Entailments of Productive Disciplinary Engagement in Technology-Enhanced Mathematics Classrooms

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Objectives

Productive disciplinary engagement (PDE; Engle & Conant, 2002) is a framework that scholars have used to capture the extent to which students' experiences embody characteristics of disciplinary practice in a variety of STEM learning environments. What existing work using PDE has been able to provide is ways of understanding how the learning environment, including the teachers' actions, shapes how students ultimately engage in disciplinary practices in classroom settings. Yet, research using PDE has applied differing definitions of *productive*, *disciplinary*, and *engagement* as well as lacked a cohesive approach to determine the quality of students' opportunities to engage in PDE. The aim of this paper is to provide an empirically derived framework that operationalizes the assessment of the quality of PDE in classroom environments.

Empirical and Theoretical Foundations

Engle and Conant (2002) define PDE through a process of considering non-empty subsets of words from the full set of three words in the phrase, defining first *engagement,* then *disciplinary engagement,* and then *productive disciplinary engagement.* Their conceptualization of engagement is primarily concerned with how students participate in discourse, how frequently that participation happens, and the quality of students' participation in a discourse (Williams-Candek, 2015).

How might we think about *engagement* in technologically enhanced learning settings? In these settings, a student could co-opt another student's diagram (or inscription, more broadly) and create a new mathematical object without having engaged in a verbal dialogue. There is a discourse inherent to the creation and use of mathematical inscriptions that students participate in, so that engagement must also consider whether and how students interact with digital objects as part of participating in discourse.

Yet another critical feature in analyzing situations for PDE is how one defines *disciplinary* engagement. Engle and Conant (2002) posit: "By using the term disciplinary engagement in a school context, we mean that there is some contact between what students are doing and the issues and practices of a discipline's discourse" (p. 402). Scholars argue for the distinction between how a discipline is represented in schools and the discipline as practiced by professionals (Civil, 2002; Stengel, 1997), so disciplinary as applied in school settings may more realistically apply to the discipline as students encounter it in schools.

Finally, there is the issue with how we define *productive* disciplinary engagement. Engle and Conant (2002) broadly construe productivity by arguing that disciplinary engagement is productive when students make "intellectual progress" (p. 403) when engaging in a task. How one defines productive clearly has implications for research methods; if developing a better understanding of the problem to be solved is an example of having made conceptual progress, then capturing students' in-the-moment interactions around a task to surface evidence about the development of understanding about the problem would be critical for determining whether conceptual progress has been made.

Our work is guided by several assumptions about PDE as it emerges in classroom settings. First, rather than defining engagement separately from disciplinary, we consider the action of "disciplinary engagement" which is students' participation in situations involving disciplinary ideas or objects. We broadly construe participation to include verbal and non-verbal engagement, as well as interaction with disciplinary ideas or objects in dialogue with other students as well as in interaction with another students' work. Further, unlike Engle and Conant (2002), our work presupposes that disciplinary engagement occurs among a community of practice, including the teacher. Finally, we follow Engle and Conant's (2002) lead in defining productive disciplinary engagement as occurring when students' participation in situations involving disciplinary ideas or objects results in intellectual progress being made. This can include tangible evidence of intellectual progress, such as a solution to a problem, as well as evidence of conceptual development.

Methods for Developing a New Framework for PDE

This work was conducted as part of a three-year, design-based research project aiming to engineer contexts for PDE to emerge through the thoughtful development of a digital middle school mathematics curriculum featuring tools for supporting collaborative problem solving.[1]

Data Sample

We first identified a pool of videos from our project data set (i.e., screen recordings) with conditions that could allow for students' PDE. First, each video needed to be relatively free from technical issues with the digital collaborative platform that students used and technical issues with equipment for recording data. Second, each video needed to include talk from multiple students in the group, with a high frequency of student talk overall. Finally, to facilitate analysis, each video clip needed to show students engaging in some task from start to finish (for example, solving one equation or answering one question from the curriculum). Overall, this process of refinement utilized 10 videos of screen recordings of small group work in a variety of problem-solving contexts.

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Analysis

Each video was separately analyzed as we developed our description of the indicators for high, medium, and low PDE. During the first stage of protocol development, we focused on three videos. Two researchers independently watched each video and noted aspects of the lesson that were related to the four embodiments of PDE. After watching all videos independently, the researchers compared and synthesized their notes to articulate common indicators for each principle of PDE. This formed the initial protocol consisting of indicators describing high quality embodiments of the four principles of PDE.

We then conducted additional iterative cycles of coding four new videos to both refine our initial criteria for high PDE episodes and expand the protocol to articulate aspects of the principles in mid-level and low-level PDE episodes. A pair of researchers individually watched the selected video clips and made decisions about areas within the list that needed to be refined. Any changes to the list of indicators was made by consensus. Based on viewing two more video clips, we developed specific descriptions for each indicator. Additionally, we sought to eliminate overlap between indicators and to make sure each indicator remained relevant to its related principle of PDE. We repeated this process with another two videos from our pool.

Finally, we sought to eliminate any indicators that could occur in the absence of students' PDE. To accomplish this, we identified one additional video that did not meet the criteria for potentially high PDE. It contained talk from dominantly one student and did not include many instances of uncertainty. Based on viewing this video, we eliminated a small number of indicators that could occur in small group interactions without students' PDE. We also re-watched all videos utilized in previous stages of development to validate the final list of indicators.

Results: The Framework and Illustrative Examples

Framework for Observing PDE in Students' Classroom Work

The protocol shown in Figure 2 is the product of the process we described previously. There are three important considerations when interpreting and using this protocol. First, not all indicators for a given principle are necessary for the episode to reflect high quality embodiments of a given principle. Second, for an episode to be considered "high PDE," it cannot contain events that could be described with any of the low indicators for any principle of PDE. Finally, a determination of whether an episode is high, mid-level, or low PDE is made by considering the "weight" of the indicators for a given episode. If the majority of indicators across all principles are low or mid-level, the entire episode would be assessed as a low or mid-level episode of PDE. Also, if only one or two principles have a majority of indicators in the high category, whereas the other principles are primarily mid-level or low, the entire episode would be assessed as either mid-level or low depending upon the number of indicators at a given level and which principles are represented.

	High	*Medium*	*Low*
Problematizing	• *Students work to understand the task conceptually and procedurally well* • *Students adopt a disciplinary perspective* • *When confused or stuck, students push to resolve the issue themselves and/or ask for help to their peers or teacher* • *Students' uncertainties concern what their conclusions mean or competing alternatives*	• *Students work to understand the task procedurally well and conceptually slightly* • *Students sometimes adopt a disciplinary perspective* • *When confused or stuck, students make small attempts to resolve the issue but might give up easily* • *Students' uncertainties concern justifying their actions or the meaning of conclusions*	• *Students work to complete the procedures but not necessarily working to understand conceptually* • *Students do not often adopt a disciplinary perspective* • *When confused or stuck, students do not attempt to resolve the issue* • *Students' uncertainties concern what action to take*
Authority	• *Most students in the group make contributions to the topic during the discussion* • *Students talk about the topic/problem with their own words* • *Many students' ideas become part of the solution path* • *Students are treated as credible experts whose ideas are often seriously considered by the group*	• *Some students in the group make contributions to the topic during the discussion* • *Students sometimes talk about the topic/problem with their own words* • *Some students' ideas become part of the solution path* • *Students are sometimes treated as credible experts whose ideas seriously considered by the group*	• *Less students in the group make contributions to the topic during the discussion* • *Students repeat the problem statements, not using their own words* • *Few students' ideas become part of the solution path* • *Students are rarely or never treated as credible experts whose ideas should be seriously considered by the group*
Accountability	• *Students thoroughly question each other and explain their own contributions until the idea make sense to everyone in the group* • *Multiple mathematical ideas and viewpoints are often debated* • *Students consistently collaborate with each other* • *Most students are involved in collaboration* • *Students are active in developing their own understanding*	• *Students attempt to question each other and explain their own contributions, but perhaps shallowly* • *Multiple mathematical ideas and viewpoints are sometimes debated* • *Students sometimes collaborate with each other* • *Some students are involved in collaboration* • *Students are somewhat active in developing their own understanding*	• *Students do not explain their contributions and blindly accept others' contributions* • *Mathematical ideas and viewpoints are rarely debated* • *Students rarely collaborate with each other* • *Few students are involved in collaboration* • *Students are passive in developing their own understanding*
Resources	• *Students actively seek multiple appropriate resources, including those outside the current problem or workspace* • *Students use resources in a way that helps them resolve uncertainties deeply*	• *Students use appropriate resources from the current problem or workspace* • *Students use resources in a way that doesn't limit their resolution of uncertainties*	• *Students make use of few or no available resources* • *Students use resources in a way that limits their resolution of uncertainties*

Figure 2: Protocol for Assessing PDE During Problem Solving in Mathematics Classrooms

Applying the Framework: Illustration of High PDE

To illustrate how this framework can be used to analyze students' small group work in mathematics, we present one group's work from the Moving Straight Ahead unit from the *Connected Mathematics Project* middle school mathematics curriculum (Lappan et al., 2006) that focuses on linear relationships. In this investigation, the mystery pouches in the Kingdom of Montarek problem introduces students to symbolic ways of solving linear equations with one unknown. Equations are represented pictorially as coins (constant term) and pouches that hold an unknown number of coins (variables). The episode focuses on representing the linear relationship of 3x=2x+12 pictorially with pouches and coins. The group consisted of four seventh grade students, each using a digital representation of the problem that allowed them to draw, mark up, change, and digitally stamp pouches and coins, as well as view group members' work.

After each student worked to solve the problem individually, the first student presented their work in the following exchange:

Student 1: So there's 12 in each. No, not 12 in each, 6 in each. So you would add 12 to [the coins], which would be 24. 24 divided by 3.
Student 3: Why would you add 12?
Student 2: Yeah, I'm confused why you added 12.
Student 3: There isn't 12 coins there [in the bags]. You only have 12 there.
Student 2: Yeah, but how can you have 6 in a bag, and later you have 8 in a bag?
Student 1: That's on the other side [of the equation].
Student 2: Oh!
Student 3: What? No...

At this point in their conversation, the group is divided in their understanding of variables (in this problem, pouches or bags) and is unsure if they maintain the same value on each side of equation. The teacher began speaking to the group to check on their progress, and the students quickly related their solution process and their (incorrect) answer of 6. Students 1 and 4 declared that the pouches were "useless" since they didn't know how many coins were in them. Referring back to an earlier problem, the teacher said, "Everybody I've talked to so far at this table has talked about cancelling out coins on the top. Like on that first one. […] How could you use that idea on this one?"

After reviewing their work, the group continued:

Student 1: Because there's 3 pouches on the left side. And if there's 2 pouches on the right side, but there's 12 coins, 12 goes in each pouch.
Student 3: Huh?
Student 2: 6. Yeah, it's 6.

Student 1: No, look it. There's 3 pouches [on this side], 2 pouches [on this side]. You need to add 1 more pouch.
Student 4: (overlapping speech) Wait, you said you can have 3…
Student 1: But [all the coins] is 1 pouch. So 12 coins. So each pouch would be 12 coins.
Student 2: That's what I said!
Student 1, So 12, 12, 12 and then 12, 12, 12. Yeah.
Student 3: I'm still confused.

The other three group members described being happy with their answer and process, and at that point the class period ended.

For this episode, the majority of the indicators for problematizing are high. The students are grappling procedurally with how to solve an equation as well as the conceptual meaning of a pouch and what it represents. Further, the main uncertainty that students were concerned with at several points during this episode was whether and how they needed to revise their work, representing a substantive uncertainty about competing alternatives (Going, Kursav, Grant, Bieda & Edson, 2017). One indicator related to problematizing was rated medium, since students did not consistently show evidence of engagement in mathematical practices which we used as indication of adopting a disciplinary perspective toward mathematics.

Authority was also assessed as high. All students in the group made mathematical contributions during this episode, and did so using their own words. Multiple students either suggested strategies or offered criticism to existing strategies, thus contributing to the solution path. Further, all students actively listened to peers and engaging respectfully.

Accountability in this episode was also assessed high. Students often questioned each other or expressed confusion, and received mathematical responses to help them resolve their questions. At the end of the episode, only one student still expressed a lack of understanding about the group's process, which may have been resolved had there been more time in the period. Further, students were consistently asking mathematical questions or verbally indicating their confusion to elicit other students' explanations.

Resources was the only principle of PDE that was assessed as medium for this episode, and both of the indicators related to resources were medium. Students used stamps and writing tools effectively to create their own work, however students did not seek supplementary resources outside of the current workspace (which would be necessary for high use of resources by our framework).

Significance

Although a considerable amount of research has focused on different aspects of PDE, with some showing features of PDE in classroom settings (Forman, Engle, Venturini, & Ford, 2014; Koretsky et al., 2014), there is a notable paucity of empirical studies exploring PDE conceptualization in STEM collaborative learning contexts (Koretsky et al., 2019). Our study contributes to existing literature by providing an empirically grounded framework that operationalizes PDE principles to describe the quality of students' engagement digitally collaborative mathematics classrooms. Additionally, as there is no common consensus on what it means to be productive, this paper takes a stance towards what it means to be productive in digital collaborative mathematics classrooms, and offers three levels, high, medium, and low that articulate student engagement with the four principles.

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