and each link travel time was reduced by 10.65%. The road became less congested with a v/c ratio 0.75, but travelers can expect an average 27.74% increase in cost to accomplish their trip.

V. CONCLUSION

In this research, we introduced an economical way to calculate the optimal E-hailing surcharge rate in order to improve travel time reliability and system performance during peak travel hours. All link-based E-hailing service congestion surcharges were calculated through a heuristic approach. The results demonstrated that the principles of UE assignment were maintained in the travelers' path choice behavior while leveraging the relationship between SO and UE using the marginal cost function. At the same time, revenue received from this surcharge can be used in various ways, such as improving transit service quality or upgrading road infrastructure which could also advance road network performance. In all applications in this paper, demand saw a significant drop from its original level. Future work can explore alternate means of estimating demand response to surcharge levels and/or fine tune the existing function.

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