1 2	Characteristics of rising tone whistler mode waves inside the Earth's plasmasphere,
3	plasmaspheric plumes and plasmatrough
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15	Key Points:
16 17	• Rising tone whistler mode waves are statistically analyzed inside the plasmasphere, plasmaspheric plumes and plasmatrough.
18 19	• Among these three regions, the occurrence rate of rising tone whistler mode waves in plasmapheric plumes is the highest.
20 21 22	• Inside the plasmasphere and plumes, rising tone whistler mode waves tend to be more field-aligned than those outside the plasmapause.

23 Abstract

- 24 Whistler mode waves, particularly rising tone emissions, are important for nonlinear interactions
- 25 with energetic electrons in the Earth's magnetosphere. In this letter, we evaluate the
- 26 characteristics of rising tone whistler mode waves in three distinct regions: 1. inside the
- 27 plasmasphere; 2. plasmapheric plumes, and 3. plasmatrough (outside the plasmapause). Our
- 28 statistical results indicate that the occurrence rate of rising tone emissions tends to increase with
- 29 increasing geomagnetic activity and is highest in plasmaspheric plumes among these three
- 30 regions. Inside the plasmasphere, rising tone emissions typically occur in the outer portion of the
- 31 plasmasphere, particularly near the dawnside. Moreover, the rising tone emissions inside the
- 32 plasmasphere and plumes tend to be more field aligned than those in the plasmatrough. Our new 33 findings of global wave properties of rising tone emissions are critical for understanding the
- 34 generation of rising tone emissions and their effects on radiation belt electron dynamics.
- 35

36 Plain Language Summary

37 Rising tone whistler mode waves are intense electromagnetic emissions and commonly present

- in the Earth's magnetosphere. They are typically observed in the plasmatrough, which is a low-
- 39 density region outside the plasmapause. By analyzing the high time resolution magnetic wave
- 40 data from Van Allen Probes measurements, we present the global distribution and characteristics
- 41 of rising tone whistler mode waves in three different regions: inside the plasmasphere,
- 42 plasmaspheric plumes and the plasmatrough. Among these three regions, the occurrence rate of
- 43 rising tone emissions is found to be highest in plasmaspheric plumes. The rising tone emissions
- 44 observed inside the plasmasphere and plumes tend to be more field-aligned than those in the
- 45 plasmatrough. Our new findings of global wave properties of rising tone emissions are critical
- 46 for understanding the generation of rising tone emissions and their effects on radiation belt
- 47 electron dynamics.

48 **1 Introduction**

49 Whistler mode chorus waves are intense coherent electromagnetic emissions exhibiting 50 discrete rising or falling tones, and typically occur in two distinct frequency bands: a lower band 51 and an upper band with a power minimum at 0.5 f_{ce} (Burtis and Helliwell, 1975; Santolík et al., 52 2004; Tsurutani and Smith, 1974), where f_{ce} is the equatorial electron cyclotron frequency. These waves play a crucial role in radiation belt electron dynamics, through effectively accelerating 53 54 100s keV electrons to MeV energies in the outer radiation belt (Horne et al., 2005; Reeves et al., 55 2013; Thorne et al., 2013) and causing precipitation of electrons to form diffuse and pulsating 56 aurora in the Earth's upper atmosphere (Ni et al., 2008, 2016; Nishimura et al., 2010; Thorne et 57 al., 2010).

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59 The generation region of chorus is known to be mostly located in the low-density region 60 outside the plasmapause, which is also called plasmatrough, near the geomagnetic equator 61 (LeDocq et al., 1998; Santolík et al., 2003). Previous statistical studies have shown that whistler 62 mode chorus waves have a high occurrence rate from the midnight to the noon sector (Li et al., 63 2000, 2011a: Maradith et al., 2002, 2012). In contrast, plasmapharia high is in trainelly observed in

63 2009, 2011a; Meredith et al., 2003, 2012). In contrast, plasmaspheric hiss is typically observed in

the high-density region inside the plasmasphere or plasmaspheric plumes, preferentially from the dawn to the dusk sector (Meredith et al., 2004, 2018; Li et al., 2015; Shi et al., 2019; Summers et 66 al., 2008; Thorne et al., 1973). Plasmaspheric hiss and chorus waves overlap in frequency (Tsurutani and Smith, 1977). However, chorus waves are typically narrowband and guasi-67 coherent (Albert et al., 2012; Artemvev et al., 2012; Bell, 1984; Bortnik et al., 2008a; Inan et al., 68 69 1978; Kellogg et al., 2010; Omura et al., 2007; Tao and Bortnik, 2010; Tao et al., 2012), while plasmaspheric hiss is normally incoherent and unstructured (Bortnik et al., 2008b; Thorne et al., 70 71 1973). The generation mechanisms of chorus and hiss are still under active investigation. 72 Although it is widely accepted that the generation of chorus waves is nonlinear (Helliwell, 1967; 73 Nunn, 1971; Omura et al., 2008; Soto-Chavez et al., 2014; Tao et al., 2017a,b; Trakhtengerts, 74 1995; Vomvoridis et al., 1982), the detailed process is still an on-going topic. Potential 75 generation mechanisms of plasmaspheric hiss include local excitation by electron cyclotron 76 instability (Solomon et al., 1988; Tsurutani et al., 2015), propagation effects of chorus waves 77 into the plasmasphere (Agapitov et al., 2018; Bortnik et al., 2008b; Chen et al., 2012; Meredith et 78 al., 2013), and origination from lightning generated whistlers (Green et al., 2005). 79

80 Plasmaspheric plumes have attracted increasing attention in recent years because of their 81 favorable conditions for generating various types of plasma waves. Some case studies reported 82 that plasmaspheric hiss is likely locally excited in plasmaspheric plumes (Laakso et al., 2015; Su 83 et al., 2018). Shi et al. (2019) proposed that the majority of whistler mode waves in plumes are 84 locally amplified in association with energetic electron injection. Hartley et al. (2019) suggested 85 that only an extremely small fraction of chorus waves can propagate into the plasmasphere 86 except for chorus waves near the edge of plasmaspheric plumes. These plume whistler mode 87 waves are found to be potentially very effective in energetic electron precipitation loss (e.g., Li et 88 al., 2019; Summers et al., 2008; Zhang et al., 2018). Case studies also show that whistler mode 89 waves in plumes exhibit either rising tones (Shi et al., 2019; Su et al., 2018) or hiss-like 90 emissions (Li et al., 2019). However, the occurrence of rising tone emissions in plasmaspheric 91 plumes has not been systematically studied vet. Although the characteristics of whistler mode 92 waves including rising and falling tones outside the plasmasphere are studied extensively (e.g., 93 Gao et al., 2014; Li et al., 2011b, 2012; Santolik et al., 2003, 2009), there is still a lack of 94 understanding about the global distribution of rising tone whistler mode waves inside the 95 plasmasphere and plasmaspheric plumes.

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97 In this letter, we aim to evaluate the global distribution and characteristics of rising tone 98 whistler mode waves in three distinct regions: (1) inside the plasmasphere, (2) in the 99 plasmaspheric plumes, and (3) in the plasmatrough (outside the plasmapause). Moreover, we 100 investigate the wave normal angle distributions of rising tone whistler mode emissions in these 101 three regions to provide insights into understanding wave generation and propagation.

102 2 Data analysis

In this study, we utilize Van Allen Probes data from October 2012 to May 2016 during which both spacecraft completed two full orbital precessions (Kessel et al., 2013; Mauk et al., 2013) and covered all magnetic local times (MLTs) twice. High time resolution measurements from burst mode of Electric and Magnetic Field Instrument Suite and Integrated Science (EMFISIS) (Kletzing et al., 2013) instrument are used to investigate rising tone whistler mode waves. A total of about 1412108 6-second burst-mode waveform data throughout the chosen period were recorded. We set the threshold of minimum magnetic power spectral density 110 to be 10^{-8} nT²/Hz in the frequency range from 0.1 f_{ce} to 0.8 f_{ce} to eliminate wave spectra without

- any or very weak signals and then select the bursts containing rising tone discrete elements by
- visually inspecting the power spectrogram. In this work, each selected 6-second waveform burst
- 113 mode data is defined as an "event". For the recorded rising tone emissions, we subsequently
- 114 categorize them into three different groups: (1) inside the plasmasphere, (2) plasmapheric 115 plumas and (2) plasmatrough. The determination of each region is mainly based on the density
- plumes, and (3) plasmatrough. The determination of each region is mainly based on the density profile, which is inferred from the High-Frequency Receiver (HFR) (Kurth et al., 2015) and from
- the Electric Field and Waves (EFW) instrument (Wygant et al., 2013), as described below.
- Following the technique of Moldwin et al. (2002), the plasmapause location is defined as the innermost steep gradient in the electron density profile when the density drops by at least a factor of 5 within a half L shell in any leg (inbound or outbound) of one orbit. If the density keeps constantly high (e.g., larger than 100 cm⁻³) throughout the leg or no plasmapause crossing is observed, the whole leg is assumed to be inside the plasmasphere. A plume is defined in a
- region outside the plasmapause. We adopted the criterion of Shi et al. (2019) to choose two
- 124 referred density values at any observed \hat{L} shell, which are $N > 1.2 \times \min(N_{lower L})$ and
- 125 $N > 2.5 \times \min(N_{lower L}) \times L_n^6 / L^6$. Here $\min(N_{lower L})$ is the minimum density between the
- 126 plasmapause crossing and the satellite location and L_n is the L shell where the minimum density
- 127 $(N_{\text{lower }L})$ is recorded. The calculation of L shell and other parameters, such as magnetic latitude
- 128 (MLat) and MLT, is based on the TS04D model (Tsyganenko and Sitnov, 2005). After applying 120 the method described above, we visually inspected the identified plasmenause leastion and
- the method described above, we visually inspected the identified plasmapause location and plume regions to ensure that the automatic identifications are reasonable. We have found 6988
- events inside the plasmasphere, 2896 in the plumes and 83879 in the plasmatrough. The majority
- 132 of rising tone whistler mode emissions are observed outside the plasmapause, but there are still a
- 133 number of them observed inside the plasmasphere and plasmaspheric plumes.

134 Figure 1 illustrates three typical examples of rising tone whistler mode waves observed 135 inside the (A) plasmasphere, (B) plasmaspheric plume, and (C) plasmatrough. Since AL index is 136 closely related to substorm activities (e.g., Gjerloev et al., 2004), we show AL (black) and AL* 137 (red) in Figures 1 A-a, B-a and C-a, where AL* is the minimum of AL in the preceding three 138 hours. In Figures 1A-a and B-a, AL* ranged between -300 and -500 nT, indicating a moderate 139 substorm activity. For the case in the plasmatrough in Figure 1C-a, AL* dropped below -500 nT, 140 indicating a strong substorm activity. In Case A, the density (Figure 1b) kept high throughout the 141 whole orbit and no abrupt density gradient was found, and thus this orbit was identified to be 142 located inside the plasmasphere. For Cases B and C, the plasmapause crossings were identified, 143 as marked by blue vertical lines. The plume regions are highlighted by the magenta line in Figure 144 1B-b. Figures 1c-e show the overview of the WFR electric and magnetic spectrogram and wave 145 normal angle (WNA) in the frequency range of 10 - 10000 Hz. Wave spectrogram and wave 146 normal angles indicate that there exist field-aligned whistler mode waves and oblique 147 magnetosonic waves in three cases. In this study, we mainly focus on rising tone whistler mode 148 emissions, which can only be identified from the burst mode data. Three vertical dashed red lines 149 denote the time of three selected burst intervals. The magnetic wave spectra and wave normal 150 angle of these continuous waveforms over 6 seconds are shown on the bottom for the three 151 occasions. Interestingly, the duration of rising tones in the high-density regions (Cases A and B) 152 is longer than that in a low-density region (Case C), which is consistent with the event reported 153 by Shi et al. (2019). Typical rising tone chorus with short duration (< 0.5 s) and a large sweep

154 rate, shown in Figure 1C-f, is not observed inside the plasmasphere and plasmaspheric plumes.

155 While a quantitative analysis of the duration of every rising tone element is beyond the scope of

this study, through visually inspecting a large number of wave emissions, we found that this is

157 likely one of the major differences for rising tone emissions in the different regions. Wave

normal angles for the three cases (Figure 1g) indicate that rising tone whistler mode waves are quasi-parallel. A statistical WNA analysis of all rising tone events is presented in section 3.

quasi-parallel. A statistical why analysis of all fising tone events is presented in section 5

160 **3 Statistical Results of Rising Tone Whistler Mode Emissions in the Three Regions**

Figure 2 shows the global distribution of occurrence rate and number of samples of rising tone whistler mode waves observed in the three regions as a function of *L* shell and MLT, as well as their dependence on AL*. The distributions are categorized by regions inside the plasmasphere (top), plasmaspheric plumes (middle) and plasmatrough (bottom) during quiet (AL* > -100 nT), moderate (-300 < AL* < -100 nT), and active (AL* < -300 nT) geomagnetic conditions. The distributions of occurrence rate, which is the ratio between the number of events and the total

167 number of burst-mode samples in each bin are shown in large panels, with the corresponding

168 number of burst-mode samples shown in small panels. Only the bins with the number of samples

169 larger than 20 are chosen to show the value of occurrence rate to eliminate the statistically

170 insignificant values.

171 The occurrence rate distribution exhibits a clear dependence on geomagnetic activity for 172 each of three regions. As geomagnetic activities enhance, the occurrence rate increases, which is 173 consistent with previous studies (e.g., Li et al., 2009; Meredith et al., 2012). The number of 174 samples recorded in the plasmaspheric plumes is much smaller than that inside the plasmasphere 175 or plasmatrough, thus the occurrence rate in the plasmaspheric plumes might be statistically less 176 significant than that in the other two regions. Nevertheless, it is very interesting to note that 177 under the same geomagnetic condition, the peak occurrence rate of rising tone emissions in 178 plasmaspheric plumes is the highest (as long as the plumes are detected), followed by the 179 emissions in the plasmatrough. The coexistence of high density cold plasma and energetic 180 particles may provide favorable conditions for rising tone whistler mode wave amplification, 181 which probably accounts for the high occurrence rate. It is worth noting that the high occurrence 182 rates in plasmaspheric plumes are mostly observed at large L shells ($\sim 5-6$). Only during active 183 geomagnetic times when the plasmapause location moves Earthward, rising tone emissions tend 184 to be observed at L < 5 near the dusk sector where the plumes are known to preferentially form

185 (e.g., Goldstein et al., 2004; Shi et al., 2019).

186 Inside the plasmasphere, while the occurrence rate of rising tone emissions is relatively 187 low, there still exist bins with the peak value of occurrence rate up to \sim 50% near the dawnside at L > 4.5 (likely outer portion of the plasmasphere) during active conditions. This may be related 188 189 to the energetic electron drift into the outer portion of the plasmasphere near the dawn-to-noon 190 sector after their initial injection near the nightside during active conditions (e.g., Li et al., 2013). 191 Note that the shape of plasmapshere is often asymmetric with a compression on the nightside and 192 an extension at large L shells on the dayside as a result of enhanced convection during active 193 times (e.g., Li et al., 2013). Correspondingly, the number of samples inside the plasmasphere is 194 smaller over the night-to-dawn sectors than that over the dawn-to-dusk sector during modest and

195 active conditions.

To compare with the occurrence rate in the above two high-density regions, the bottom panels in Figure 2 show the distribution of rising tone chorus observed in the plasmatrough. Rising tones preferentially occur from the midnight to the afternoon sector, especially during active times, consistent with previous statistical studies (e.g., Li et al., 2009; Meredith et al., 2003, 2012). As expected, the occurrence rate of rising tone chorus increases with increasing geomagnetic activity up to tens of % from the night to the afternoon sector.

202 Figure 3 presents the normalized probability distribution function (PDF) of f_{pe}/f_{ce} and density in the three different regions, where f_{pe}/f_{ce} is the ratio of plasma frequency to local 203 204 electron gyrofrequency. Interestingly, there is no clear difference in the density and f_{pe}/f_{ce} 205 distribution between rising tone emissions inside the plasmasphere and plasmaspheric plumes, 206 probably because rising tone emissions detected inside the plasmasphere are distributed in the 207 outer portion of the plasmasphere (at relatively large L shells). However, the difference between 208 inside the plasmaphere and outside the plasmapause is very clear. The f_{pe}/f_{ce} distribution in the 209 plasmatrough is much narrower than that inside the plasmasphere or in plumes, and has a 210 pronounced peak at about 4-5, which is consistent with Meredith et al. (2003). On the other 211 hand, in the high-density regions, both inside the plasmasphere and plasmaspheric plumes, the f_{pe}/f_{ce} distribution is much broader and peaks around 11. Since the density and f_{pe}/f_{ce} are closely 212 related, the PDF values of density for the three groups show a similar trend as that of f_{pe}/f_{ce} . 213 214 Inside the plasmasphere and plasmaspheric plumes, the density distribution is much broader and peaks at a higher value ~ 40 cm^{-3} . However, outside the plasmasphere, the density distribution is 215 very narrow, peaking at ~ 5 cm⁻³. Previous studies demonstrated that rising tone emissions are 216 preferentially observed in the region with low values of f_{pe}/f_{ce} (Meredith et al., 2003; Li et al., 217 218 2012), where the local electron acceleration is the most efficient (Horne et al., 2003). 219 Interestingly, our study indicates that rising tone emissions also exist in the region with high 220 values of f_{pe}/f_{ce} , both inside the plasmasphere and plumes.

Wave normal angle distributions of chorus waves have been extensively studied over the past few decades. Rising tone chorus is shown to be mostly field-aligned (Burton and Holzer, 1974; Cornilleau-Wehrlin et al., 1976; Li et al., 2011b; Taubenschuss et al., 2014), while falling tone chorus typically has a wave normal angle close to the resonance cone (Artemyev et al., 2016; Burton and Holzer, 1974; Cornilleau-Wehrlin et al., 1976; Li et al., 2011b). In this study, we exclude the falling tones and focus on the wave normal angle distribution of rising tone whistler mode waves alone in both upper and lower bands in the three different regions.

228 To evaluate the detailed wave normal angle distribution, we present the rising tone 229 statistics as a function of wave normal angle and normalized wave frequency in Figure 4. The 230 top panels show the PDF of wave occurrence in three regions. The PDF value of upper and lower 231 band observed in the plasmatrough is close, while the PDF values of upper band observed inside 232 the plasmasphere and plumes are very low. A possible reason could be that the excitation mechanism of upper band chorus waves depends on plasma density or f_{pe}/f_{ce} . The wave normal 233 234 angle distribution shows that rising tone whistler mode waves both inside the plasmasphere and 235 plasmaspheric plumes tend to peak at about 10° . However, in the plasmatrough, the WNA 236 distribution peaks at $< 20^{\circ}$ for the wave frequency less than 0.3 f_{ce} , and tends to peak at larger wave normal angles for the wave frequency higher than $0.3 f_{ce}$. In all three regions, the wave 237 238 normal angles of upper band tend to have smaller values at higher frequency, and exhibit a 239 broader distribution just above $0.5 f_{ce}$. Li et al. (2016) showed that lower band chorus has a

- quasi-parallel mode and a quasi-electrostatic mode, and the upper band chorus wave normal
- angle varies between 0° and the resonance cone (Taubenschuss et al., 2014, 2015). The second
- peak near the resonance cone angle is not very evident in Figure 4 possibly because of the
- exclusion of falling tone chorus waves, which are typically very oblique, close to the resonance
- cone (Li et al., 2011b). The middle and bottom panels in Figure 4 show the mean value of
 magnetic and electric power spectral density in each wave frequency and normal angle bin. The
- magnetic/electric power spectral density in each wave frequency and normal angle off. The magnetic/electric wave power of upper band in the plasmasphere and plumes are much lower
- than that of lower band. The magnetic wave power is stronger in plasmaspheric plumes than that
- inside the plasmasphere, consistent with the findings by Shi et al. (2019). The bottom panels in
- Figure 4 show that rising tone emissions observed inside the plasmasphere and plasmaspheric
- 250 plumes have weaker electric wave power compared to chorus waves in the plasmatrough.

251 4 Summary and Discussion

In this letter, we performed a statistical analysis to evaluate the characteristics of rising tone whistler mode emissions in three different regions: inside the plasmasphere, plasmaspheric plumes, and plasmatrough (outside the plasmapause) using Van Allen Probes waveform data. The main findings are summarized below:

- 1. Rising tone whistler mode emissions are observed not only in the plasmatrough, but also
- 257 inside the plasmasphere and plasmaspheric plumes. The occurrence rate of rising tone emissions
- tends to increase as geomagnetic activity increases. Interestingly, among these three regions, the
- 259 occurrence rate is highest in plasmaspheric plumes (once the plumes are observed), particularly
- from the dawn to the dusk sector during active geomagnetic conditions. Inside the plasmasphere, the accurrence rate follows a similar pattern as that in the plasmatrouch but with smaller values
- the occurrence rate follows a similar pattern as that in the plasmatrough, but with smaller values. Moreover, the occurrence rate is higher in the outer portion of the plasmasphere, not deep inside
- 263 the plasmasphere, and is higher from the post-midnight sector to the dawn sector, which is
- 264 consistent with the electron drift trajectory after injection.
- 2. The probability distribution of total electron density and f_{pe}/f_{ce} indicates that in the highdensity region, both inside the plasmasphere and plumes, the peak values of density (f_{pe}/f_{ce}) are similar, around 40 cm⁻³ (10). However, in the plasmatrough, the peak value of density (f_{pe}/f_{ce}) is much lower, dropping to 10 cm⁻³ (5).
- 269 3. The wave normal angle distributions of rising tone whistler mode emissions both inside the
- 270 plasmasphere and in plumes exhibit only one peak, near the field-aligned direction, whereas
- rising tone chorus in the plasmatrough tend to have more oblique wave normal angles,
- 272 particularly at the frequency above $0.3 f_{ce}$. The magnetic wave power of rising tone whistler
- mode emissions in plumes is stronger than that inside the plasmasphere. The electric wave power
- outside the plasmapause is evidently stronger than that inside the plasmasphere and plumes,
- especially at higher wave normal angles. Different from rising tone chorus emissions observed in the plasmatrough, very limited upper band rising tone emissions are observed inside the
- 270 the plasmatrough, very finited upper band rising tone emiss 277 plasmasphere and plumes.
 - It is important to note that the effects of rising tone emissions, observed in the plasmatrough and plasmasphere/plumes, on energetic electrons are very different, since they

- 280 occur in the regions with distinct values of f_{pe}/f_{ce} , which is a key factor determining the resonant
- electron energy. Moreover, the rising tone elements inside the plasmasphere and plasmaspheric
- plumes appear to last longer than typical chorus waves observed outside the plasmapause, as an example shown in Figure 1, as well as in many other events (not shown). This interesting feature
- suggests that the duration of rising tone emissions is likely to be closely related to the plasma
- density or $f_{\rm pe}/f_{\rm ce}$. The quantitative evaluation of the duration of each rising tone emission is
- beyond the scope of this present study, and is left for future investigations. Nevertheless, our
- 287 systematic analysis of rising tone whistler mode waves in three distinct regions provides
- 288 important information on the wave distribution and characteristics and thus is critical for
- understanding the generation mechanism of rising tone whistler mode waves and their potential
- roles in energetic electron dynamics in the Earth's magnetosphere.
- 291

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- 298 https://emfisis.physics.uiowa.edu/data/index/, and from EFW were obtained from
- 299 http://rbsp.space.umn.edu/data/rbsp/.
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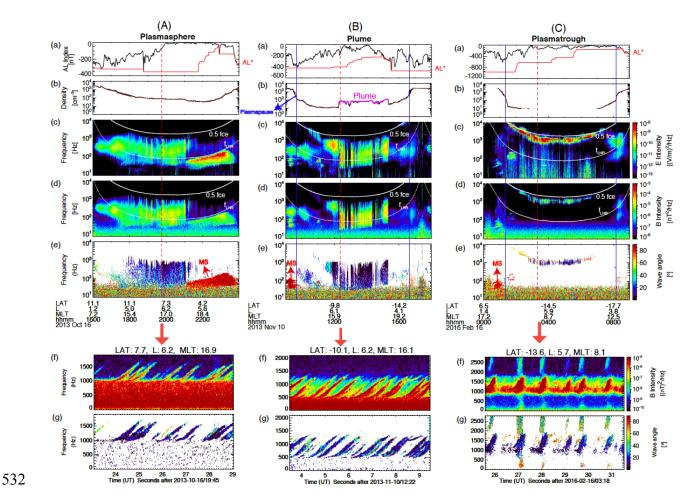
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533 Figure 1. Three examples of rising tone whistler mode waves observed (A) inside the

plasmasphere, (B) plasmaspheric plume, and (C) plasmatrough. (a) AL (black) and AL* (red). (b)

Plasma density inferred from the upper hybrid resonance line, where the magenta line
 corresponds to the density in plume regions. (c) Electric spectral density, (d) magnetic spectral

density, and (e) wave normal angle. In Figures 1c and 1d, the white lines represent 0.5 f_{ce}

(dashed) and f_{LHR} (dotted), where f_{ce} and f_{LHR} are electron cyclotron frequency and lower hybrid

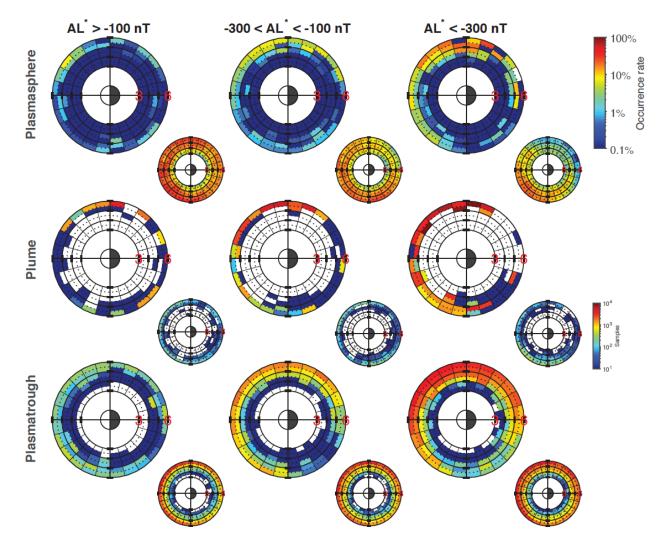
539 resonance (LHR) frequency respectively. The vertical blue lines indicate the plasmapause

540 crossing. The vertical red dashed lines denote the three occasions when the burst mode data were

541 captured. The bottom panels show (f) the magnetic wave spectra and (g) wave normal angle

542 using burst mode data.

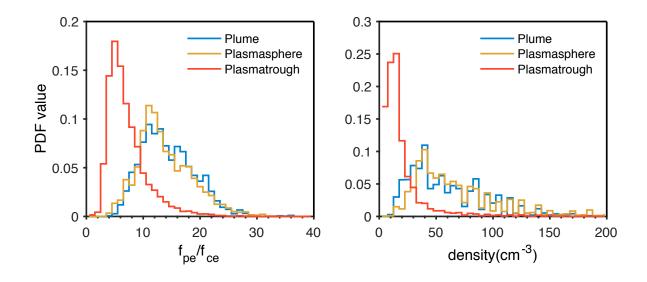
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- 545 **Figure 2.** The L-MLT distribution of the occurrence rate of rising tone whistler mode waves
- 546 observed inside the plasmasphere (top), plasmaspheric plumes (middle), and plasmatrough
- 547 (bottom) for different levels of AL*. The corresponding distribution of number of samples is
- 548 shown in the small panels. The bin size is 1 hour in MLT and 0.5 in L.

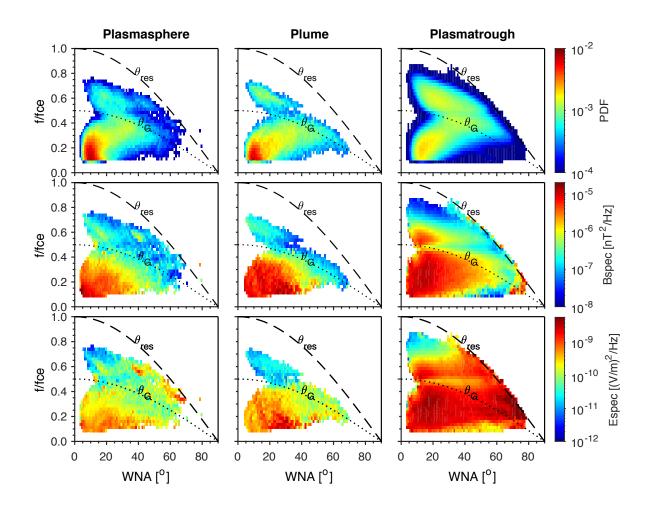
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Figure 3. The probability distribution function of density and the ratio of plasma frequency (f_{pe}) to electron cyclotron frequency (f_{ce}) for rising tone whistler mode wave events in three different

regions: inside the plasmasphere (yellow), plasmaspheric plumes (blue) and plasmatrough (red).



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Figure 4. The normalized frequency and wave normal angle distribution of rising tone whistler mode waves in three different regions: inside the plasmasphere (left column), in plasmaspheric plumes (middle column), and plasmatrough (right column). The top to bottom rows are the probability distribution function (PDF) of wave occurrences (top), the average magnetic (middle) and electric power spectral density (bottom). The dashed lines indicate the resonance cone angle estimated using $\arccos(f/f_{ce})$, and the dotted lines indicate the Gendrin angle calculated using

561 $\operatorname{arccos}(2f/f_{ce})$.