The Role of Strategic Load Participants in Two-Stage Settlement Electricity Markets

Pengcheng You, Dennice F. Gayme, and Enrique Mallada

Abstract— **Two-stage electricity market clearing is designed to maintain market efficiency under ideal conditions, e.g., perfect forecast and nonstrategic generation. This work demonstrates that the individual strategic behavior of inelastic load participants in a two-stage settlement electricity market can deteriorate efficiency. Our analysis further implies that virtual bidding can play a role in alleviating this loss of efficiency by mitigating market power of strategic load participants. We use real-world market data from New York ISO to validate our theory.**

I. INTRODUCTION

Electricity markets are designed to complement physical power systems by utilizing prices or other monetary incentives to motivate efficient system operation. Wholesale electricity markets generally consist of two-stage settlement. The first stage is a day-ahead market where participants buy or sell electricity through bids or offers on an hourly basis. An independent system operator (ISO) determines the hourly generation and load schedules along with the corresponding day-ahead clearing prices for the next day. The second stage is a real-time market where participants trade in the same way at the real-time clearing prices on a smaller timescale, usually every five minutes, to offset any discrepancy between day-ahead commitments and actual generations/loads.

The day-ahead and real-time markets are tightly coupled via time-varying supply, demand and prices [1]. The two-stage settlement is designed to maintain equal day-ahead and real-time prices such that no speculator is able to perform arbitrage, i.e., to enforce the so-called *no-arbitrage* condition. However, the two stages are settled separately in practice and identical prices in the day-ahead and real-time markets are therefore not directly enforced [2]. The difference between a day-ahead price and its real-time counterpart is technically termed a price *spread*. Any nonzero spread is generally considered a loss of efficiency [3]. Situations that result in systematic nonzero spreads include external factors, such as load forecast errors [4], non-scheduled generator shutdowns or line maintenance, as well as internal market power generally exercised by strategic generators [5].

Transactions that are not intended for physical fulfillment in real time but holding financial positions for arbitrage are referred to as *virtual bids*. Virtual bids primarily consist of *decrement* bids similar to load bids that buy electricity in the day-ahead market with the obligation to sell back the same amount in the real-time market, as well as *increment* offers similar to generation offers that work exactly in the opposite way [6]. See [7]–[10] for various examples of virtual bidding strategies. Virtual bidding is a valuable component of the two-stage settlement design that contributes to increasing market liquidity and mitigating market power by allowing extra asset-free participants to compete in electricity markets. This practice has proven, through both real observation [11] and theoretical analysis [3], [12]–[14], to improve market efficiency by driving the day-ahead and real-time prices to converge.

Despite the aforementioned studies, little attention has yet been paid to strategic behavior of load participants in electricity markets, which may also play a role in degrading market efficiency. The load side is usually less regulated due to its inelasticity, which leaves load participants more freedom to make strategic decisions. Conceptually, even with inelastic demand, a load participant still enjoys the flexibility of two-stage settlement, which potentially enables it to exercise market power.

In this paper, we look at the role of strategic inelastic load participants that take advantage of the two-stage settlement mechanism. We first establish a simple two-stage settlement market model that assumes (fully regulated) nonstrategic generation to characterize the inherent connection between the day-ahead and real-time markets. The strategic behavior of load participants is then analyzed through a Cournot game. We further extend the framework to accommodate decrement bids in virtual bidding as a special case of strategic inelastic load participation in electricity markets. Real-world market data from New York ISO (NYISO) are employed for validation.

Our analysis has multiple implications. *First*, the proposed market model unveils the underlying mechanism that relates the no-arbitrage condition with market efficiency while maintaining realistic market settlement conditions such as the day-ahead cleared load being approximately equal to the total load for efficiency. *Second*, we identify adverse impacts of strategic behavior by inelastic load participants that induces negative spreads and deteriorates efficiency in electricity markets, despite perfect forecast and nonstrategic generation. *Third*, we show that virtual bidding is an effective solution to alleviating the loss of market efficiency caused by strategic load participants.

The rest of the paper is organized as follows. Section II introduces our electricity market model. The role of strategic behavior by inelastic load participants is then analyzed in Section III. The empirical validation using real-world data follows in Section IV. Section V concludes the paper.

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II. Electricity Markets Model

In this section we describe the proposed electricity market model for the two-stage settlement mechanism. Consider an electricity market that consists of a day-ahead market and a real-time market. Assume that the generation side is highly regulated and all generators are non-strategic, i.e., they reveal their true cost functions[1]. The generators are categorized into two sets based on whether they are sufficiently fast to participate in the real-time market. Let $\mathcal{F}$ and $\mathcal{S}$ respectively denote the two sets of fast-responsive and slow-responsive generators. Slow-responsive generators can only participate in the day-ahead market while fast-responsive generators are able to participate in both markets. For a fast-responsive generator $i \in \mathcal{F}$ that outputs $x_i^f \geq 0$ amount of power, we assume a quadratic cost function of the form

$$C_i^f(x_i^f) := \frac{\alpha_i^f}{2} x_i^{f2} + \beta_i^f x_i^f, \tag{1}$$

where $\alpha_i^f > 0$ and β_i^f are constant cost coefficients. Similarly, we denote the cost function of a slow-responsive generator $j \in \mathcal{S}$ by

$$C_j^s(x_j^s) := \frac{\alpha_j^s}{2} x_j^{s2} + \beta_j^s x_j^s, \tag{2}$$

where $x_j^s \geq 0$, $\alpha_j^s > 0$ and β_j^s are defined accordingly. Let $x^f := (x_i^f, i \in \mathcal{F})$ and $x^s := (x_j^s, j \in \mathcal{S})$.

A. Two-Stage Settlement

The two-stage settlement mechanism meets a total inelastic load of $d > 0$ by clearing it separately in the day-ahead market and the real-time market, both in an efficient way. Denote the day-ahead cleared portion as d^{DA} and the real-time cleared portion as d^{RT}, which satisfy $d = d^{DA} + d^{RT}$. In the slow-timescale day-ahead market, all the generators in $\mathcal{F}$ and $\mathcal{S}$ are involved to clear the load d^{DA} based on the following:

Day-ahead market clearing problem

$$\min_{x^f, x^s \geq 0} \quad \sum_{i \in \mathcal{F}} C_i^f(x_i^f) + \sum_{j \in \mathcal{S}} C_j^s(x_j^s) \tag{3a}$$

$$\text{s.t.} \quad \sum_{i \in \mathcal{F}} x_i^f + \sum_{j \in \mathcal{S}} x_j^s = d^{DA} \;\; : \;\; \lambda^{DA}, \tag{3b}$$

where λ^{DA} is the dual Lagrange multiplier for the equality constraint (3b). Due to strong convexity, (3) has a unique minimizer which we denote by (x^{f*}, x^{s*}). Since (3b) is affine, the KKT conditions suggest that all of the participating generators should have an identical marginal cost that equals the optimal dual Lagrange multiplier[2]:

$$\lambda^{DA} = \alpha_i^f x_i^{f*} + \beta_i^f = \alpha_j^s x_j^{s*} + \beta_j^s, \quad \forall i \in \mathcal{F}, j \in \mathcal{S}, \tag{4}$$

[1]In real electricity markets, piecewise linear generation offers are made as a proxy for true generation cost functions, which are assumed to be known by the ISO here for ease of analysis.

[2]For illustration purposes, throughout this paper we restrict our considerations to the case where the constraints $x_i^f \geq 0, i \in \mathcal{F}$ and $x_j^s \geq 0, j \in \mathcal{S}$ are not binding.

where we abuse λ^{DA} to denote its optimum. λ^{DA}, technically termed the *shadow price* in economics [15], is the minimum price to incentivize generators to output the desired amount of power, which captures marginal generation cost.

By combining (3b) and (4), we are able to extract

$$\lambda^{DA} = \alpha^{DA} d^{DA} + \beta^{DA}, \tag{5a}$$

where

$$\alpha^{DA} := \left(\sum_{i \in \mathcal{F}} \frac{1}{\alpha_i^f} + \sum_{j \in \mathcal{S}} \frac{1}{\alpha_j^s} \right)^{-1}, \tag{5b}$$

and

$$\beta^{DA} := \left(\sum_{i \in \mathcal{F}} \frac{1}{\alpha_i^f} + \sum_{j \in \mathcal{S}} \frac{1}{\alpha_j^s} \right)^{-1} \left(\sum_{i \in \mathcal{F}} \frac{\beta_i^f}{\alpha_i^f} + \sum_{j \in \mathcal{S}} \frac{\beta_j^s}{\alpha_j^s} \right). \tag{5c}$$

Here α^{DA} and β^{DA} serve as the aggregate pricing coefficients. The expressions in (5) implicitly reflect the *elasticity of supply*, defined as the responsiveness of the quantity of power supplied to a change in its price, in the day-ahead market. Basically, given the market price λ^{DA}, the generators are willing to output d^{DA} amount of power in total. In other words, to clear the day-ahead load d^{DA} in the market, the clearing price needs to be set at λ^{DA}.

The fast-timescale real-time market clears in the same way as the day-ahead market except that only fast-responsive generators in $\mathcal{F}$ are involved. Note that these generators have also participated in the day-ahead market and already been scheduled to output x^{f*}. Therefore, in order to clear the load d^{RT}, the real-time market solves the following optimization problem:

Real-time market clearing problem

$$\min_{\delta x^f} \quad \sum_{i \in \mathcal{F}} C_i^f(x_i^{f*} + \delta x_i^f) \tag{6a}$$

$$\text{s.t.} \quad \sum_{i \in \mathcal{F}} \delta x_i^f = d^{RT} \;\; : \;\; \lambda^{RT}. \tag{6b}$$

Here δx_i^f denotes the scheduled output adjustment from x^{f*} for generator $i \in \mathcal{F}$ and $\delta x^f := (\delta x_i^f, i \in \mathcal{F})$. λ^{RT} is the (optimal) dual Lagrange multiplier for the equality constraint (6b). Note that the cost of fast-responsive generators in the day-ahead market, i.e., $\sum_{i \in \mathcal{F}} (\frac{\alpha_i^f}{2} x_i^{f*2} + \beta_i^f x_i^{f*})$, should be subtracted from the objective function to represent exactly the total cost for clearing the real-time load d^{RT}. We ignore this constant term for brevity.

We denote the unique minimizer of (6) as δx^{f*} and deduce the following from the KKT conditions:

$$\begin{aligned} \lambda^{RT} &= \alpha_i^f (x_i^{f*} + \delta x_i^{f*}) + \beta_i^f \\ &= \alpha_i^f \delta x_i^{f*} + \lambda^{DA}, \quad \forall i \in \mathcal{F}, \end{aligned} \tag{7}$$

where the second equality follows directly from (4). Substituting (7) into (6b) yields

$$\lambda^{RT} = \alpha^{RT} d^{RT} + \beta^{RT}, \tag{8a}$$

where

$$\alpha^{RT} := \left(\sum_{i\in\mathcal{F}} \frac{1}{\alpha_i^f}\right)^{-1} \tag{8b}$$

and

$$\beta^{RT} := \lambda^{DA}. \tag{8c}$$

Here α^{RT} and β^{RT} are the aggregate pricing coefficients that embody the elasticity of supply in the real-time market. Meanwhile, (8) also unveils the inherent correlation between the day-ahead and real-time prices: the latter deviates from the former to account for the real-time cleared load. See Fig. 1. Formally, the price spread between the day-ahead and real-time prices is

$$\lambda^{DA} - \lambda^{RT} = -\left(\sum_{i\in\mathcal{F}} \frac{1}{\alpha_i^f}\right)^{-1} d^{RT}. \tag{9}$$

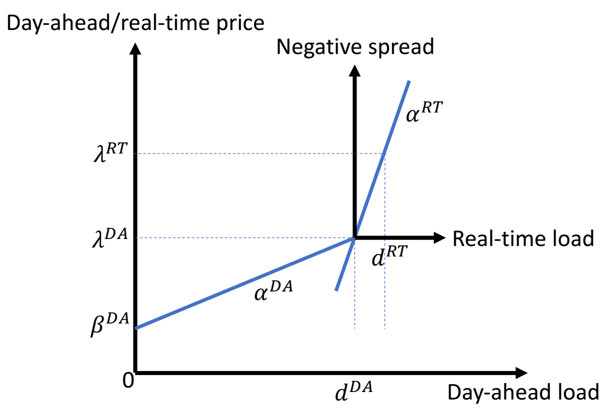

Fig. 1: Correlation between day-ahead and real-time prices.

***Remark** 1:* Notably, according to (5), (8), we always have $\alpha^{RT} > \alpha^{DA} > 0$, as Fig. 1 illustrates, due to a smaller subset of generators involved in the real-time market. This is consistent with the observation that real-time prices are more volatile than day-ahead prices, since the real-time market typically has a smaller price elasticity of supply than the day-ahead market, i.e., the quantity of power supply in the real-time market is less sensitive to a change in its price than that in the day-ahead market.

B. Market Efficiency

We next formalize our definition of market efficiency. Given all the available generators in $\mathcal{F}$ and $\mathcal{S}$, we define market efficiency as the minimization of social cost to meet the total inelastic load d, which is specifically realized by solving the following:

Social cost minimization problem

$$\min_{x^f, x^s \geq 0} \quad \sum_{i\in\mathcal{F}} C_i^f(x_i^f) + \sum_{j\in\mathcal{S}} C_j^s(x_j^s) \tag{10a}$$

$$\text{s.t.} \quad \sum_{i\in\mathcal{F}} x_i^f + \sum_{j\in\mathcal{S}} x_j^s = d \ : \ \lambda, \tag{10b}$$

i.e., jointly optimizing the dispatch of all the generators across the two markets. We define the (optimal) dual Lagrange multiplier λ for the equality constraint (10b) and denote the unique minimizer of (10) by $(x^{f\#}, x^{s\#})$. The KKT conditions require

$$\lambda = \alpha_i^f x_i^{f\#} + \beta_i^f = \alpha_j^s x_j^{s\#} + \beta_j^s, \quad \forall i \in \mathcal{F}, j \in \mathcal{S}, \tag{11}$$

i.e., equal marginal cost for all the participating generators, to achieve efficiency.

Recall that the day-ahead price equals the marginal cost of slow-responsive generators in (4) while the real-time price equals the marginal cost of fast-responsive generators in (7). By comparing them with the indicator of market efficiency (11), we are able to conclude the following theorem:

***Theorem** 1:* In the two-stage settlement electricity market, efficiency can only be realized at

$$\lambda^{DA} = \lambda^{RT} = \lambda \tag{12}$$

when the day-ahead and real-time prices equalize, which further implies

$$d^{DA} = d, \quad d^{RT} = 0. \tag{13}$$

Theorem 1 matches exactly the intuition of the two-stage settlement design: all (forecast) load should be cleared in the day-ahead market while the real-time market accounts for any load deviation from the forecast. It also suggests that efficiency is consistent with the no-arbitrage condition between the two-stage markets, guaranteed by the zero spread from (12), which is necessary for the market model to be realistic.

Similar models for the two-stage settlement mechanism have been used in [4], [16], [17]. However, our simple model further addresses several issues that are missing in these previous works, e.g., the fact that the day-ahead cleared load should amount to the total load is not accounted for in [16], [17]; the correlation between the no-arbitrage condition and market efficiency is not demonstrated in [4].

III. Strategic Load Participant

Given the two-stage settlement mechanism, an electricity market should clear all load in the day-ahead market and zero load in the real-time market in order to achieve efficiency. However, we observe in the NYISO market that there is an obvious positive bias for real-time loads throughout the year of 2018, as shown in Fig 2, which cannot be accounted for by uncertainties. We attribute this loss of efficiency to the strategic behavior by inelastic load participants and next investigate their market power by taking advantage of the two-stage settlement mechanism. Ideal assumptions of perfect forecast and nonstrategic generation are made to focus our attention on the impact of strategic load. As we will see below, our analysis extends naturally to accommodate the role of virtual bidding.

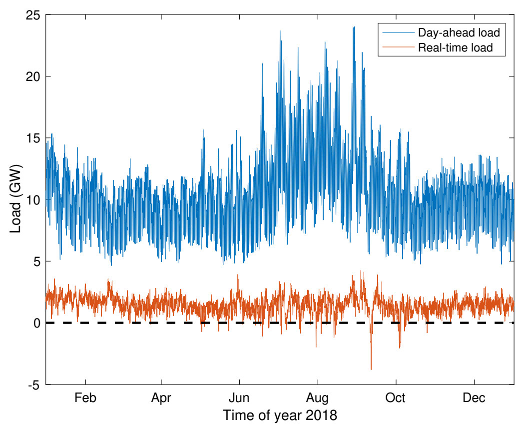

Fig. 2: Day-ahead and real-time cleared loads, NYISO, 2018.

A. Single Load

We start with the simplest case where there is only one single (ensemble of) inelastic load d to be cleared. It has the option to participate in one or both of the day-ahead and real-time markets to meet its demand. The participation of the load in the two markets affects the market clearing prices, which in turn determine its cost. For analysis purposes, we assume the load has full knowledge of the supply elasticity of both markets, i.e., it knows the exact values of $\alpha^{DA}, \beta^{DA}, \alpha^{RT}, \beta^{DA}$, e.g., through estimates based on long-term historical data[3].

A strategic load will anticipate the impact of its decision on the two markets and minimize accordingly the expenditure of purchasing electricity to meet its demand. Formally, it solves the following:

Single load cost minimization problem[4]

$$\min_{d^{DA}\geq 0, d^{RT}} \quad \lambda^{DA}(d^{DA})\cdot d^{DA} + \lambda^{RT}(d^{DA}, d^{RT})\cdot d^{RT} \tag{14a}$$

$$\text{s.t.} \quad d^{DA} + d^{RT} = d. \tag{14b}$$

***Theorem** 2:* The optimal load participation for a single load in the two-stage settlement electricity market is uniquely determined by

$$d^{DA*} = \left(1 - \frac{\alpha^{DA}}{2\alpha^{RT}}\right) d, \quad d^{RT*} = \frac{\alpha^{DA}}{2\alpha^{RT}}\, d. \tag{15}$$

Therefore, $\lambda^{RT} > \lambda^{DA}$ and a strictly negative spread, as defined in (9), follow.

Proof: First of all, we relax the constraint $d^{DA} \geq 0$ and show that it is not binding at optimum. Given the explicit expressions of $\lambda^{DA}(d^{DA})$ and $\lambda^{RT}(d^{DA}, d^{RT})$ in (5) and (8), we can substitute (14b) into (14a) to reorganize the objective function in terms of d^{DA} only:

$$\begin{aligned}
&\lambda^{DA}(d^{DA})\cdot d^{DA} + \lambda^{RT}(d^{DA}, d^{RT})\cdot d^{RT}\\
=\ &\lambda^{DA}(d^{DA})\cdot d^{DA} + (\alpha^{RT} d^{RT} + \lambda^{DA}(d^{DA}))\cdot d^{RT}\\
=\ &(\alpha^{DA} d^{DA} + \beta^{DA}) d + \alpha^{RT}(d - d^{DA})^2\\
=\ &\alpha^{RT} d^{DA2} + (\alpha^{DA} d - 2\alpha^{RT} d) d^{DA} + \beta^{DA} d + \alpha^{RT} d^2.
\end{aligned}$$

The unique minimizer to the above unconstrained optimization is straightforwardly obtained by the first-order optimality condition, i.e., (15). Recall $\alpha^{RT} > \alpha^{DA} > 0$, which implies $d > d^{DA*} > \frac{1}{2}d > 0$ and $\frac{1}{2}d > d^{RT*} > 0$. The relaxed constraint is satisfied and (15) is also the unique optimum of (14). ■

***Remark** 2:* The negative spread indicates the loss of market efficiency caused by the strategic behavior of a single inelastic load participant in the two-stage settlement electricity market. Meanwhile, the strictly positive load participation in the real-time market coincides with the observation of the positive bias for real-time loads in Fig. 2.

The single-load case serves as a toy example. Next we proceed to characterize the general case with market competition and analyze its impact on efficiency.

B. Load-Side Cournot Competition

We extend the above analysis to the case with multiple individual strategic loads, e.g., different local utility companies in a market. Let $\mathcal{L} := \{1, 2, \ldots, L\}$ be the set of these loads. Each load $l \in \mathcal{L}$ can independently determine its participation, $\vec{d_l} := (d_l^{DA} \geq 0, d_l^{RT})$, in the day-ahead and real-time markets in order to satisfy its inelastic demand d_l with

$$d_l^{DA} + d_l^{RT} = d_l, \quad l \in \mathcal{L}. \tag{16}$$

Let $\vec{d} := (\vec{d_l}, l \in \mathcal{L})$ be the aggregate decisions of all the loads. Further denote the aggregate decisions of all the other loads except load l as $\vec{d}_{-l}$. Suppose that all the loads are aware of the mechanism that determines market prices, i.e.,

$$\lambda^{DA} = \alpha^{DA} d^{DA} + \beta^{DA}, \quad \lambda^{RT} = \alpha^{RT} d^{RT} + \beta^{RT}, \tag{17}$$

where $d^{DA} := \sum_{l\in\mathcal{L}} d_l^{DA}$ and $d^{RT} := \sum_{l\in\mathcal{L}} d_l^{RT}$. Define $d := \sum_{l\in\mathcal{L}} d_l$ as the total load to be cleared. Each load l will aim to minimize its expenditure of purchasing electricity from the two markets to meet demand given other loads' decisions, i.e.,

$$\min_{\vec{d_l}} \quad c_l(\vec{d_l}; \vec{d}_{-l}) := \lambda^{DA}(\vec{d})\cdot d_l^{DA} + \lambda^{RT}(\vec{d})\cdot d_l^{RT} \tag{18a}$$

$$\text{s.t.} \quad (16). \tag{18b}$$

These loads compete in quantities of participation in the two markets that affect market clearing prices and seek to minimize individual cost, which can be formalized as a Cournot game:

Load-side Cournot game

Players: each load $l \in \mathcal{L}$;

Strategies: load participation $\vec{d_l}$ in the day-ahead and real-time markets to satisfy (16);

Costs: expenditure of purchasing electricity $c_l(\vec{d_l}; \vec{d}_{-l})$.

[3]Since the set of participating generators in an electricity market and their cost functions are usually stable subject to subtle changes in the long run, it is reasonable to argue those coefficients that characterize the dependence of market prices on the amount of cleared load are approximately constant and easy to estimate.

[4]Note that on the load side we constrain nonnegative load participation in the day-ahead market to maintain the identity as a load.

Definition 1: $\vec{d}^*$ is a Nash equilibrium of the load-side Cournot game if it satisfies $c_l(\vec{d}^*) \leq c_l(\vec{d}_l; \vec{d}^*_{-l})$, $\forall l \in \mathcal{L}$.
At a Nash equilibrium, no load has the incentive to deviate from its current decision unilaterally, given other loads' decisions. In order to characterize the Nash equilibrium of the load-side Cournot game, we first propose the following lemma:

Lemma 1: There do not exist equilibria of the load-side Cournot game where $d_l^{DA*} = 0$ for some $l \in \mathcal{L}$.
Refer to the appendix for the proof of Lemma 1. Given Lemma 1, the possibility of Nash equilibria with any of the constraints $d_l^{DA} \geq 0,\ l \in \mathcal{L}$ binding is excluded and we next prove the existence and uniqueness for the Nash equilibrium of the load-side Cournot game by ignoring these constraints:

Theorem 3: In the two-stage settlement electricity market, there exists a unique Nash equilibrium of the load-side Cournot game, characterized by

$$
\begin{aligned}
d_l^{DA*} &= \left(1 - \frac{L\alpha^{DA}}{(L+1)\alpha^{RT}}\right) d_l + \frac{\alpha^{DA}}{(L+1)\alpha^{RT}} \sum_{k \in \mathcal{L}\setminus\{l\}} d_k, \\
d_l^{RT*} &= \frac{L\alpha^{DA}}{(L+1)\alpha^{RT}} d_l - \frac{\alpha^{DA}}{(L+1)\alpha^{RT}} \sum_{k \in \mathcal{L}\setminus\{l\}} d_k,
\end{aligned}
\tag{19}
$$

for $\forall l \in \mathcal{L}$.

Proof: Given (16) and (17), the expenditure function $c_l(\vec{d}_l; \vec{d}_{-l})$ of each load l in (18) can be rewritten explicitly in terms of d_l^{DA} only as follows.

$$
\begin{aligned}
& \lambda^{DA} d_l^{DA} + \lambda^{RT} d_l^{RT} \\
= \ & (\alpha^{DA} \sum_{k \in \mathcal{L}} d_k^{DA} + \beta^{DA}) d_l^{DA} \\
& + (\alpha^{RT}(d - \sum_{k \in \mathcal{L}} d_k^{DA}) + \beta^{RT})(d_l - d_l^{DA}) \quad (20a) \\
= \ & (\alpha^{DA} \sum_{k \in \mathcal{L}} d_k^{DA} + \beta^{DA}) d_l \\
& + \alpha^{RT}(d - \sum_{k \in \mathcal{L}} d_k^{DA})(d_l - d_l^{DA}) \quad (20b) \\
= \ & \alpha^{DA} d_l d_l^{DA} + \alpha^{RT}(d_l - d_l^{DA})^2 \\
& + \alpha^{RT} \sum_{k \in \mathcal{L}\setminus\{l\}} (d_k - d_k^{DA})(d_l - d_l^{DA}) \\
& + \alpha^{DA} d_l \sum_{k \in \mathcal{L}\setminus\{l\}} d_k^{DA} + \beta^{DA} d_l, \quad (20c)
\end{aligned}
$$

where (20b) follows from $\beta^{RT} = \lambda^{DA}$. Given Lemma 1 and the strict convexity of the expenditure function $c_l(\vec{d}_l; \vec{d}_{-l})$ in d_l^{DA}, the Nash equilibrium of the load-side Cournot game can be characterized by imposing the first-order optimality condition on all the loads, i.e., for $\forall l \in \mathcal{L}$,

$$
\alpha^{DA} d_l - 2\alpha^{RT}(d_l - d_l^{DA*}) - \alpha^{RT} \sum_{k \in \mathcal{L}\setminus\{l\}} (d_k - d_k^{DA*}) = 0, \tag{21}
$$

or equivalently,

$$
d_l^{DA*} = \left(1 - \frac{\alpha^{DA}}{2\alpha^{RT}}\right) d_l + \frac{1}{2} \sum_{k \in \mathcal{L}\setminus\{l\}} (d_k - d_k^{DA*}). \tag{22}
$$

Note that the first term on the right-hand side of (22) is exactly the individual optimum without any competitors in (15) while the second term represents the influence of competition. Intuitively, if other loads participate more in the real-time market, load l will increase its participation in the day-ahead market to hedge against the rising real-time price.

Combining (22) for all $l \in \mathcal{L}$ naturally yields the unique solution (19). We can readily observe $d_l^{DA*} > 0$, which corroborates Lemma 1. The theorem follows. ■

By summing (19) over $\mathcal{L}$ and reorganizing the expression, we are able to derive the following:

Corollary 1: At the Nash equilibrium of the load-side Cournot game, the total day-ahead load and real-time load are respectively

$$
\begin{aligned}
\sum_{l \in \mathcal{L}} d_l^{DA*} &= \left(1 - \frac{\alpha^{DA}}{(L+1)\alpha^{RT}}\right) \sum_{l \in \mathcal{L}} d_l, \\
\sum_{l \in \mathcal{L}} d_l^{RT*} &= \frac{\alpha^{DA}}{(L+1)\alpha^{RT}} \sum_{l \in \mathcal{L}} d_l,
\end{aligned}
\tag{23}
$$

which implies $d > d^{DA*} > \frac{L}{L+1} d$ and $\frac{1}{L+1} d > d^{RT*} > 0$. Therefore, $\lambda^{RT} > \lambda^{DA}$ and a strictly negative spread follow.

Remark 3: Notably, the optimal load participation (15) in the single-load case is a special case of (23) where $L = 1$. Corollary 1 generalizes the conclusion to multi-load cases that the strategic behavior of load participants even with inelastic demand hurts market efficiency by taking advantage of the two-stage settlement mechanism. However, as the number of load participants L increases, the total day-ahead load approaches the total load and the spread diminishes towards zero, meaning the restoration of market efficiency. This is consistent with the intuition that when there are infinite participants, the individual impact on market prices becomes negligible and therefore the market power of each strategic load vanishes, which drives the market to be competitive.

C. The Role of Virtual Bidding

Virtual bidding is an essential part of efficient electricity markets as it mitigates market power. Virtual bidders profit from arbitrage on nonzero spreads. As analyzed above, we have identified systematic negative spreads that result from the strategic behavior of load participants. However, through an extended analysis of the prior load-side Cournot competition, we now show decrement bids in virtual bidding that act like load participation play an important role in driving these spreads to zeros.

In particular, consider a set $\mathcal{V} := \{1, 2, \ldots, V\}$ of virtual bidders. They individually determine their participation $(d_v^{DA}, d_v^{RT}), v \in \mathcal{V}$ to compete in the day-ahead and real-time markets in pursuit of arbitrage. However, they differ from real load participants $l \in \mathcal{L}$ in that no real demand needs to be satisfied, i.e., $d_v = 0$, $v \in \mathcal{V}$. The following theorem characterizes the involvement of these virtual bidders in the load-side Cournot game:

***Theorem** 4:* In the two-stage settlement electricity market, there exists a unique Nash equilibrium of the load-side Cournot game with virtual bidders, where the virtual bids are given by

$$\begin{aligned} d_v^{DA*} &= \frac{\alpha^{DA}}{(L+V+1)\alpha^{RT}} \sum_{l\in\mathcal{L}} d_l, \\ d_v^{RT*} &= -\frac{\alpha^{DA}}{(L+V+1)\alpha^{RT}} \sum_{l\in\mathcal{L}} d_l, \end{aligned} \tag{24}$$

for $\forall v \in \mathcal{V}$.
Here $d_v^{DA*} > 0$ represents a decrement bid.

***Corollary** 2:* At the Nash equilibrium of the load-side Cournot game with virtual bidders, the total day-ahead load and real-time load are respectively

$$\begin{aligned} \sum_{l\in\mathcal{L}} d_l^{DA*} + \sum_{v\in\mathcal{V}} d_v^{DA*} &= \left(1 - \frac{\alpha^{DA}}{(L+V+1)\alpha^{RT}}\right) \sum_{l\in\mathcal{L}} d_l, \\ \sum_{l\in\mathcal{L}} d_l^{RT*} + \sum_{v\in\mathcal{V}} d_v^{RT*} &= \frac{\alpha^{DA}}{(L+V+1)\alpha^{RT}} \sum_{l\in\mathcal{L}} d_l, \end{aligned} \tag{25}$$

which implies $d > d^{DA*} > \frac{L+V}{L+V+1} d$ and $\frac{1}{L+V+1} d > d^{RT*} > 0$. As the number of virtual bidders V goes to infinity, the total day-ahead load is driven to approach the total load and the spread converges to zero.

***Remark** 4:* Virtual bidders have the incentive to arbitrage in the two-stage settlement electricity market, which in turn contributes to alleviating the loss of market efficiency resulting from strategic behavior of load participants by driving the two-stage market prices to equalize.

***Remark** 5:* From (25), the real demand from load participants in the day-ahead market remains positive but actually decreases with the number of virtual bidders V, as captured below:

$$\sum_{l\in\mathcal{L}} d_l^{DA*} = \left(1 - \frac{(V+1)\alpha^{DA}}{(L+V+1)\alpha^{RT}}\right) \sum_{l\in\mathcal{L}} d_l. \tag{26}$$

IV. Real-World Data Validation

We employ real-world electricity market data from NYISO to verify the extent to which our model and analysis reflect real market conditions.

A. Electricity Market Model

Day-ahead loads and prices as well real-time loads and prices are collected for the whole year of 2018[5]. Note that uniform *energy clearing prices* are adopted instead of locational marginal prices since emphasis is laid on the two-stage settlement mechanism rather than physical constraints. Fig. 3 is a scatterplot of day-ahead prices with respect to day-ahead loads. As (5) suggests, a day-ahead price should be linear in the corresponding day-ahead load. The linear regression result in Table I shows that both of the pricing coefficients α^{DA} and β^{DA} are statistically significant. Fig. 4 is a scatterplot of negative spreads, i.e., $\lambda^{RT} - \lambda^{DA}$, with respect to real-time loads to justify the connection between the day-ahead and real-time prices, identified in (9). A multiple linear regression of real-time prices on real-time loads and day-ahead prices is carried out with the result in Table II that confirms that the linearity approximately holds. As analyzed, the coefficient γ for day-ahead prices is almost 1 and $\alpha^{RT} > \alpha^{DA}$ is observed. However, the proposed model cannot account for the negative intercept δ. This could be caused by factors that our analysis neglects, such as strategic generation.

[5]Note that several periods of time, such as Jan. 1-9 and May 21-31, that exhibit extremely abnormal price elasticity of supply are removed.

TABLE I: Linear regression for $\lambda^{DA} = \alpha^{DA} d^{DA} + \beta^{DA}$

	Estimate	Standard error	p-value	RMSE	R^2
α^{DA}	2.4535	0.0208	< 0.001	5.7128	0.6518
β^{DA}	0.7848	0.2253	< 0.001		

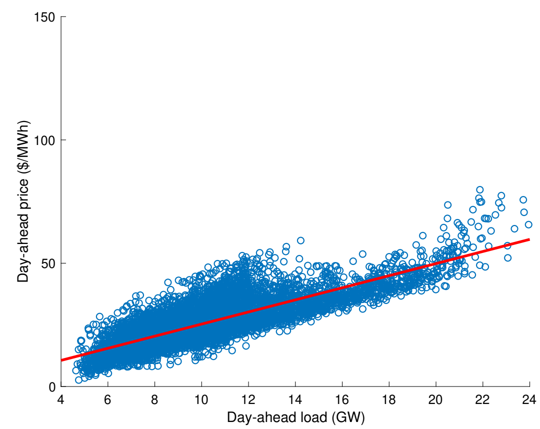

Fig. 3: Day-ahead price with respect to day-ahead load, NYISO, 2018.

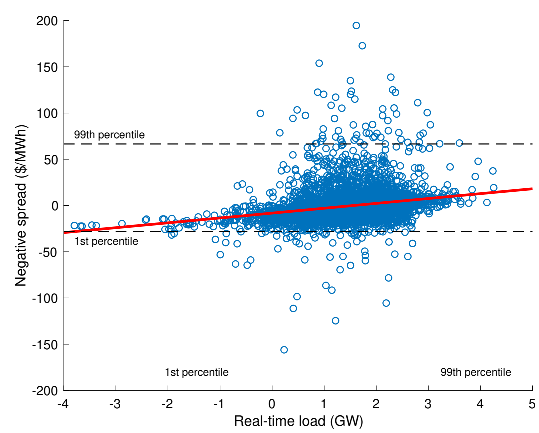

Fig. 4: Negative spread with respect to real-time load, NYISO, 2018.

TABLE II: Linear regression for $\lambda^{RT} = \alpha^{RT} d^{RT} + \gamma \lambda^{DA} + \delta$

	Estimate	Standard error	p-value	RMSE	R^2
α^{RT}	5.2658	0.1833	< 0.001	10.2941	0.4444
γ	1.0009	0.0132	< 0.001		
δ	-8.2569	0.4980	< 0.001		

B. Virtual Bidding

To corroborate our analysis of strategic load participants, we use the special case of virtual bidding due to its significant and verifiable impact on market clearing. The mechanism of virtual bidding was officially introduced into the NYISO market in November, 2001 [18]. We collected available data of real loads cleared in the day-ahead market and total actual loads for the several months around that time point to validate the deduction in Remark 5. It is reasonable to assume $V = 0$ prior to the introduction of virtual bidding while $V > 0$ thereafter. As a result, the proportion $1 - \frac{(V+1)\alpha^{DA}}{(L+V+1)\alpha^{RT}}$ is anticipated to diminish with virtual bidding introduced, which is precisely captured by the sudden drop in Fig. 5[6].

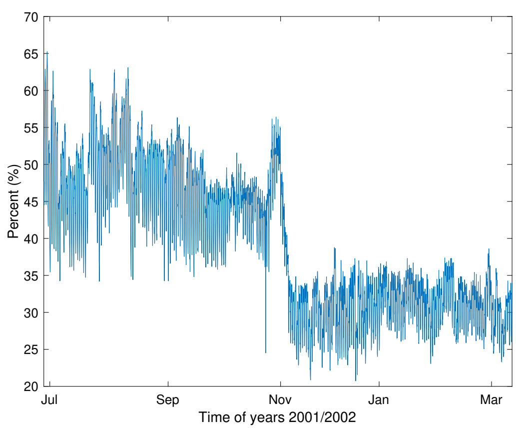

Fig. 5: Percentage of day-ahead real load in total load, NYISO, 2001/2002. The introduction of virtual bidding in November, 2001 caused a sudden drop in this percentage.

V. Concluding Remarks

This paper develops a model for two-stage settlement electricity markets that explicitly characterizes the interconnection between day-ahead and real-time markets. Given the model, we attribute systematic negative spreads in electricity markets to the strategic behavior of inelastic load participants that takes advantage of the two-stage settlement mechanism. We therefore argue that strategic load participation in electricity markets should be taken into account in the characterization of nonzero spreads, in addition to empirical factors like load forecast errors or market power of strategic generators. Our analysis generalizes to accommodate virtual bidding and demonstrates its role in improving market efficiency by mitigating market power. Real-world market data from NYISO justify our argument.

Our model and analysis focus on strategic behavior by inelastic load participants only and are thus not able to account for other factors that can also result in loss of market efficiency. A more comprehensive framework is the subject of ongoing work.

[6]Note that the total load in the NYISO market includes a significant part that is cleared through bilateral transactions outside the market. Therefore, the overall percentage is low.

References

[1] J. Pang, P. You, and M. Chen, "Temporally networked Cournot platform markets," in *Proc. of 51st Hawaii International Conf. on System Sciences (HICSS)*, pp. 3427–3436, 2018.

[2] New York ISO, "Energy market & operational data." https://www.nyiso.com/energy-market-operational-data. Accessed: 2019-03-17.

[3] W. W. Hogan, "Virtual bidding and electricity market design," *The Electricity Journal*, vol. 29, no. 5, pp. 33–47, 2016.

[4] W. Tang, R. Rajagopal, K. Poolla, and P. Varaiya, "Model and data analysis of two-settlement electricity market with virtual bidding," in *Proc. of 55th IEEE Conf. on Decision and Control (CDC)*, pp. 6645–6650, 2016.

[5] N. A. Ruhi, K. Dvijotham, N. Chen, and A. Wierman, "Opportunities for price manipulation by aggregators in electricity markets," *IEEE Trans. on Smart Grid*, vol. 9, no. 6, pp. 5687–5698, 2018.

[6] PJM Interconnection, "Virtual transactions in the PJM energy market." https://www.pjm.com/-/media/committees-groups/committees/mc/20151019-webinar/20151019-item-02-virtual-transactions-in-the-pjm-energy-markets-whitepaper.ashx. Accessed: 2019-03-08.

[7] S. Baltaoglu, L. Tong, and Q. Zhao, "Algorithmic bidding for virtual trading in electricity markets," *IEEE Trans. on Power Systems*, vol. 34, no. 1, pp. 535–543, 2019.

[8] E. Mashhour and S. M. Moghaddas-Tafreshi, "Bidding strategy of virtual power plant for participating in energy and spinning reserve markets—Part I: Problem formulation," *IEEE Trans. on Power Systems*, vol. 26, no. 2, pp. 949–956, 2011.

[9] E. Mashhour and S. M. Moghaddas-Tafreshi, "Bidding strategy of virtual power plant for participating in energy and spinning reserve markets—Part II: Numerical analysis," *IEEE Trans. on Power Systems*, vol. 26, no. 2, pp. 957–964, 2011.

[10] M. Rahimiyan and L. Baringo, "Strategic bidding for a virtual power plant in the day-ahead and real-time markets: A price-taker robust optimization approach," *IEEE Trans. on Power Systems*, vol. 31, no. 4, pp. 2676–2687, 2016.

[11] L. Hadsell, "The impact of virtual bidding on price volatility in New York's wholesale electricity market," *Economics Letters*, vol. 95, no. 1, pp. 66–72, 2007.

[12] A. G. Isemonger, "The benefits and risks of virtual bidding in multi-settlement markets," *The Electricity Journal*, vol. 19, no. 9, pp. 26–36, 2006.

[13] M. Celebi, A. Hajos, and P. Q. Hanser, "Virtual bidding: The good, the bad and the ugly," *The Electricity Journal*, vol. 23, no. 5, pp. 16–25, 2010.

[14] J. Mather, E. Bitar, and K. Poolla, "Virtual bidding: Equilibrium, learning, and the wisdom of crowds," *IFAC-PapersOnLine*, vol. 50, no. 1, pp. 225–232, 2017.

[15] D. S. Kirschen and G. Strbac, *Fundamentals of power system economics*. John Wiley & Sons, 2018.

[16] H. Bessembinder and M. L. Lemmon, "Equilibrium pricing and optimal hedging in electricity forward markets," *the Journal of Finance*, vol. 57, no. 3, pp. 1347–1382, 2002.

[17] S. Zhang, C. Chung, K. Wong, and H. Chen, "Analyzing two-settlement electricity market equilibrium by coevolutionary computation approach," *IEEE Trans. on Power Systems*, vol. 24, no. 3, pp. 1155–1164, 2009.

[18] B. de Mello and N. Bouchez, "Virtual bidding in the nyiso." https://www.caiso.com/Documents/VirtualBiddinginNYISO-Presentation.pdf. Accessed: 2019-03-17.

APPENDIX

PROOF OF LEMMA 1

Proof: We prove this lemma by contradiction. Assume there exists a Nash equilibrium where $d_l^{DA*} = 0$ for some $l \in \mathcal{L}$. Define $\mathcal{L}'$ as the set of the remaining loads with $d_l^{DA*} > 0$ and let $L' := |\mathcal{L}'|$. Note that the expenditure function $c_l(\vec{d}_l; \vec{d}_{-l})$ of each load $l \in \mathcal{L}$ can be rewritten as a strictly convex function in terms of d_l^{DA} only; see (20). For each load $l \in \mathcal{L} \backslash \mathcal{L}'$ with $d_l^{DA*} = 0$, the first-order optimality condition does not hold, i.e.,

$$\begin{aligned} d_l^{DA*} = 0 &\geq (1 - \frac{\alpha^{DA}}{2\alpha^{RT}}) d_l + \frac{1}{2} \sum_{k \in \mathcal{L} \backslash \{l\}} (d_k - d_k^{DA*}) \\ &= \frac{\alpha^{RT} - \alpha^{DA}}{2\alpha^{RT}} d_l + \frac{1}{2} \sum_{k \in \mathcal{L}} d_k - \frac{1}{2} \sum_{k \in \mathcal{L}'} d_k^{DA*}. \end{aligned} \tag{27}$$

However, the first-order optimality condition holds for the loads $l \in \mathcal{L}'$, which can be expressed as

$$d_l^{DA*} = \frac{\alpha^{RT} - \alpha^{DA}}{2\alpha^{RT}} d_l + \frac{1}{2} \sum_{k \in \mathcal{L}} d_k - \frac{1}{2} \sum_{k \in \mathcal{L}' \backslash \{l\}} d_k^{DA*}, \tag{28}$$

where we replace $\mathcal{L}$ with $\mathcal{L}'$ since $d_l^{DA*} = 0, l \in \mathcal{L} \backslash \mathcal{L}'$. Summing (28) over $\mathcal{L}'$ and reorganizing the expression lead to

$$\begin{aligned} \sum_{l \in \mathcal{L}'} d_l^{DA*} &= \left(1 - \frac{\alpha^{DA}}{(L'+1)\alpha^{RT}}\right) \sum_{l \in \mathcal{L}'} d_l + \frac{L'}{L'+1} \sum_{l \in \mathcal{L} \backslash \mathcal{L}'} d_l \\ &< \sum_{l \in \mathcal{L}} d_l - \frac{\alpha^{DA}}{(L'+1)\alpha^{RT}} \sum_{l \in \mathcal{L}'} d_l \ < \ \sum_{l \in \mathcal{L}} d_l. \end{aligned} \tag{29}$$

Substituting (29) into (27) yields a contradiction:

$$0 \geq \frac{\alpha^{RT} - \alpha^{DA}}{2\alpha^{RT}} d_l + \frac{1}{2} \sum_{k \in \mathcal{L}} d_k - \frac{1}{2} \sum_{k \in \mathcal{L}'} d_k^{DA*} > 0, \tag{30}$$

where the second inequality also uses $\alpha^{RT} > \alpha^{DA}$; recall Fig. 1. The preliminary assumption is rejected and therefore the lemma is proved. ■