

# Pre-compensation of servo tracking errors through data-based reference trajectory modification

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## ABSTRACT

This paper presents a new dynamic error compensation approach with novel data-based closed-loop tuning scheme to enhance tracking accuracy of machine tool feed-drives. Both servo dynamics and friction disturbance induced positioning errors are pre-compensated by modifying the reference trajectory. Velocity and acceleration profiles of reference trajectory are modulated to achieve perfect tracking. Reference position profile is modified based on the pre-sliding friction regime to eliminate quadrant glitches. Optimal error compensation is achieved by a digital trajectory pre-filter whose parameters are tuned automatically by making on-the-fly iterative adjustments. Effectiveness of proposed compensation approach is validated experimentally in multi-axis feed-drive systems.

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## 1. Introduction

Productivity and overall accuracy of machine tools depend on speed and accuracy of its feed-drive systems [1]. Feed-drives suffer from quasi-static volumetric and dynamic tracking errors. Volumetric errors can be measured and compensated through reference trajectory modification without interfering with servo controller [2]. Dynamic tracking errors occur constantly as servo controller tries to compensate for inertial and viscous damping forces and struggles to overcome non-linear friction and process induced disturbances [3].

Machine tool literature focuses on two major strategies to mitigate dynamic feed-drive errors: (i) exhausting limits of feedback (FB) control through design of non-linear, robust adaptive [1,3,4,5] controllers with modal damping [4,6], and (ii) compensating rest of the dynamic errors through feedforward (FF) control, which is not only used to cancel known servo dynamics [3,7], but also disturbances such as the non-linear friction [3], pitch error induced loads and harmonic cogging forces [1,4]. If the dynamics to be compensated is represented accurately by causal, stable models; FF compensation is safe and effective [3,7]. However, modelling stage is typically by-passed in practice, and FF compensator gains are hand-tuned via trial-and-error. Drive dynamics vary with both position and workpiece inertia [1,5]. Friction dynamics vary by time, and hence compensator re-tuning is necessary to regain the lost dynamic accuracy, which is time consuming and costly.

Relying solely on FB and FF control is insufficient. Recently, benefits of reference trajectory generation are recognized as they

are optimized to avoid residual vibrations [8] or pre-compensate for the servo lag to enhance machine's contouring performance [9]. Most NC systems provide digital trajectory pre-filters [2] and external trajectory command input functions [10] to help end-users exploit limits of their machines, and also integrate with the industry 4.0.

This paper presents a novel strategy to pre-compensate dynamic feed-drive errors through reference trajectory modification. Overall scheme is shown in Fig. 1. Reference velocity and acceleration commands are modified to pre-compensate servo dynamics induced errors. Whereas, reference position profile is modified locally to cancel pre-sliding (stick/slip) friction induced positioning errors to achieve near-perfect tracking. Two key contributions are presented. Firstly, pre-compensation of the trajectory is realized by a fixed-structure digital pre-filter whose parameters are identified automatically via machine-in-the-loop learning. Secondly, identification of the machine specific trajectory pre-filter is posed as a convex optimization problem, which provides rapid, safe and reliable auto-tuning. The proposed approach is simple and does not require any expertise. Once tuned, the pre-filter can improve dynamic accuracy of older or newer CNC machines either through offline or online trajectory modification, and its effectiveness is demonstrated experimentally.

As shown in Fig. 1 reference trajectory  $x_R$  is modified by two pre-compensation filters,  $F_N$  and  $F_L$ . The modified trajectory  $x_M = x_R + F_L(x_R) + F_N(x_R)$  is then sent to the closed-loop servo control system so that final axis position  $x$  perfectly follows the command  $x_R$ . The pre-compensation filter  $F_L$  is used to cancel feed-drive's linear closed loop dynamics. Whereas,  $F_N(\cdot)$ , is designed to cancel non-linear stick/slip friction induced errors by offsetting reference trajectory during velocity reversals.

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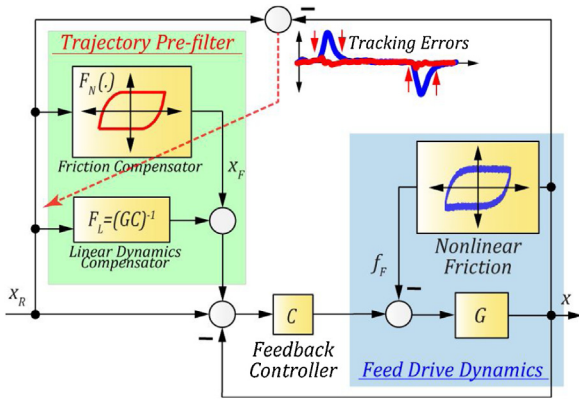


Fig. 1. Proposed trajectory modification strategy.

## 2. Pre-compensation of linear servo dynamics

Machine tool feed-drives are mostly controlled by cascade P-PI or PID controllers tuned carefully to avoid exciting structural resonances [1]. As a result, typical feed-drive tracking response is dominated by rigid-body plant  $G(s)$  and FB controller  $C(s)$  dynamics, which induces large servo errors during rapid acceleration and deceleration. If the original trajectory is modified by a pre-filter  $F_L$  (See Fig. 1) whose transfer function in Laplace (s) domain is selected as:

$$F_L = \frac{1}{GC} = \frac{ms^3 + bs^2}{K_D s^2 + K_P s + K_I} \rightarrow \frac{x}{x_R} = (1 + F_L) \underbrace{\left( \frac{GC}{1 + GC} \right)}_{G_{CL}} \simeq 1 \quad (1)$$

closed loop servo dynamics  $G_{CL}(s)$  can be compensated to achieve near-perfect tracking ( $x \approx x_R$ ). Above pre-filter structure satisfies 2 key assumptions. Firstly, the numerator amplifies original acceleration and velocity profiles only by the uncompensated mass ( $m$ ) and viscous friction ( $b$ ) amount. Secondly, the denominator tries to compensate for the FB controller dynamics. Note that all linear servo controllers can be implemented by mapping their parameters into basic PID gains,  $K_R$ ,  $K_D$  and  $K_I$ . [4,6]. Therefore, proposed pre-filter design can capture dynamics of most industrial servo systems.

Initial pre-filter parameters are identified automatically from a single closed-loop tracking experiment. The machine is instructed by a simple back-and-forth trajectory and the resulting error profile  $e_L$  is recorded. The error dynamics is governed by:

$$\begin{aligned} \frac{e_L}{x_R} &= \frac{1}{1 + GC} = \frac{ms^3 + bs^2}{ms^3 + (K_D + b)s^2 + K_P s + K_I} \\ e_L &= x_R - \frac{1}{m} \left( (K_D + b) \int_0^t e_L d\tau + K_P \int_0^t \int_0^t e_L d\tau + K_I \int_0^t \int_0^t \int_0^t e_L d\tau + b \int_0^t x_R d\tau \right) \end{aligned} \quad (2)$$

Eq. (2) is sampled at servo loop period  $T_s$  and mass-normalized ( $m$ ) pre-filter parameters are identified from least squares (LS) fitting:

$$\min_{\theta} \frac{1}{2} \|\Phi\theta - \gamma\|_2^2 \quad \text{subject to: } \theta \geq 0 \quad (3)$$

$$\begin{aligned} \gamma &= \begin{bmatrix} x_R(0) - e_L(0) \\ x_R(T_s) - e_L(T_s) \\ \vdots \\ x_R(MT_s) - e_L(MT_s) \end{bmatrix}, \quad \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} = \begin{bmatrix} K_D + b \\ K_P \\ K_I \\ b \end{bmatrix} \frac{1}{m} \\ \Phi &= \begin{bmatrix} e_i(0) & e_{ii}(0) & e_{iii}(0) & -x_{Ri}(0) \\ e_i(T_s) & e_{ii}(T_s) & e_{iii}(T_s) & -x_{Ri}(T_s) \\ \vdots & \vdots & \vdots & \vdots \\ e_i(MT_s) & e_{ii}(MT_s) & e_{iii}(MT_s) & -x_{Ri}(MT_s) \end{bmatrix} \end{aligned}$$

where  $e_i$ ,  $e_{ii}$ ,  $e_{iii}$  are digitally integrated error  $e_L$  profiles in Eq. (2) over  $0 \dots M$  samples.

In practice, above transfer function fitting approach may not provide the most suitable compensator parameters. If the feed-drive suffers from strong non-linear guideway friction, identified rigid-body parameters could be biased [11], or even the pre-filtered trajectory may excite resonances [7]. To alleviate those practical problems following machine-in-the-loop fine-tuning approach is developed, and numerator of the pre-filter is updated.

The machine is commanded by an NC code containing non-stop speed changes shown in Fig. 2(a). Pre-filter parameters are updated to minimize the tracking errors around acceleration transients, which helps to eliminate non-linear friction bias and at the same time penalizes excitation of higher order error dynamics. This can be postulated by the following optimization problem:

$$\min_{m, b} \left( J_L = \frac{1}{2} \|\mathbf{e}_L\|_2^2 = \frac{1}{2} \mathbf{e}_L^T \mathbf{e}_L \right), \quad \text{subject to: } m \geq 0, b \geq 0 \quad (4)$$

where  $\mathbf{e}_L$  is the tracking error vector. Based on Eq. (4) pre-filter parameters are adjusted; compensated trajectory is sent to the machine again, and resulting error trend is used to update the filter parameters for the next run as shown in Fig. 2(b). Machine-in-the-loop iterations are continued until convergence is achieved.

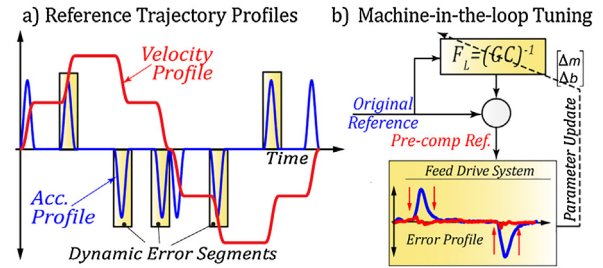


Fig. 2. Machine-in-the-loop tuning of trajectory pre-filter  $F_L$ .

This approach mimics manual trial-and-error tuning by an expert. Safe and reliable convergence is crucial. Automatic parameter update is achieved by solving Eq. (4). This is a convex optimization problem, and thus utilizing gradient (Jacobian) of its cost function  $\nabla J_L = [\partial J_L / \partial m, \partial J_L / \partial b]^T = \mathbf{e}_L^T \nabla \mathbf{e}_L$ , and the positive definite Hessian matrix  $\nabla^2 J_L = \nabla \mathbf{e}_L^T \nabla \mathbf{e}_L$  enables us to rapidly and safely guide the search to global minimum. Notice that  $\mathbf{e}_L$  is already available as a measurement data, and its gradient  $\nabla \mathbf{e}_L$  can be obtained by filtering reference trajectory  $x_R$  or the original uncompensated response  $x$  from Eq. (1) as:

$$\begin{aligned} e_L &= x_R - x = \left( \frac{1 - F_L GC}{1 + GC} \right) x_R \rightarrow \nabla e_L = \begin{bmatrix} \frac{\partial e_L}{\partial m} \\ \frac{\partial e_L}{\partial b} \end{bmatrix} = -\nabla F_L \underbrace{\left( \frac{GC}{1 + GC} \right)}_x x_R \\ \text{where } \nabla F_L &= \begin{bmatrix} s^3 \\ s^2 \end{bmatrix} \frac{1}{K_D s^2 + K_P s + K_I} \end{aligned} \quad (5)$$

Filter parameters are updated by simply making use of Newton's second order iteration scheme:

$$\begin{bmatrix} m \\ b \end{bmatrix}^{k+1} = \begin{bmatrix} m \\ b \end{bmatrix}^k - \alpha (\nabla^2 J_L)^{-1} (\nabla J_L) \quad (6)$$

where  $\alpha$  is the learning gain, and  $k$  is the iteration number. Notice that above iteration scheme is a discrete linear dynamic system. Range of learning gains for reliable convergence can be analyzed [12] and  $1 > \alpha > 0$  provides reliable convergence. Inequality constraints are included using primal-dual interior point algorithm [13].

### 2.1. Experimental validation

Proposed algorithm is tested on a cartesian micro-machine tool shown in Fig. 3. X and Y axes are controlled by cascade P-PI controllers with hand-tuned FF compensators on the deltaTau NC

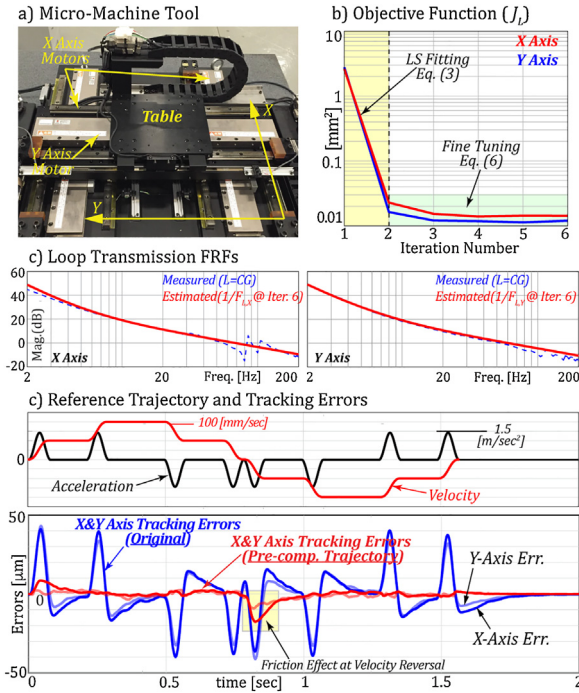


Fig. 3. Experimental validation of servo dynamics compensation.

system. Axes are moved by a single G-code back-and-forth at 50 and 100[mm/sec]. Reference trajectory and error profiles are recorded at 1 kHz. As shown in Fig. 3(c), tracking errors peak around acc/dcc sections of reference trajectory. Proposed trajectory prefiltering is applied by tuning (training) the prefilter  $F_L$  using Eqs. (3) and (6) on the same trajectory. As shown in Fig. 3(b) best filter parameters are identified within 5–6 iterations with  $\alpha = 0.75$ , and tracking errors are reduced to less than  $\pm 3[\mu\text{m}]$ . Remaining tracking errors are due to non-linear friction disturbance. They are eliminated by the reference trajectory pre-filtering technique presented in the following section.

Once the trajectory pre-filter  $F_L$  is trained for the machine's feed drive dynamics, it can be applied to any trajectory for perfect tracking. This is demonstrated by eliminating contouring errors along a curved tool-path presented in the following Section 3.1.

### 3. Pre-compensation of nonlinear pre-sliding friction

Another major source of dynamic servo errors is the non-linear friction. As shown in previous section, industrial servo controllers cannot effectively reject rapidly changing friction forces, which leaves quadrant glitches and surface location errors during velocity reversals [3,4,5,7]. This section shows how reference trajectory can be modified to compensate for them.

Trajectory pre-filter  $F_L$  is used to cancel linear servo dynamics. Therefore, only friction induced errors are apparent at velocity reversals and can be predicted from closed-loop dynamics as:

$$e = f_F \left( \frac{G}{1+GC} \right) - x_F \left( \frac{GC}{1+GC} \right) \rightarrow e = \left( \frac{f_F}{C} - x_F \right) \left( \frac{GC}{1+GC} \right) \quad (7)$$

where  $x_F$  is the pre-compensation command generated by  $F_N(\cdot)$  (see Fig. 1),  $D(s)$  is the disturbance TF, and  $f_F$  is the nonlinear friction disturbance. Eq. (7) reveals an important fact: If the reference trajectory is offset by controller  $C(s)$  filtered friction forces  $x_F = x_R + C^{-1}f_F$ , it can cancel friction induced errors as well. This requires (i) knowledge of controller dynamics  $C(s)$ , which is already identified from LS fitting in Eq. (3), and (ii) an accurate estimation of friction forces. The following describes how nonlinear stick/slip friction could be identified automatically (adaptively) from closed-loop experiments and cancelled by the pre-compensation signal generated by trajectory prefilter,  $F_N(\cdot)$ .

Generalized Maxwell-slip (GMS) model [3,14] accurately captures hysteretic stick/slip induced pre-sliding friction dynamics with non-local memory. However, it must be tuned either manually [3,14] or using non-linear optimization [15] to compensate or cancel friction disturbances. To facilitate automatic tuning, a modified GMS form is proposed here as:

$$f_{GMS} = \sum_{i=1}^K k_i z_i, \quad \frac{dz_i}{dt} = \begin{cases} \dot{x}_R = v_R, & \text{if stick (i.e. } z_i \leq |P_i/2|) \\ 0, & \text{if slip (i.e. } z_i > |P_i/2|) \end{cases} \quad (8)$$

where  $K$  is the number of MS blocks that stick and slip (See Fig. 4), and  $k_i$  is  $i^{\text{th}}$  block's spring coefficient while sticking. As blocks undergo local micro-translation ( $z_i$ ), they slip only when they exceed their break-away (stick) distance,  $P_i$ . The fundamental difference of above modified GMS formulation (Eq. (8)) from the conventional one is that; here, the stick/slip conditions are governed by break-away distances rather than break-away forces [14]. Stick distance of feed-drives range 10–100[ $\mu\text{m}$ ] [14], which can be observed from a simple tracking experiment. The stick distance is divided and distributed to individual MS block's stick distances as;  $P_1 < P_2 < \dots < P_K = P$ . As a result, only unknowns in this modified GMS model are spring coefficients  $k_i$  that can be identified simply in the sense of linear least squares (LS). Furthermore, above GMS formulation preserves all the properties of original GMS including the non-local hysteresis memory [3,14].

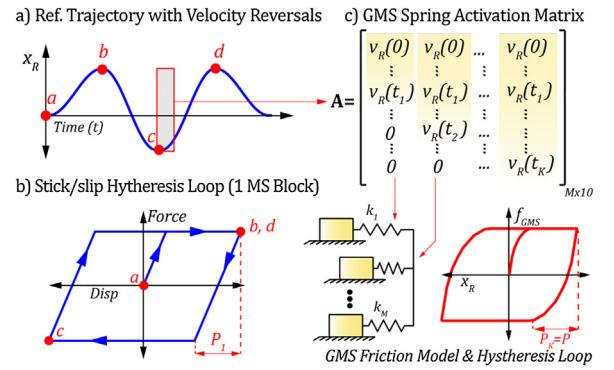


Fig. 4. Modified GMS Model and the Spring Activation Matrix (A).

Eq. (8) can be put in a simple linear matrix-vector form as:

$$\frac{d}{dt} f_{GMS} = \mathbf{A} \mathbf{k}, \quad \text{where } \mathbf{A} = [\dot{z}_1 \quad \dot{z}_2 \quad \dots \quad \dot{z}_K] \quad (9)$$

where,  $\mathbf{k} = [k_1 \dots k_K]^T$  contains spring constants, and  $\mathbf{A}$  is the *spring activation matrix*, which contains stick/slip velocities  $v_R$  for each individual block. Its structure is illustrated in Fig. 4 for a sinusoidal trajectory. Each column of  $\mathbf{A}$  represents individual MS block's stick/slip kinematics during velocity reversals. For instance, first MS block sticks during a velocity reversal ( $t=0$ ) until  $x_r = P_1$ , which occurs at  $t=t_1$ . The next block sticks a little longer, until  $x_R = P_2$ ,  $t=t_2$ . The  $\mathbf{A}$  matrix is generated from reference trajectory, and it is “velocity reversal dependent”. Combined with spring constants, it captures the friction induced hysteresis behavior shown in Fig. 4.

Identification of GMS spring constants to compensate (minimize) friction errors is postulated by the following optimization problem:

$$\min_{\mathbf{k}} \left( J_N = \frac{1}{2} \|\mathbf{e}_v\|_2^2 = \frac{1}{2} \mathbf{e}_v^T \mathbf{e}_v \right), \quad \text{subject to: } \mathbf{k} \geq 0 \quad (10)$$

where  $\mathbf{e}_v$  is the tracking error vector recorded around a velocity reversal. Similar to the previous section, Eq. (10) depicts an optimization problem with quadratic objective  $J_N$ , and it can be solved utilizing its gradient  $\nabla J_N = [\partial J_N / \partial k_1 \dots \partial J_N / \partial k_K]^T = \mathbf{e}_v^T \nabla \mathbf{e}_v$  and Hessian  $\nabla^2 J_N = \nabla \mathbf{e}_v^T \nabla \mathbf{e}_v$ ,  $\nabla \mathbf{e}_v$  that are available analytically by



filtering measured tracking error data by plugging Eq. (9) into (7):

$$e = (f_F - f_{GMS}) \frac{1}{C} \left( \frac{GC}{1+GC} \right) \rightarrow \nabla e = -\nabla f_{GMS} \frac{1}{C} \left( \frac{GC}{1+GC} \right) \quad (11)$$

where:  $\nabla f_{GMS} = \int_{t=0}^{MT_s} A dt$ , and  $\nabla = \left[ \frac{\partial}{\partial k_1} \quad \dots \quad \frac{\partial}{\partial k_K} \right]^T$

where  $\nabla f_{GMS}$  is the friction pre-compensation signal, and  $\nabla f_{GMS}$  is its discretized vector containing  $M$  samples.

As shown, proposed formulation of GMS friction model (Eq. (8)) allows us to define friction induced error gradient explicitly as a function of the reference trajectory. Machine-in-the-loop iterations are used to safely train the friction pre-compensator parameters  $F_N = f_{GMS}/C$  using Newton iterations:

$$\mathbf{k}^{k+1} = \mathbf{k}^k - \alpha (\nabla^2 J_N)^{-1} (\nabla J_N) \quad (12)$$

### 3.1. Experimental validation

Effectiveness of the proposed trajectory pre-filtering scheme for friction compensation is tested on the same linear motor driven micro-machine tool (See Fig. 3(a)). A spiral curved tool-path shown in Fig. 5(a) is commanded at a feed of 50[mm/sec]. The linear  $F_L$  filter (tuned previously) is used to compensate for servo dynamics. As a result, large quadrant glitches occur around velocity reversals due to X–Y drive's friction disturbances, and they are visible in the contour error trend in Fig. 5(c). Friction compensator pre-filter  $F_N$  is trained using Eq. (12) at each velocity reversal (iteration) *on-the-fly* to offset the trajectory and cancel friction. 10 GMS blocks are used to capture the pre-sliding friction regime of the feed-drive. As shown in Fig. 5, after the 15th velocity crossing, stick/slip hysteresis curve is automatically identified, and reference trajectory is modified to perfectly cancel quadrant glitches as shown in Fig. 5 (a) and c. Contour errors are reduced down to 1[μm]. By combining both linear servo dynamics and the friction pre-compensator filters, near-perfect tracking could be achieved. Once trained, the trajectory compensation filters can be used at any trajectory to enhance the contouring performance.

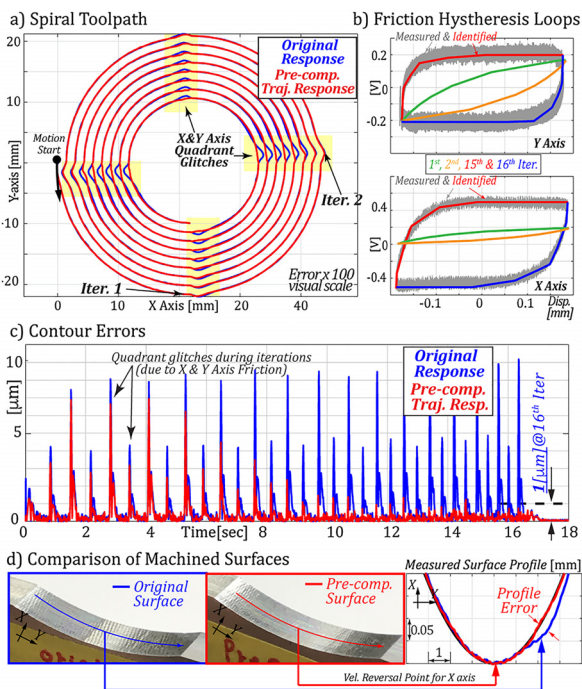


Fig. 5. Experimental compensation of stick/slip friction.

Finally, micro milling experiments are conducted to showcase the importance of stick/slip pre-sliding friction compensation in surface finish quality in Fig. 5(d). A concave surface is cut on A5052 workpiece at 5[mm] depth and 0.1[mm] radial immersion with 15[krpm] spindle speed and 0.02[mm/rev-tooth] feed. X-axis reverses its motion direction at the bottom of surface. At that instant pre-sliding friction starts kicking in and causes surface profile errors over 20[μm] (Fig. 5(d), measured by Surfcom Flex-50A). Notice that profile errors occur slightly delayed from velocity reversal point due to friction breakaway distance. Proposed pre-compensated trajectory cancels friction errors and thereby enables machining of near-ideal surface profile.

## 4. Conclusions

A novel error pre-compensation scheme is proposed, which greatly improves dynamic accuracy of machine tools by compensating for the closed-loop servo and stick/slip (pre-sliding) friction induced errors by modifying reference trajectory. Proposed scheme does not need prior knowledge of feed-drive dynamics nor it requires complicated open-loop identification experiments. It is implemented by a digital pre-filter whose parameters are tuned automatically from simply moving the axes back-and-forth by standard G-code, or even on-the-fly as machine travels along a known tool-path. It requires no expertise. Safe and reliable auto-tuning is achieved by formulating the tuning problem based on iterative learning and convex optimization. Overall, proposed technique can improve dynamic accuracy of newer or older machines equipped with a modern NC system.

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## References

- [1] Altintas Y, Verl A, Brecher C, Uriarte L, Pritschow G (2011) Machine Tool Feed Drives. *Annals of the CIRP* 60(2):779–796.
- [2] Kono D, Matsubara A, Yamaji I, Fujita T (2008) High-Precision Machining by Measurement and Compensation of Motion Error. *IJMTM* 48(10):1103–1110.
- [3] Jamaludin Z, Van Brussel H, Pipeleers G, Swevers J (2008) Accurate Motion Control of XY High-Speed Linear Drives Using Friction Model Feedforward and Cutting Forces Estimation. *Annals of the CIRP* 57(1):403–406.
- [4] Erkorkmaz K, Kamalzadeh A (2006) High Bandwidth Control of Ball Screw Drives. *Annals of the CIRP* 55(1):393–398.
- [5] Van Brussel H, Van den Braembussche P (1998) Robust Control of Feed Drives with Linear Motors. *Annals of the CIRP* 47(1):325–328.
- [6] Dumanli A, Sencer B (2018) Optimal High-Bandwidth Control of Ball-Screw Drives with Acceleration and Jerk Feedback. *Precision Engineering* 54:254–268.
- [7] Matsubara A, Nagaoka K, Fujita T (2011) Model-Reference Feedforward Controller Design for High-Accuracy Contouring Control of Machine Tool Axes. *Annals of the CIRP* 60(1):415–418.
- [8] Sencer B, Dumanli A (2018) Spline Interpolation with Optimal Frequency Spectrum for Vibration Avoidance. *Annals of the CIRP* 67(1):377–380.
- [9] Altintas Y, Khoshdarregi MR (2012) Contour Error Control of CNC Machine Tools with Vibration Avoidance. *Annals of the CIRP* 61(1):335–338.
- [10] Siemens Sinumerik 840D/810D/FM-NC OEM package MMC User's Manual, (1997), 12a ed. .
- [11] Erkorkmaz K, Altintas Y (2001) High Speed CNC System Design. Part II: Modeling and Identification of Feed Drives. *IJMTM* 41(10):1487–1509.
- [12] Meulen SH, Tousain RL, Bosgra OH (2008) Fixed Structure Feedforward Controller Design Exploiting Iterative Trials: Application to a Wafer Stage and a Desktop Printer. *Journal of Dynamic Systems Measurement and Control* 130 (5):051006.
- [13] Boyd S, Vandenberghe L (2004) *Convex Optimization*, Cambridge University Press.
- [14] Yoon JY, Trumper DL (2014) Friction Modelling, Identification, And Compensation Based on Friction Hysteresis and Dahl Resonance. *Mechatronics* 24 (6):734–741.
- [15] Amthor A, Zschack S, Ament C (2010) High Precision Control Using an Adaptive Friction Compensation Approach. *IEEE Automatic Control* 55 (1):274–278.