

Does the resolution of marine magnetic anomalies affect geomagnetic reversal statistics?

B. Buffett¹ and M. Avery¹

¹Department of Earth & Planetary Science, University of California, Berkeley

Key Points:

- Marine magnetic anomalies show a deficit of short polarity intervals relative to a Poisson process.
- Stochastic models also predict a deficit of short polarity intervals when the time resolution is 30 kyr or longer.
- High-resolution observations should reveal more short polarity intervals than a comparable Poisson process.

Abstract

Observations of polarity intervals from marine magnetic anomalies suggest that geomagnetic reversals depart from a Poisson process. A deficit of short polarity intervals favors a distribution described by a Weibull probability density function with a shape parameter $k > 1$. Polarity intervals from a stochastic model for the paleomagnetic field also obey a Weibull distribution, but the shape parameter is controlled by the temporal averaging used to approximate the finite resolution of geological observations. Temporal averaging in excess of 30 kyr yields polarity intervals that are well described with a shape parameter $k > 1$. Conversely, a shorter averaging time requires a probability distribution with $k < 1$. This behavior suggests that short polarity intervals are not captured by marine magnetic anomalies. Instead, these events may be recorded in paleomagnetic measurements from high-resolution sediment cores. Geomagnetic excursions with changes in magnetic direction of 180° may be examples of the missing polarity intervals. All of these short polarity intervals are expected to occur when the dipole field is weak. A correspondence between the duration of these short events and the strength of the dipole field offers a test of the suggestion that the geodynamo preferentially produces short polarity intervals relative to the predictions of a Poisson process.

1 Introduction

Marine magnetic anomalies (MMA) over the past 160 Myr provide a detailed record of geomagnetic polarity transitions. The chronology of these transitions serves the basis of the geomagnetic polarity timescale (Ogg, 2012). All polarity intervals 30 kyr and longer are thought to be captured in the MMA record (Gee & Kent, 2015), although shorter, and previously unrecognized, polarity intervals are occasionally proposed (Krijgsman & Kent, 2004; Roberts & Lewin-Harris, 2000). Counter arguments usually invoke fluctuations in paleointensity to account for the anomalous paleomagnetic observations (Bowles, Tauxe, Gee, McMillan, & Cande, 2003; Lanci & Lowrie, 1997). Distinguishing between these two interpretations is important because it affects our assessment of the statistics of geomagnetic reversals.

An early investigation of the MMA record represented geomagnetic reversals as a Poisson process (Cox, 1969). The expected probability distribution for the polarity interval, τ , is governed by an exponential distribution, which requires the occurrence of short polarity intervals. A subsequent study (Naidu, 1971) favored a gamma distribution to account for the perceived deficit of short polarity intervals. The gamma distribution,

$$g(\tau) = \frac{\lambda^k}{\Gamma(k)} \tau^{k-1} e^{-\lambda\tau}, \quad (1)$$

is specified by two parameters; λ defines the average rate of the process and k is a shape parameter that describes deviations from an exponential distribution. Here $\Gamma(k)$ is the gamma function. An exponential distribution is recovered by setting $k = 1$. Fewer short polarity intervals are expected when $k > 1$, whereas more short polarity intervals are predicted when $k < 1$ (see Figure 1). The current MMA record favors $k > 1$ (Naidu, 1971), and this result is supported by arguments about the finite time required for polarity transitions to occur (Merrill & McFadden, 1994). This line of reasoning is taken as evidence that geomagnetic reversals cannot be strictly a Poisson process.

Use of dynamo models to assess reversal statistics has not yet offered much insight. Even the most realistic simulations (e.g. Schaeffer, Jault, Nataf, & Fournier, 2017) are still very far from Earth-like conditions, and these models are presently too computationally demanding to run for long enough to collect statistics on geomagnetic reversals. Instead, the numerical models must adopt very high values for fluid viscosity to allow longer integration, necessary to produce a sufficient sample of polarity transitions (e.g. Driscoll & Olson, 2009). Such high viscosities restrict the fluid motions to large scales,

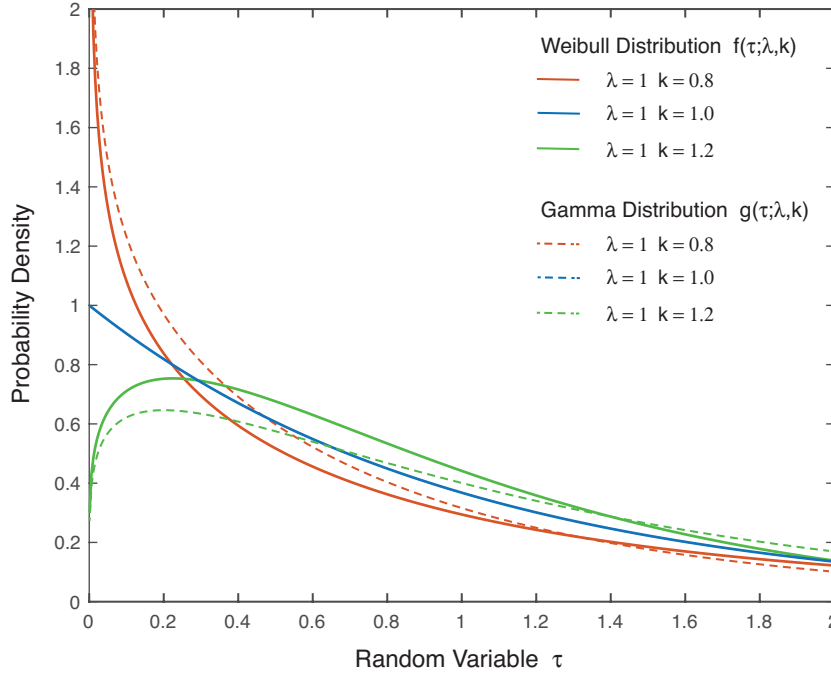


Figure 1. Gamma and Weibull probability density functions. These closely related distributions are specified by a rate parameter, λ , and a shape parameter, k . Short polarity intervals occur more (less) often when $k < 1$ ($k > 1$). An exponential distribution ($k = 1$) describes the polarity intervals from a Poisson process.

which raises questions about the capability of these models to produce short polarity intervals. The study of Driscoll and Olson (2009) reported no polarity intervals shorter than about 2 dipole diffusion times, which corresponds to roughly 100 kyr.

An alternative approach relies on stochastic models for the paleomagnetic field (e.g. Brendel, Kuipers, Barkema, & Hoyng, 2007). These models have a physical basis (Scullard & Buffett, 2018), and they are capable of reproducing many observed properties of the paleomagnetic field (Buffett & Puranam, 2017). For this reason we use stochastic models to address the question of reversal statistics and to quantify the extent to which these statistics are affected by the temporal resolution of the geological record.

2 Stochastic Model for the Paleomagnetic Field

Fluctuations in the axial dipole moment, $x(t)$, can be represented conceptually by the motion of a particle in a double potential well (see Figure 2). The particle settles toward the base of one of the potential wells, but random noise continuously disturbs the motion. The source of noise is intended to reflect the influence of random convective fluctuations on the dipole moment. Occasionally, a series of random fluctuations push the particle across the barrier between the two potential wells, causing a polarity transition. The amplitude of the random noise and the general form of the potential well can be estimated from paleomagnetic observations. Here, we adopted the double potential well, $U(x)$, and the noise amplitude, $D(x)$, recovered by Buffett and Puranam (2017) from stacks of relative paleointensity measurements from the past 2 Myr (Ziegler, Constable, Johnson, & Tauxe, 2011), supplemented with higher-resolution measurements from the past 10 kyr (Constable, Korte, & Panovska, 2016). We treat the noise amplitude as a

constant, which allows us to recover a representative estimate from the 10-kyr CALS10k2 model. The potential is based on the 2-Myr PADM2M model using the functional form given in equation (27) from Buffett and Puranam (2017). Realizations of the stochastic model are computed by numerically integrating the stochastic differential equation

$$dx = -\left(\frac{\partial U}{\partial x}\right) dt + \sqrt{2D} dW, \quad (2)$$

using an Euler-Maruyama method (Risken, 1989) with a discrete time step of $\Delta t = 1$ kyr. Here dW represents uncorrelated (white) noise drawn from a normal distribution with mean of 0 and variance of Δt .

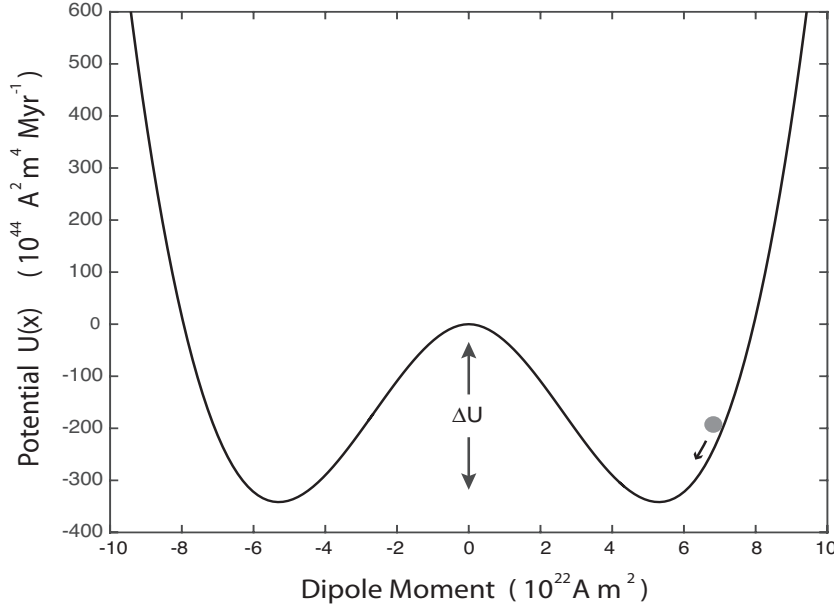


Figure 2. Motion of a particle in a double potential well, $U(x)$, is analogous to the stochastic model for the dipole moment. The particle settles toward the bottom of a potential well, but random noise continuously disturbs the state x . A sequence of disturbances can occasionally send the particle over the barrier at $x = 0$, causing a polarity transition.

We assess the statistics of polarity intervals using the mean, median and standard deviation of τ recovered from realizations of the stochastic model. Dividing the median and standard deviation by the mean yields quantities that are independent of the rate, λ . This choice allows us to focus attention on the shape parameter k . A drawback of interpreting the stochastic model in terms of the gamma distribution is that the median for $g(\tau)$ has no closed-form expression. It is preferable to adopt the closely related Weibull distribution

$$f(\tau) = k\lambda^k \tau^{k-1} e^{-(\lambda\tau)^k}, \quad (3)$$

where λ and k are the rate and shape parameters (see Figure 1). Both the Weibull and gamma distributions revert to the exponential distribution when $k = 1$. We also note that the gamma and Weibull distributions have very similar dependence on τ at short polarity intervals. The normalized median is

$$\tilde{m} = \frac{(\ln 2)^{1/k}}{\Gamma(1 + 1/k)}, \quad (4)$$

and the normalized standard deviation is

$$\tilde{\sigma} = [\Gamma(1 + 2/k)/\Gamma(1 + 1/k)^2 - 1]^{1/2}. \quad (5)$$

Both of these expressions are independent of the rate parameter λ .

Variations in $\tilde{m}$ and $\tilde{\sigma}$ with k for the Weibull distribution are shown in Figure 3. Higher values of k yield lower $\tilde{\sigma}$ and higher $\tilde{m}$. Conversely, lower values of k give higher $\tilde{\sigma}$ and lower $\tilde{m}$. An exponential distribution ($k = 1$) has $\tilde{\sigma} = 1$ and $\tilde{m} = \ln 2$. Observations of the normalized median and standard deviation from the MMA record are superimposed on the predictions for the Weibull distribution in Figure 3. Each point in Figure 3 is computed from the sample mean, median and standard deviation in a single 10-Myr time interval. This interval is shifted back in time in increments of 1 Myr to produce a set of estimates that extend back to 48 Ma. The scatter of points fall on top of the Weibull distribution between $k = 0.9$ to 1.2, although the majority of points lie between $1 < k < 1.2$. This preference for $k > 1$ is consistent with a deficit of short polarity intervals.

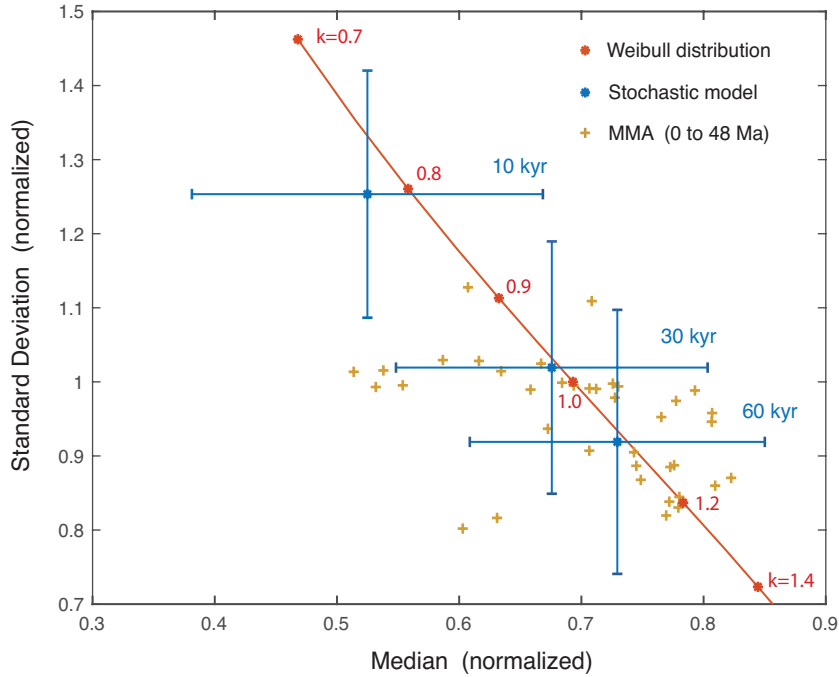


Figure 3. Normalized median, $\tilde{m}$, and standard deviation, $\tilde{\sigma}$, for a Weibull distribution (red). Statistics for the observed polarity intervals are computed from 10-Myr intervals of the MMA record. The cloud of points is generated by shifting the 10-Myr interval back in time at increments of 1 Myr. Polarity intervals from the stochastic model follow a Weibull distribution, where the shape parameter, k , is set by the temporal resolution of the realization. The error bars represent the 1-sigma uncertainty from two hundred 10-Myr realizations.

We compare this result with the predictions of the stochastic model by running a series of 10-Myr realizations. We recover a single estimate for $\tilde{m}$ and $\tilde{\sigma}$ from each realization. Combining the results of 200 realizations gives a reliable sample mean and variance. Each realization is filtered in time using a running average to account for the limited temporal resolution of the geological observations. Distinct results for the reversal statistics are recovered with different choices for averaging time. Calculations with an averaging time of 10 kyr, 30 kyr and 60 kyr are shown in Figure 3, together with the 1-sigma error bars. The normalized median and standard deviation from the stochastic model follow the trend from the Weibull distribution, where the averaging time appears to correspond to a particular value of k . An averaging time of 10 kyr corresponds roughly

to $k \approx 0.8$, whereas an averaging time of 60 kyr corresponds to $k \approx 1.1$. Interestingly, the stochastic model with a time average of 30 kyr has reversal statistics very close to the predictions for an exponential distribution ($k = 1$).

It appears that the reversal statistics from the stochastic model approximate a Weibull distribution, where the shape parameter can be interpreted as an indication of the temporal resolution of record. We cannot strictly equate the averaging time to a temporal resolution because about 12% of the polarity intervals from the stochastic model are shorter than the averaging time. Still, we can expect the temporal resolution to be proportional to the averaging time. A longer averaging time yields a deficit of short polarity intervals relative to a Poisson process, so we might reasonably expect a low resolution record to yield statistics with $k > 1$. This outcome is not entirely surprising, but we can now quantitatively assess how the statistics change when the temporal resolution is altered.

3 Consequences of Temporal Resolution

The preponderance of short polarity intervals in the stochastic model at low averaging times (e.g. $k < 1$) suggests that the geological record is missing short polarity intervals. The direct influence of averaging time on the rate of polarity transitions is shown in Figure 4. We use a large ensemble of 10-Myr realizations to compute the mean reversal rate and its uncertainty. The error bars are mainly determined by the duration of the record; longer realizations produce smaller error bars. A 10-Myr realization is chosen to match our analysis of the observed polarity intervals. As we reduce the averaging time from 80 kyr to 10 kyr there is a steady increase in the rate of reversals. An estimate of the reversal rate from Kramers' formula (see equation (28) in Buffett & Puranam, 2017) is also shown in Figure 4. Kramers' formula can be interpreted as the mean time it takes for a particle at the bottom of one potential well to reach the top of the barrier. The slightest nudge sends this particle sliding down into the second potential well. The prediction of Kramers' formula agrees well with the outcome of model realizations when the averaging time is roughly 30 to 40 kyr. The reason for this correspondence is related to the typical duration of polarity transitions.

When a particle is placed at the top of the barrier it will eventually make its way to the bottom of a potential well. We can use the stochastic model to predict how long this descent takes, at least on average. An estimate can be recovered from a large number of realizations, or the problem can be formulated in terms of a solution of the Fokker-Planck equation (Risken, 1989). In either case, we compute a recovery time of about 28 kyr (Buffett & Puranam, 2017), which is in good agreement with the average value inferred from the PADM2M model of Ziegler et al. (2011). In other words, we can expect the dipole moment to return to its time-averaged amplitude roughly 28 kyr after a reversal. At this point the dipole begins its next attempt to cross the barrier from a position near the bottom of a potential well. More complicated trajectories are possible if we focus on the evolution of the dipole while it remains near the top of the barrier after a reversal. For example, a particle might return to the original polarity shortly after moving into the reversed polarity. Most of the details of these trajectories are suppressed by averaging over a timescale that is comparable to the recovery time (see Figure 5). We record only the dipole arriving at the bottom of a potential after roughly 30 kyr (on average). Starting the next reversal process from the bottom of a potential well is precisely the context that Kramers' formula was intended to approximate.

We now consider the more complicated trajectories that become possible when the temporal averaging is reduced. A relatively small fluctuation is sufficient to send the dipole back over the barrier when the time after a reversal is too short to allow the dipole to settle into a stable polarity. As we decrease the averaging time below the recovery time of 28 kyr, we begin to identify reversals in the stochastic model that occur before the amplitude of the dipole has fully recovered. Once the averaging time drops below 10 kyr,

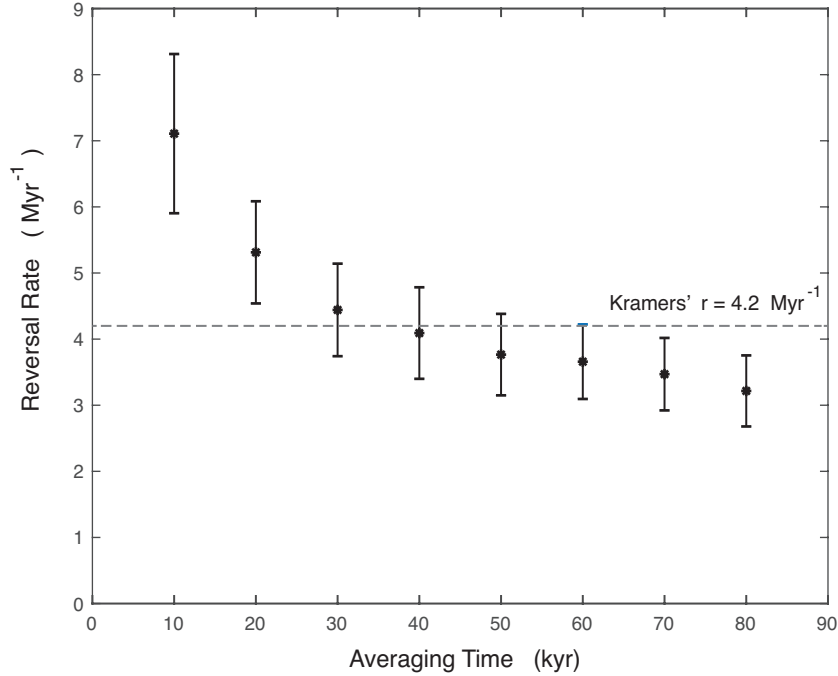


Figure 4. Average reversal rate, r , from two hundred 10-Myr realizations of the stochastic model. Changes in temporal averaging cause a systematic trend in r . An estimate of r from Kramers' formula (Risken, 1989) corresponds most closely to an averaging time of 30 kyr to 40 kyr. A shorter averaging time causes a marked increase in r .

we have nearly doubled the rate of reversals, and all of these additional reversals occur when the geomagnetic field starts in a weak state relative to its long-term average. Detecting these reversals in the geological record would be difficult for two reasons. First, we need sufficient temporal resolution to identify short polarity transitions. Second, we need to detect these transitions when the amplitude of the dipole field is weak. Contributions from the non-dipole part of the field become more important when the dipole is weak, so short polarity intervals are liable to have a complex geographic expression in paleomagnetic observations (Brown, Holme, & Bargery, 2007).

On strictly theoretical grounds we expect the reversal rate in the stochastic model to increase without limit as the averaging time decreases. This surprising outcome is actually consistent with the predictions of the Weibull distribution. To understand this behavior we note that the stochastic differential equation in (2) describes a Wiener process near $x = 0$ because the gradient of the potential vanishes. If we discretize the solution by taking fixed time steps (say $\Delta t = 1$ kyr), then the number of times $x(t)$ crosses zero, on average, is proportional $\sqrt{n}$, where n is the number of 1-kyr time steps through the transition (DasGupta & Rubin, 1998). Here the transition duration is taken to mean the time needed for the dipole to settle into a stable polarity. Decreasing the time step improves the temporal resolution, but this change increases the number of time steps through the transition; it also increases the number of zero crossings. All of these additional zero crossings occur at the shortest time interval permitted by the refinement in the time step. In other words we are accumulating zero crossings with small τ , consistent with the expected correspondence between a small temporal resolution (Δt) and a small shape parameter $k < 1$. In the limit that Δt and k go to zero, the normalized median in (4) also goes to zero. This means that the polarity intervals cluster without limit near $\tau = 0$.

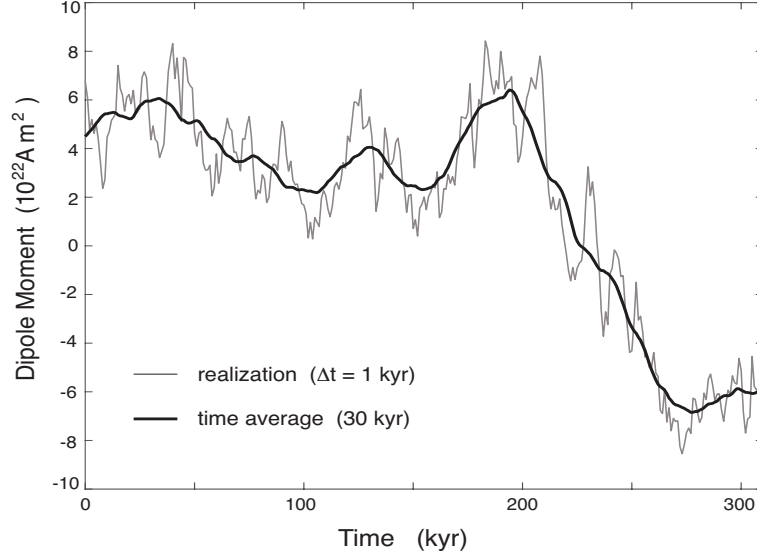


Figure 5. Realization of the stochastic model through a polarity transition. A smooth transition between stable polarities occurs when a time average of 30 kyr is applied to account for the temporal resolution of the MMA record (black). The actual realization (gray) exhibits several zero crossing when the time-averaged dipole moment is weak. Fluctuations in the dipole moment during the transition can reach $x = \pm 3 \times 10^{22} \text{ A m}^2$.

In practice we cannot allow the time step in the stochastic model to decrease without limit because the noise term would no longer be uncorrelated. Numerical dynamo models (e.g. Buffett & Matsui, 2015; Olson, Christensen, & Driscoll, 2012) suggest the correlation time of convective fluctuations is shorter than the overturn time, $L/V_{rms} = 140$ years, where $L = 2260$ km is the thickness of the outer core and $V_{rms} = 0.5 \text{ mm s}^{-1}$ is an estimate of the convective velocity (Holme, 2015). We can think of the correlation time as a limit on the temporal resolution of a standard stochastic model. Such a model could plausibly permit polarity intervals as short as a few hundred years if there were no other restrictions on our ability to record the dipole field. Of course, the amplitude of the dipole field between these transitions would be small, and such events would not conform to our usual view of geomagnetic reversals. However, there is no simple way to separate the continuum of behavior between low-amplitude and short-period polarity intervals and the more conventional view of geomagnetic reversals as a transition between two states of stable polarity. In fact, our conventional view may be partially shaped by the coincidence of dipole recovery time with the temporal resolution of the MMA record. Under these conditions the dipole moves smoothly from one polarity to the other without much complexity during the transition.

4 Discussion

Several lines of evidence point to short polarity intervals in the paleomagnetic record. Confirmation of new polarity transitions in high-resolution sediment cores, like those pro-

posed by Roberts and Lewin-Harris (2000), would shift the statistics of geomagnetic reversals toward lower values for k . Geomagnetic excursions are another type of brief event that sometimes exhibit directional changes of approximately 180° before returning to the direction established prior to the excursion (e.g. Laj, Kissel, & Roberts, 2006). Laj and Channell (2015) prefer to describe these events as microchrons because their duration is less than 10 kyr, which is too short to be recorded by MMA. Including these events in an assessment of reversal statistics would also shift k toward lower values. There are also indications of precursors and rebound features in magnetic directions around the time of well-established polarity transitions (Valet, Fournier, Courtillot, & Herrero-Bevera, 2012). Magnetic directions can change by 90° or more before and after the main transition. It is possible that some of these features represent short-period polarity transitions during times when the dipole is in a weakened state. It is common for the stochastic model to exhibit three or more sign changes during a transition from one stable state to another (see Figure 5). The main question is whether the amplitude of the dipole field between these sign changes is large enough to be detected in magnetic directions.

A weak dipole field allows magnetic directions to be strongly influenced by the non-dipole field. These directions become increasingly aligned with the dipole when the strength of the dipole rises above the level of the non-dipole field. The strength required to align magnetic directions with the dipole depends on the amplitude of non-dipole components and on the geographic location of the observations (Quidelleur, Gillot, Carlut, & Courtillot, 1999). The study of Brown et al. (2007) used the non-dipole components from the CALS7k.2 model (Korte & Constable, 2005) to show that magnetic directions during a polarity transition cluster around the final direction once the dipole moment reaches $x = 2.5 \times 10^{22}$ A m². Some geographic locations have magnetic directions aligned with the dipole when axial dipole moment is as small as $x = 2 \times 10^{22}$ A m² (Brown & Korte, 2016; Valet & Plenier, 2008). We can use the time required for the dipole to reach to a suitable threshold to establish the duration of detectable polarity transitions. An approximate expression for the average recovery time, τ_t , is (Buffett, 2015)

$$\tau_t = \frac{4x_t^2}{D\pi^2}, \quad (6)$$

where x_t is the prescribed threshold and D was previously defined as the amplitude of the noise term. This simple expression overestimates τ_t when the threshold is set at the time-averaged magnetic field (i.e. $x_t = 5.3 \times 10^2$ A m²). We obtain 33 kyr from (6) compared with 28 kyr from a full solution of the Fokker-Planck equation using $D = 0.34 \times 10^{44}$ A² m⁴ kyr⁻¹ (Buffett & Puranam, 2017). However, this error is substantially reduced when the threshold is lowered. For $x_t = 2.5 \times 10^{22}$ A m² we obtain $\tau_t = 7.5$ kyr, which is comparable to values commonly cited for the duration of a reversal in magnetic direction (Clement, 2004). More rapid transitions are allowed because τ_t represents an average value. On the other hand, consistently shorter durations would require a lower threshold to establish the new magnetic direction. Taking $x_t = 2 \times 10^{22}$ A m² gives an average duration of 4.8 kyr. Any specific instance of the stochastic process could plausibly give durations that were 50% higher or lower. This means that we could establish a new magnetic direction on timescales as short as 2.4 kyr if a dipole moment of $x_t = 2 \times 10^{22}$ A m² was sufficient to define the new magnetic direction.

Detectable polarity intervals with durations shorter than 10 kyr are possible with a weak dipole field (nominally $x < 3 \times 10^{22}$ A m²). Substantially shorter durations are possible with even weaker dipole fields, but the paleomagnetic expression of these events might be too spatially complex to interpret. A precursor event that shifts magnetic directions by 90° could plausibly be explained by a transient drop of the dipole moment below the threshold where the dipole no longer defines the magnetic direction. All of these short period events are ultimately associated with a weak dipole field. A testable prediction of the stochastic model is that the duration of the shortest polarity intervals should be correlated with the initial amplitude of the dipole field. Including these events in our

discussion of reversals statistics suggests that the short-period intervals are preferentially generated by the geodynamo.

5 Conclusions

A statistical analysis of the observed polarity intervals in the MMA record supports the view that geomagnetic reversals are not represented by a Poisson process. The observed record reveals a deficit of short polarity intervals relative to the predictions of a Poisson process. Instead the observations are better represented by a Weibull probability density with a shape parameter $k > 1$. Polarity intervals from a stochastic model are also found to obey a Weibull distribution. However, the shape parameter is controlled by the temporal averaging applied to the stochastic model to represent the finite resolution of geological observations. Applying an averaging time of 30 kyr to the stochastic model yields a distribution of polarity intervals that closely follows a Poisson process. Adopting a lower time average (higher resolution) produces polarity intervals that follow a Weibull distribution with a shape parameter $k < 1$. This result means that short polarity intervals are preferentially generated by the stochastic model relative to the predictions of a Poisson process. We suggest that these short polarity intervals are not captured in the MMA record, but they may be present in paleomagnetic observations from high-resolution sediments core. Brief geomagnetic excursions that shift magnetic directions through 180° may be representative examples of this type of short event. All of these short events should occur when the dipole field is weak, and we expect a correspondence between the duration of these events and the initial strength of the dipole field before the transition.

Acknowledgments

This work is partially supported a grant (EAR-164464) from National Science Foundation to B.B. and by Postdoctoral Fellowship from the National Science Foundation to M.A.

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