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TIME DELAY CONTROL OF A HIGH-DOF ROBOT MANIPULATOR THROUGH FEEDBACK LINEARIZATION BASED PREDICTOR

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ABSTRACT

We formulate a predictor-based controller for a high-DOF manipulator to compensate a time-invariant input delay during a pick-and-place task. Robot manipulators are widely used in telemanipulation systems on the account of their reliable, fast, and precise motions while they are subject to large delays. Using common control algorithms on such delay systems can cause not only poor control performance, but also catastrophic instability in engineering applications. Therefore, delays need to be compensated in designing robust control laws. As a case study, we focus on a 7-DOF Baxter manipulator subject to three different input delays. First, delay-free dynamic equations of the Baxter manipulator are derived using the Lagrangian method. Then, we formulate a predictor-based controller, in the presence of input delay, in order to track desired trajectories. Finally, the effects of input delays in the absence of a robust predictor are investigated, and then the performance of the predictor-based controller is experimentally evaluated to reveal robustness of the algorithm formulated. Simulation and experimental results demonstrate that the predictor-based controller effectively compensates input delays and achieves closed-loop stability.

1 Introduction

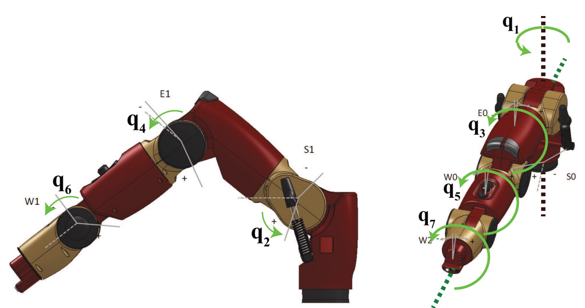
Robot manipulators are widely used in various applications to track desired trajectories, particularly in telemanipulation systems, on the account of their reliable, fast, and precise motions in executing tasks such as moving debris and turning valves [1]. Remote manipulators provide the capability of executing tasks safely at an unreachable/dangerous location while they are subject to large input delays as with many engineering systems. Interest in delay, as a common dynamic phenomenon, is driven by applications in modeling and control of traffic systems [2], teleoperators [3–5], vehicles [6], and robot manipulators [7, 8].

The detrimental impact of time delay is well-established, which plays the most significant role in degrading remote perception and manipulation. Large input delays often arise from communication delay between sensor and actuator, or from time-consuming computational burden of multi-agent networks. For instance, the foremost concern of vision-based control is tackling the delay introduced by image acquisition and image processing. One of the earliest challenges in engineering has been the control of systems subject to delays. Note that a common approach to tackle and handle this problem is the use of predictive algorithms. In 1959, Smith presented the delay compensator known as the Smith predictor [9]. However, in some cases, the Smith predictor – a modification of a nominal controller designed to stabilize the delay-free system – may fail to achieve the closed-loop stability when the plant is unstable [10].

Many studies in recent years were carried out for linear systems subject to the input delays [11–20]. In addition to many researches on linear systems, the further recent developments

TABLE 1. Baxter's Denavit-Hartenberg Parameters

Link/Joint	a_i	d_i	α_i	θ_i
$1/S_0$	0.069	0.27035	$-\pi/2$	θ_1
$2/S_1$	0	0	$\pi/2$	$\theta_2+\pi/2$
$3/E_0$	0.069	0.36435	$-\pi/2$	θ_3
$4/E_1$	0	0	$\pi/2$	θ_4
$5/W_0$	0.010	0.37429	$-\pi/2$	θ_5
$6/W_1$	0	0	$\pi/2$	θ_6
$7/W_2$	0	0.3945	0	θ_7



(a) (b)

FIGURE 1. The 7-DOF Baxter manipulator: (a) The joints configuration; (b) sagittal view

of predictor-based control laws for nonlinear systems with input delays can be found in [21–30]. Motivated by the harmful consequences of input delays on the stability and performance of such control systems, we formulate and implement a predictor-feedback controller [31] for the compensation of large input delays in a multi-input highly nonlinear system – the 7-DOF Baxter manipulator as a case study. We reasonably assume that all input channels induce the same delay due to the fact that it is practically impossible to have different delays for the robot with highly coupled dynamics.

This paper is organized as follows. We begin with a brief mathematical modeling of the system in Section 2, along with deriving dynamics equations, in order to formulate the predictor-feedback control law. In Section 3, we present the global asymptotic stability of the closed-loop system using the predictor-feedback control law and necessary assumptions. Finally, Section 4 is devoted to the results of experiments (pick-and-place task) in order to reveal the significance of predictor for the system stabilization in the presence of three different input delays.

2 Mathematical Modeling

The redundant Baxter manipulator, which is being studied here, has seven degrees of freedom, see Fig. 1. The Denavit-Hartenberg parameters for this manipulator are determined based on the specifications provided by the manufacturer, shown in Table 1.

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)=\tau \tag{1}$$

where, $q\in\mathbb{R}^7$, $\dot{q}\in\mathbb{R}^7$, and $\ddot{q}\in\mathbb{R}^7$ are angles, angular velocities, and angular accelerations of joints, respectively, and $\tau\in\mathbb{R}^7$ indicates the vector of joint driving torques. Also, $M(q)\in\mathbb{R}^{7\times7}$, $C(q,\dot{q})\in\mathbb{R}^{7\times7}$, and $G(q)\in\mathbb{R}^7$ are the mass, Coriolis, and gravitational matrices, respectively, which are symbolically derived using the Euler-Lagrange equation [32–35]. Note that the inertia matrix $M(q)$ is symmetric, positive definite, and consequently invertible. This property is used in the subsequent development. The multi-input nonlinear system (1) can be written as 14^{th}-order ODEs with the following general state-space form,

$$\dot{X}=f_0(X,U) \tag{2}$$

where $X=[q_1,\cdots,q_7,\dot{q}_1,\cdots,\dot{q}_7]^T\in\mathbb{R}^{14}$ is the vector of states and $U=\tau_{7\times1}\in\mathbb{R}^7$ is the input of nonlinear system (2).

Since we intend to design a predictor-based controller leading to perfect tracking, we derive error dynamics and then design the controller to stabilize the error dynamics making the origin asymptotically stable.

$$\dot{E}=f(E,U) \tag{3}$$

where, $E=[e_1^T,e_2^T]^T\in\mathbb{R}^{14}$ is the vector of error states and $e_1(q,t)$, $e_2(q,\dot{q},t)\in\mathbb{R}^7$ are defined as

$$e_1=q_{\text{des}}-q \tag{4}$$
$$e_2=\dot{e}_1+\alpha e_1 \tag{5}$$

where $\alpha\in\mathbb{R}^{7\times7}$ is a constant positive definite matrix, and the following assumption is held for the desired joint trajectories.

Assumption 1. *The desired joint trajectories* $q_{\text{des}}(t)\in\mathbb{R}^7$ *and their derivatives* $\dot{q}_{\text{des}}(t)$, $\ddot{q}_{\text{des}}(t)\in\mathbb{R}^7$ *exist and are bounded for all* $t\geq0$.

3 Designing the Predictor-Based Controller

Dealing with highly nonlinear and coupled dynamic equations could cause a complicated problem of designing computationally efficient control scheme to avoid the large delay. Therefore, we derive a predictor-based controller for a multi-input nonlinear system, in the presence of input delay, to stabilize the closed-loop system. In order to demonstrate the generality of our approach, consider the following general multi-input nonlinear system with m inputs, n states, and constant input delay D,

$$\dot{E}(t) = f\left(E(t), U_1(t-D), \cdots, U_m(t-D)\right) \tag{6}$$

where, $E \in \mathbb{R}^n$ is the vector of states, $U_1, \cdots, U_m \in \mathbb{R}$ are the control inputs, $D > 0$ is an input delay, and $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is a locally Lipschitz vector field. We assume that a feedback law $U_i(t) = \kappa_i(E(t))$ is known such that the functions $\kappa_i : \mathbb{R}^n \to \mathbb{R}$ globally asymptotically stabilize the delay-free system – the closed-loop system $\dot{E}(t) = f(E(t), \kappa(E(t)))$ is globally asymptotically stable in the absence of delay. Therefore, in the delay system, the control law needs to be as follows:

$$U_i(t-D) = \kappa_i(E(t)) \tag{7}$$

which can be expressed as

$$U_i(t) = \kappa_i(E(t+D)) = \kappa_i(P(t)) \tag{8}$$

where $P(t)$ is the D-time units ahead predictor of $E(t)$. The predictor law for the system (6) is given by,

$$P(t) = E(t) + \int_{t-D}^{t} f\Big(P(\theta), U_1(\theta), U_2(\theta), \cdots, U_m(\theta)\Big)\, d\theta \tag{9}$$

with the following initial conditions for the integral (9),

$$P(\theta) = E(0) + \int_{-D}^{\theta} f\Big(P(s), U_1(s), U_2(s), \cdots, U_m(s)\Big)\, ds \tag{10}$$

where $\theta \in [-D, 0]$. Note that $P(t)$ is defined in terms of its past values, however a solution $P(t)$ to (9) does not always exist since the control applied after $t = D$ has no effect on the plant over the time interval $[0, D]$; consequently the system (6) can exhibit finite escape before $t = D$. Therefore, in order to ensure the global existence of the predictor state, we need to be sure that, for all initial conditions and all locally bounded input signals, the system's solutions exist for all time. This property is the so-called "forward completeness". For designing the predictor-based controller, we utilize the results of [31] in the following theorem.

Theorem 1. *Consider the closed-loop system consisting of the plant* (6) *with input delay. If there exists control laws* (8)-(9) *such that $\dot{E}(t) = f(E(t), \kappa(E(t)))$ becomes asymptotically stable, subject to the assumptions of open-loop system forward completeness and the Input-to-State Stability (ISS) of closed-loop system $\dot{E}(t) = f(E(t), \kappa(E(t)) + \omega)$ with respect to ω, the following holds for all $t \geq 0$,*

$$\Omega(t) \leq \beta\left(\Omega(0), t\right) \tag{11}$$

where

$$\Omega(t) = |E(t)| + \sum_{i=1}^{n} \sup_{t-D \leq \theta \leq t} |U_i(\theta)| \tag{12}$$

3.1 Error System Development

The control objective includes converging joint position and velocity errors to zero implying the generalized coordinates track the desired time-varying joint trajectories, $q_{\text{des}}(t) \in \mathbb{R}^7$. A state-space model for the tracking error (Eq. (3)) is developed based on Eqs. (4) and (5). Then a controller is formulated to improve tracking performance indices, converging errors to zero, subject to the assumption of knowing the system's dynamics, as mentioned earlier.

A state-space model, based on the tracking error, is formulated through premultiplying the inertia matrix by the time derivative of Eq. (5), while Eqs. (1) and (4) are substituted,

$$-M\dot{e}_2 + (M\alpha - C)e_2 + (-M\alpha^2 + C\alpha)e_1 + M\ddot{q}_{\text{des}} + C\dot{q}_{\text{des}} + G = \tau \tag{13}$$

which yields,

$$\dot{e}_2 = \alpha e_2 + h - M^{-1}\tau \tag{14}$$

where $h \in \mathbb{R}^7$ is a nonlinear function defined as

$$h = \ddot{q}_{\text{des}} - \alpha^2 e_1 + M^{-1}(C\dot{q}_{\text{des}} + G + C\alpha e_1 - Ce_2) \tag{15}$$

and the state-space model of error dynamics becomes,

$$\dot{E} = f(E, \tau) = \begin{bmatrix} e_2 - \alpha e_1 \\ \alpha e_2 + h - M^{-1}\tau \end{bmatrix} \tag{16}$$

As we mentioned through Theorem 1, the forward completeness and ISS properties of the nonlinear system need to be established. The forward-complete systems include all linear systems both stable and unstable, as well as various nonlinear systems with bounded nonlinearities. The mathematical model of robot manipulators contains trigonometric nonlinearities as a result of rotational motions, which implies that $q(t)$ and consequently $e_1(t)$ do not escape to infinity within a finite time. Therefore, robot manipulators are the forward-complete nonlinear systems [22].

We can also utilize the following theorem [36] to establish the forward completeness of the system,

Theorem 2. *System $\dot{x} = f(x,d)$ is forward complete if and only if there exists a proper and smooth function $V : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ such that the following exponential growth condition is verified:*

$$DV(x)f(x,d) \leq V(x), \quad \forall x \in \mathbb{R}^n, \forall d \in \mathscr{D} \tag{17}$$

We now establish (17) by considering the following Lyapunov function,

$$V(E) = \frac{1}{2}e_1^T e_1 + \frac{1}{2}e_2^T e_2 \tag{18}$$

We easily have,

$$\begin{aligned} \dot{V} &= e_1^T(e_2 - \alpha e_1) + e_2^T(\alpha e_2 + h - M^{-1}\tau) \\ &= e_1^T e_2 - e_1^T \alpha e_1 + e_2^T \alpha e_2 + e_2^T h - e_2^T M^{-1}\tau \end{aligned} \tag{19}$$

Since M, M^{-1}, and C include trigonometric functions, we get,

$$\begin{aligned} e_1^T e_2 - \ e_1^T \alpha e_1 + e_2^T \alpha e_2 \leq \frac{1}{2}\left(e_1^T e_1 + e_2^T e_2\right) \\ -\lambda_m e_1^T e_1 + \lambda_M e_2^T e_2 \end{aligned} \tag{20}$$

$$\begin{aligned} e_2^T h \leq \ & e_2^T \Big(\ddot{q}_{\text{des}} - \alpha^2 e_1 + M^{-1}C\dot{q}_{\text{des}} + \\ & M^{-1}G + M^{-1}C\alpha e_1 - M^{-1}Ce_2 \Big) \\ \leq \ & \frac{1}{2}\left(e_2^T e_2 + \ddot{q}_{\text{des}}^T \ddot{q}_{\text{des}}\right) + \frac{\gamma_1}{2}\left(e_2^T e_2 + e_1^T e_1\right) \\ & + \frac{\gamma_2}{2}\left(e_2^T e_2 + \dot{q}_{\text{des}}^T \dot{q}_{\text{des}}\right) + \frac{\gamma_3}{2}\left(e_2^T e_2 + \Gamma^2\right) \\ & + \frac{\gamma_4}{2}\left(e_2^T e_2 + e_1^T e_1\right) - \gamma_5\left(e_2^T e_2\right) \\ & -e_2^T M^{-1}\tau \leq \frac{\gamma_6}{2}\left(e_2^T e_2 + \tau^T \tau\right) \end{aligned} \tag{21}$$

where λ_M and λ_m denotes the maximum and minimum eigenvalues of matrix α, respectively. Also, $\gamma_i's > 0$ ($i = 1,2,3,4,5,6$) and Γ is the L_2-norm of gravitational vector. Substituting Eqs. (20) and (21) into Eq. (19) yields,

$$\begin{aligned} \dot{V} \leq \ & (1 - 2\lambda_m + \gamma_1 + \gamma_4)\left(\frac{1}{2}e_1^T e_1\right) \\ & + (2 + 2\lambda_M + \gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 - 2\gamma_5 + \gamma_6)\left(\frac{1}{2}e_2^T e_2\right) \\ & + \frac{1}{2}\left(\ddot{q}_{\text{des}}^T \ddot{q}_{\text{des}}\right) + \frac{\gamma_2}{2}\left(\dot{q}_{\text{des}}^T \dot{q}_{\text{des}}\right) + \frac{\gamma_3}{2}\Gamma^2 + \frac{\gamma_6}{2}|\tau|^2 \\ \leq \ & \gamma_6\left(\frac{1}{2}e_1^T e_1 + \frac{1}{2}e_2^T e_2\right) + \gamma_7 \end{aligned} \tag{22}$$

where,

$$\begin{aligned} \gamma_6 = \ \max\Big\{ & \Big(1 - 2\lambda_m + \gamma_1 + \gamma_4\Big), \\ & \Big(2 + 2\lambda_M + \gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 - 2\gamma_5 + \gamma_6\Big)\Big\} \end{aligned} \tag{23}$$

$$\frac{1}{2}\left(\ddot{q}_{\text{des}}^T \ddot{q}_{\text{des}}\right) + \frac{\gamma_2}{2}\left(\dot{q}_{\text{des}}^T \dot{q}_{\text{des}}\right) + \frac{\gamma_3}{2}\Gamma^2 + \frac{\gamma_6}{2}|\tau|^2 \leq \gamma_7 \tag{24}$$

since τ, $\ddot{q}_{\text{des}}$, and $\dot{q}_{\text{des}}$ are bounded, we get,

$$\dot{V} \leq \gamma_6 V(E) + \gamma_7 \tag{25}$$

Consequently, $(V(E) + \frac{\gamma_7}{\gamma_6})^{\frac{1}{\gamma_6}}$ is a smooth Lyapunov function satisfying (17). With this function, we have established that the following assumption of Theorem 1 is verified.

Assumption 2. *The system $\dot{E} = f(E, \tau_1, \cdots, \tau_m)$ is forward complete.*

The forward-completeness ensures that, for every initial condition and locally bounded input signal, the corresponding solution is defined for all $t \geq 0$.

Then, we design a predictor feedback law for (16), which achieves global asymptotic stability for the delay-free system, as mentioned in Theorem 1 – the closed-loop system $\dot{E}(t) = f(E(t), \kappa(E(t)))$ should be asymptotically stable. Since the dynamics of system (1) is known, the controller is formulated, based on Eq. (14), as

$$\tau = \kappa(E) = M(h + (\beta + \alpha)e_2) \tag{26}$$

where $\beta \in \mathbb{R}^{7\times 7}$ is a constant positive definite matrix. Substituting Eq. (26) into Eq. (14) and using the invertible property of inertia matrix result in the closed-loop error signal for $e_2(t)$ as

$$\dot{e}_2 = -\beta e_2 \tag{27}$$

Finally, the state-space model of closed-loop system, with respect to Eqs. (5) and (27), is derived as follows,

$$\dot{E} = f(E, \kappa(E)) = AE(t) \tag{28}$$

where $A \in \mathbb{R}^{14\times 14}$ is defined as

$$A = \begin{bmatrix} -\alpha & I_{7\times 7} \\ 0_{7\times 7} & -\beta \end{bmatrix} \tag{29}$$

where $I_{7\times 7}$ and $0_{7\times 7}$ are identity and zero matrices, respectively. Since A is an upper triangular block matrix and is also Hurwitz for any positive definite α and β matrices, (28) is hence exponentially stable.

Note that the control law is formulated such that the error dynamics becomes exponentially and subsequently asymptotically stable. Now the only property we need to establish, before using Theorem 1, is the input-to-state (ISS) stability of the following closed-loop system with respect to $\omega = [\omega_1, \cdots, \omega_m]^T$.

$$\dot{E} = f(E, \kappa(E) + \omega) = \begin{bmatrix} e_2 - \alpha e_1 \\ \alpha e_2 + h - M^{-1}(\kappa(E) + \omega) \end{bmatrix}$$

$$= AE(t) - \begin{bmatrix} 0_{7\times 7} \\ M^{-1}\omega \end{bmatrix} \tag{30}$$

The ISS property can be shown using the following Lemma [37].

Lemma 1. *Suppose $\dot{x} = f(t, x, u)$ is continuously differentiable and globally Lipschitz in (x, u), uniformly in t. If the unforced system $\dot{x} = f(t, x, 0)$ has a globally exponentially stable equilibrium point at the origin, then the system is input-to-state stable.*

Due to the fact that $\dot{E} = f(E, \kappa(E) + \omega)$ is continuously differentiable and globally Lipschitz in (E, ω), the closed-loop system (30) is therefore ISS with respect to ω using Lemma 1. Hence, the following assumption for our system is verified,

Assumption 3. *The system $\dot{E} = f(E,\ \kappa_1(E) + \omega_1,\ \cdots,\ \kappa_m(E) + \omega_m)$ is Input-to-State Stable (ISS) with respect to $\omega = [\omega_1, \cdots, \omega_m]^T$.*

Finally, as Assumptions 2 and 3 are held for our system, we can employ Theorem 1 to design a predictor-based controller to compensate any large input delay, asymptotically stabilize the error, and make the robot to follow the desired joint trajectories.

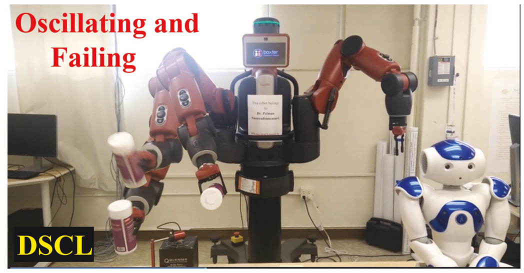


FIGURE 2. The robot fails to track the desired trajectory without a predictor in the presence of input delay

FIGURE 3. A stable obstacle-avoidance pick-and-place task with input delay using the predictor-based controller

4 Experimental Results

We experimentally implement the predictor-based controller for the 7-DOF Baxter manipulator as a case study, through a pick-and-place task, while input delays are reasonably similar in all input channels [38]. We reveal the destabilizing effect of input delay on the control of the manipulator, as shown in Fig. 2, and also discuss the effect of incremental delay on the stability of the robot. We intentionally apply the following input delays and then operate the manipulator without any predictor:

$D = 0.01s$: indicates minimum feasible input delay with respect to the sampling rate ($t_s = 0.01s$) of Baxter.

$D = 0.02s$: the increased delay to determine a crucial value causing the robot operational failure.

$D = 0.04s$: the increased delay to study the significant effect of a relatively large input delay.

The joint trajectories and torques, for the three cases mentioned above, are presented in Figs. 4 and 5, respectively, and also compared with the experimental results of delay-free system. Note that we did not plot the joint trajectories of delay-free system since the manipulator almost perfectly tracks the desired trajecto-

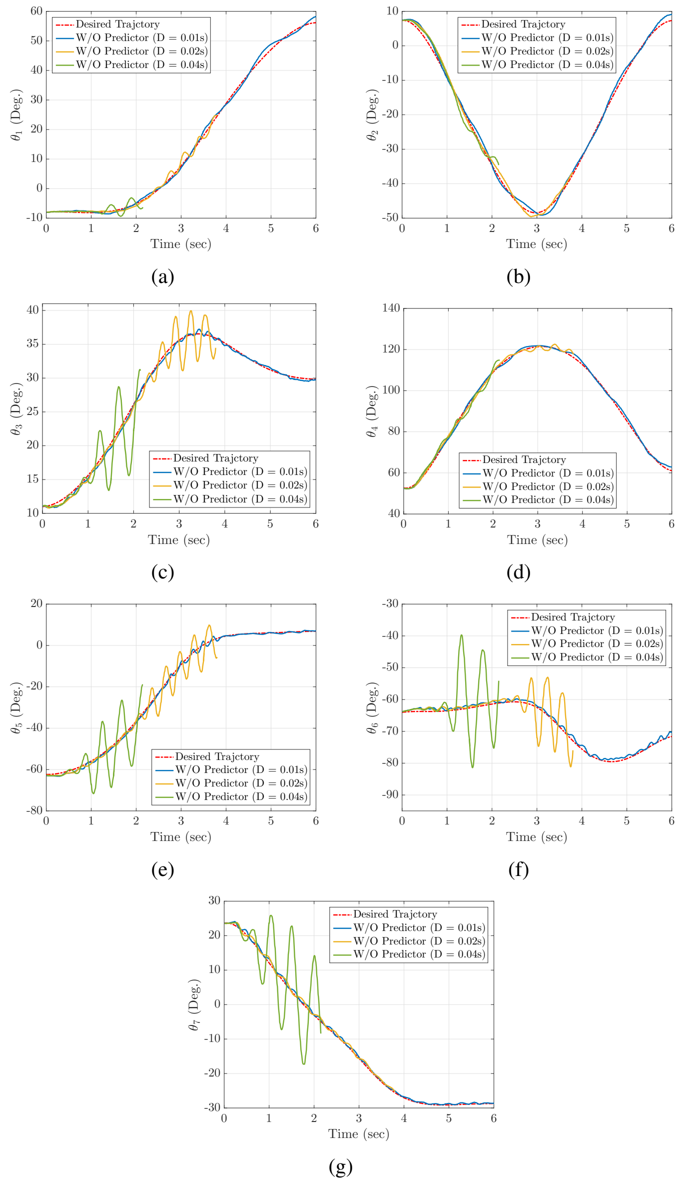


FIGURE 4. The experimental (a) S_0, (b) S_1, (c) E_0, (d) E_1, (e) W_0, (f) W_1, and (g) W_2 joint trajectories in the presence of $D = 0.01s$ (blue line), $D = 0.02s$ (orange line), and $D = 0.04s$ (green line) input delays without a predictor

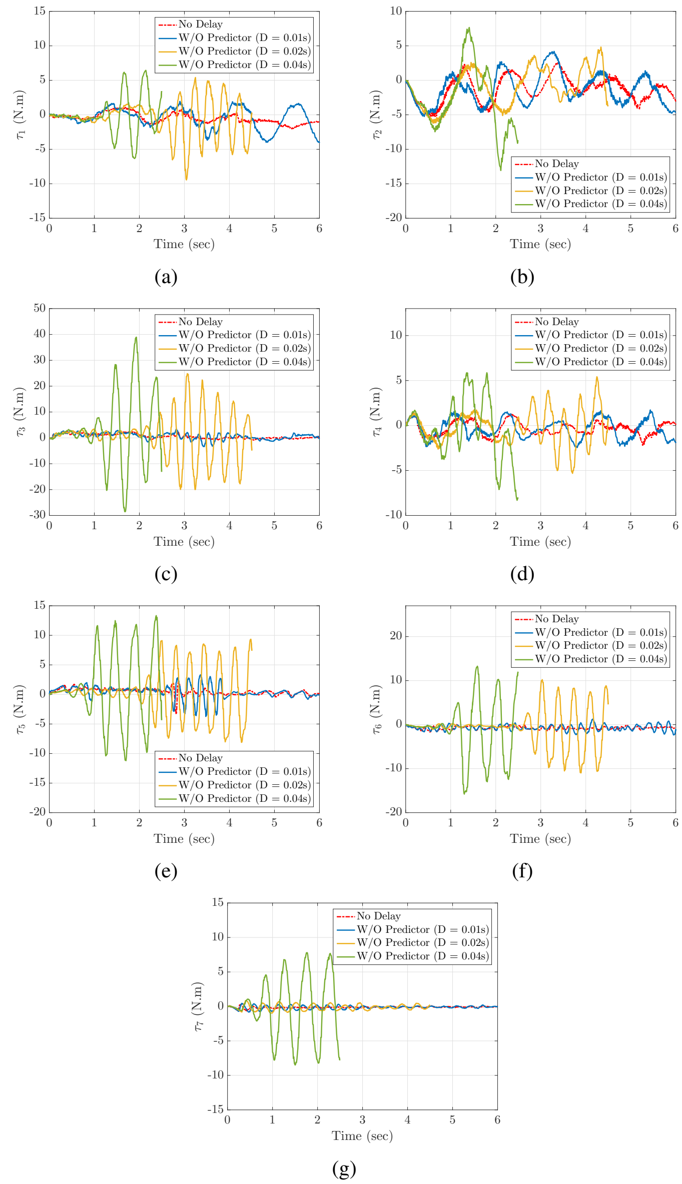


FIGURE 5. The experimental joint torques of (a) S_0, (b) S_1, (c) E_0, (d) E_1, (e) W_0, (f) W_1, and (g) W_2 in the presence of $D = 0.01s$ (blue line), $D = 0.02s$ (orange line), and $D = 0.04s$ (green line) input delays without a predictor

ries. As shown in Fig. 4, for $D = 0.01s$, the manipulator can still follow the desired trajectories while the joint torques are more than those of the delay-free system (Fig. 5). The results also reveal that the joint torques, in particular for the joint 5 (Fig. 5(e)), oscillate since the manipulator approaches its singular configuration while passing over the obstacle ($2.5s \leq t \leq 3.5s$). By increasing the delay from $0.01s$ to $0.02s$, the manipulator becomes unstable and expectedly cannot follow the desired trajectories (Fig. 4). Note that the joint 2 (S_1) does not oscillate like the other ones because of the supporting spring mounted at this joint (Fig. 1). We also examine the robot's performance in the presence of $0.04s$ input delay. The results illustrate that the manipulator harmfully oscillates and then fails to properly operate. Therefore, the robot, as expected, becomes unstable within a shorter time interval through increasing the amount of delay. It is clear that the instability of one link results in the robot failure due to the highly dynamic interconnections among the links.

It is worth mentioning that we operate the manipulator using joint torque control mode, as an advanced control scheme, which grants the access to the lowest control levels and puts much responsibility on the control algorithm. Consequently, for both $0.02s$ and $0.04s$ input delays, we could not capture more data since Baxter moves stochastically leading to the catastrophic malfunction. The AVI files of the experiments are accessible through our Dynamic Systems and Control Laboratory (DSCL) website.

In summary, as shown in Fig. 4, the closed-loop system becomes unstable for small input delays. Therefore, implementing the predictor-based controller is a necessity to be carried out. We hence take the advantage of the predictor-based controller, using Theorem 1, in order to globally asymptotically stabilize the manipulator due to the fact that all the assumptions are valid for the robot's arm. We formulate the predictor along with the controller, and then thoroughly investigate their performances in compensating the destabilizing input delays. In order to examine the effects of delay's magnitude, experiments are carried out in the presence of three different large input delays: $0.8s$, $0.9s$, and $1.0s$. Note that exposing the robot to the input delays more than $1.0s$ is not logical since the whole operational time is $6.0s$. Shown in Figs. 6 and 7 are the joint angles and torques, respectively.

As shown in Fig. 7, there is no control torque before $t = D$ and consequently, the robot remains stationary (Fig. 6). Therefore, the errors expectedly emerge within $t \in [0, D)$, in particular for the joints 2 (S_1), 4 (E_1), and 7 (W_2) (Fig. 6). At $t = D$, the manipulator begins following the desired trajectories using the predictor-based controller by applying high amounts of torques. Figs. 3 and 6 present an acceptable performance of the predictor-based controller since the tracking errors converge to zero after $4.0s$.

From another aspect, Figs. 6(a) and 6(e) reveal that the tracking errors of the joints 1 (S_0) and 6 (W_1) are not considerably high, for $0 \leq t \leq D$, despite the other ones. It is obvious that the less tracking error typically demands the less control torque to be applied with respect to the ranges of joint rotation angles. Increasing the input delay expectedly imposes higher tracking errors at the onset of the robot operation and consequently, much more control torques are needed to be applied (Fig. 7). After $t = D$, the manipulator begins to perfectly track the desired trajectories using the considerable initial control torques. The control torques peak at $t = D$ and then decline by the decremental tracking errors (Fig. 7). As mentioned earlier, the manipulator approaches its singular configuration around $t = 3.0s$, which subsequently results in the incremental oscillation-like joint torques, in particular for the joint 5 (W_0), as shown in Fig. 7(e).

As shown in Fig. 7, comparing the control torques at $t = D$ reveals that τ_2, τ_3, and τ_4 take higher values than the other ones since the joints 2 (S_1), 3 (E_0), and 4 (E_1) are subject to more

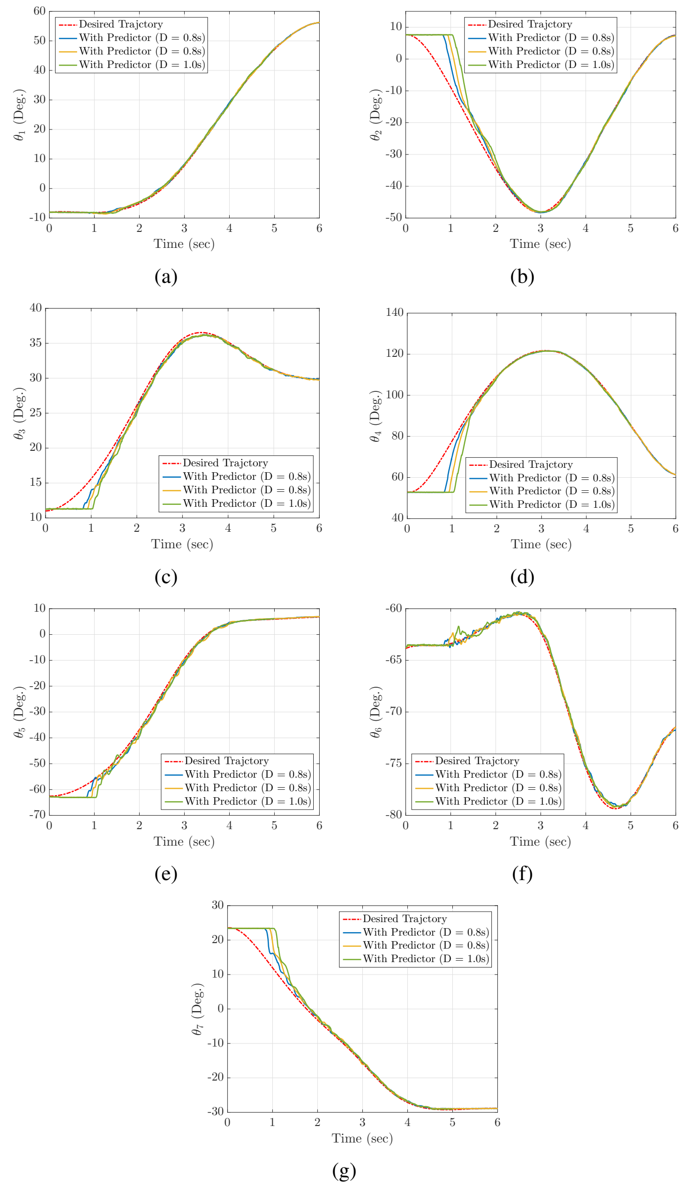


FIGURE 6. The experimental (a) S_0, (b) S_1, (c) E_0, (d) E_1, (e) W_0, (f) W_1, and (g) W_2 joint trajectories in the presence of $D = 0.8s$ (blue line), $D = 0.9s$ (orange line), and $D = 1.0s$ (green line) input delays using the predictor-based controller

tracking errors and loads (based on the manipulator structure) with respect to the other joints. Finally, comparing Figs. 5 and 7 implies that even an uncompensated small delay results in harmful torques and therefore, the manipulator expectedly fails to track the desired trajectories. As can be observed in Fig. 6, the tracking errors begin to decrease after $t = D$, due to the fact that Theorem 1 guarantees the asymptotic convergence of the tracking errors to zero subject to any large delay. Fig. 8 presents the

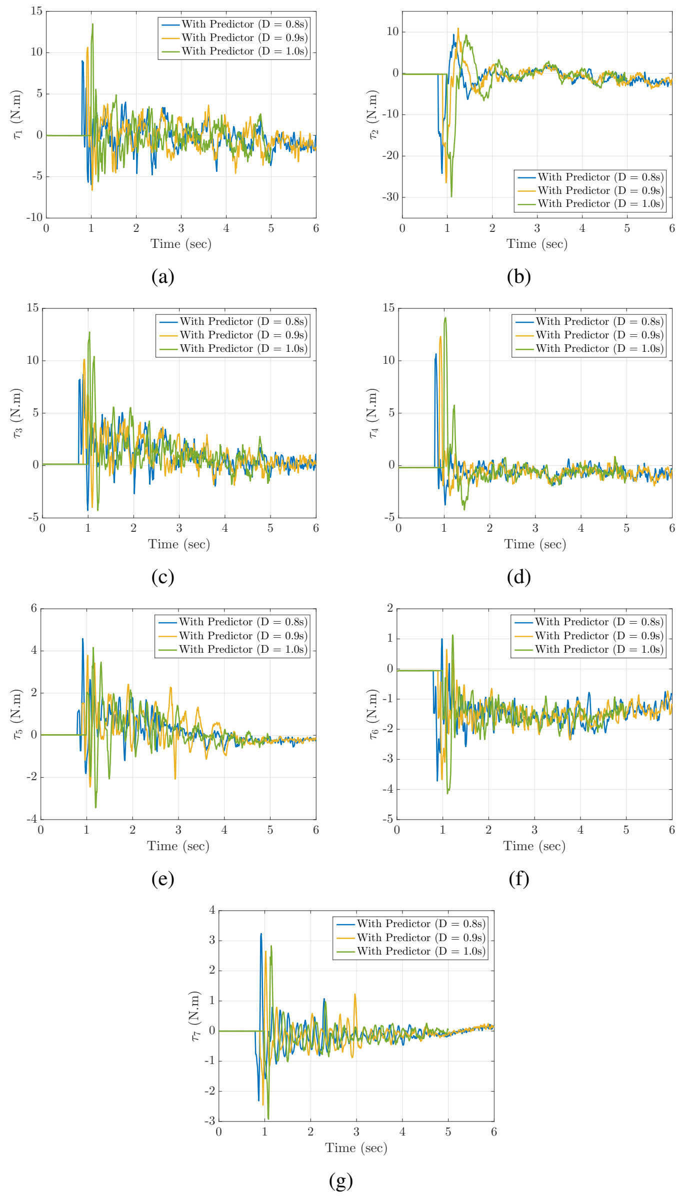


FIGURE 7. The experimental joint torques of (a) S_0, (b) S_1, (c) E_0, (d) E_1, (e) W_0, (f) W_1, and (g) W_2 joints in the presence of $D = 0.8s$ (blue line), $D = 0.9s$ (orange line), and $D = 1.0s$ (green line) input delays using the predictor-based controller

experimental tracking errors for $D = 0.8s$.

The negligible experimental tracking errors mainly root on the inaccuracy of sensors and actuators. We experimentally verified the model in a back-and-forth procedure [32, 35, 38] and there is an acceptable correlation between our model and Baxter's dynamics. Moreover, unmodeled dynamics, such as friction in joints or external disturbances, may result in the prediction offset from the actual path, which we carefully considered through

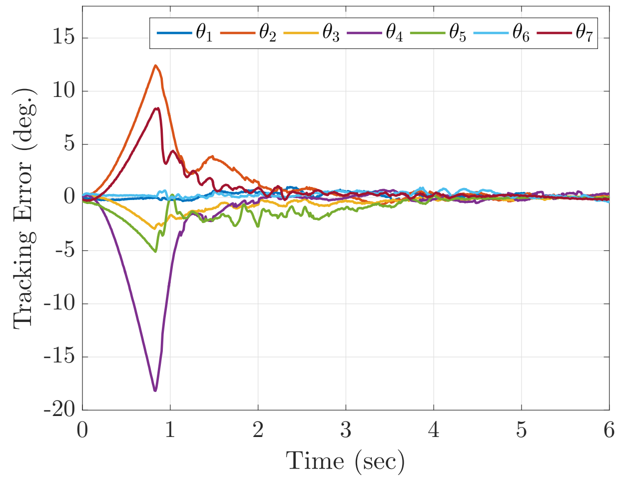


FIGURE 8. The experimental tracking errors subject to the predictor-based controller in the presence of $0.8s$ input delay

designing the controller. The algorithm measures the robot's joint angles at each iteration and hence making predictions begins from that measurement – the state $E(t)$ in Eq. (9) is measured in each iteration. Providing this measurement to the predictor endows robustness against small uncertainties and avoids any cumulative error caused by uncertainties or unmodeled dynamics. Moreover, we established that the closed-loop system is ISS, which in turn provides the control robustness against any bounded disturbance. Based on the data provided by Baxter's manufacturer, the series elastic actuators act as filters helping to reduce both the friction and backlash through low-cost gearbox. Therefore, as can be seen in Fig. 8, the tracking errors asymptomatically converge to zero. Also, Fig. 9 presents the simulation results for $D = 0.8s$ revealing that the tracking errors asymptomatically converge to zero, as expected.

5 Conclusions

Throughout this paper, we designed a predictor-based controller for a general highly interconnected nonlinear system subject to the time-invariant input delay. We investigated the destabilizing effects of three different input delays shown in Figs. 4 and 5, and then the controller was implemented for the 7-DOF Baxter manipulator as a case study. Toward designing the controller, we established the forward completeness of the open-loop system and Input-to-State Stability (ISS) properties of the closed-loop system. We then formulated the predictor-based controller to asymptotically stabilize the system employing Theorem 1, and then investigated the effects of large input delays on the control of Baxter robot.

The experimental results revealed that the predictor-based controller, in the presence of large input delays, makes the robot asymptomatically stable, and the robot tracks the desired joint

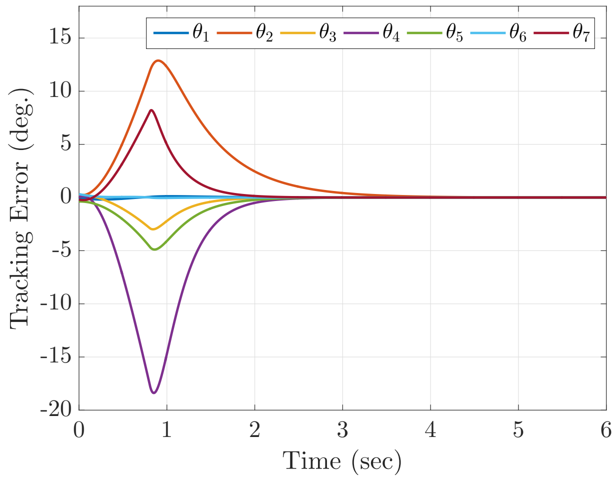


FIGURE 9. The simulated tracking errors subject to the predictor-based controller in the presence of $0.8s$ input delay

trajectories, as expected. We also established that the tracking errors, subject to the predictor-based controller, asymptotically converge to zero. The negligible amounts of the tracking errors, shown in Fig. 8, mainly root on the inaccuracy of sensors and actuators. The simulation results also presented the asymptotic convergence of the tracking errors to zero guaranteed through Theorem 1, as shown in Fig. 9. The principal results of this research work can be summarized as follows:

- The minimum input delay destabilizes the robot.
- Using Theorem 1, the stability of the system for any large input delay is guaranteed.
- The predictor-based controller analytically and experimentally compensates the input delay and achieves the closed-loop asymptotic stability.

It is worth mentioning that although our controller is robust against a time-invariant delay and small uncertainties, but time-varying input delays and enormous uncertainties may affect the control performance; this problem has not yet been addressed. Therefore, we are currently focusing our efforts on designing a nonlinear adaptive time-delay control scheme with application to high-DOF robotic manipulators.

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References

[1] Mostafa Bagheri, Arash Ajoudani, Jinoh Lee, Darwin G Caldwell, and Nikos G Tsagarakis. Kinematic analysis and design considerations for optimal base frame arrangement of humanoid shoulders. In *2015 IEEE International Conference on Robotics and Automation (ICRA)*, pages 2710–2715, May 26-30 2015.

[2] Oren Solomon and Emilia Fridman. New stability conditions for systems with distributed delays. *Automatica*, 49(11):3467–3475, 2013.

[3] Alper Denasi, Dragan Kostić, and Henk Nijmeijer. Time delay compensation in bilateral teleoperations using impact. *IEEE Transactions on Control Systems Technology*, 21(3):704–715, 2013.

[4] Yen-Chen Liu and Nikhil Chopra. Control of semi-autonomous teleoperation system with time delays. *Automatica*, 49(6):1553–1565, 2013.

[5] Farzad Hashemzadeh, Iraj Hassanzadeh, and Mahdi Tavakoli. Teleoperation in the presence of varying time delays and sandwich linearity in actuators. *Automatica*, 49(9):2813–2821, 2013.

[6] Abdelkader Abdessameud and Abdelhamid Tayebi. Formation control of vtol unmanned aerial vehicles with communication delays. *Automatica*, 47(11):2383–2394, 2011.

[7] Yangquan Chen, Zhiming Gong, and Changyun Wen. Analysis of a high-order iterative learning control algorithm for uncertain nonlinear systems with state delays. *Automatica*, 34(3):345–353, 1998.

[8] N Fischer, A Dani, Nitin Sharma, and Warren E Dixon. Saturated control of an uncertain nonlinear system with input delay. *Automatica*, 49(6):1741–1747, 2013.

[9] Otto JM Smith. A controller to overcome dead time. *ISA J.*, 6:28–33, 1959.

[10] Miroslav Krstić. Compensation of infinite-dimensional actuator and sensor dynamics. *IEEE Control Systems*, 30(1):22–41, 2010.

[11] Nikolaos Bekiaris-Liberis and Miroslav Krstić. Delay-adaptive feedback for linear feedforward systems. *Systems & Control Letters*, 59(5):277–283, 2010.

[12] Nikolaos Bekiaris-Liberis and Miroslav Krstić. Stabilization of linear strict-feedback systems with delayed integrators. *Automatica*, 46(11):1902–1910, November 2010.

[13] Miroslav Krstić. Lyapunov stability of linear predictor feedback for time-varying input delay. *IEEE Transactions on Automatic Control*, 55(2):554–559, February 2010.

[14] Nikolaos Bekiaris-Liberis and M. Krstić. Lyapunov stability of linear predictor feedback for distributed input delay. *IEEE Transactions on Automatic Control*, 56(3):655–660, March 2011.

[15] Delphine Bresch-Pietri and Miroslav Krstić. Adaptive trajectory tracking despite unknown input delay and plant parameters. *Automatica*, 45(9):2074–2081, 2009.

[16] Nikolaos Bekiaris-Liberis, Mrdjan Jankovic, and Miroslav Krstić. Adaptive stabilization of lti systems with distributed input delay. *International Journal of Adaptive Control and Signal Processing*, 27(1-2):46–65, 2013.

[17] Iasson Karafyllis and Miroslav Krstić. Delay-robustness of linear predictor feedback without restriction on delay rate. *Automatica*, 49(6):1761–1767, 2013.

[18] Yang Zhu, Hongye Su, and Miroslav Krstić. Adaptive backstepping control of uncertain linear systems under unknown actuator delay. *Automatica*, 54:256–265, 2015.

[19] Yang Zhu and Miroslav Krstić. Adaptive output feedback control for uncertain linear time-delay systems. *IEEE Transactions on Automatic Control*, 62:545–560, 2017.

[20] Daisuke Tsubakino, Miroslav Krstić, and Tiago Roux Oliveira. Exact predictor feedbacks for multi-input lti systems with distinct input delays. *Automatica*, 71:143–150, 2016.

[21] Miroslav Krstić. On compensating long actuator delays in nonlinear control. In *IEEE American Control Conference, 2008*, pages 2921–2926, June 11-13 2008.

[22] Miroslav Krstić. *Delay Compensation for Nonlinear, Adaptive, and PDE Systems (Systems & Control: Foundations & Applications)*. Springer, 2009.

[23] Miroslav Krstić. Input delay compensation for forward complete and strict-feedforward nonlinear systems. *IEEE Transactions on Automatic Control*, 55(2):287–303, 2010.

[24] Iasson Karafyllis, Miroslav Krstić, Tarek Ahmed-Ali, and Francoise Lamnabhi-Lagarrigue. Global stabilisation of nonlinear delay systems with a compact absorbing set. *International Journal of Control*, 87(5):1010–1027, 2014.

[25] Nikolaos Bekiaris-Liberis and Miroslav Krstić. Compensation of state-dependent input delay for nonlinear systems. *IEEE Transactions on Automatic Control*, 58(2):275–289, 2013.

[26] Nikolaos Bekiaris-Liberis and Miroslav Krstić. *Nonlinear control under nonconstant delays*, volume 25. Siam, 2013.

[27] Nikolaos Bekiaris-Liberis and Miroslav Krstić. Robustness of nonlinear predictor feedback laws to time-and state-dependent delay perturbations. *Automatica*, 49(6):1576–1590, 2013.

[28] Iasson Karafyllis and Miroslav Krstić. Numerical schemes for nonlinear predictor feedback. *Mathematics of Control, Signals, and Systems*, 26(4):519–546, 2014.

[29] Nikolaos Bresch-Pietri and Miroslav Krstić. Delay-adaptive control for nonlinear systems. *IEEE Transactions on Automatic Control*, 59:1203–1217, 2014.

[30] Joon-Young Choi and Miroslav Krstić. Compensation of time-varying input delay for discrete-time nonlinear systems. *International Journal of Robust and Nonlinear control*, 26:1755–1776, 2016.

[31] Nikolaos Bekiaris-Liberis and Miroslav Krstić. Predictor-feedback stabilization of multi-input nonlinear systems. *IEEE Transactions on Automatic Control*, 62(2):516–531, 2017.

[32] Mostafa Bagheri and Peiman Naseradinmousavi. Novel analytical and experimental trajectory optimization of a 7-dof baxter robot: global design sensitivity and step size analyses. *The International Journal of Advanced Manufacturing Technology*, 93(9-12):4153–4167, December 2017.

[33] Mostafa Bagheri, Peiman Naseradinmousavi, and Rasha Morsi. Experimental and novel analytical trajectory optimization of a 7-dof baxter robot: Global design sensitivity and step size analyses. In *ASME 2017 Dynamic Systems and Control Conference*, page V001T30A001. American Society of Mechanical Engineers, American Society of Mechanical Engineers, October 11-13 2017.

[34] Mostafa Bagheri, Miroslav Krstić, and Peiman Naseradinmousavi. Joint-space trajectory optimization of a 7-dof baxter using multivariable extremum seeking. In *IEEE American Control Conference (ACC 2018)*, pages 2176–2181, Milwaukee, WI, June 27-29 2018.

[35] Mostafa Bagheri, Miroslav Krstić, and Peiman Naseradinmousavi. Multivariable extremum seeking for joint-space trajectory optimization of a high-degrees-of-freedom robot. *Journal of Dynamic Systems, Measurement, and Control*, 140(11):111017, 2018.

[36] David Angeli. Input-to-state stability of pd-controlled robotic systems. *Automatica*, 35(7):1285–1290, 1999.

[37] Hassan K. Khalil. *Noninear Systems*. Prentice Hall, third edition, 2002.

[38] Mostafa Bagheri, Miroslav Krstić, and Peiman Naseradinmousavi. Analytical and experimental predictor-based time delay control of baxter robot. In *ASME 2018 Dynamic Systems and Control Conference*, pages V001T04A011–V001T04A011. American Society of Mechanical Engineers, 2018.