Efficient Resource Distribution by Adaptive Inter-agent Spacing in Multi-agent Systems

Tamzidul Mina[1], Maliha Hossain[2], Jee Hwan Park[1] and Byung-Cheol Min[3]

***Abstract*— In multi-agent systems, limited resources must be shared by individuals during missions to maximize the group utility of the system in the field. In this paper, we present a generalized adaptive self-organization process for multi-agent systems featuring fast and efficient distribution of a consumable and refillable on-board resource throughout the group. An adaptive inter-agent spacing (AIS) controller based on individual resource levels is proposed that spaces out high resource bearing agents throughout the group including the group boundary extrema, and allows low resource bearing agents to adaptively occupy the in-between spaces receiving resource from the high resource bearing agents without over-crowding. Experimental results for cases with and without the proposed AIS controller validate faster convergence of individual resource levels to the group mean resource level using the proposed AIS controller. The generalized approach of the self-organizing process allows flexibility in adapting the proposed AIS controller for various multi-agent applications.**

I. INTRODUCTION

Multi-agent systems have a rich potential for application in monitoring and surveillance [1], exploration [2], search and rescue [3] etc. In many situations, agents are required to share/distribute certain on-board resources with/to other agents in the group. In the robotics arena, such on-board resources have been defined as consumable and refillable resources that robots use during any task execution [4].

A multi-agent system of vacuum cleaning robots is considered as an example. Each robot uses on-board battery energy to move around the vacuuming area and fills up its dust storage with absorbed dust. For this application, the on-board energy and the available dust storage space for each robot can be considered its on-board resource [4]. Individuals may collect different amounts of dust and use different battery levels while vacuuming certain areas. Without any immediately available resource replenishing stations for the system, the maximum group potential of the multi-robot system for the task can only be utilized until the robot with the lowest available resource level exhausts its resource.

Designing physical mechanisms of sharing such individual internal resources with other agents in the group is a challenging problem, but so is the self-organization of the multi-agent system to ensure an efficient process. Numerous problems have been identified in literature on the topic, including delays in fully connected networks for coordination involving limited communication bandwidth [5], space limitations and interference with other agents in locally connected networks for internal resource sharing [6]. Over-crowding of zero/low resource bearing agents on high resource bearing agents can potentially reduce resource sharing/distribution efficiency [7]. Inept distribution of high resource bearing agents throughout the group may also result in a slow resource distribution process [8].

Energy sharing/distribution between agents to extend the working life of multi-robot groups have previously been proposed in [9], [10]. A recharging process of multi-agents in the field using a traveling tanker approach was proposed in [11]. Some relevant models for efficient resource distribution can also be found among disaster relief distribution models [12][13]. An optimal dispatching model of point-of-distribution locations, and effective models for relief distribution from such locations to victims were simulated with detailed performance analysis in [8]. Several such multi-agent organization models proposed in literature provide effective results in efficient resource distribution, but very few are able to adapt to changing circumstances in the field over time once initially deployed.

In this paper, we present a generalized adaptive self-organization process for a multi-agent system to ensure fast and efficient inter-agent resource sharing/distribution. An Adaptive Inter-agent Spacing (AIS) controller dependent on current individual resource levels is proposed that:

- ensures distribution of higher resource bearing agents throughout the group including group boundary extrema;
- prevents over-crowding of low resource bearing agents on high resource bearing agents;
- converges agents to an equilibrium inter-distance at group resource equilibrium.

The generalized setup also allows adaptation of the proposed AIS controller for disaster relief distribution, task allocation and various other applications in multi-agent systems.

II. PROBLEM SETUP

We consider N fully actuated point mass mobile agents randomly distributed on a planar surface with dynamics of the form:

$$\dot{q}_i = v_i \text{ and } \dot{v}_i = u_i, \qquad i \in \{1,2,..,N\} \tag{1}$$

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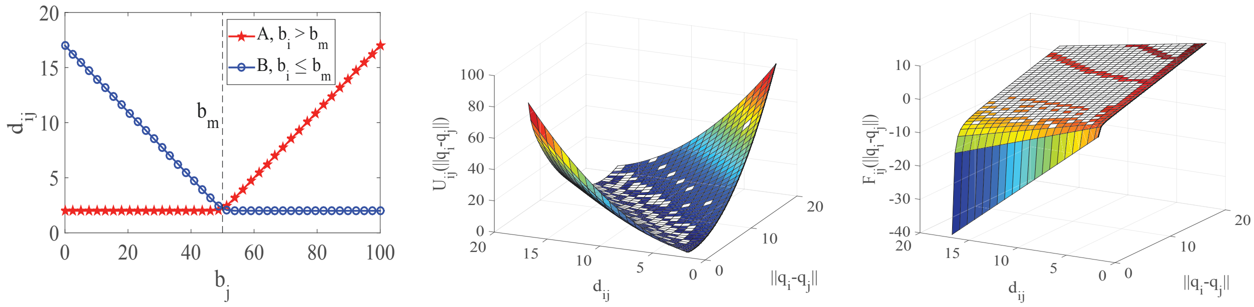
[1]Tamzidul Mina and Jee Hwan Park are with the SMART Lab, Department of Computer and Information Technology, Purdue University, and with the School of Mechanical Engineering, Purdue University, West Lafayette, IN 47907, USA tmina,park458@purdue.edu

[2]Maliha Hossain is with the School of Electrical and Computer Engineering, Purdue University, West Lafayette, IN 47907, USA mhossain@purdue.edu

[3]Byung-Cheol Min is with the SMART Lab, Department of Computer and Information Technology, Purdue University, West Lafayette, IN 47907, USA minb@purdue.edu



(a) Inter-agent spacing function plot for agent i in set A or set B, based on agent j resource level b_j with $\rho = 0.3$, $b_m = 50$ and $d_e = 2$.

(b) Interaction potential $U_{ij}(||q_{ij}||)$ vs. $||q_{ij}||$ plot for increasing inter-agent distance d_{ij} with $b_m = 50$, $\alpha = 0.5$ and $\beta = 0.5$.

(c) Scalar force $F_{ij}(||q_{ij}||)$ vs. $||q_{ij}||$ plot for increasing inter-agent distance d_{ij}, $b_m = 50$, $\alpha = 0.5$ and $\beta = 0.5$ without velocity damping.

Fig. 1: Inter-agent spacing is dictated by the group mean resource level b_m and the resulting classification of agent i of itself and its neighbor j into set A or set B. U_{ij} reaches a minimum along the derived $d_{ij} = q_{ij}$ line with zero force magnitude.

in which $q_i \in \mathbb{R}^p$, $v_i \in \mathbb{R}^p$, and $u_i \in \mathbb{R}^p$ denote the position, velocity and control input of robot i. We denote the on-board resource level on each agent to be distributed as $0 < b_i \leq 100$, $i \in \{1,2,..,N\}$, and the mean resource level of the group of N agents as b_m. At any given time, agent $i \in (\{A,B\} \ni A \cup B = \Lambda,\ A \cap B = \emptyset)$, where A is defined as the set of agents with $b_i > b_m$, and B as the set of agents with $b_i \leq b_m$.

Maintaining generality, we assume that each agent is able to transmit/receive resource b to/from other agents within a specified radius r_b, at a rate dependent on the number of agents within r_b due to space and bandwidth limitations. We model generalized resource sharing for agent i as:

$$\delta b_i = \gamma \frac{\sum \Delta b_{ij}}{n} \qquad i, j \in (\{1,2,..,N\} | q_{ij} \leq r_b) \tag{2}$$

where γ is a positive scalar control gain, $\Delta b_{ij} = b_j - b_i$, and n is its number of neighboring agents within radius r_b.

The objective is for all N agents to adaptively self-organize in the group based on current on-board resource levels b_i, $i \in \{1,2,..,N\}$ to converge to the group mean resource level b_m and equilibrium inter-distance d_e. We base our AIS controller framework on the multi-robot segregation model previously proposed by Santos *et al.* in [14] following initial work by Kumar *et al.* on the *differential potential concept* in [15].

III. Proposed Solution

A. Control Law

For efficient resource distribution in a group of N mobile agents governed by the dynamics model in (1), we base our proposed control law on the previously established multiple heterogeneous units segregation solution put forward in [**?**]:

$$u_i = -\sum_{j \neq i} \nabla_{q_i} U_{ij}(||q_{ij}||) - \sum_{j \neq i} (v_i - v_j) \tag{3}$$

where $U_{ij}(||q_{ij}||)$ is an artificial potential function defining the interaction between agents $i, j \in \{1,2,..,N\}$, dependent on the Euclidean norm of the vector $q_{ij} = q_i - q_j$. The second term acts as a velocity damping force such that, agents match their velocities to counter large variations in potential differences among agents that cause chaotic movements.

The artificial potential field U_{ij} is defined as a function of current and target relative distances between a pair of agents:

$$U_{ij}(||q_{ij}||) = \frac{1}{2}\alpha(||q_{ij}|| - d_{ij})^2 + \beta(\ln||q_{ij}|| + \frac{d_{ij}}{||q_{ij}||}) \tag{4}$$

where α and β are positive scalar control gains, and d_{ij} is a positive inter-agent target distance parameter described as a function of b_j and b_m in Section III-B. We assume that at the initial time instant $||q_{ij}|| \neq 0$, for which (4) is undefined; i.e. agents i, j do not collide. The corresponding interaction force can therefore be defined as $F_{ij}(||q_{ij}||) = \nabla U_{ij}(||q_{ij}||)$.

B. Adaptive Inter-agent Spacing for Efficient Resource Distribution

For efficient distribution of resource b, we design the inter-agent target distance parameter d_{ij} as a continuous function based on individual agent resource levels with the following properties:

- At system equilibrium, all agents converge to the mean resource level b_m and maintain the equilibrium agent inter-distance d_e.
- Agents in set A maintain inter-distances proportional to the other's resource level above b_m, such that the higher the amount of resource to be distributed by the pair for equilibrium, the higher the number of agents from set B that can occupy the created in-between space to receive the distributed resource.
- Agents in set B maintain inter-distances inversely proportional to the other's resource level equal to or below b_m, such that adequate spacing is available between agents in set B dependent on their resource levels to prevent crowding on set A agents around them.

We propose the following continuous function $d_{ij}(b_j)$ for agents classifying themselves into A or B and interacting with agents in Λ, derived from a smoothed approximation of the rectified linear unit (ReLU) function, satisfying the above set requirements:

$$d_{ij}(b_j) = \begin{cases} \rho\ \ln(1 + e^{b_j - b_m}) + d_e & \text{if } i \in A,\ j \in \Lambda \\ \rho\ \ln(1 + e^{b_m - b_j}) + d_e & \text{if } i \in B,\ j \in \Lambda \end{cases} \tag{5}$$

where ρ is a positive scalar control gain. Figure 1a illustrates the distance relationship between agents for the two cases for $\rho = 0.3$, $d_e = 2$ and $b_m = 50$. The corresponding potential function and scalar force plots are shown in Fig. 1b and 1c for $\alpha = 0.5$ and $\beta = 0.5$.

The significance of the proposed function $d_{ij}(b_j)$ is that it ensures that agents with large on-board resources for distribution spread out throughout the group without accumulating together. This is particularly important for efficient resource distribution in systems where the initial resource distribution is skewed on certain areas of the group. Furthermore, the design of the spacing between agents with lower resources ensures that no agent with resource level larger than the mean is over-crowded with agents with lower resource level agents at any given time. This allows fast and efficient resource transfer between agents within r_b when transfer limits are present dependent on the number of connecting agents.

C. Controller Analysis

To investigate the stability and the convergence of the multi-agent system to equilibrium inter-agent distance d_e using the proposed control law, we define the Lyapunov function as,

$$V(q,v) = U(q) + \frac{1}{2}v^{\mathrm{T}}v \tag{6}$$

where $q \in \mathbb{R}^{Np}$ and $v \in \mathbb{R}^{Np}$ are stacked position and velocity vectors of N robots in the system, and $U(q) : \mathbb{R}^{Np} \longrightarrow \mathbb{R}_{>0}$ is the collective potential energy of the system written as,

$$U(q) = \frac{1}{2}\sum_{i\in A}\sum_{j\neq i} U_{ij}(||q_{ij}||) + \frac{1}{2}\sum_{i\in B}\sum_{j\neq i} U_{ij}(||q_{ij}||) \tag{7}$$

where the first term represents the total potential for pairs of agents $i \in A,\ j \in \Lambda$ and the second term for pairs of agents in $i \in B,\ j \in \Lambda$. The collective dynamics of the system is written as,

$$\dot{q} = v, \qquad \dot{v} = -\nabla U(q) - \hat{L}(q)v \tag{8}$$

where $\hat{L}(q)$ is the Kronecker product of the fully connected system's graph Laplacian $L(q)$ and identity matrix I_p.

The proposed controller setup is similar to the multi-agent segregation controller proposed by Santos *et al.* in [14]. In the segregation controller, each agent utilizes a binary classifier on its interacting agent to check if it is in the same partition to define its interaction with other agents, as opposed to the controller proposed in this paper, where each agent classifies itself into either set A or B to define its interaction with other agents in the group.

Therefore, with the total system energy defined as (6), collective dynamics as (8), resource sharing and control defined as (2) and (3) respectively for the proposed system, we refer to the controller analysis shown by Santos *et al.* in [14], and conclude that the multi-agent system with agents exclusively in sets A or B asymptotically converges to the set equilibrium inter-agent distance d_e without any collision where the system's collective potential reaches a local minimum. We also conclude that the velocity of each agent is bounded and all velocities match at equilibrium.

IV. VALIDATION

A. Setup

To validate our proposed concept, we show that a group of agents randomly distributed on a flat planar surface each having a different resource level $0 < b_i \leq 100$, successfully converges to b_m faster with AIS rather than without. We define the base case for our comparison as all agents rendezvous with agent inter-distance d_e and share resources with all neighbors within r_b. With such a setup, where all aspects of the experiment are held constant except for the proposed d_{ij} function, we isolate the effectiveness of the proposed AIS control law on system performance.

The validation process is set up with four specific scenarios to study the performance and effectiveness of our proposed resource distribution method in comparison to the defined base method. The scenarios include N randomly distributed agents on the xy plane initially having:

- Scenario 1 ($S1$): Left-skewed
- Scenario 2 ($S2$): Bi-modal
- Scenario 3 ($S3$): Normal
- Scenario 4 ($S4$): Random

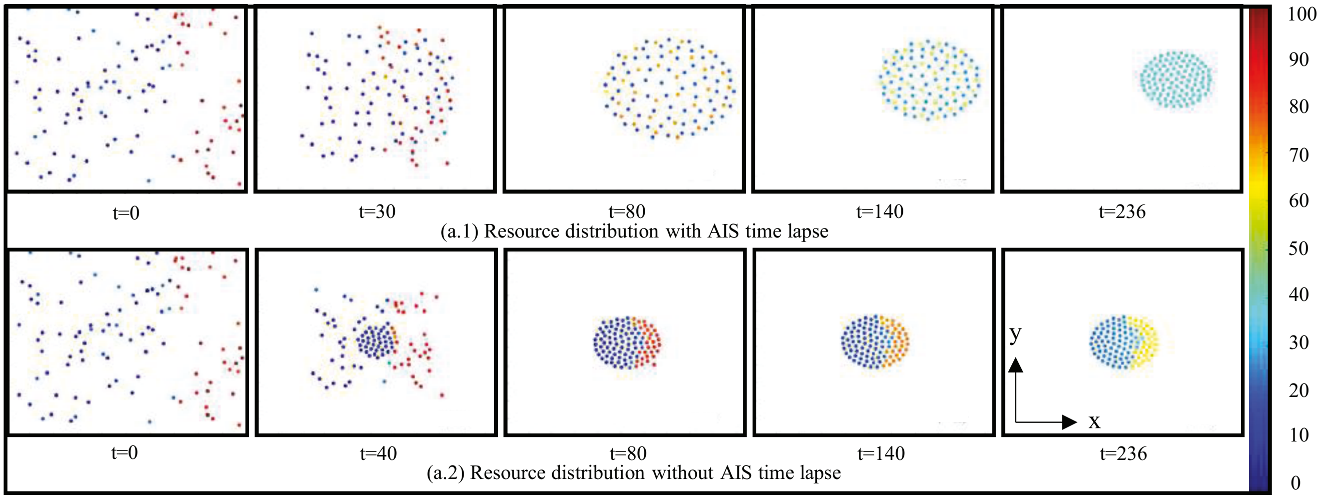
distributed resource levels on agents along the planar x-axis, each being compared to its corresponding base approach solution. The left-skewed initial distribution is designed such that all agents on the right one-quarter area of the x-y plane have $80 \leq b \leq 100$, while all others have $0 < b \leq 20$. The bi-modal initial distribution is designed such that all agents on the left one-quarter and right one-quarter area of the x-y plane have $80 \leq b \leq 100$ and while all others have $0 < b \leq 20$. The normal initial distribution is designed such that all agents on the middle one-quarter area of the x-y plane have $80 \leq b \leq 100$, while all others have $0 < b \leq 20$. Lastly, the random initial distribution allows all agents in the x-y plane to have a random resource level $5 < b \leq 100$.

Each set of experiments consist of $N = 100$ robots, each with zero initial velocity, with exaggerated parameters $d_e = 2$, $r_b = 3$, $\alpha = 0.8$, $\beta = 0.4$, $\gamma = 0.1$ and $\rho = 0.8$ for brevity of the simulations.

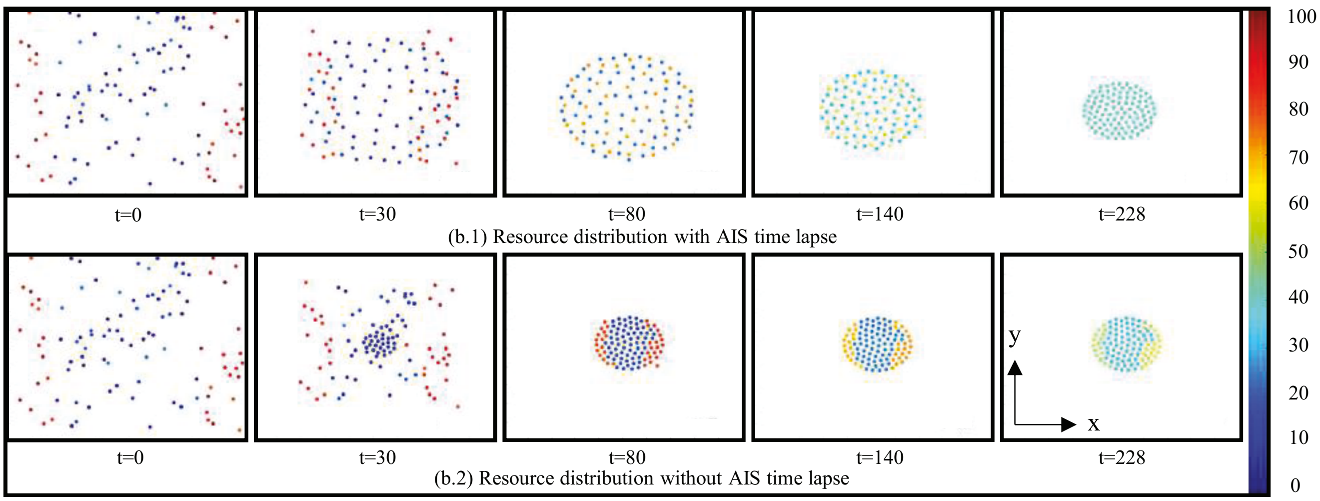
B. Experiment

Figure 2a, 2b and 2c each illustrate sets of simulation time step sequences using the proposed method and its corresponding base method (a.1, a.2), (b.1, b.2) and (c.1, c.2) for scenarios $S1$, $S2$ and $S3$ respectively.

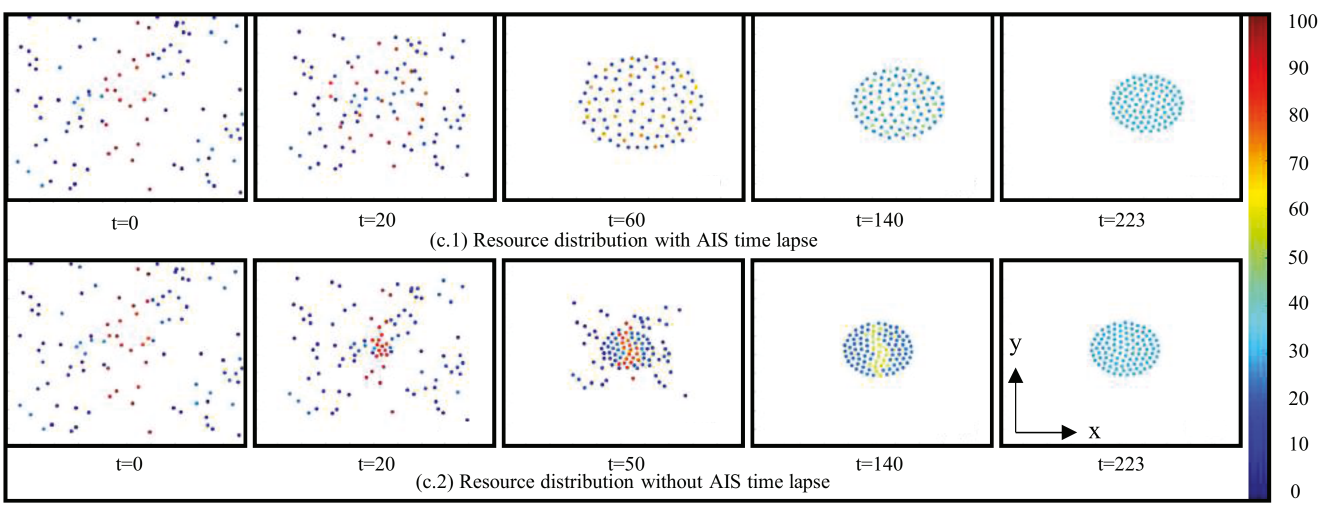
Following the d_{ij} inter-distance formulation of the proposed AIS controller, agents with high resource levels from the right in $S1$ Fig. 2a (a.1), start to diffuse in to the rest of the group at $t = 30$ and 80. Similar behavior was observed in $S2$ Fig. 2b (b.1) and $S3$ Fig. 2c (c.1) with the proposed method, where agents with high resource levels from the ends and center respectively, diffuse throughout the group at $t = 30$, 80 and $t = 20$, 60. With high resource level agents placed throughout the group, the resource distribution following (2) reaches the group mean resource level equilibrium $b_m =$ 36.29, 40.72, 29.83 at $t = 236$, 228 and 223 time steps for $S1$, $S2$ and $S3$ respectively.



(a) S1 - Left skewed initial resource distribution along x-axis

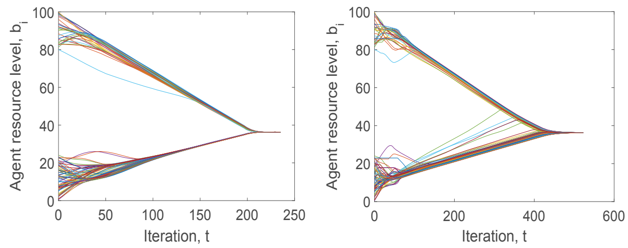


(b) S2 – Bi-modal initial resource distribution along x-axis



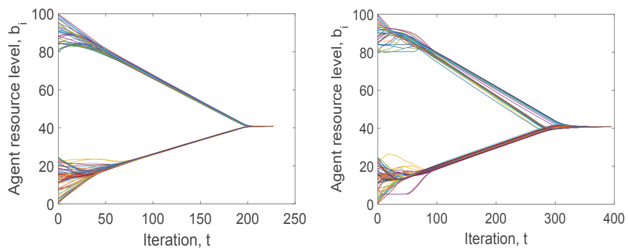
(c) S3 – Normal initial resource distribution along x-axis

Fig. 2: Time lapse comparison of scenarios (a) $S1$: Left-skewed, (b) $S2$: Bi-modal, (c) $S3$: Normal initial resource distributions using the proposed AIS control law for resource distribution and their corresponding base methods for $N = 100$ agents. Resource level of individual agents $0 < b_i \leq 100$ represented as color bar on the right.



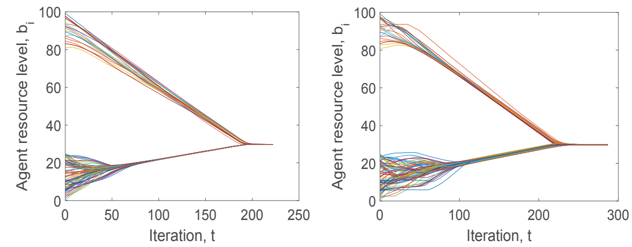
(a) $S1$: Left-skewed - convergence with AIS. (b) $S1$: Left-skewed - convergence without AIS.

Fig. 3: $S1$: Left-skewed initial resource distribution - individual agent resource level convergence vs. iteration time.



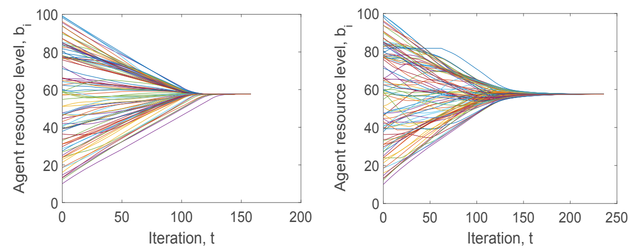
(a) $S2$: Bi-modal - convergence with AIS. (b) $S2$: Bi-modal - convergence without AIS.

Fig. 4: $S2$: Bi-modal initial resource distribution - individual agent resource level convergence vs. iteration time.



(a) $S3$: Normal - convergence with AIS. (b) $S3$: Normal - convergence without AIS.

Fig. 5: $S3$: Normal initial resource distribution - individual agent resource level convergence vs. iteration time.



(a) $S4$: Random - convergence with AIS. (b) $S4$: Random - convergence without AIS.

Fig. 6: $S4$: Random initial resource distribution - individual agent resource level convergence vs. iteration time.

In contrast, using the base method in each of the scenarios $S1$ Fig. 2a (a.2), $S2$ Fig. 2b (b.2) and $S3$ Fig. 2c (c.2) respectively, all agents rendezvous to a minimal potential state at inter-distance d_e regardless of each other's resource level. In $S1$ Fig. 2a (a.2), due to the left-skewed initial resource distribution, agents with higher resource levels clump together on the right. Similarly, agents with higher resource levels clump together on the ends for a bi-modal and center for a normal initial distribution in $S2$ Fig. 2b (b.2) and $S3$ Fig. 2c (c.2) respectively. At $t = 236$, 228 and 223 for $S1$, $S2$ and $S3$, while the system has already reached the group mean resource level equilibrium using AIS, the base case was yet to reach equilibrium as seen from the simulation time step in each of the scenarios. The adaptive placement of high resource bearing agents throughout the group using AIS ensured fast and efficient resource equilibrium attainment over the base method.

The convergence of resource levels of each agent to the group mean resource level for each of the described scenarios $S1$, $S2$ and $S3$ using the proposed AIS controller and the base method approaches are illustrated in Fig. 3, 4 and 5 respectively. The system converges to the mean group resource level at $t = 524$, 396 and 288 without using AIS for each of the $S1$, $S2$ and $S3$ scenarios. Therefore, 55%, 42% and 23% performance improvements in equilibrium attainment convergence time were obtained using the proposed AIS controller over the base method in $S1$, $S2$ and $S3$ respectively.

The experiment was repeated for $S4$ with a random initial resource distribution $5 \leq b \leq 100$ for all agents. The system converged to $b_m = 57.57$ at $t = 159$ using AIS and at $t = 237$ without using AIS, showing 33% performance improvement in equilibrium attainment time over the base method, proving the effectiveness of the formulated d_{ij} function in efficient resource distribution in a group. Fig 6 shows the resource level convergence of each agent to b_m using the proposed and base methods for $S4$.

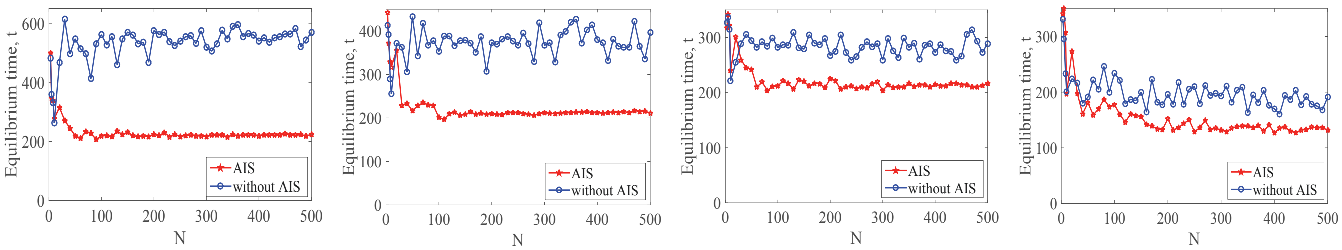
C. Discussion - Robustness & Scalability

The significance of the proposed method in this paper is that it adaptively distributes agents with higher individual resource levels throughout the entire group regardless of initial group resource distribution. The process is continuous over time and thus the adapting inter-distance is always maintained throughout the group based on inter-agent resource levels at all time instances until equilibrium.

1) Observation 1: The AIS based resource distribution method out-performed the base method in all scenarios $S1$, $S2$, $S3$ and $S4$ for $N = 100$ agents. Therefore, we conclude that the proposed resource distribution method is robust to extreme initial group resource distributions.

2) Observation 2: At any given time, each individual classifies itself and others as either having resource level above or below the group mean to determine d_{ij}; i.e. the proposed method performs independent of the number of agents in the entire group at any given time. Hence, AIS resource distribution is robust to dynamic changes in the number of agents in the group during the resource distribution process assuming no agent is incapacitated or faulty.

Observation 2 also supports scalability of the proposed method. To further investigate, we demonstrate the scalability by repeating scenarios $S1$, $S2$, $S3$ and $S4$ for $N = 3$, 5, 10 and then with increments of 10 up to $N = 500$, and plotting the



(a) $S1$: Skewed initial distribution scalability.
(b) $S2$: Bi-modal initial distribution scalability.
(c) $S3$: Normal initial distribution scalability.
(d) $S4$: Random initial distribution scalability.

Fig. 7: System resource convergence time to b_m with and without using AIS with increasing N for $S1$, $S2$, $S3$, and $S4$.

time steps required for the system to reach the resource equilibrium for each case. For each N in the scalability plots, we consider the average time step of 5 simulation runs. Figure 7 shows the experiment results obtained with and without the proposed AIS controller. The proposed AIS based resource distribution method consistently yielded shorter convergence times for increasing N. The most effective difference is seen in initially skewed resource distributions and the closest difference is seen with the random initial resource distribution.

In two dimensional euclidean space, the highest density lattice arrangement of circles is the hexagonal packing [16]. In most cases for $N \leq 6$, the base method of without using AIS performed better since all robots converged together into a small enough group for fast resource distribution; whereas with AIS, larger inter-distances between robots meant larger traveling times until resource equilibrium attainment. Therefore, we conclude that the proposed AIS resource distribution method is effective for $N > 6$.

The scalability plots with increasing N show a diminishing convergence time for small N and then gradually reaches a steady state. This is a consequence of using artificial potential functions to model the dynamics of the system. With larger N, a larger amount of resource transfer occurs resulting in longer convergence times. However, the total potential energy of the system is higher as well, with each agent experiencing larger attraction and repulsion forces resulting in faster movements in the environment. This contributes to smaller convergence times. As a result of cancelling effects of the two phenomenons, a steady convergence time is observed in all scenarios shown in Fig. 7 regardless of N.

A video of the simulations is available for reference at https://youtu.be/xDd-S5ZRPG4.

V. Conclusion

In this paper, an adaptive inter-agent spacing control law based on resource levels of robot pairs is proposed as an efficient self-organization process for resource distribution in multi-agent systems. Experimental results validate improved resource distribution performance with AIS for any initial resource distribution cases.

The improved performance by the proposed AIS controller complements self-sustainability of various multi-agent systems on autonomous missions such as search and rescue, surveillance, monitoring etc. The current system assumes that agents are aware of the position and velocity of all other agents in the group, which might not be applicable in a real life scenario. Further work on adapting the AIS controller for a decentralized approach is currently underway.

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