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Bremsstrahlung emission and collisional damping rate for Langmuir waves

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Abstract
The emission of electrostatic Langmuir waves by collisional process, termed electrostatic bremsstrahlung emission, and the collisional damping of Langmuir waves, which can be considered as the inverse electrostatic bremsstrahlung process, are rigorously discussed. Some inaccuracies in the previous formalisms are also corrected. It is shown that the improved formulae in the case of Maxwellian particle distributions are given in forms where they satisfy Kirchhoff's law in the balanced form.

Keywords: kinetic theory of plasmas, plasma emission, Langmuir waves, weak turbulence theory

(Some figures may appear in colour only in the online journal)

1. Introduction

Bremsstrahlung and its inverse process are common in diverse physical situations, which at first glance may not appear to be related. One application relates to the laser-metal-plasma interaction problem, where a high intensity laser incident on the metal target first liquifies and vaporizes the material, forming a partially ionized high-density plasma cloud surrounding the target. The laser ablated plasma blob absorbs subsequently incident laser energy by collisional damping and inverse bremsstrahlung, thus preventing further laser-metal interaction [1, 2].

Bremsstrahlung radiation, which is known in the solar and astrophysics community as free-free emission, plays a pivotal role in solar x-ray emission. It is well known that thermal electrons emit soft x-rays, which contribute to the thermal emission spectra of the quiet Sun. The hard x-ray emissions associated with the flare acceleration region, on the other hand, are generated by non-thermal electron distribution. On the observational side there have been immense developments of various aspects of solar x-ray emissions thanks to many spacecraft missions [3].

On the theoretical side, standard emission formula available in the literature [4, 5] are still being employed in order to fit or interpret spacecraft data or in modeling laser-metal-plasma interactions. The implications of using a more rigorous approach based upon advanced plasma kinetic theory has not yet been considered in the literature. In the presence of plasma, bremsstrahlung emissions become part of the many particle interactions rather than binary collisions. Consequently, proper description may require a self-consistent theory.

The purpose of the present paper is to lay down the first layer of a theoretical foundation towards a more rigorous theory of plasma bremsstrahlung and its inverse process, which requires electromagnetic treatment. In the present paper, we instead resort to an electrostatic problem, and consider the emission of an electrostatic plasma wave from a binary collisional process—that is, electrostatic bremsstrahlung emission, and absorption by the same wave from binary collisional damping. The electrostatic bremsstrahlung and collisional damping, which can be equivalently termed the inverse bremsstrahlung absorption rate, have been formulated on the basis of rigorous plasma kinetic theory [6] and further discussed in a couple of subsequent papers [7, 8]. However, it turns out that the previously published

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formalisms are not entirely accurate. Consequently, the purpose of the present paper is to revisit the basic formulae discussed in our earlier papers, and to improve the mathematical expressions. While our theoretical approach is limited in that we work under electrostatic approximation, a future more complete theoretical framework can be built upon this work.

It should be noted, however, that the classical theory of bremsstrahlung, which assumes collisions between point-like charged particles, has been improved along a different direction, namely the problem of bremsstrahlung radiation involving charged objects with internal structures. This area of research is active and ongoing, and the subject matter is known as polarization bremsstrahlung [9–11]. When the bremsstrahlung from plasma is discussed in this context [10], the effects of Debye shielding of ions are taken into account rather than the full collective effects. In [12], the collective effects of plasma particles on bremsstrahlung are discussed. This reference mainly addresses the plasma effects on the bremsstrahlung emission of electrostatic modes in plasma, which is similar to the present work, as well as our previously published papers [6–8]. However, the approaches taken in [12] versus [6–8] are not entirely equivalent. Whereas [6–8] start from generalized nonlinear kinetic theory that includes non-eigenmode contributions and systematically deduce terms that correspond to the collective damping term and bremsstrahlung emission term, [12] begins the discussion with the consideration of the dipole moment associated with the emission of longitudinal waves as in the traditional discussions of bremsstrahlung. The bremsstrahlung emission formula is then modified by considering the collective effects. The equivalence of the two approaches has not been established yet. In the present paper, we rely on the formalism developed in [6–8].

The organization of the present paper is as follows: in section 2 we initiate the discussion by laying out the scientific backdrop of the present work. Then in section 3 we present the detailed mathematical formalism. Section 4 presents numerical analysis and the findings of the present paper are summarized and the ramification discussed in section 5.

2. Further scientific background

The standard expression for the electron-ion collision frequency ν_{ei} found in the literature, known as the *Spitzer* formula, is given by

$$\nu_{ei} = \frac{4}{3}\left(\frac{2\pi}{m_e}\right)^{1/2}\frac{Z^2 e^4 n_i \ln\Lambda}{T^{3/2}}, \tag{1}$$

where m_e, e, n_i, and T stand for electron mass, unit electric charge, ion density, and plasma temperature, respectively and $\ln\Lambda$ represents the Coulomb logarithm. The above expression is widely used in the literature. For instance, in the laser-plasma interaction problem [1, 2], the laser absorption is associated with the spatial damping rate k_i, where the collision frequency ν_{ei} is incorporated into the spatial attenuation rate of the incident laser light,

$$k_i = \frac{2\omega}{c}\,\mathrm{Im}\left(1 - \frac{\omega_p^2}{\omega^2(1 + i\nu_{ei})}\right)^{1/2}. \tag{2}$$

Equation (2) corresponds to equation (15) of [1]. except that we have used the collision frequency ν_{ei} instead of the electron-ion collision time τ_{ei}. In equation (2), ω is the frequency of the laser light, c stands for the speed of light in vacuum, $\omega_p = (4\pi n_0 e^2/m_e)^{1/2}$ is the plasma frequency—n_0 being the ambient density –, and ν_{ei} is the Spitzer collisional frequency defined in equation (1). Consequently, in the standard approach, the collisional damping rate of laser radiation is associated with ν_{ei}. In the solar flares context, the model that treats x-ray generating electrons by considering only the collisional dynamics is known as the thick target model, e.g., [13]. However, there are some discussions in the literature that emphasize the importance of Langmuir wave dynamics in that context, e.g., [14–18]. In such an approach, the collisional damping of Langmuir waves is also treated in terms of the Spitzer formula (1). This can be seen, for instance, in equation (3) of [14] and in equation (2) of [15].

However, there are reasons to believe that formulae (1) is not completely accurate and can be improved. This is because the customary collisional damping frequency ν_{ei} implies that any plasma wave, regardless of its frequency (wavelength) or polarization, will be damped at the *same* rate, which depends only on macroscopic parameters, T, n_e, and n_i. This is conceptually too simplistic. The damping of a plasma wave, such as a Langmuir wave for example, is governed by the microscopic wave-particle interaction, satisfying the resonance condition, $\omega - \mathbf{k}\cdot\mathbf{v} = 0$. For collision-free plasmas, such an interaction leads to Landau damping, which depends on the velocity-space distribution function and its slope in velocity space, $\partial f_e(\mathbf{v})/\partial\mathbf{v}$. Here, $f_e(\mathbf{v})$ denotes the electron velocity distribution function. For plasmas subject to collisions, the electron distribution $f_e(\mathbf{v})$ will be modified by electron-ion collisions, but other than that, the damping of a plasma wave in collisional plasma is no different than Landau damping in a fundamental sense. The only difference is that for collisional plasmas the particle distribution function is modified by collisions. The correct collisional damping formula must therefore reflect the collisional modification of the distribution, hence, the formula must be inherently microscopic and also nonlinear in its character.

In [6], the correct formulation of the collisional damping rate for an electrostatic plasma wave, including that of a Langmuir wave, was carried out under the generalized weak turbulence theory. According to [6] the formal (and correct) expression for the collisional damping rate for a Langmuir wave is given by

$$\gamma_L^{\text{collision}}(\mathbf{k}) = -\sum_a \frac{4e_a^2}{\pi}\frac{1}{\epsilon'(\mathbf{k},\omega_\mathbf{k}^L)}\int d\mathbf{k}'\int d\mathbf{v} \times \frac{1}{k'^2|\epsilon(\mathbf{k}',\mathbf{k}'\cdot\mathbf{v})|^2} \times \mathrm{Im}\,\mathscr{P}\,\frac{2\{\chi^{(2)}(\mathbf{k}',\mathbf{k}'\cdot\mathbf{v}|\mathbf{k}-\mathbf{k}',\omega_\mathbf{k}^L-\mathbf{k}'\cdot\mathbf{v})\}^2}{\epsilon(\mathbf{k}-\mathbf{k}',\omega_\mathbf{k}^L-\mathbf{k}'\cdot\mathbf{v})} f_a(\mathbf{v}), \tag{3}$$

where $\epsilon(\mathbf{k}, \omega)$ stands for the linear dielectric function, $\epsilon'(\mathbf{k}, \omega) = \partial \mathrm{Re}\, \epsilon(\mathbf{k}, \omega)/\partial\omega$ being its derivative with respect to angular frequency, and $\chi^{(2)}(\mathbf{k}_1, \omega_1|\mathbf{k}_2, \omega_2)$ represents the (second-order) nonlinear susceptibility. Explicit expressions for these quantities appear in section 3, where the theoretical formulation is presented in more detail. In equation (3) the quantity $f_a(\mathbf{v})$ denotes the one-particle distribution function normalized to the ambient number density ($\int d\mathbf{v} f_a(\mathbf{v}) = n_a$). The notation $\mathscr{P}$ denotes the principal value. Notice that the above expression indicates that the collisional damping rate not only depends on the wave frequency, but is also a function of the wave vector, which contrasts with the standard Spitzer formula (1). Moreover, it involves the nonlinear response of the plasma. The above formula was subsequently analyzed by Tigik *et al* [7], and it was found that the widely used Spitzer formula for the collisional damping rate greatly over-estimates the actual damping rate. In that paper, however, the approximations employed for the linear and nonlinear plasma response functions were not entirely satisfactory, which left room for further improvements. The present paper will address this issue and complement the work by Tigik *et al* [7].

Continuing with further scientific background, in the standard literature, whether in the context of the laser-plasma interaction problem or in solar/astrophysics, the bremsstrahlung effects are discussed on the basis of the textbook thermal bremsstrahlung emissivity [4, 5],

$$\kappa_\nu = \frac{1}{3\pi^2}\left(\frac{\pi}{6}\right)^{1/2} \frac{Z^2 e^6}{(\pi c)^3 m_e^2}\left(\frac{m_e}{T}\right)^{1/2} g_{ff}\, n_i n_e \exp\left(-\frac{h\nu}{T}\right), \tag{4}$$

where Ze is the atomic number of the ion, g_{ff} is known as the Gaunt factor, which is calculated from quantum mechanics but is set equal to unity in the classical limit, and h is the Planck constant. For plasmas with many particle collective interactions, the standard approach of calculating the bremsstrahlung emissivity may be insufficient. In the textbook approach, one considers a single electron encountering an ion, which leads to the dipole radiation. Then, the radiation emissivity is superposed over many electrons and ions, resulting in formula (4). In plasmas, on the other hand, first, the single particle encounter between an electron and ion is not common, since the Debye shielding makes such a binary encounter rare. Second, when the radiation emission takes place, it is reabsorbed by other electrons and re-emitted. In the process, the particle distribution function will be modified. This collective behavior is encapsulated in the formula derived in [6], which for a Langmuir wave, is given by

$$\begin{aligned} P_L(\mathbf{k}) = \sum_{a,b} \frac{48 e_a^2 e_b^2}{\pi} \frac{1}{|\epsilon'(\mathbf{k}, \omega_{\mathbf{k}}^L)|^2} \int d\mathbf{k}' \int d\mathbf{v} \int d\mathbf{v}' \\ \times \frac{|\chi^{(2)}[\mathbf{k}', \mathbf{k} - \mathbf{k}' | \mathbf{k}' \cdot \mathbf{v}, \omega_{\mathbf{k}}^L - \mathbf{k}' \cdot \mathbf{v}]|^2}{k'^2 |\mathbf{k} - \mathbf{k}'|^2 |\epsilon(\mathbf{k}', \mathbf{k}' \cdot \mathbf{v})|^2 |\epsilon[\mathbf{k} - \mathbf{k}', \omega_{\mathbf{k}}^L - \mathbf{k}' \cdot \mathbf{v}]|^2} \\ \times \delta[\omega_{\mathbf{k}}^L - \mathbf{k}' \cdot \mathbf{v} - (\mathbf{k} - \mathbf{k}') \cdot \mathbf{v}'] f_a(\mathbf{v}) f_b(\mathbf{v}'). \end{aligned} \tag{5}$$

Note that the above formula describes the bremsstrahlung emission of Langmuir waves, not the actual radiation. In order to extend the formalism to radiation emission one must generalize the theory of [6] to fully electromagnetic formalism, which is yet to be done.

The correct plasma bremsstrahlung theory is not very well developed in the literature. There are some early works, see, e.g., [19–23]. The approach taken in [6], if generalized to fully electromagnetic formalism, could in principle, extend to all these prior works. In the meantime, [8] made use of the electrostatic bremsstrahlung theory of [6] as well as the collisional damping theory of [7] in order to address the origin of suprathermal electron population, which is presumed to exist at the base of the coronal exosphere in the Sun. The presence of suprathermal particles is an important ingredient in the so-called velocity filtration model of coronal heating theory [24], but its origin is not clear. In this regard, [8] made a potentially important contribution. Nonetheless, the electrostatic bremsstrahlung emission formula in [8], which is an approximate version of the formal result (5), is again, not entirely satisfactory, as with the case with collisional damping rate discussed in [7]. For this reason, modified and improved versions of the formulae are called for. The above mentioned issues have motivated the present paper. In the rest of this paper, we present the derivation of improved collisional damping rate and electrostatic bremsstrahlung emission formula for Langmuir waves.

3. Theoretical analysis

We now discuss the use of the improved approximations for the collisional damping rate (3) and electrostatic bremsstrahlung emission formula (5). These quantities are two ingredients that contribute to the balance of transfer equation for Langmuir waves, which also includes the mechanisms of spontaneous emission and Landau damping,

$$\begin{aligned} \left.\frac{dI_L(\mathbf{k})}{dt}\right|_{\mathrm{coll}} = 2[\gamma_L^{\mathrm{collision}}(\mathbf{k}) + \gamma_L^{\mathrm{Landau}}(\mathbf{k})] I_L(\mathbf{k}) \\ + [P_L(\mathbf{k}) + S_L(\mathbf{k})], \end{aligned} \tag{6}$$

where $I_L(\mathbf{k})$ represents the electrostatic field energy density associated with the Langmuir waves, and $\gamma_L^{\mathrm{Landau}}(\mathbf{k})$ and $S_L(\mathbf{k})$ stand for the Landau damping rate and spontaneous emission term, respectively,

$$\begin{aligned} \gamma_L^{\mathrm{Landau}}(\mathbf{k}) = \frac{\pi \omega_{\mathbf{k}}^L \omega_{pe}^2}{2k^2} \int d\mathbf{v}\, \delta(\omega_{pe} - \mathbf{k} \cdot \mathbf{v}) \mathbf{k} \cdot \frac{\partial f_e}{\partial \mathbf{v}}, \\ S_L(\mathbf{k}) = \frac{\omega_{pe}^2 n e^2}{k^2} \int d\mathbf{v}\, \delta(\omega_{pe} - \mathbf{k} \cdot \mathbf{v}) f_e. \end{aligned} \tag{7}$$

However, before obtaining the steady state solution of equation (6), one can investigate what would be the Langmuir spectrum if the collisional processes could be considered as the dominant mechanisms, by the use of the following approximation,

$$\left.\frac{dI_L^{\mathrm{collision}}(\mathbf{k})}{dt}\right|_{\mathrm{coll}} = 2\gamma_L^{\mathrm{collision}}(\mathbf{k}) I_L(\mathbf{k}) + P_L(\mathbf{k}). \tag{8}$$

If we consider the steady state, equation (8) leads to the so-called Kirchhoff's law in the balanced form, namely,

$$I_L^{\mathrm{collision}}(\mathbf{k}) = -\frac{P(\mathbf{k})}{2\gamma_L^{\mathrm{collision}}(\mathbf{k})}, \tag{9}$$

where the resulting expression for $I_L(\mathbf{k})$ should be well defined in the mathematical sense.

To begin the detailed discussion of this approximation, let us first make use of the following definitions relevant for the linear dielectric response function [25]:

$$\epsilon(\mathbf{k},\omega) = 1 - \sum_a \frac{\omega_{pa}^2}{k^2 v_{Ta}^2} Z'\left(\frac{\omega}{k v_{Ta}}\right),$$
$$\epsilon'(\mathbf{k},\omega_\mathbf{k}^L) = \frac{\partial\,\mathrm{Re}\,\epsilon(\mathbf{k},\omega_\mathbf{k}^L)}{\partial\omega_\mathbf{k}^L} \approx \frac{2}{\omega_\mathbf{k}^L},$$
$$\omega_\mathbf{k}^L = \omega_{pe}\left(1+\frac{3}{4}\frac{k^2 v_{Te}^2}{\omega_{pe}^2}\right), \tag{10}$$

where $v_{Ta} = (2T_a/m_a)^{1/2}$ is the thermal speed of particles of species a, $Z(\zeta) = \pi^{-1/2}\int_{-\infty}^{\infty}(x-\zeta)^{-1}e^{-x^2}dx + \pi\sigma i$ ($\sigma = 0$ for Im $\zeta > 0$, $\sigma = 1$ for Im $\zeta = 0$, and $\sigma = 2$ for Im $\zeta < 0$), is the plasma dispersion function, and $\omega_\mathbf{k}^L$ represents the long-wavelength approximation for the angular frequency of Langmuir waves. The quantity of relevance is the second-order nonlinear susceptibility with appropriate arguments [25],

$$\begin{aligned}
&\chi^{(2)}(\mathbf{k}',\mathbf{k}'\cdot\mathbf{v}|\mathbf{k}-\mathbf{k}',\omega_\mathbf{k}^L-\mathbf{k}'\cdot\mathbf{v})\\
&= -\frac{i}{2}\sum_a\frac{e_a}{m_a}\frac{\omega_{pa}^2}{k\,k'|\mathbf{k}-\mathbf{k}'|}\\
&\times\int d\mathbf{u}\,\frac{1}{\omega_\mathbf{k}^L-\mathbf{k}\cdot\mathbf{u}}\left[\mathbf{k}'\cdot\frac{\partial}{\partial\mathbf{u}}\right.\\
&\times\left(\frac{(\mathbf{k}-\mathbf{k}')\cdot\partial F_a/\partial\mathbf{u}}{\omega_\mathbf{k}^L-\mathbf{k}'\cdot\mathbf{v}-(\mathbf{k}-\mathbf{k}')\cdot\mathbf{u}}\right)\\
&\left.+(\mathbf{k}-\mathbf{k}')\cdot\frac{\partial}{\partial\mathbf{u}}\left(\frac{\mathbf{k}'\cdot\partial F_a/\partial\mathbf{u}}{\mathbf{k}'\cdot(\mathbf{v}-\mathbf{u})}\right)\right],
\end{aligned} \tag{11}$$

which upon partial integrations can be alternatively written as

$$\begin{aligned}
&\chi^{(2)}(\mathbf{k}',\mathbf{k}'\cdot\mathbf{v}|\mathbf{k}-\mathbf{k}',\omega_\mathbf{k}^L-\mathbf{k}'\cdot\mathbf{v})\\
&= -\frac{i}{2}\sum_a\frac{e_a}{m_a}\frac{\omega_{pa}^2}{k\,k'|\mathbf{k}-\mathbf{k}'|}\\
&\times\int d\mathbf{u}\,\frac{F_a(\mathbf{u})}{\mathbf{k}'\cdot(\mathbf{v}-\mathbf{u})[\omega_\mathbf{k}^L-\mathbf{k}'\cdot\mathbf{v}-(\mathbf{k}-\mathbf{k}')\cdot\mathbf{u}](\omega_\mathbf{k}^L-\mathbf{k}\cdot\mathbf{u})}\\
&\times\left(\frac{k'^2[\mathbf{k}\cdot(\mathbf{k}-\mathbf{k}')]}{\mathbf{k}'\cdot(\mathbf{v}-\mathbf{u})}\right.\\
&\left.+\frac{|\mathbf{k}-\mathbf{k}'|^2(\mathbf{k}\cdot\mathbf{k}')}{\omega_\mathbf{k}^L-\mathbf{k}'\cdot\mathbf{v}-(\mathbf{k}-\mathbf{k}')\cdot\mathbf{u}}+\frac{k^2[\mathbf{k}'\cdot(\mathbf{k}-\mathbf{k}')]}{\omega_\mathbf{k}^L-\mathbf{k}\cdot\mathbf{u}}\right),
\end{aligned} \tag{12}$$

Note that F_a represents the distribution function normalized to unity ($\int d\mathbf{v}F_a = 1$) such that it is defined by $f_a = n_a F_a$. Upon making use of the Langmuir wave property, namely, $\omega_\mathbf{k}^L \gg \mathbf{k}\cdot\mathbf{v}$, we may approximate (12) by ignoring the second and third terms within the large parenthesis in the last line on the right-hand side,

$$\begin{aligned}
&\chi_a^{(2)}(\mathbf{k}',\mathbf{k}'\cdot\mathbf{v}|\mathbf{k}-\mathbf{k}',\omega_\mathbf{k}^L-\mathbf{k}'\cdot\mathbf{v})\\
&= -\frac{i}{2}\sum_a\frac{e_a}{m_a}\frac{\omega_{pa}^2}{(\omega_\mathbf{k}^L)^2}\frac{k'[\mathbf{k}\cdot(\mathbf{k}-\mathbf{k}')]}{k|\mathbf{k}-\mathbf{k}'|}\int d\mathbf{u}\\
&\times\frac{F_a(\mathbf{u})}{[\mathbf{k}'\cdot(\mathbf{v}-\mathbf{u})]^2},
\end{aligned} \tag{13}$$

Henceforth we are interested in thermal distribution, hence, the quantity of interest is

$$\begin{aligned}
\int d\mathbf{u}\,\frac{F_a(\mathbf{u})}{[\mathbf{k}'\cdot(\mathbf{v}-\mathbf{u})]^2} &= \frac{1}{\pi^{3/2}v_{Ta}^3}\int d\mathbf{u}\,\frac{e^{-u^2/v_{Ta}^2}}{[\mathbf{k}'\cdot(\mathbf{v}-\mathbf{u})]^2}\\
&= \frac{1}{k'^2 v_{Ta}^2}Z'\left(\frac{\mathbf{k}'\cdot\mathbf{v}}{k' v_{Ta}}\right),
\end{aligned} \tag{14}$$

which leads to

$$\begin{aligned}
&\chi_a^{(2)}(\mathbf{k}',\mathbf{k}'\cdot\mathbf{v}|\mathbf{k}-\mathbf{k}',\omega_\mathbf{k}^L-\mathbf{k}'\cdot\mathbf{v})\\
&= -\frac{i}{4}\sum_a\frac{e_a}{T_a}\frac{\omega_{pa}^2}{(\omega_\mathbf{k}^L)^2}\frac{[\mathbf{k}\cdot(\mathbf{k}-\mathbf{k}')]}{k\,k'|\mathbf{k}-\mathbf{k}'|}Z'\left(\frac{\mathbf{k}'\cdot\mathbf{v}}{k'v_{Ta}}\right).
\end{aligned} \tag{15}$$

Inserting equations (15) to (3) and (5) we obtain

$$\begin{aligned}
\gamma_L^{\mathrm{collision}}(\mathbf{k}) &= \sum_a\frac{e_a^2}{2\pi}\frac{1}{\epsilon'(\mathbf{k},\omega_\mathbf{k}^L)}\int d\mathbf{k}'\int d\mathbf{v}\\
&\times\frac{1}{k'^2|\epsilon(\mathbf{k}',\mathbf{k}'\cdot\mathbf{v})|^2}\\
&\times\frac{\mathrm{Im}\,\epsilon^*(\mathbf{k}-\mathbf{k}',\omega_\mathbf{k}^L-\mathbf{k}'\cdot\mathbf{v})}{|\epsilon(\mathbf{k}-\mathbf{k}',\omega_\mathbf{k}^L-\mathbf{k}'\cdot\mathbf{v})|^2}\\
&\times\left[\sum_b\frac{e_b}{T_b}\frac{\omega_{pb}^2}{(\omega_\mathbf{k}^L)^2}\frac{[\mathbf{k}\cdot(\mathbf{k}-\mathbf{k}')]}{kk'|\mathbf{k}-\mathbf{k}'|}Z'\left(\frac{\mathbf{k}'\cdot\mathbf{v}}{k'v_{Tb}}\right)\right]^2 f_a(\mathbf{v}),
\end{aligned} \tag{16}$$

$$\begin{aligned}
P_L(\mathbf{k}) &= \sum_{a,b}\frac{3e_a^2e_b^2}{\pi}\frac{1}{|\epsilon'(\mathbf{k},\omega_\mathbf{k}^L)|^2}\int d\mathbf{k}'\int d\mathbf{v}\int d\mathbf{v}'\\
&\times\frac{1}{k'^2|\mathbf{k}-\mathbf{k}'|^2|\epsilon(\mathbf{k}',\mathbf{k}'\cdot\mathbf{v})|^2|\epsilon(\mathbf{k}-\mathbf{k}',\omega_\mathbf{k}^L-\mathbf{k}'\cdot\mathbf{v})|^2}\\
&\times\left[\sum_c\frac{e_c}{T_c}\frac{\omega_{pc}^2}{(\omega_\mathbf{k}^L)^2}\frac{[\mathbf{k}\cdot(\mathbf{k}-\mathbf{k}')]}{k\,k'|\mathbf{k}-\mathbf{k}'|}Z'\left(\frac{\mathbf{k}'\cdot\mathbf{v}}{k'v_{Tc}}\right)\right]^2\\
&\times\delta[\omega_\mathbf{k}^L-\mathbf{k}'\cdot\mathbf{v}-(\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}']f_a(\mathbf{v})f_b(\mathbf{v}')
\end{aligned} \tag{17}$$

At this point, let us note that the most important contribution to the velocity integral $\int d\mathbf{v}$ in both equations (16) and (17) comes from those regions where the distribution function $f_a(\mathbf{v})$ takes on the maximum value, that is, in the vicinity of $\mathbf{v} \sim 0$. This means that we may simply approximate the situation by allowing the velocity distribution $f_a(\mathbf{v})$ to be replaced by a cold delta function,

$$f_a(\mathbf{v}) \sim n\delta(\mathbf{v}), \tag{18}$$

where $n = n_e = n_i$. This reduces equations (16) and (17) to

$$\gamma_L^{\text{collision}}(\mathbf{k}) = \sum_a \frac{n e_a^2}{2\pi} \frac{1}{\epsilon'(\mathbf{k}, \omega_{\mathbf{k}}^L)} \int d\mathbf{k}' \times \frac{1}{k'^2 |\epsilon(\mathbf{k}', 0)|^2} \frac{\mathrm{Im}\, \epsilon^*(\mathbf{k}-\mathbf{k}', \omega_{\mathbf{k}}^L)}{|\epsilon(\mathbf{k}-\mathbf{k}', \omega_{\mathbf{k}}^L)|^2} \times \left(\sum_b \frac{e_b}{T_b} \frac{\omega_{pb}^2}{(\omega_{\mathbf{k}}^L)^2} \frac{[\mathbf{k}\cdot(\mathbf{k}-\mathbf{k}')]}{k k' |\mathbf{k}-\mathbf{k}'|^2} Z'(0) \right)^2, \tag{19}$$

$$P_L(\mathbf{k}) = \sum_{a,b} \frac{3n e_a^2 e_b^2}{\pi} \frac{1}{|\epsilon'(\mathbf{k}, \omega_{\mathbf{k}}^L)|^2} \times \int d\mathbf{k}' \frac{1}{k'^2 |\mathbf{k}-\mathbf{k}'|^2 |\epsilon(\mathbf{k}', 0)|^2 |\epsilon(\mathbf{k}-\mathbf{k}', \omega_{\mathbf{k}}^L)|^2} \times \left| \sum_c \frac{e_c}{T_c} \frac{\omega_{pc}^2}{(\omega_{\mathbf{k}}^L)^2} \frac{[\mathbf{k}\cdot(\mathbf{k}-\mathbf{k}')]}{k k' |\mathbf{k}-\mathbf{k}'|} Z'(0) \right|^2 \times \int d\mathbf{v}'\, \delta[\omega_{\mathbf{k}}^L - (\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}'] f_b(\mathbf{v}'). \tag{20}$$

Upon making use of

$$Z'(0) \approx -2, \quad \epsilon(\mathbf{k}', 0) \approx 1 + \frac{2\omega_{pe}^2}{k'^2 v_{Te}^2}\left(1 + \frac{T_e}{T_i}\right), \tag{21}$$

and trivially redefining the dummy integral variable, $\mathbf{k} - \mathbf{k}' \to \mathbf{k}'$, we may proceed to the next step. In doing so, we also ignore the terms of order m_e/m_i or lower. Note that approximation (21) assumes finite T_i. Further, we also make use of various expressions and properties of the linear dielectric response function,

$$\mathrm{Im}\, \epsilon^*(\mathbf{k}', \omega_{\mathbf{k}}^L) = -\pi^{1/2} \frac{2\omega_{pe}^2}{k'^2 v_{Te}^2} \frac{\omega_{\mathbf{k}}^L}{k' v_{Te}} \exp\left(-\frac{(\omega_{\mathbf{k}}^L)^2}{k'^2 v_{Te}^2}\right). \tag{22}$$

Finally, we assume the Gaussian distribution for $f_b(\mathbf{v}')$,

$$f_b(\mathbf{v}') = \frac{n}{\pi^{3/2} v_{Tb}^3} \exp\left(-\frac{v'^2}{v_{Tb}^2}\right). \tag{23}$$

This leads to the desired expressions,

$$\gamma_L^{\text{collision}}(\mathbf{k}) = -\frac{g\omega_{pe}}{4\pi^{5/2}} \frac{\omega_{pe}^4}{k^2 v_{Te}^2 (\omega_{\mathbf{k}}^L)^2} Q(\mathbf{k}), \tag{24}$$

$$P_L(\mathbf{k}) = \frac{6g\omega_{pe}}{\pi^{7/2}} \frac{T_e}{8\pi} \frac{\omega_{pe}^4}{k^2 v_{Te}^2 (\omega_{\mathbf{k}}^L)^2} Q(\mathbf{k}), \tag{25}$$

where the plasma parameter g is defined by

$$g = \frac{1}{n\lambda_{De}^3} = \frac{(2\pi)^{3/2} n^{1/2} e^3}{T_e^{3/2}}. \tag{26}$$

Note that both $\gamma_L^{\text{coll}}(\mathbf{k})$ and $P_L(\mathbf{k})$ are defined via a common function $Q(\mathbf{k})$, given by

$$Q(\mathbf{k}) = \int d\mathbf{k}' \frac{v_{Te}^2}{\omega_{pe}^2} \frac{1}{D(\mathbf{k}-\mathbf{k}')} \times \frac{1}{|\epsilon(\mathbf{k}', \omega_{\mathbf{k}}^L)|^2} \frac{(\mathbf{k}\cdot\mathbf{k}')^2}{k'^4} \exp\left(-\frac{(\omega_{\mathbf{k}}^L)^2}{k'^2 v_{Te}^2}\right),$$

$$D(\mathbf{k}-\mathbf{k}') = \left| 1 + \frac{T_e}{T_i} + \frac{(\mathbf{k}-\mathbf{k}')^2 v_{Te}^2}{2\omega_{pe}^2} \right|^2,$$

$$\epsilon(\mathbf{k}', \omega_{\mathbf{k}}^L) = 1 - \frac{\omega_{pe}^2}{k'^2 v_{Te}^2} Z'\left(\frac{\omega_{\mathbf{k}}^L}{k' v_{Te}}\right). \tag{27}$$

Equations (24) and (25) constitute the major findings of the present paper, and these formulae supersede the previous approximate formulae adopted in [7] and [8]. With the use of these expressions, the approximated form of the Langmuir wave transfer equation (8) is obtained, where the collective effects have been ignored, which leads to a steady-state level of the collisionally excited Langmuir wave spectrum which is given as follows,

$$I_L^{\text{collision}}(\mathbf{k}) = -\frac{P_L(\mathbf{k})}{2\gamma_L^{\text{collision}}(\mathbf{k})} = \frac{3T_e}{2\pi^2}. \tag{28}$$

Of course, as mentioned previously, the steady state Langmuir fluctuation spectrum should include the collective counterparts, and is given by the steady state solution of equation (6),

$$I_L(\mathbf{k}) = -\frac{S_L(\mathbf{k}) + P_L(\mathbf{k})}{2\gamma_L^{\text{Landau}}(\mathbf{k}) + 2\gamma_L^{\text{collision}}(\mathbf{k})}, \tag{29}$$

where

$$\gamma_L^{\text{Landau}}(\mathbf{k}) = -\frac{\pi^{1/2}\omega_{pe}^2(\omega_{\mathbf{k}}^L)^2}{k^3 v_{Te}^3} \exp\left(-\frac{(\omega_{\mathbf{k}}^L)^2}{k^2 v_{Te}^2}\right),$$

$$S_L(\mathbf{k}) = \frac{T_e}{2\pi^{3/2}} \frac{\omega_{pe}^4}{k^3 v_{Te}^3} \exp\left(-\frac{(\omega_{\mathbf{k}}^L)^2}{k^2 v_{Te}^2}\right). \tag{30}$$

It is interesting to note that if we ignore contributions from collisional processes altogether, then the steady state spectrum Langmuir wave fluctuations that arise solely from the collective processes is given by

$$I_L^{\text{collective}}(\mathbf{k}) = -\frac{S_L(\mathbf{k})}{2\gamma_L^{\text{Landau}}(\mathbf{k})} = \frac{T_e}{4\pi^2} \frac{\omega_{pe}^2}{(\omega_{\mathbf{k}}^L)^2}. \tag{31}$$

4. Numerical analysis

In the previous section we have shown that the improved expressions for the collisional damping rate and the electrostatic bremsstrahlung emission formula for Langmuir waves are defined by a common spectral function $Q(\mathbf{k})$ such that in the steady state, assuming that we may ignore the contributions from collective processes, one may obtain a constant Langmuir fluctuation intensity. In the present section, we use numerical integration to examine the spectral profile for the function $Q(\mathbf{k})$ in detail. We also apply the same method to analyze the spectra of the collisional damping rate and electrostatic bremsstrahlung emission in the improved approximation in order to show how the new expressions alter the form of the steady-state spectrum of Langmuir fluctuations when collisional effects are included in addition to collective effects.

For the numerical analysis, equations (24) and (25) are written in terms of suitable dimensionless quantities

$$\mathbf{q} = \frac{\mathbf{k} v_e}{\omega_{pe}} = \mathbf{k}\sqrt{2}\,\lambda_{De}, \quad z_{\mathbf{q}}^L = \frac{\omega_{\mathbf{k}}^L}{\omega_{pe}} \simeq 1 + \frac{3}{4}q^2. \tag{32}$$

The resulting dimensionless equations for the collisional damping and electrostatic bremsstrahlung are respectively given by

$$\gamma_L^{\mathrm{CD}}(\mathbf{q}) = -\frac{4}{\sqrt{\pi}}\frac{g}{(z_{\mathbf{q}}^L)^2}\frac{1}{q^2}\int d\mathbf{q}'\frac{(\mathbf{q}\cdot\mathbf{q}')^2}{q'^5\,|\epsilon(\mathbf{q}',z_{\mathbf{q}}^L)|^2} \times \exp\left(-\frac{(z_{\mathbf{q}}^L)^2}{q'^2}\right) \times \left[2\left(1+\frac{T_e}{T_i}\right)+|\mathbf{q}-\mathbf{q}'|^2\right]^{-2}, \tag{33}$$

$$P_L(\mathbf{q}) = \frac{24}{\sqrt{\pi}}\frac{g^2}{(z_{\mathbf{q}}^L)^2}\frac{1}{q^2}\int d\mathbf{q}'\frac{(\mathbf{q}\cdot\mathbf{q}')^2}{q'^5\,|\epsilon(\mathbf{q}',z_{\mathbf{q}}^L)|^2}\exp\left(-\frac{(z_{\mathbf{q}}^L)^2}{q'^2}\right) \times \left[2\left(1+\frac{T_e}{T_i}\right)+|\mathbf{q}-\mathbf{q}'|^2\right]^{-2}, \tag{34}$$

where the plasma parameter is redefined as $g = 1/[2^{3/2}(4\pi)^2 n\lambda_{De}^3]$, and the dimensionless linear dielectric function has the following form

$$\epsilon(\mathbf{q}', z_{\mathbf{q}}^L) = 1 + \frac{2}{q'^2}\left[1 + \frac{z_{\mathbf{q}}^L}{q'}Z\left(\frac{z_{\mathbf{q}}^L}{q'}\right)\right]. \tag{35}$$

From equations (33) and (34), one can easily identify the normalized expression for the common spectral function

$$Q(\mathbf{q}) = \int d\mathbf{q}'\frac{(\mathbf{q}\cdot\mathbf{q}')^2}{q'^5\,|\epsilon(\mathbf{q}',z_{\mathbf{q}}^L)|^2}\exp\left(-\frac{(z_{\mathbf{q}}^L)^2}{q'^2}\right) \times \left[|\mathbf{q}-\mathbf{q}'|^2 + 2\left(1+\frac{T_e}{T_i}\right)\right]^{-2}. \tag{36}$$

Part of the integral in equation (36) can be carried out analytically by using spherical coordinates, assuming azimuthal symmetry. Then, after integrating over the angular variables, we obtain an equation that involves a single integral in q':

$$Q(\mathbf{q}) = \int_0^\infty \frac{dq'}{q'^3}\frac{1}{|\epsilon(\mathbf{q}',z_{\mathbf{q}}^L)|^2}\exp\left(-\frac{(z_{\mathbf{q}}^L)^2}{q'^2}\right) \times \left[\frac{2B^2-A^2}{B^2-A^2} - \frac{B}{A}\ln\left(\frac{A+B}{-A+B}\right)\right], \tag{37}$$

where A and B are given by

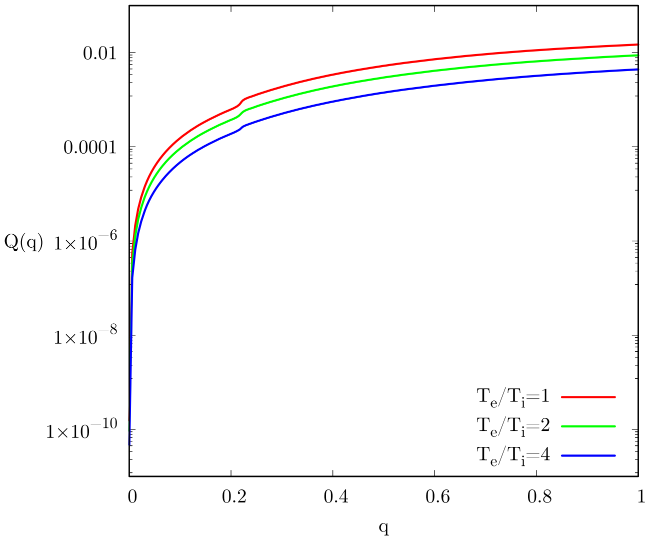

Figure 1. Common spectral function $Q(\mathbf{q})$ in logarithm scale, versus the normalized wave number q, for three different values of T_e/T_i.

$$A = -2qq'$$
$$B = 2\left(1+\frac{T_e}{T_i}\right) + q^2 + q'^2. \tag{38}$$

For the numerical integration, we wrote a Fortran code, in double precision. Equation (37) was then solved for three different values of the ratio between electron and ion temperatures $T_e/T_i = 1$, $T_e/T_i = 2$ and $T_e/T_i = 4$. In figure 1 we see that the spectral profile of $Q(\mathbf{q})$ has an inverse relation with T_e/T_i, with the most intense spectrum being the curve for $T_e/T_i = 1$. We also can notice a 'bump' right after $q = 0.2$. This is the region where both the collisional damping and the electrostatic bremsstrahlung suddenly change their behavior and start tending to zero, as can be seen in the two panels of figure 2.

Looking at equations (37) and (38) we notice that $Q(\mathbf{q})$, and coefficients A and B are exactly the same as the ones that appear in the normalized expression for the collisional damping rate of Langmuir waves in [7]. The main difference between the two expressions is in the quantities that multiply the integral, with the most important change being the square of the dispersion relation that now appears in the denominator. However, when it comes to the electrostatic bremsstrahlung equation, if we compare our expression with the non-normalized equation appearing in [8], we notice that the integral in the previous approximation is more complex because, in that occasion, we did not make the cold delta assumption (equation (18)). This assumption makes the electrostatic bremsstrahlung equation simpler and more suitable for numerical analysis.

This new approach also includes improvements in the numerical scheme. The code was rewritten using double precision and the convergence parameter of the subroutine that performs the numerical integration was finely adjusted in order to resolve the region around $q = 0.2$. This region is prone to numerical instabilities (see [7]) due to a sudden change in the spectrum, as can be seen in both panels of figure 2.

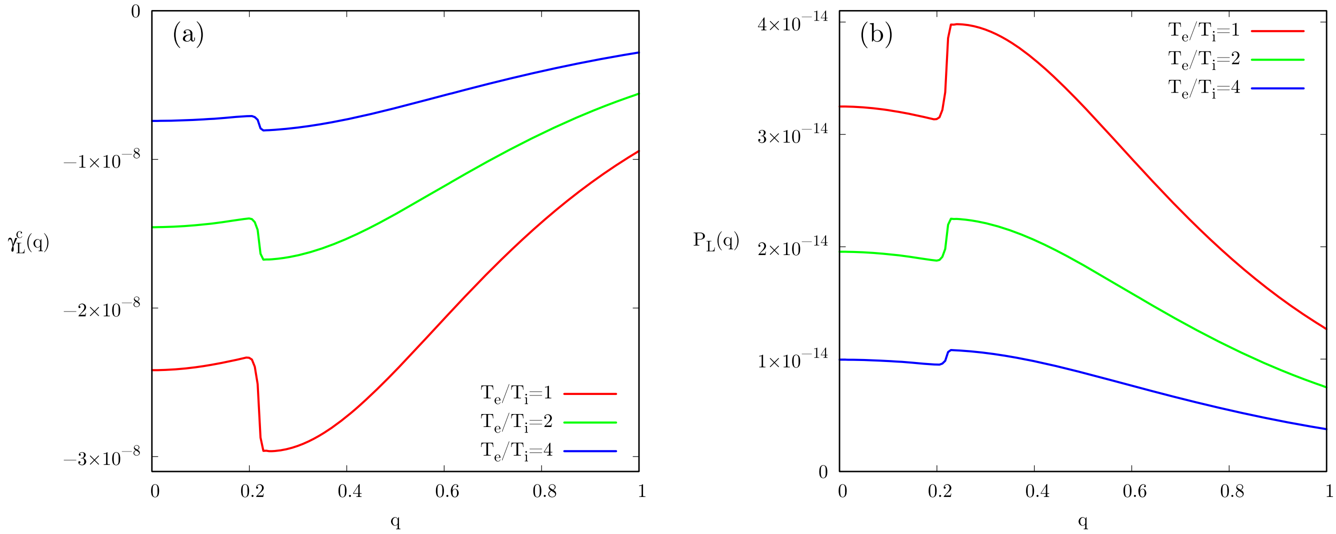

Figure 2. Solution of equations (33) and (34), for three different values of T_e/T_i, with plasma parameter $g = 1 \times 10^{-4}$. Panel (a) depicts the spectral behavior associated with the collisional damping rate, and panel (b) shows the electrostatic bremsstrahlung spectrum. It is important to emphasize here that the considerable difference in scale between both effects is due to the fact that the electrostatic bremsstrahlung is proportional to g^2, while the collisional damping rate is proportional to g.

With this slight change in the expression for the collisional damping and the new, simpler formulation of the electrostatic bremsstrahlung equation, we ended up showing that both effects are inversely related to each other, with the first one depicting an absorption process in the same wave number region that the latter emits. Such a relation becomes clear from the symmetry between the two panels in figure 2, where we also see that T_e/T_i, $\gamma_L(\mathbf{q})$ and $P_L(\mathbf{q})$ have the same behavior as dictated by the common spectral function $Q(\mathbf{q})$, which is expected.

The normalized equation for the steady-state of Langmuir waves is given by

$$\varepsilon_L(\mathbf{q}) = -\frac{S_L(\mathbf{q}) + P_L(\mathbf{q})}{\gamma_L^{\mathrm{LD}}(\mathbf{q}) + 2\gamma_L^{\mathrm{CD}}(\mathbf{q})}, \tag{39}$$

where $\varepsilon_L(\mathbf{q}) = \frac{(2\pi)^2 g}{m_e v_{Te}^2} I_L(\mathbf{k})$ is the normalized Langmuir wave intensity, and $S_L(\mathbf{q})$ and $\gamma_L^{\mathrm{LD}}(\mathbf{q})$ are the dimensionless forms of the spontaneous emission and Landau damping expressions, which are respectively given by

$$S_L(\mathbf{q}) = \sqrt{\pi}\,\frac{g}{q^3}\exp\left(-\frac{(z_{\mathbf{q}}^L)^2}{q^2}\right), \tag{40}$$

$$\gamma_L^{\mathrm{LD}}(\mathbf{q}) = -2\sqrt{\pi}\,\frac{(z_{\mathbf{q}}^L)^2}{q^3}\exp\left(-\frac{(z_{\mathbf{q}}^L)^2}{q^2}\right). \tag{41}$$

In figure 3, we see that the presence of collisional processes, in addition to collective effects, creates a plateau region for $q \lesssim 0.2$ that is approximately three times the value for the steady-state spectrum solved only for collective processes in the same region. For $q \gtrsim 0.2$, the two curves are exactly the same, which means that, for Langmuir waves, the action of collisional processes is limited to small values of wave number, or large wavelengths.

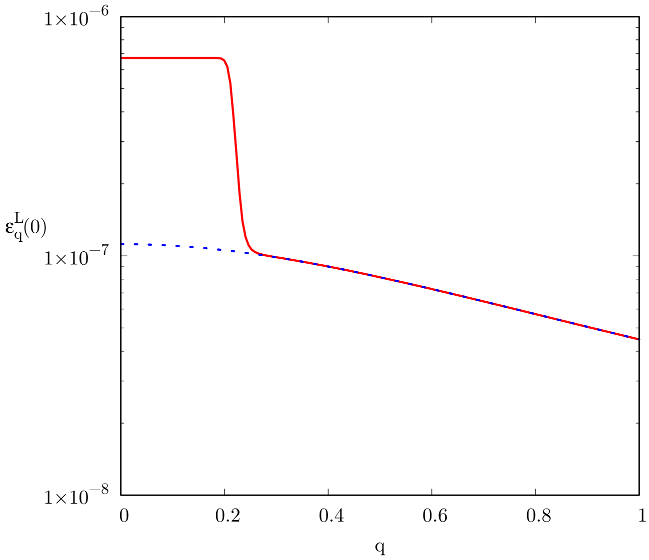

Figure 3. Steady-state of the Langmuir wave spectrum, versus the normalized wave number q. The red line depicts the steady-state given by equation (39), which takes into account collective and collisional effects. The blue dashed curve represents the steady-state spectrum only in the presence of collective processes.

5. Summary and discussion

In the present paper, we reanalyzed the problem of electrostatic bremsstrahlung emission and collisional damping of Langmuir waves. By doing so, we have corrected some inaccuracies in the previous formalisms [7, 8]. It is shown that the improved formulae in the case of Maxwellian particle distributions are given in forms where they satisfy Kirchhoff's law in the balanced form. The present discussion, which pertains to electrostatic formalism, may form a foundation for future electromagnetic theory of plasma bremsstrahlung and its inverse process, which are intimately associated with fundamental processes of the laser-metal-plasma interaction problem as well as for solar and astrophysical x-ray emission problems.

Before we close, we consider the relationship of the present discussion and the customary collisional damping rate found in the literature, known as the Spitzer formula, equation (1). For this purpose, let us examine the spectral function Q again. It is advantageous to express the result in dimensionless form in terms of normalized quantities,

$$\tau = \frac{T_e}{T_i}, \qquad \kappa = k\lambda_{De} = \frac{k v_{Te}}{\sqrt{2}\,\omega_{pe}}. \tag{42}$$

Without loss of generality, we may assume $\mathbf{k} = k\hat{\mathbf{z}}$, and after some straightforward manipulations, it is possible to express Q as

$$\begin{aligned} Q = \int_0^\infty ds \Bigg(& \frac{(\Delta s^2 + 1)^2 + (2\Delta + \kappa^2)\kappa^2 s^4}{(\Delta s^2 + 1)^2 + 2\Delta\kappa^2 s^4 + \kappa^4 s^4 - 2\kappa^2 s^2} \\ & + \frac{\Delta s^2 + 1 + \kappa^2 s^2}{4\kappa s} \ln \frac{\Delta s^2 + (\kappa s - 1)^2}{\Delta s^2 + (\kappa s + 1)^2} \Bigg) \\ & \times \frac{1}{|\Lambda(\kappa, s)|^2} \exp\left(-\frac{1 + 3\kappa^2}{2} s^2\right), \\ \Lambda(\kappa, s) = {} & 1 + s^2 \left[1 + \sqrt{\frac{1 + 3\kappa^2}{2}}\, s\, Z\left(\sqrt{\frac{1 + 3\kappa^2}{2}}\, s\right)\right], \\ \Delta = {} & 1 + \tau, \end{aligned} \tag{43}$$

where we have redefined the integral variable as $s = 1/\kappa\zeta$. Considering that the s integral has an overall Gaussian weighting factor $\exp[-(1 + 3\kappa^2)s^2/2]$, it becomes quite reasonable to assume that the most important contribution to the s integral arises from the vicinity around $s = 0$. Consequently, it is interesting to consider the expansion of the integrand near $s = 0$,

$$\begin{aligned} & \frac{(\Delta s^2 + 1)^2 + (2\Delta + \kappa^2)\kappa^2 s^4}{(\Delta s^2 + 1)^2 + (2\Delta + \kappa^2)\kappa^2 s^4 - 2\kappa^2 s^2} \\ & + \frac{\Delta s^2 + 1 + \kappa^2 s^2}{4\kappa s} \ln \frac{\Delta s^2 + (\kappa s - 1)^2}{\Delta s^2 + (\kappa s + 1)^2} \approx \frac{\kappa^2 s^2}{3}. \end{aligned} \tag{44}$$

As a result, we may perform the s integral in closed form. This leads to the approximate form of the collisional damping rate,

$$\frac{\gamma_L^{\text{collision}}(\mathbf{k})}{\omega_{pe}} = -\frac{g}{6\pi^{3/2}} \frac{1}{(1 + 3k^2\lambda_{De}^2)^3}. \tag{45}$$

This result is in contrast to the heuristic Spitzer collisional damping rate,

$$\gamma_{\text{Spitzer}} = -\frac{\pi n e^4 \ln\Lambda}{m_e^2 v_{Te}^3}, \tag{46}$$

which in normalized form, is given by

$$\frac{\gamma_{\text{Spitzer}}}{\omega_{pe}} = -\pi g \ln[2^{3/2}(4\pi g)]. \tag{47}$$

When compared with the approximate formula (45) the heuristic formula (47) does contain some resemblances, in that both are roughly proportional to g. However, while the Spitzer formula is independent of wave number, formula (45), although approximate, still has a weak k dependence. The basic assumption made in the derivation of the approximate formula (45) is that the most important contribution to the s integral arises at the vicinity of $s = 0$. Since s is the inverse of $k\zeta$, this approximation implies that the most significant contribution to collisional damping comes from short wavelength collisional processes. This assumption is not entirely valid for Langmuir waves in that for low values of k, the approximate formula (45) is expected to be invalid, and indeed, numerical comparison with figure 2 (a) does indeed show that the approximate formula (45) grossly over-exaggerates the damping rate for low k regimes. Nevertheless, by employing the approximation that led to (45) it is possible to make some qualitative connection to the heuristic collisional damping rate widely adopted in the literature.

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