

PV Power Generation Credit Sharing towards Sustainable Community Solar

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Abstract—This paper presents a new conceptual framework of PV power generation credit sharing by leveraging social tie in residential community to maximize the financial benefit of community solar programs. Social tie-driven credit sharing schemes for residential community are designed to manage the dynamic allocation of solar PV power production ratio and credit among community members, so as to avoid unnecessary devaluation of solar PV power production by electric utilities. Along this avenue, a community solar management system that incorporates and integrates social tie network and credit sharing schemes is then developed.

Index Terms—Community solar, PV power generation, virtual metering, shared renewable.

I. INTRODUCTION

With an aim to fulfill the renewable portfolio standards of the nation, renewable generation (wind and solar) assets have been rapidly developed and integrated into U.S. power grids [1]. Particularly, the size of installed and planned solar PV generation has increased exponentially in the past decades, targeted at 20,000 MW by 2020 [1]. According to Business Insider [2], solar is getting cheaper as the expansion of PV manufacturing achieves considerable scales of economy. Along this effort, community solar systems have been adopted as an alternative to traditional rooftop solar PV systems, and are regarded by NREL as the most viable solution to future residential solar PV systems [3]. Specifically, a community solar system is a solar-electric system that has a centralized solar PV garden (as opposed to separate PV systems at individual households) that is typically collocated with a residential community. Community solar systems have been regarded as preferable over traditional separate rooftop PV systems by residential customers, electric utilities and third-party renewable energy development companies [1]. As illustrated in Fig. 1, for community solar systems, there is no need to install PV panels at individual residential customer's premise. Utility can use one single revenue meter and/or phasor measurement unit to monitor the real-time power production and operating condition of the solar PV system; this is impossible for the case of separate rooftop PV systems. In addition, by adopting community solar as a replacement technology, these "behind-the-meter" separate rooftop PV systems become visible to utility operator, which significantly increases the situational awareness of power system operations for distributed energy resources. Further, renewable energy development projects

could save electronic devices (only one inverter is needed for centralized community solar) and maintenance cost.

Through one single inverter and one single revenue meter, community solar provides power and financial benefit to multiple community members. Residential members who invest in community solar can receive significant financial benefit. Through the virtual net metering programs adopted by many pilot community solar programs in U.S. [3], residential customers are allowed to use their share of solar PV generation to offset its load, i.e., individual residential customer is charged in real-time manner for its net-load at retail price. Particularly, when its net-load is negative, residential member will receive credit from solar PV generation. However, it is expected that these pilot net metering programs will not be sustainable under the current business model. The main reason is that by adopting net metering, electric utilities are factually paying at retail price for residential customer's PV generation, while electric utilities typically buy bulk power from wholesale power market at a much less wholesale price. Therefore, in future when the penetration scale of solar PV generation is sufficiently high, electric utilities will have to de-value residential customers' solar PV generation. Indeed, many existing solar PV programs have adopted or switched to feed-in tariff (which is lower than retail price) to residential customers' solar PV generation, and some other programs impose stringent restrictions on the period over which the credit received in a certain metering interval (5 min to an hour) can be rolled.

Shared renewables refer to the investment, development and operations of renewable generation facility in a cooperative and shared manner. Community solar is one example of shared renewables, in which residential customer invests in a solar PV facility and is allocated a ratio of the PV power generation. The ratio is typically fixed [1], which could be assigned based on the amount of the investment from individual residential customer. With the assigned ratio, individual residential customer gets credit/payment from utility company at a fixed price as per power purchase agreement between utility and residential customers.

In recent research and literature, share renewables also refer to the cooperative participation of renewable energy producers in power market through a renewable energy aggregator (which could be an electric utility or third-party company). In this cooperative effort, renewable energy generation offers low marginal cost, making the renewable power producers

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able to bid a lower price and hold a competitive standing in electric power market. In order to overcome the uncertainty inherent in intermittent wind and solar energy sources, so as to avoid the risk of non-deliverable renewable generation bid and consequent penalty [4] in electric power market, dispersed renewable energy generations can be aggregated before they participate in electric power market. The benefit of aggregating dispersed renewable energy sources lies in the fact revealed by recent studies [5], [6] that geographical diversity can reduce the magnitude of uncertainty when aggregated, as renewable energy sources in different geographical location can have negative correlation. Along this venue, a consideration amount of effort has been directed toward investigating the performance of aggregated renewables in whole power market and designing the payment sharing mechanisms for [7]–[14]. Particularly, game-theoretic approaches [10], [14] have been developed to ensure that individual renewable energy producers in this aggregate gets higher payment than the case that they participate in the power market separately. It is worth mentioning that aggregate renewables v.s. individual renewables would not have impact on the results power market clearance or the reliable operations of power systems. The only difference is that renewable energy producers get different payments in these two different mechanisms.

In a nutshell, community solar is a shared investment among residential customers, and aggregate renewables has shared payment for individual renewable energy producers. In this project, a new dimension is added to the concept of shared renewables, by incorporating the philosophy of sharing economy, i.e., by sharing renewable generation resource and associated credit into community solar. It is worth mentioning that the shared credit of community solar in the proposed research is totally different from the shared payment of aggregate renewable generation. Further, the scenarios considered in this project is different from those in the aggregate renewable generation literature [7]–[14]. Specifically, individual renewable energy producers are typically large power producers (e.g., a wind farm, solar farm, etc.) to participate in wholesale power market. Further, the scenarios in these literatures [7]–[14] could be unrealistic, as the geographical diversity of these renewable energy producers could usually prevent them from participating in the power market as an aggregate, as the transmission and operational limits may prevents some of them from being deliverable. In contrast, the scenarios considered in this project are well grounded by the existing practice of community solar. One important impact of the substantial financial benefits of credit-shared community solar brought to residential customer is that community solar development will be among the top people’s choice for clean energy development, and thus is potential to attract more investment to the solar clean energy sector.

The rest of the paper is organized as follows. The concept of PV generation credit sharing in the context of community solar is introduced in Section II. Section III presents the proposed approach to social tie-driven credit sharing in community solar. Numerical experiments are carried out in Section IV. Finally,

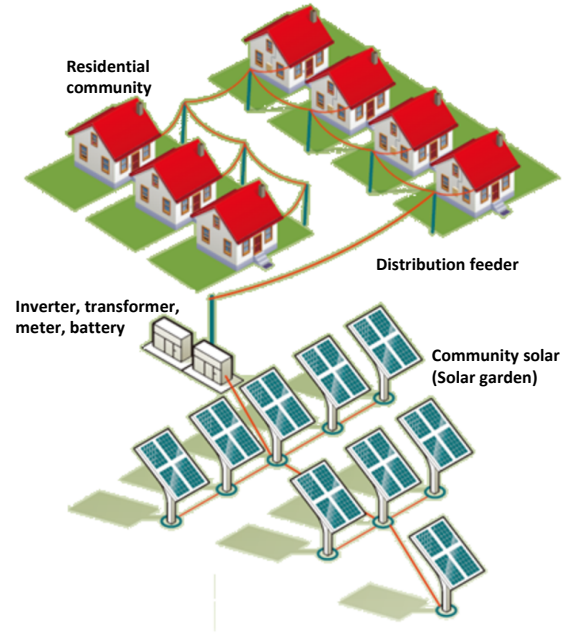


Fig. 1. Residential community solar [15].

conclusions are given in Section V.

II. CONCEPT OF CREDIT SHARING IN COMMUNITY SOLAR

The shared credit concept in community solar exploits the diversity in load profiles of individual residential customers in a practical scenario that dynamic retail pricing (e.g., time-of-use (TOU) pricing and the trending real-time pricing (RTP) [16]) is adopted. It is observed that individual residential customer may have mismatched load profiles, due to the difference in vocation and life styles [17], which would offer load profile diversity under TOU pricing. Further, in the scenario of RTP mechanisms, the retail rates can change as frequently as whole-sale rates, i.e., up to every 5 minutes. Therefore, under the RTP scenarios, the exact time of power-extensive appliances (heater, AC, etc.) of individual households can be quite different. Consider the following case illustrated in Fig. 2, in which two residential customers (e.g., Alice and Bob) have different load profile. Assume that the two households invested the same in the community solar and were assigned the same ratio. Then, the allocated PV power productions over time and the resultant credit are the same for the two household. Particularly, it can occur to Alice that she has power surplus, i.e., her allocated PV generation exceed her load during certain TOU or RTP pricing periods. Meanwhile, it is possible that Bob coincidentally has power deficit. Then, in the philosophy of sharing economy, Alice could transfer her PV power surplus to Bob. This would reduce the total bills to Alice and Bob, as otherwise, the utility company will charge Bob at retail price, while pay credit to Alice based on a lower power purchase agreement price. It is worth mentioning that there could be occasions that Bob pays his “debt” by sharing his PV power surplus and credit with Alice.

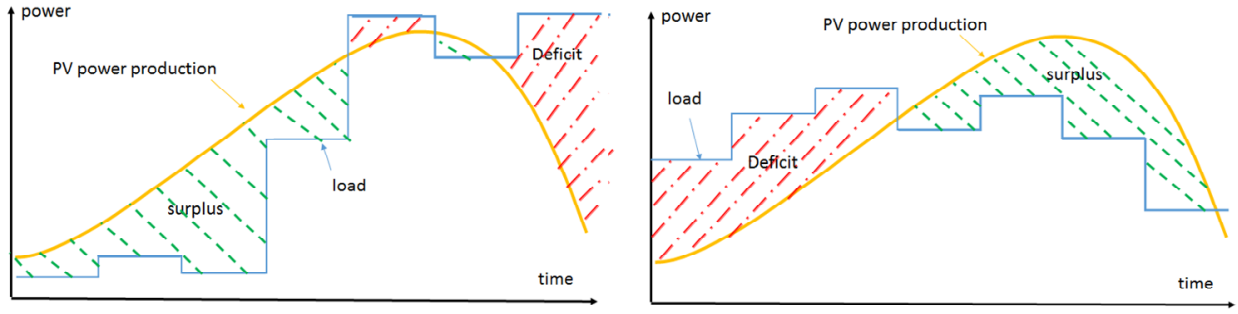


Fig. 2. An example of load profile diversity and potential credit sharing: (left) Alice (right) Bob.

From the above example, the benefit of credit-shared community solar could be seen. For this concept, one natural concern could be whether utility company would allow residential customer to dynamical manage their ratio of allocation. To address this concern, it is known that community solar has only one meter to the grid, as can be seen from Fig. 1. In any way, utility will need to use a certain allocation ratio of individual customer to calculate its credit, despite that the ratio is fixed or dynamically allocated. Further, the credit sharing among individual residential customers will not change the overall PV power production and injection to utility's distribution grid, and thus it has no impact on reliable operations of power systems. Therefore, it is reasonable that credit sharing among individual customers could be adopted, and a realistic scenario for this practice would be utility would like certain time windows for allocation ratio to be changed, for example, 1 hour before the term-of-use prices changes.

From the above introduction and explanation, it can be seen that, in a strict sense, the proposed credit sharing in community solar is an exact practice of sharing economy, i.e., sharing physical resources; while the community solar by itself is essentially a joint investment, and aggregate renewable generation does not involve sharing of any physical resources. It is also worth mentioning that concept of credit sharing has been practiced according to NREL's case studies [3], e.g., in the "local flavor" partnership case and the apartment group billing business model, where business partners or apartment residents share solar PV generation credit in a manner by equally splitting their net electricity bills. However, these cases are not applicable to generic residential communities, in which one household may have social ties with only a few neighbors, and there does not exist an apartment contract and manager enforcing the equally splitting of electricity bills. With this insight, it is thus proposed to leverage the social tie network for credit sharing in community solar.

III. SOCIAL TIE-DRIVEN CREDIT SHARING IN COMMUNITY SOLAR

A. Credit-shared community model

Assume that the residents that opt to participate in credit-shared community solar program use an online/mobile app with an agent to manage their credit, as shown in Fig. 3. Once the PV generation and residential load are realized

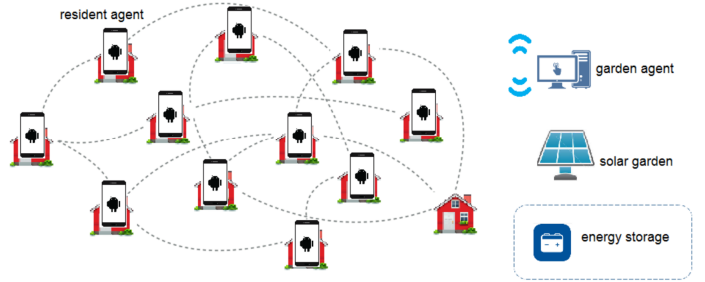


Fig. 3. Social tie network and credit-shared community solar (with or without energy storage).

in a given billing time window, resident agent with surplus will choose the sharing decisions on behalf of residential customer and notify this the garden agent (which could be on the billing server of utility company). After gathering all sharing decisions, the garden agent will recalculate the ratio and credit allocated to each residential customer, based on the surplus/deficit amount of each resident. A simplistic yet effective allocation rule could be characterized by the following two cases: 1) when the overall surplus is less than the overall requested, the garden agent will assign the overall surplus to each receiving resident proportionally to their request; 2) otherwise, the garden agent will return the unassigned surplus to the sharing resident proportionally to their surplus. A detailed credit calculation under this allocation rule would be provided in the case study of Section V.

B. An Ising model for sharing decision

It is observed that the sharing decision of individual customer are binary decision: share or not to share. This decision would be effected by the social tie, in the sense that, if a resident has surplus, he will be inclined to share if multiple or all of his/her acquainted neighbors are in deficit. With this insight, a Ising model [18] is used to characterize the social tie for credit sharing. Let X_i denote the sharing decision of resident agent i , the meaning of is as follows:

- $X_i = -1$ for a energy-deficit resident, and 2) for an energy-surplus household, $X_i = +1$ represents that it will share its surplus PV generation;
- $X_i = -1$ represents that it will not share and simply receive credit from utility.

Then the Ising model is given by:

$$\Pr(\mathbf{X}) = \frac{1}{Z} \exp(-V(\mathbf{X})), \quad (1)$$

in which Z is the partition function, and $V(\mathbf{X})$ is the potential function given by:

$$V(\mathbf{X}) = \sum W_{ij} X_i X_j. \quad (2)$$

The above model is a modified Ising model in the sense that $V(\mathbf{X})$ contains only pairwise clique functions, and regarding W_{ij} : 1) the pairwise weight W_{ij} takes value 0 if residents resident i and resident j have no social tie, and 2) the pairwise weight W_{ij} takes negative values if otherwise. The rationale behind for negative values of W_{ij} is that it would be highly likely that the decisions between neighbors with social ties would be complementary (a surplus resident will have a donor agent if neighbors are receiving agents), and thus W_{ij} has to take negative value for the potential function takes smaller values, which is inspired by the fact of low-energy state has higher probability to occur as described by the Ising model. There are two technical issues towards using this modified Ising model for credit sharing decision making: 1) the topology, and 2) the weights of the social tie network within the community, which would be addressed below.

C. Social tie network discovery

The social tie network topology and weights could be determined by allowing residents in the community to identify their social ties, or could be discovered through social network data mining, and both options should be offered to residential customers. For the latter option, state-of-the-art social network data mining techniques [19] could be adopted, yet a key question is what data source should be utilized. Instead of major social networks (e.g., Facebook, Twitter), the proposed research plan to utilized data from neighborhood social network as the data source. The rationale behind this is that neighborhood social network contains data regarding community's everyday life (see Fig. 4 for example in Nextdoor), and thus is more relevant to the quantification of weights for community solar credit sharing. Based on the example shown in Fig. 4, a naive method to identifying the topology and quantifying the weight could be a simple count C of 'Thanks' between any pairs of residents over unit time period T , i.e., $W_{ij} = C/T$. Note that it is rather the relative values between the weights W_{ij} than their absolute value matters for the probability of a sharing decision to occur, and thus even the above method would be effective in capturing the community social ties. More sophisticated method leveraging natural language processing and side information on the type of posts (e.g., a request, a discussion or a general question) could also be used.

D. Credit sharing decision making

By following the above method, an Ising model for community social network could be obtained off-line using historical data, and keep updated over time. Using the given Ising model,

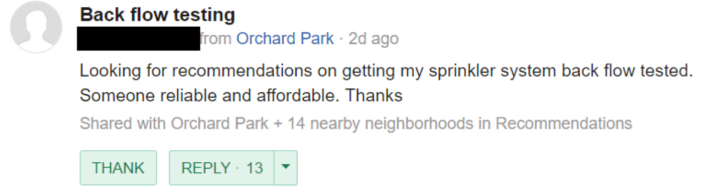


Fig. 4. Community social network data sample).

resident agents could take the following process to determine its sharing decision:

- Step 1-1: resident agents of deficit household set sharing decision to -1;
- Step 1-2: resident agents of surplus household randomly chose a sharing decision;
- Step 2: all resident agents broadcast their decisions
- Step 3: using the sharing decision of all other residents, each resident agents of surplus household updates its decision, by adopting a decision of +1 with a probability given below, then go to Step 2.

$$\Pr(X_i = 1 | X \setminus X_i) = \frac{e^{-\sum W_{ij} X_j}}{e^{-\sum W_{ij} X_j} + e^{\sum W_{ij} X_j}} \quad (3)$$

Remark: the above procedure used message passing algorithms over a probabilistic graphic model, and thus is guaranteed to converge, and in converged states, the decisions of all residents makes sure that the resultant potential function take small values (convergence to low energy state of Ising modeling), indicating that maximum sharing between the groups of surplus and deficit household that have social ties is achieved. In this way, the resident agent following the above procedure will automatically deliver consistent decisions to the case of manmade decision by the residents themselves. Further, it is noted that by using the above procedure, a household could receive credit sharing from ones who have no existing social tie with it (although very unlikely). This sharing could promote new social ties to be built among the residents of the community.

E. Credit-shared community microgrid with energy storage

It is thus very clear that the above credit sharing procedure can only take place during daytime, i.e., when solar PV generation is available. Battery energy storage systems could be deployed in conjunction with community solar to eventually become a microgrid, that is able to supply power to the community during nighttime. In this way, there is one additional option for the decision making of resident agent - charge surplus power to energy storage system. With this insight, a tristate Potts model [20] could be adopted as a replacement for the Ising model in (1). Specifically, in the new Potts model, $X_i=0$ would represent the case that resident agent chooses to charge the surplus energy to energy storage for future use.

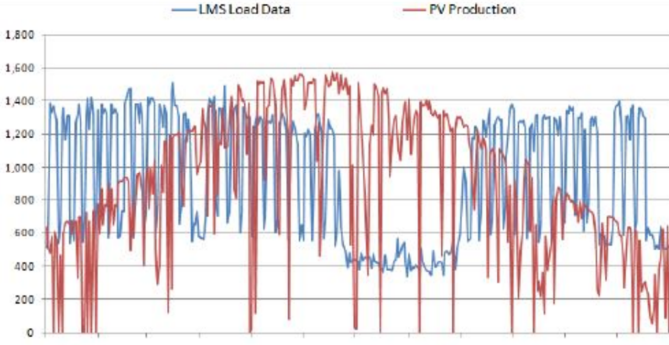


Fig. 5. A household load and PV power production).

IV. CASE STUDY

A. Test System and Data

It is assumed that a community has $N=100$ households, and each household is allocated with the same PV power production from the community solar garden. The social network among the N households is generated by using the Erdos-Renyi model [21] with the average degree $\langle k \rangle$ being the control parameter. Data of household load, PV generation, and electric tariffs are obtained from OpenEI - U.S. DOE Open Data Catalog [22]. Once sample trace of household load and PV power production could be found in Fig. 5. In the context of community solar, it is reasonable to assume all households have the same PV power production profile. Load data of different shapes and characteristics from the database [22] are chosen for individual household load.

B. Credit and Payment Calculation

TABLE I
NOTATION

D_i	energy demand (kWh) of resident i
G_i	PV power generation (kWh) of resident i
M_i	payment of resident i
r_t	retail power price (\$/kWh) in timeslot t
c	unit PV power production credit (\$/kWh)
X_i	indicator of PV power doner (+1) or receiver (-1)

By complying the credit sharing rule described in Section III.A, as well as the sharing decision making procedure in Section III.D, the total energy surplus to be shared and requested could be calculated. For brevity, the index of time t is neglected below. Specifically, the total energy surplus, P_s , that is to be shared by participating residents is given by:

$$P_s = \sum_{X_i=+1} (G_i - D_i), \quad (4)$$

the total energy requested, P_r , is

$$P_r = \sum_{X_i=-1} (D_i - G_i)^+, \quad (5)$$

where the superscript $^+$ represents the positive part. Taking the positive part is because $X_i=-1$ could correspond to a resident

with energy surplus yet does not want to share. Then, there are two cases to be considered for credit and payment calculation. 1) Case 1: when $P_r \geq P_s$, i.e., requested is more than shared. Then, according to the credit sharing rule described in Section III.A, each requesting resident receives an amount proportional to its' request. Therefore, for $X_i=-1$ and $D_i \geq G_i$, the overall payment of a requesting resident i to utility is then

$$M_i = (D_i - G_i)(1 - P_s/P_r)r_t. \quad (6)$$

Further, it is easy to see that the overall payment of a sharing resident i to utility is $M_i=0$.

2) Case 2: when $P_r < P_s$, i.e., requested is less than shared. Then, according to the credit sharing rule described in Section III.A, each requesting resident receives an full amount, and unassigned surplus will be returned to the sharing resident proportionally to their surplus. Therefore, for $X_i=+1$, the overall credit of a sharing resident i obtained from utility is then

$$M_i = (D_i - G_i)(1 - P_r/P_s)c. \quad (7)$$

Further, it is easy to see that the overall payment of a receiving resident i to utility is $M_i=0$. For both Case 1 and Case 2, the overall credit (with a sign of -) of a surplus but not sharing resident i is

$$M_i = (D_i - G_i)c. \quad (8)$$

C. Benchmark Systems

Two benchmark systems are considered. The first one is the realistic system, i.e., a system without considering PV credit sharing. Then, it is easy to see that in this system, surplus resident receives $-(D_i - G_i)c$ and deficit resident pays $(D_i - G_i)r_t$. The second benchmark system is hypothetical, in the sense that all the residents unlimitedly share the total power production and split their utility bill equally, it is thus easy to see that under this system model, when system is overall surplus, each resident receives a credit given by

$$M_i = \frac{1}{N}(\sum_{i=1}^N D_i - \sum_{i=1}^N G_i)c, \quad (9)$$

and when system is overall deficit, each resident pays an amount of

$$M_i = \frac{1}{N}(\sum_{i=1}^N D_i - \sum_{i=1}^N G_i)r_t, \quad (10)$$

It is observed that from the perspective of overall community payment to utility, benchmark system two is the best, the overall community payment is less than benchmark system 1 and the proposed credit-sharing system. This is simply because in benchmark system two, the financial benefit of PV power production is maximized, i.e., PV power production is maximally utilized to offset aggregate load, realizing a value of r_t \$/kWh. However, if PV power production is purchased by utility, the realized values is only c \$/kWh. Since r_t is typically greater than c , the hypothetical system 2 has least overall payment.

D. Performance Evaluation

From the previous discussion, a performance metric with regard to aggregate community financial benefit is defined as follows:

$$\eta = \frac{r_t \sum_{i=1}^N D_i - \sum_{i=1}^N M_i}{r_t \sum_{i=1}^N G_i}, \quad (11)$$

which has a physical meaning of actual realized value of per unit PV power production (relative to the maximum realized value r_t). Thus, an $\eta=1$ indicates that all PV power production has realized maximum value r_t , i.e., all PV power production has been utilized to offset load.

TABLE II
PERFORMANCE EVALUATION ()

	Credit-sharing	Benchmark 1	Benchmark 2
η	0.96	0.91	0.99
$\langle k \rangle$	2	5	10
η	0.92	0.96	0.98

Using the real-world load and PV power data collected from [22], simulations are carried out. The performance metric η is then calculated for all three systems. It is observed from Table. II that the proposed credit-sharing system outperforms the Benchmark 1 system (the realistic system), and is very close to the Benchmark 2 system. It is worth mentioning that η is not 1 for Benchmark 2 system, since there are occasions that total PV power production exceeds overall community load, and thus not all PV power production can realize a value of r_t . Further, the control parameter, $\langle k \rangle$, i.e., the average degree of the social network of the community is changed among values 2, 5, and 10. It is worth noting that $\langle k \rangle$ represents and average number of neighbors that have social tie with a resident. Therefore, when $\langle k \rangle$ is larger, a resident would have more neighbors with social ties, and is thus more likely to act as a sharing when the resident has surplus as well as more neighbors are requesting. This conclusion has been verified by the results summarized in Row 3-4 of Table. II, where η improves as $\langle k \rangle$ becomes larger.

V. CONCLUSION

This paper presents a novel concept of PV power credit sharing in community solar. A graphical model-based credit sharing mechanism is then derived. Although this mechanism is very simplistic, yet it has been proven to be very effective to realize the increased values of PV power production through credit sharing between residents with social ties. In the case study, real-world data is used to quantify the performance of the proposed credit sharing system, with comparison to a realistic system without credit sharing and a hypothecae system with unrestricted sharing. The impact of average degree on the performance is also investigated. In future, this work could be extended to the scenarios with peer-to-peer debt limit and more sophisticated sharing mechanism.

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