Optimal Deformation Modes for Estimating Soil Properties

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ABSTRACT

Accurate estimation of soil mechanical properties represents a crucial step for most engineering applications. Both in situ and laboratory testing fundamentally rest on mechanically deforming (actuating) the material and simultaneously measuring its response in terms of displacements and stresses (reactions). Facing this widely adopted scheme, key questions remain unanswered: 1) what is the optimal type and/or mode of actuation that can most effectively extract soil properties; 2) what types of measurements are most useful for inferring material constants? As a first step in the investigation of these questions, an inverse model for the direct simple shear (DSS) test is constructed, wherein measurable responses are used to back-calculate soil properties. Specimens with two different aspect ratios are considered to study the influence of the deformation mode. The effect of the choice of measurements (i.e., which displacements and/or stresses are observed) is explored by assessing inverse model performance considering the DSS test as a boundary value problem, with variable displacement and stress fields, versus the conventional interpretation as an elemental test. Parameter sensitivities and correlation coefficients are employed as quantifiable metrics to compare material characterization based on different aspect ratios and types of measurements, and to interpret the performance of inverse analysis.

INTRODUCTION

Current geotechnical practice has many established methods of determining material parameters. Among all methods, both *in situ* and laboratory, there is little indication as to the optimal method to test materials. The term “optimal” in this context implies that the maximum amount of information can be extracted from a single test, and parameters can be determined with the greatest degree of accuracy. Should we be unlimited by the apparatus and the modes in which we can deform materials, we might expect that there is an optimal method in which we can extract the maximum amount of information from a deformation. This could be in the form of applying a combination of tractions and displacements to the boundaries of an arbitrary volume of material and measuring the material response, as shown in Figure 1(a).

In order to provide justification that it is possible to define an optimal mode of deformation, this work assesses various idealizations of an existing laboratory test: the direct simple shear (DSS) test. Towards this end, numerical simulations of the DSS test are performed, and an inverse model to estimate soil properties from measurable responses in the simulations, cast as an optimization problem, is constructed and analyzed.

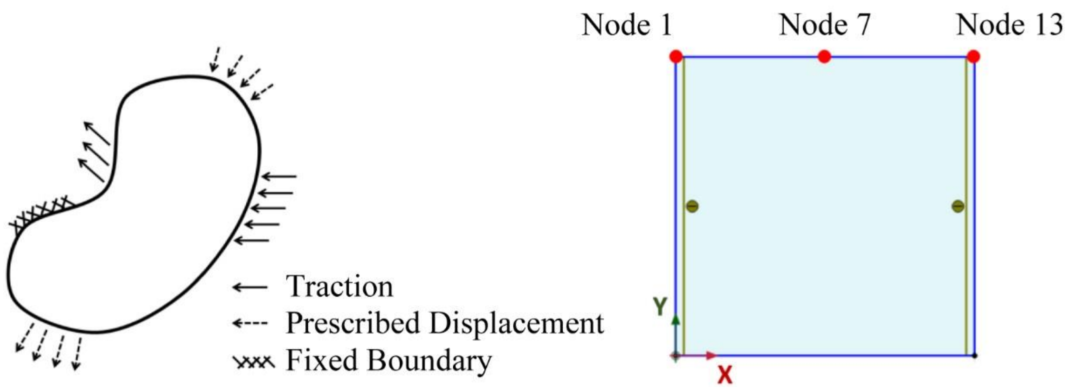

Figure 1. (a) Arbitrary volume of material. (b) Simulations of DSS test with Plaxis 2D (aspect ratio of *w*/*h* = 1).

The DSS test is chosen due to its somewhat controversial traditional treatment as an elemental test, where a set of average shear and normal tractions measured at boundaries are assumed to be representative of a uniform stress field distributed throughout the soil specimen. In reality it has been proven, both numerically and experimentally (Wood et al., 1979, Stroud 1971, Budhu, 1984 & 1988, Moon and Hashash, 2015, Dounias and Potts, 1993, Doherty and Fahey 2011) that this test is indeed a boundary value problem, and the material response varies significantly across the sample. In particular, Wood et al. (1979), Budhu (1984) and Moon and Hashash (2015) explicitly exploited the non-uniformity of a sample tested by a direct simple shear device to benefit the extraction of the stress-strain behavior.

In this work, the purpose is not to study the typical setup of the DSS test, but rather to use the non-uniform stress and strain fields induced in the test as a means of enhancing the interpretation of material properties. We define the “simplified interpretation model” (SIM) as the test where only the resultant forces are measured and the “extended interpretation model” (EIM) as one where stresses (or displacements) are measured at multiple locations along the boundary. These two models are theoretically analyzed and compared to explore whether one is able of producing a better estimation of soil properties from this simple laboratory test by capturing more information from the heightened non-uniform conditions, e.g., through the future development of techniques for sensing stress, strain, and local displacement. In the DSS test, the deformation depends on the aspect ratio of the specimen and the imposed displacement. Both the SIM and the EIM are therefore constructed for two different aspect ratios as a means of exploring, in a simplified fashion, the role of the deformation mode in the estimation of soil properties.

SIMULATION OF DSS TEST

Simulations are completed using Plaxis 2D assuming the soil is idealized as an isotropic elastic perfectly-plastic material characterized by Young’s modulus (E), Poisson’s ratio (v), cohesion (c), friction angle (ϕ), and dilation angle (ψ). The simulated DSS test consists of a dry soil body surrounded by four rigid plates, as shown in Figure 1(b). The height of the specimen (along the y-direction in the figure) is denoted by h, and its width (along the x-direction) is denoted by w. Two width-to-height ratios are considered: $w/h = 3$ and 1. The base plate is fixed, and the two side plates are free to rotate. All plate connections are pinned to allow for free rotation. Frictionless interfaces are assumed along the sides ($x = 0$ and $x = w$), as shown by the superposed vertical lines in Figure 1(b), in a manner consistent with Dounias and Potts (1993). The use of frictionless boundaries intensifies the non-uniformities in a sample (Vucetic, 1981) and thus generates more information for estimating soil properties. The DSS test is simulated by applying a displacement to the top plate in the *x*-direction, denoted by u, while maintaining zero displacement in the *y*-direction. A total displacement of $u = 0.05h$ is applied in increments of $\Delta u = 0.01h$. In this section, normalized quantities are used to present the results so as to avoid the arbitrary specification of units. In general, however, inverse analysis deals with the evaluation of parameters in a particular system of units. Therefore, in subsequent sections, kilopascals (kPa) and meters (m) are used, and for the sake of convenience we specify a specimen height of $h = 1$ m. Initial stresses are arbitrarily taken as $\sigma_{xx,0} = 50$ kPa and $\sigma_{yy,0} = 100$ kPa.

Four distinct types of simulation are completed for this study. Two correspond to the simplified interpretation model (SIM), and the other two relate to the extended interpretation model (EIM). In each case, the two specimen aspect ratios of $w/h = 3$ and 1 are analyzed. For the SIM, normal and tangential forces acting on the top plate are divided by the plate area to deduce average normal and shear stresses respectively, as presented in Figure 2. In contrast, for the EIM, a set of normal and shear tractions at multiple nodes on the top soil boundary are used to estimate soil properties. Observations in the form of tractions are evaluated at 13 nodes at the top boundary for the specimen aspect ratio of $w/h = 3$, and at 9 nodes for the aspect ratio of $w/h = 1$. In all models a "medium" mesh with a coarseness factor of 0.25 (PLAXIS, 2018) was used.

The non-uniformity of the stress field in the DSS test can be highlighted by looking at the SIM and EIM outputs. Figure 2 compares normal and shear stresses computed from both models with aspect ratio of $w/h = 1$, for material parameters given in Table 1. In these plots, the displacement u is the prescribed displacement of the top plate in the Plaxis simulation, where $u \leq 0.05h$. The node numbers indicated in Figure 2 represent the nodes in the EIM. The three selected nodes are equally spaced along the top of the model, as shown in Figure 1(b), and the nodes at the corners are placed directly adjacent to the corners. It can be seen that the average stress values are a good representation of the material behavior at the center of the sample, however these average values neglect appreciable non-uniformity. Nodes 1 and 13 represent the corners of the sample, and in both cases the stress components vary significantly from those at the center of the sample.

This sample variation is further assessed through the comparison of the relative shear in the soil along model top boundary, presented in Figure 3. Relative shear stress, denoted by τ_{rel}, is a measure of the closeness of the current stress state to the yield surface. When $\tau_{rel} = 1$, the stress state is at yielding (PLAXIS, 2018). As seen in Figure 3, when the displacement of the top plate is relatively small ($u/h = 0.01$ and 0.02), the majority of the material along the top boundary remains in the elastic state. When $u/h = 0.03$, most of this boundary has reached yielding. The distribution of stress at the various stages is provides further evidence that the DSS test is not an elemental test. It is also interesting to note that, as the shear deformation increases, some of the locations along this boundary which were previously plastic undergo elastic unloading, in particular at Nodes 5 and 6, during later stages.

PARAMETER SENSITIVITY AND CORRELATION ANALYSIS

To assess the performance of the models (SIM and EIM) for estimating soil properties, a "synthetic case" is defined. The responses (tractions) computed from this synthetic case are treated as observations that are analogous to measurements from laboratory or *in situ* tests. Inverse analysis as carried out in this study refers to the procedure by which the soil properties that best match the observations are computed. By considering synthetic data, the difference between the estimated soil properties and the true (synthetic) properties provides a quantitative measure of the performance of inverse analysis. In this paper, for simplicity and in view of the space available, only one synthetic case is considered, and the assumed soil properties are presented in Table 1.

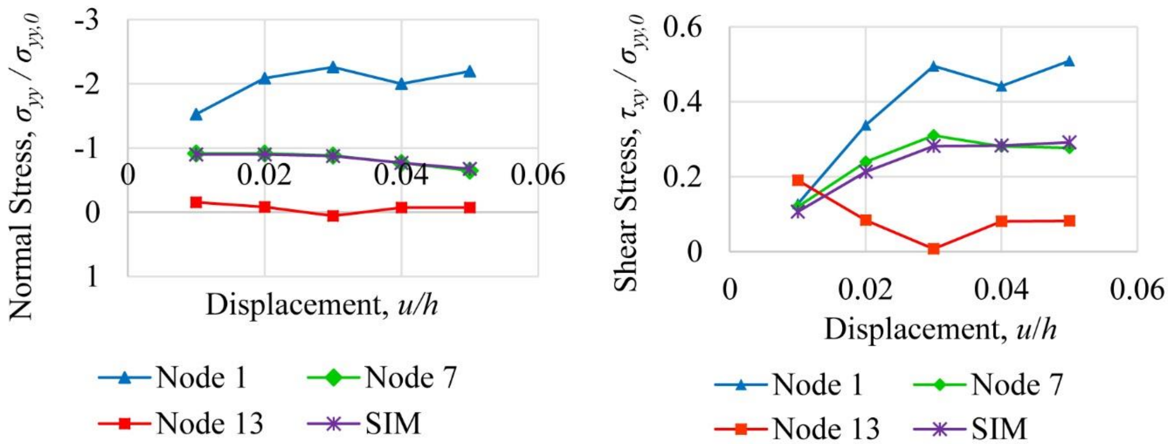

Figure 2. Stress distribution comparisons (see Figure 1(b) for node locations).

Table 1. Input Parameters.

Parameter	Synthetic	Starting Guess		
		#1	#2	#3
Young's modulus, E (kPa)	3000	2500	1500	4000
Poisson's ratio, v	0.25	0.25	0.25	0.25
Cohesion, c (kPa)	1	1	10	2
Friction angle, ϕ (°)	30	25	22	38
Dilation angle, ψ (°)	0	0	0	0

A parameter sensitivity analysis is completed in order to determine the parameters which are most likely to significantly impact the convergence and are therefore most efficient to optimize. An analysis of the correlation between parameters is conducted to determine whether multiple parameters can be optimized simultaneously (Poeter and Hill, 1997 & 1998). The composite scaled sensitivity (CSS) and parameter correlation coefficient (PCC) are both calculated as per the procedure outlined by Poeter and Hill (1997 & 1998).

Figure 4(a) presents the CSS for all four of the analyzed models. In all models, the friction angle has the highest sensitivity, closely followed by Young's modulus, and these are therefore determined as the critical parameters. Although Poisson's ratio is the next most sensitive parameter, the cohesion is selected as another parameter to be further analyzed. This is because the Poisson's ratio of soils often varies within a relatively narrow range, and accordingly a reasonable estimation can be readily made. Figure 4(b) gives the computed values of PCC considering Young's modulus, cohesion and friction angle. A large value of PCC identifies a pair of parameters that have higher correlation, and thus are hard to identify simultaneously. This analysis suggests that the simultaneous identification of Young's modulus with either cohesion or friction angle through optimization would have a higher chance of retrieving the true parameters than that of an attempt to identify cohesion and friction angle simultaneously.

An additional feature of this analysis is to identify which of the two models, EIM and SIM, is preferable for discovering the true soil properties. From Figure 4, one can infer that the EIM most likely performs better than the SIM, and the aspect ratio of w/h = 1 is expected to perform slightly better than w/h = 3. For both aspect ratios, the CSS is generally higher for the EIM, and the PCC is lower for all parameters, which should indicate a better ability to identify the true parameters.

OBJECTIVE FUNCTION

In order to conduct the inverse analysis, an objective function must be defined which evaluates the error between the observed (synthetic) observations and the observations corresponding to a trial set of soil parameters. The plot of the objective function for a single parameter can be used to give an indication of the ability to optimize that particular parameter. A smooth objective function with a single, well-defined minimum is desired as there is less chance for the optimization to diverge, or to converge to a local minimum. Here the analysis of the objective function is restricted to single-parameter optimization due to the computational demands of the multi-parameter counterparts.

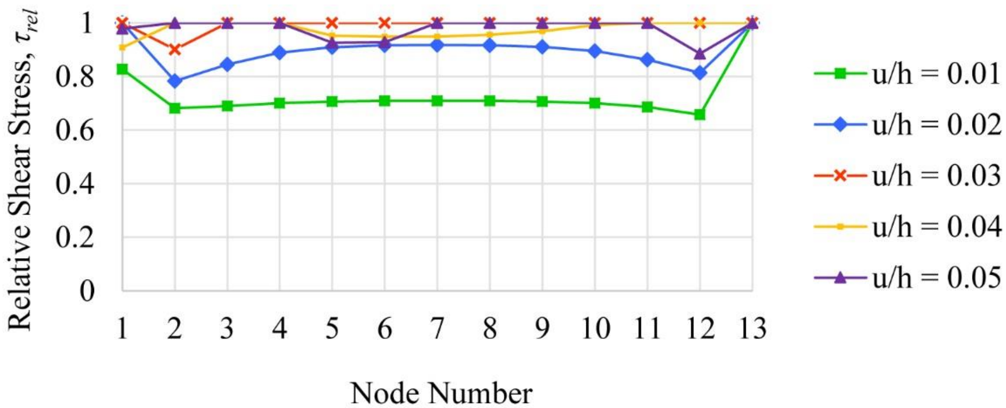

Figure 3. Sample relative shear.

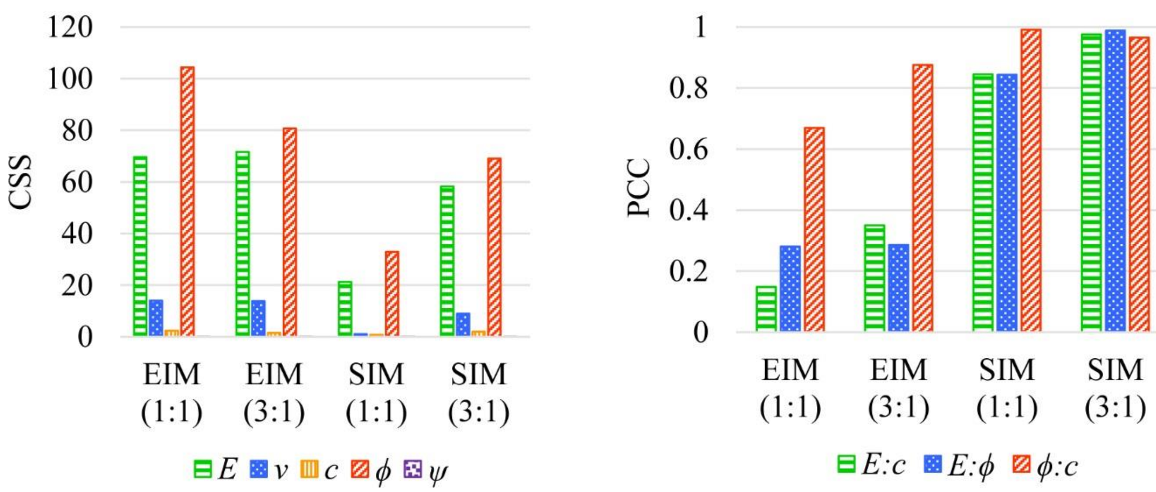

Figure 4. (a) Sensitivities. (b) Correlation Coefficients.

The objective function in this work takes the following specific form:

$$f(y_i) = \sqrt{\sum_{i=1}^{N} w_i (y_i - y_i')^2} \tag{1}$$

where y_i is the i^{th} observation from the current calculation based on the trial soil parameters, y_i' is the i^{th} prediction from the synthetic case, N is the total number of observations, and w_i is the weight applied to observations. In this analysis all weights were equal to unity, as all measurements are of the same type and are assumed to have equal accuracy (Hill, 1998). The index i is used to designate observations varying with respect to the position, direction and the analysis stage.

The objective functions can be calculated for all models and all parameters, but in this work they are limited to the previously identified critical parameters (i.e., E and ϕ) for the four models (see Figure 5). Note that all objective functions exhibit a global minimum corresponding to the true parameters (Table 1) used to generate the synthetic data. When the objective function is assessed for its dependence on the specimen aspect ratio, one may observe that the objective function is smoother for all parameters assessed in all models with the aspect ratio of w/h = 1. In all instances the objective function for a single parameter is well-defined with no significant local minima and constant gradients, indicating that single-parameter optimization should converge for all models.

OPTIMIZATION TECHNIQUE AND RESULTS

The inverse analysis technique adopted in this work estimates soil properties by formulating an optimization problem in which the soil parameters are iteratively changed to reduce the objective function defined in Eq. (1). Optimization is performed using the sequential quadratic programming algorithm embedded in the FMINCON solver of MATLAB, where Python is used as an interface between MATLAB and Plaxis 2D, as illustrated in Figure 6. Convergence is defined when the step size or first-order optimality was less than a specified tolerance (1×10^{-6} in this work). This inverse analysis technique has been extensively used within geotechnical engineering, including the work of Calvello and Finno (2004) and Rechea et al. (2004).

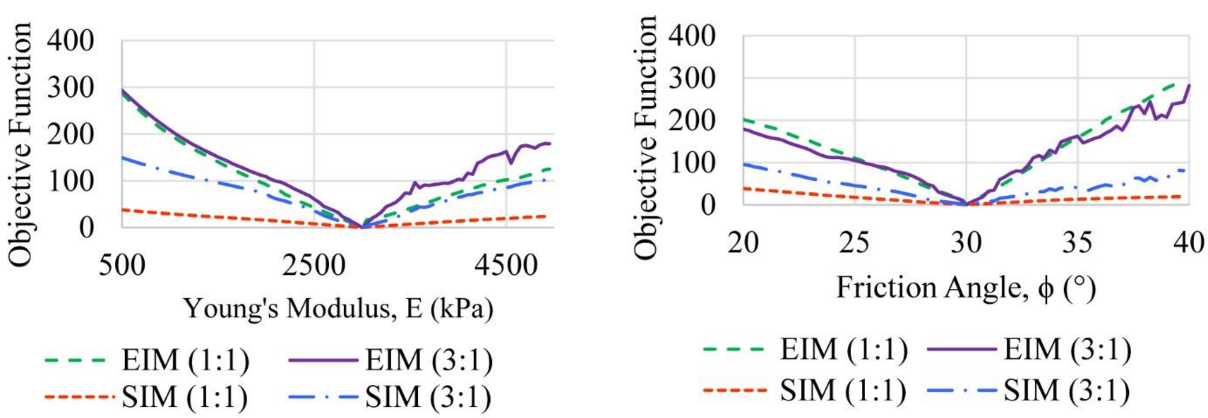

Figure 5. (a) Young's Modulus. (b) Friction Angle.

In order to test the ability of the different models to complete an optimization, and thus investigate the optimal aspect ratio (mode of deformation) and types of measurements, three types of analysis are assessed: single-parameter, two-parameter, and three-parameter optimization. In single-parameter optimization, each model's performance is assessed for its ability to estimate Young's modulus and the friction angle individually (i.e., holding all other parameters fixed at the true values). The models are then tested for their ability to estimate these two parameters simultaneously. Finally, cohesion was also included in the optimization to form a three-parameter optimization problem. As shown in Figure 6, a starting guess is required in each analysis. Many starting guesses were used in this work in order to determine trends, but only results for the starting guesses shown in Table 1 are presented in this paper. All three starting guesses in the table are used to investigate single-parameter optimization. Starting Guess #1 is used for the two-parameter optimization, whereas Starting Guess #2 and Starting Guess #3 are used for the three-parameter optimization.

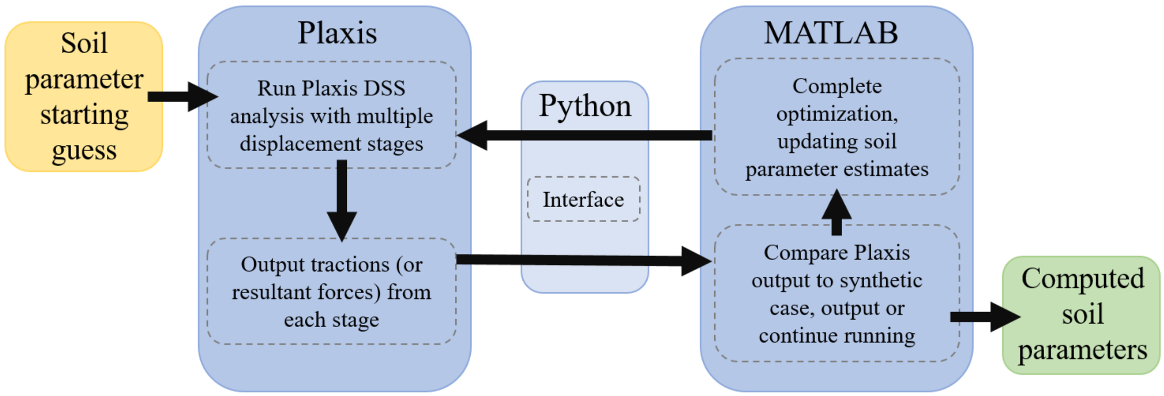

Figure 6. Optimization Procedure.

Single-parameter Optimization: For the case of single-parameter optimization, all models converge back to the true (synthetic) parameters for the three different starting guesses shown in Table 1. This finding is consistent with the character of the objective function observed in Figure 7 and suggests, if only in principle, that a single DSS test can identify Young's modulus, friction angle, or cohesion when only one parameter is unknown.

Two-parameter Optimization: Two-parameter optimization is completed for the simultaneous determination of Young's modulus and friction angle. For varying starting guesses of the parameters, the models presented consistent behavior. The normalized error of the optimization procedure is presented in Figure 7 for Starting Guess #1 (Table 1), as this is representative of the behavior for all starting guesses investigated. The normalized errors for all models are calculated as the absolute value of the difference between the true and estimated (converged) parameters, which is then divided by the true parameter. In all cases the Young's modulus converged to the true parameter, and the error can be deemed negligible. The friction angle converged to a value close to the true parameter in the case of the EIM but not the case of the SIM, suggesting that the estimation of soil parameters can benefit from feeding more information, in this case the variation of stresses, into the inverse analysis.

The better convergence of Young's modulus defies the expectation from parameter sensitivity analysis, where friction angle has a higher CSS value. This response can be attributed to the fact that plasticity is activated along specimen top boundary only when shear deformation is relatively large. In contrast, elasticity influences the entire loading course, as previously detailed in the section entitled "Simulation of DSS Test." In other words, the estimation of Young's modulus benefits from a richer set of observations, one obtained over the whole history of deformation. The effect of the aspect ratio for the EIM remains inconclusive, while for the SIM the aspect ratio of w/h = 1 presents a better recovery of the true parameters.

Three-parameter Optimization: The three-parameter optimization is completed by simultaneously optimizing Young's modulus, cohesion and friction angle. Again, multiple inputs are tested and in no case do all three parameters converge to the synthetic case. Two cases are selected as representative of the results: Starting Guesses #2 and #3. As seen by the normalized error plots in Figure 8, the convergence is heavily dependent on the starting guesses (Table 1). In all cases, the cohesion does not converge back to the true parameter. However, the EIM gives an accurate estimate of the friction angle for Starting Guess #3. Similar to two-parameter optimization, the effect of the specimen aspect ratio is inconclusive aside from the better recovery of the true parameters for the SIM for w/h = 1 with a Starting Guess #3.

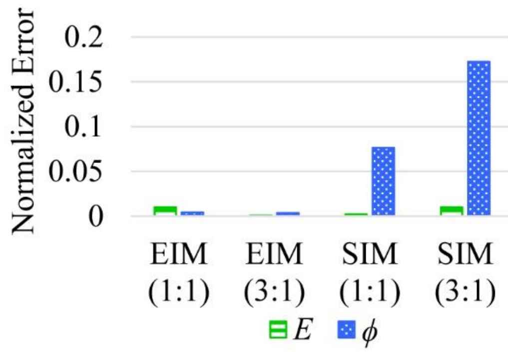

Figure 7. Normalized error for two-parameter optimization.

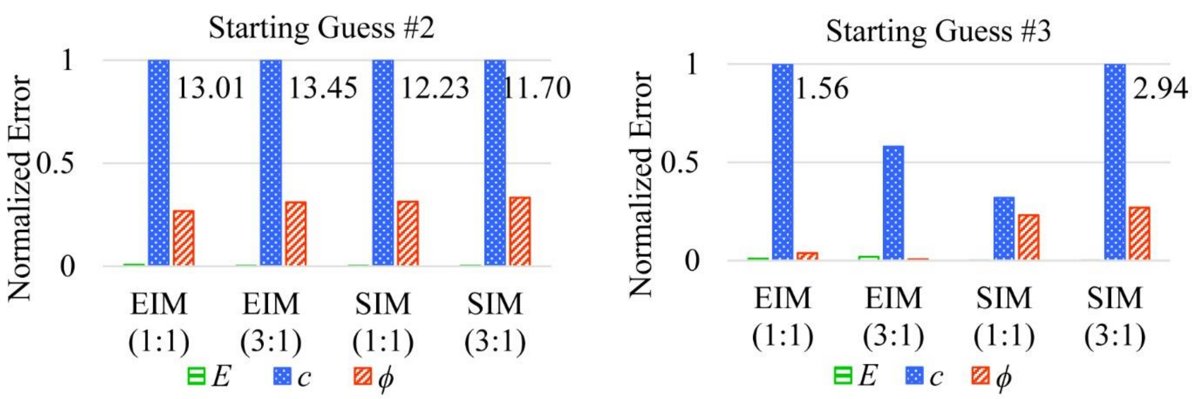

Figure 8. Normalized error for three-parameter optimization (the numbers in the plot show the values of variables that are outside the range of axis).

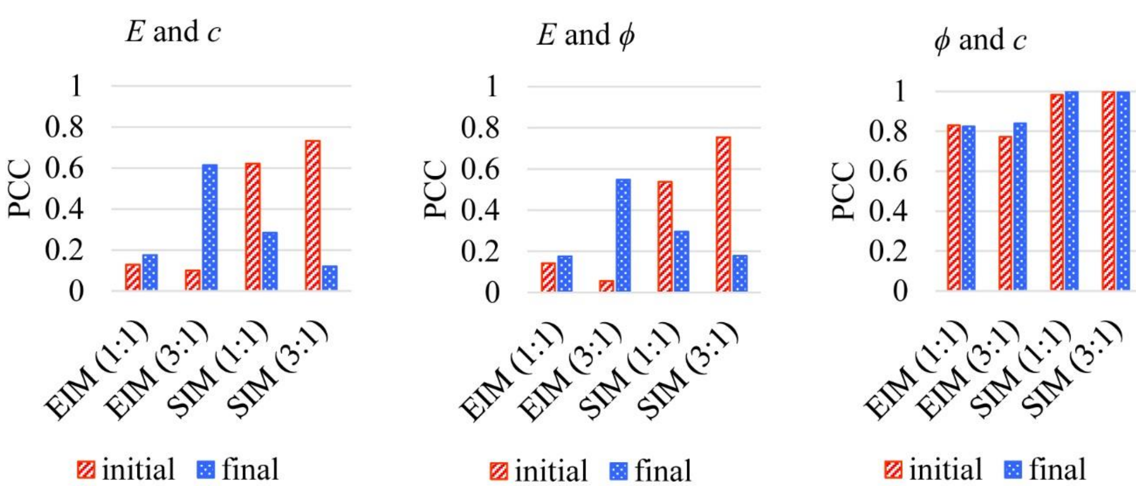

Figure 9. Parameter correlation coefficient for three-parameter optimization.

The PCC is calculated for both the input parameters and the converged parameters for

Starting Guess #2, as shown in Figure 9. It can be seen that the convergence of these parameters is heavily dependent on the PCCs. Due to the high relative correlation of friction angle and cohesion when compared to the other PCCs, the optimization is not able to accurately estimate these parameters simultaneously.

CONCLUSION

It is demonstrated in this paper that the treatment of a DSS test as a boundary value problem rather than the traditional elemental test can potentially lead to the more accurate determination of soil parameters. These differing treatments are referred to as the "extended interpretation model" (EIM) and "standard interpretation model" (SIM), respectively. For the cases considered, the analysis of the parameter correlation coefficients (PCCs) indicates that the EIM performs better than the SIM. Additional evidence for the superiority of the EIM is observed when two parameters are simultaneously optimized: the EIM recovers the synthetic (true) parameters more accurately than the SIM. In particular, the EIM correctly outputs the parameters for the synthetic case, whereas the SIM gives an error of approximately 8-20% for the friction angle.

Expectations prior to undertaking this study were that the specimen aspect ratio would impact the optimization through the alternation of the mode of deformation. The analysis of the PCCs indicates that there is a slight difference between the aspect ratios considered (w/h = 1 and 3). However, aspect ratio had no apparent impact on the accuracy of the parameters determined through the inverse analysis, at least for the particular cases investigated here. The objective function is impacted greatly by the specimen aspect ratio. This may ultimately imply that one aspect ratio is preferable to another, although this remains unproven from the analyses conducted.

For the case of single-parameter optimization, all models were repeatedly capable of converging from the initial guesses to the synthetic (true) parameters. The optimization for three parameters presented a convergence issue regardless of the models used. It was identified that the high correlation between the friction angle and cohesion could be limiting the optimization. Moreover, the performance of simultaneously estimating three parameters was highly dependent on the input parameters. In order to assess this impact, future work will look into including multiple starting guesses in combination with local (gradient-based) minimization as a means of removing the dependence on input parameters, as well as consider alternative optimization techniques.

The analysis presented in this paper can be considered a first step in the quantitative evaluation of laboratory and *in situ* testing configurations that allow for the most accurate resolution of the fundamental material properties of interest. Serving the purpose of proving the concept, we have introduced multiple simplifications (e.g., only frictionless interfaces between the soil and DSS box are considered, and the soil is assumed to have zero dilation). More complicated scenarios will be pursued in future work, where the influence of other factors on the inverse analysis and estimated soil properties will be explored.

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