Transportation Research Part C 107 (2019) 490–509

Contents lists available at ScienceDirect

Transportation Research Part C

journal homepage: www.elsevier.com/locate/trc

Managing traffic with raffles

Pinchao Zhang[a], Zhen (Sean) Qian[a,b,*]

[a] Department of Civil and Environmental Engineering, Carnegie Mellon University, Pittsburgh, PA 15213, United States
[b] Heinz College, Carnegie Mellon University, Pittsburgh, PA 15213, United States

ARTICLE INFO

Keywords:
Raffle
Risk aversion
Morning commute
Bottleneck model
Exponential utility function

ABSTRACT

Raffles have been widely used in commercial markets to influence consumers' behaviors. Instead of charging users a fee or issuing a credit to users as incentives, the raffle-based scheme incentivizes users to change their choices by giving out credits that are used to enter a lottery. Comparing to tolls or credits, raffles may be less controversial since it is based on voluntary participation. The promises of a raffle-based scheme is that users are more risk-seeking, thus willing to participate in the lottery, when stakes are low. This would particularly fit the traffic management where the cost for a user to change his/her choices may be relatively insignificant comparing to the reward of a raffle. This paper examines the effect of raffle or lotteries for travel demand management, particularly in the morning commute. We establish the utility-based user equilibrium by incorporating risks and uncertainties in users' choices. A class of Flat Raffle schemes is proposed where a constant lottery winning chance is set for a period of time during the morning commute. We show that a marginal user who is indifferent from participating the lottery or not would exist for arbitrary utility functions. Furthermore, we analytically derive and solve for the equilibrium using exponential utility functions, and subsequently the optimal travel profiles under four potential system management objectives. We find that minimizing the total generalized travel cost (TTC) seems a reasonable goal for an optimal Flat Raffle possibly resulting in approximately minimum commuting duration and minimum total queuing delay at the same time. Under the minimum TTC, there is no queuing delay for the user who is indifferent from participating the lottery or not. Most interestingly, a Flat Raffle scheme works the best and outperforms the optimal one-step toll, when the population is overall slightly risk-seeking. This is particularly appealing since the raffle reward is relatively significant comparing with a not-so-significant cost increase associated with users' behavioral change.

1. Introduction

Traffic congestion results from excessive demand using limited roadway capacity in a particular time of day and locations. The whole idea of travel demand management is to incentivize travelers to shift their choices (e.g., departure time choices and location choices). As a result, excessive demand can be eased so as to reduce congestion. Many incentive schemes are being designed and/or tested, which can be generally categorized into surcharge and credit. Surcharge, such as congestion tolls and parking dynamic pricing, is to charge travelers a fee during a designated time/location to discourage travels. Though proven to be practically effective in reducing congestion, it raises concerns regarding travelers' willingness to pay and social equity, thus is not generally favorable by travelers. On the other hand, credit, such as tradable credits or parking cash-out programs, is either challenging to implement in

* Corresponding author at: Department of Civil and Environmental Engineering, Carnegie Mellon University, Pittsburgh, PA 15213, United States.
E-mail addresses: pinchaoz@andrew.cmu.edu (P. Zhang), seanqian@cmu.edu (Z.S. Qian).

https://doi.org/10.1016/j.trc.2019.09.002
Received 18 March 2019; Received in revised form 2 September 2019; Accepted 2 September 2019

practice or its effectiveness remains to be explored.

This paper proposes a new incentive scheme, namely raffles or lotteries, for travel demand management. Instead of charging users a fixed fee, the raffle-based scheme incentivizes users to change their choices by giving out credits that are used to enter a lottery. The chance of winning the lottery is relatively low, but the reward, upon winning, is substantial. The main idea of a raffle-based scheme is that people are more risk-seeking when stakes are low (Haisley et al., 2008). For instance, two gambles are provided where the first gamble is getting paid $1 for sure or 1% chance to win $100, and the second gamble is getting paid $100 for sure or 1% chance to win $10,000. Most people would prefer a lottery in the first game and walk away with the fixed credit $100 in the second one. This is because most people can afford risks at a low stake, but not so much when the stake is high. Consider its applications in travel demand management. Paying every single traveler $1 to change his/her choice may be ineffective because the stake is so low. However, a low-stake lottery may be effective in shifting a fraction of travelers who are potentially risk-seeking.

Because designing and implementing incentives in reality is complex, we do not intend to praise raffles in this paper. Rather, we propose the concept of raffle applications in travel demand management, study the effectiveness of raffles in various settings, compare and contrast raffles with toll charges, and identify when raffles may be more effective than other incentive schemes in terms of several specific goals. We argue that raffles may be an effective incentive that can complement existing travel demand management strategies.

Using raffles to ease excessive travel demand has been implemented and tested in the real world (Merugu et al., 2009; Yue et al., 2015), which shows great promises to complement existing demand management strategies. This paper would provide some theoretical discussions regarding raffle applications in traffic demand management.

Lottery incentives has been proved to be a good tool to incentive people and been used by various business, educational institutes and governments for a long time. Thanks to the emerging techniques, it becomes possible to apply raffle-based policies for public management (Loiseau et al., 2011) and regulation of citizens' behaviors. The raffle-based policy may be easily implementable and robust (Loiseau et al., 2011) comparing to traditional policies. It is also considered to be acceptable to users because it is based on voluntary participation. Very little work has been done to explore the potential of raffle-based policies to alleviate traffic congestion. The goal of this paper is to study how a raffle-based policy could affect users' travel behavior and henceforth benefit the entire transportation system in several ways, and how this effect is different from conventional traffic management policies/strategies. More specifically, we will propose a class of raffle-based policies to alleviate morning commuting congestion and present how and when it may work.

To our best knowledge, the idea of using raffle-based schemes in transportation systems was first proposed by Morgan (2000). In this pioneering paper, the lottery was proven to be a good tool for maximizing the utility of publicly owned goods. Loiseau et al. (2011) focused on how the raffle-based scheme performs for allocating shared resources. The paper showed that a raffle-based scheme can be a useful tool for general congestion mitigation. Later, Loiseau et al. (2011) extended the work to Internet congestion management. Very little work was done in the area of traffic management. Merugu et al. (2009) conducted an experiment in Bangalore, India, using a lottery-based mechanism to encourage bus commuters to shift their commuting time from peak hours to off-peak. That paper showed that, as a result of the lottery-based reward scheme, the shuttle bus for commuters could be much less crowded and thus operated more effectively. Rey et al. (2016) provides a theoretical evidence that lottery-based revenue-neutral incentives can be efficient in shifting transit demand to off-peak time.

In this paper, we will take a theoretical approach to study the effect of a general raffle scheme in morning commute. The primary consideration is on how raffle can effective change commuters' departure time from home and thus improve the entire transportation system during morning commute. Different from Rey et al. (2016), the lottery mechanism is established by the public sector, and not revenue-neutral. We use the classical Vickrey's bottleneck model (Vickrey, 1969) to set up our network and travel demand. The Vickrey's model has been expanded or revised in the past few decades for studying various policy questions, including with elastic demand (Arnott et al., 1993), heterogeneous users (van den Berg and Verhoef, 2011; Lindsey, 2004), stochastic demand and capacity (Arnott et al., 1999; Li et al., 2008). This paper follows classical settings to assume a fixed roadway capacity, identical values of times among users, and a fixed commuting demand, which will be adequate to provide policy insights regarding raffle applications in traffic management.

To alleviate morning commuting congestion, various strategies were proposed such as congestion toll (Arnott et al., 1993; Yang and Huang, 1997; Tscharaktschiew and Evangelinos, 2019), tradable credit scheme (Verhoef et al., 1997; Akamatsu and Wada, 2017; Yang and Wang, 2011; Tian et al., 2013), parking management (Qian et al., 2011, 2012; Qian and Rajagopal, 2015), personalized information or incentives (Xiong et al., 2019; Zhu et al., 2019). Incentives mechanism design is also widely applied in other transportation systems, e.g., shared mobility services (Angelopoulos et al., 2018) and freight deliveries (Holguín-Veras et al., 2017). The key difference between a raffle-based scheme and these existing schemes is that users voluntarily participate in the incentive program with an uncertain outcome. In existing schemes, travelers are certain about what they will receive once they make their decisions of their choices. In a raffle-based scheme, travelers are aware of the sets of possible outcomes for a decision with some information regarding the probabilities of each outcome. Therefore, it is critical to understand how travelers respond to those uncertainties. It is likely that the response may vary substantially by time of day, by locations, or by trip purposes. In light of this, this paper would use different travel behavioral model from typical morning commute studies to incorporate travelers' decisions in response to uncertain outcomes. Instead of using a deterministic generalized travel cost in most morning commute studies, we use the probabilistic utility (disutility) to encapsulate the choice decisions. We define a (dis)utility function that maps the monetary value/cost of outcomes with certain probabilities to a numerical value that represents the perceived utility of a decision made upon those possible outcomes. A commonly used form of such utility functions is a linear function. A consumer with a linear utility function would be indifferent from: 1) a choice of receiving 500 dollars for sure and; 2) a raffle with a 1% chance of winning 50,000 dollars and a 99% chance of receiving nothing. However, this is usually not the case in real world for most consumers. Consumers have different risk attitudes towards uncertainty (Hillson and Murray-Webster, 2004; Zhu et al., 2018). Provided with the same outcomes associated with the same respective probabilities, some may prefer uncertainty with a potentially high rewards, also known as risk-seeking users, while others may prefer certainty with a small reward, also known as risk-averse users. Users who are indifferent between the two types of decisions are risk-neutral users. Pålsson (1996) showed that household characteristics are related to consumers' degree of relative risk aversion. Holt et al. (2002) proposed several paired lottery choices as experiments to measure individuals' risk attitudes. In this paper, we assume that users' risk attitudes follow a general probability distribution that can be tuned based on real-world applications. The form of such a probability distribution can differ and calibrated from area to area. Thus, the performance of a raffle-based scheme is likely to depend on the population characteristics. We are interested in finding out under what distributions of population risk attitudes and why raffles may outperform other management strategies, and by how much.

The paper is organized as below. Section 2 introduces the basic settings of the problem comparing to the classical morning commute model, with the emphasis on the general settings of a raffle-based scheme and the utility function for individuals' choices. We will also define a specific raffle scheme in Section 2 that can be practically implemented and called a Flat Raffle Scheme. In Section 3, we develop theoretical properties of the Flat Raffle Scheme. We present how travelers make departure time choices under an equilibrium under raffle-based schemes. The advantages and disadvantages of a raffle-based scheme is discussed in Section 3 as well. Results of numerical experiments are presented in Section 4 with a comparative study of the raffle scheme with other existing schemes. We show under what conditions raffles are more advantageous than others and by how much. Section 5 concludes the paper and discusses further work.

2. The model

2.1. The classic morning commute model

We first introduce the classic morning commute model (also known as the bottleneck model) initially proposed by Vickrey (1969). Consider a highway connecting a residential area and a workplace where commuters travel to the workplace in the morning. The highway represents an aggregation of all alternative roads for morning commute, and it contains a bottleneck with a limited capacity. Commuters choose a departure time to maximize their respective utilities (or minimize their disutilities). Denote the number of commuters and the bottleneck capacity as N and s respectively.

When the inflow rate exceeds the bottleneck capacity, a queue will form before the bottleneck, resulting in queuing delay. A user's travel time consists of two parts: free flow travel time t_f and waiting time at the queue (i.e. congestion delay) $w(t)$ where t is the departure time of the user from the origin. Note that the value of $w(t)$ not only depend on t but also other users' departure time, since the congestion is due to the choice of all road users. The free flow travel time is assumed to be zero and constant, without loss of generality. Thus, $t + w(t)$ is the arrival time to the office of a user departing home at t. All users have an identical desired arrival time to the office t^*. α is the monetary value of queuing time. β and γ are the monetary vales of schedule early delay and schedule late delay, respectively. According to Small (1982), we would assume $\gamma > \alpha > \beta > 0$. The generalized travel cost (or equivalently disutility in this case) of a commuter who departs home at t is,

$$c(t) = \alpha w(t) + \max\{\beta(t^* - t - w(t)), \gamma(t + w(t) - t^*)\} \tag{1}$$

Users choose their departure time to minimize their generalized cost. Under User Equilibrium, all users would experience identical generalized cost and no one can reduce the cost by unilaterally making another choice of departure time. Consequently, arrival curve to the office, departure curve from home, as well as time-varying queuing delay, can be analytically derived to provide policy implications.

2.2. Utility functions and risk attitudes

In general, we assume that people make decisions based on their perceived utility or disutility rather than the generalize travel cost. Utility is a numeric measure of preferences over a combination of goods and/or service for a consumer. If a choice A leads to a greater utility for a consumer than a choice B, than he/she will prefer A to B if both choices are available. It has been argued that a utility function only gives the ordinal preference of possible choices (Harsanyi, 1953; Koopmans, 1960; High and Bloch, 1989). For instance, choice A is preferred than B for the utility of A is greater than B, but the utility associated with A and B do not necessarily carry quantitative information. A direct implication following this argument is that utilities cannot be compared between consumers. Note that the individual utility is more general than and different from the generalized travel cost that is used in classic morning commute model. Though consumers are not always rational or may not be aware of his/her own utility (Hirshleifer and Teoh, 2003; Becker, 1962; Kahneman, 1994), in this paper, we assume that every user is rational based on his/her ordinal utilities. Their decisions are made through maximizing their expected utilities.

A typical utility function can be represented as $U(c, g)$. c is a measure of expenditure or generalized cost, and g is a measure of the quantities of various goods a consumer receives. In the morning commute model, we only consider the generalized travel cost $c(t)$ at time t and consider the quantities of goods, namely the service provided by road infrastructure, are identical for all users. Thus, a user's utility function can be simplified to $U(c)$.

Assumption 1 (*Monotonic utility*). For a particular user, his/her utility monotonically decreases with respect to the generalized travel cost.

$$\frac{\partial U}{\partial c} < 0 \tag{2}$$

In general, different users would have different utility functions, thus we need a subscript to indicate the utility function for each particular user. To present the ideas clearly, in the remaining context, we will omit the user indicator subscript unless otherwise specified. But note that our utility presentation works for each individual, not for the population. While computing the generalized travel costs c that users experience for their respective trips, we assume they are homogeneous in terms of value of time for simplicity. Namely, they share the same values of α, β and γ in Eq. (1). While the utility function $U(c)$ that maps a travel cost to a perceived utility vary by users. In the real world, commuters have different values of time, especially those with different risk attitudes. Here we work with an average value of time and value of schedule delay for all commuters, for deriving analytical results. Users' heterogeneity is assumed to stem entirely from their heterogeneous risk attitudes that translate travel costs to utilities differently. In other words, each commuter with a distinct risk attitude would have a general perception of how his/her travel cost varies by different departure time, which is encapsulated by the homogeneous generalized travel cost function, but the cost would represent different utilities due to her/her distinct risks. This would allow us to separate the effect of generalized travel cost from the effect of risk attitudes, and obtain analytical solutions for policy insights.

As an example, a simple utility (disutility) function with respect to the generalized travel cost could be

$$U(c) = -c \tag{3}$$

which ensures $U'(c) < 0$. This is precisely the case for the classic morning commute model. This utility function is linear in terms of travel cost. As a result, a user who makes decision based on the utility function in Eq. (3) will be indifferent between the two choices of the same expected travel costs. While dealing with uncertainty with respect to the cost, this user would make decisions based on the (negative) expected travel cost. This is a risk neutral user. The classic morning commute model implicitly assumes all users are risk neutral.

$$\mathbb{E}(U(c)) = -\mathbb{E}(c) \tag{4}$$

In the real world, most users have non-neutral risk attitudes. For example, gambles or lotteries always have negative expected returns, and people who participate in those are risk seeking. In general, each person usually has various risk attitudes under different stakes. The risk attitude varies substantially by users as well. We will work with utility functions to characterize risk attitudes for a population of users, and for a user at different stakes.

The Arrow-Pratt measure of absolute risk-aversion (ARA) (Arrow, 1965; Pratt, 1992) is a commonly used measure of risk attitudes for a user with a utility function $U(c)$ with respect to the user's cost c. It is also known as degree of absolute risk aversion $A(c)$ and is defined as,

$$A(c) = \frac{U''(c)}{U'(c)} \tag{5}$$

A linear utility function such as Eq. (3) gives $ARA = 0$, which implies that all users with this utility function are risk-neutral at any cost c. A positive ARA implies a risk-aversion user, whereas a negative ARA implies a risk-seeking user. Recall that $\frac{\partial U}{\partial c} < 0$ always holds under Assumption 1. Therefore, when the utility function $u(c)$ is convex for some c, then $A(c) < 0$ and the user is risk-seeking with this cost. When $u(c)$ is concave for some c, then this user becomes risk-aversion. The degree of risk aversion $A(c)$ is a function of c, namely the risk attitude changes with the travel cost for a particular user. We assume that all users (with their respective utility functions $u_1(c)$, $u_2(c)$, $\cdots, u_i(c)$, $\cdots$) has a consistent order of the degree of risk aversion which is defined as the following.

Assumption 2 (*A consistent order of the degree of risk aversion at any cost*). For any two users in the population whose degrees of risk aversion read $A_1(c)$ and $A_2(c)$, respectively, if $A_1(c') < A_2(c')$ for some cost c', then for any cost c'' we have $A_1(c'') \leqslant A_2(c'')$.

Although the risk attitude of a user could change with respect to his stakes, Assumption 2 ensures that he/she will always be more risk-averse (seeking) than another user at any particular cost, if for some cost c he/she is more risk-averse (seeking) than that user. This implies that all users in the population can be consistently ordered in terms of their risk attitudes, regardless of any particular stake. Having this property enables us to compare the general risk attitudes of two users, and ensures the consistency that one user is more risk seeking or averse than another user.

For instance, one of the many utility functions that preserves the order of the degree of risk aversion is the exponential utility function that has a constant ARA. The use of this utility function was initially proposed in Arrow (1971). The exponential utility given the degree of risk aversion of a user λ,

$$U(c, \lambda) = \begin{cases} \frac{1-\exp\{\lambda c\}}{\lambda} & \lambda \neq 0 \\ -c & \lambda = 0 \end{cases} \tag{6}$$

One can verify $ARA = \lambda$ for the exponential utility function. Fig. 1 plots the exponential utility function with different values of λ. The utility declines with the increase in the generalized travel cost c. The utility curve of a risk-seeking user ($\lambda = -0.01$) is convex,

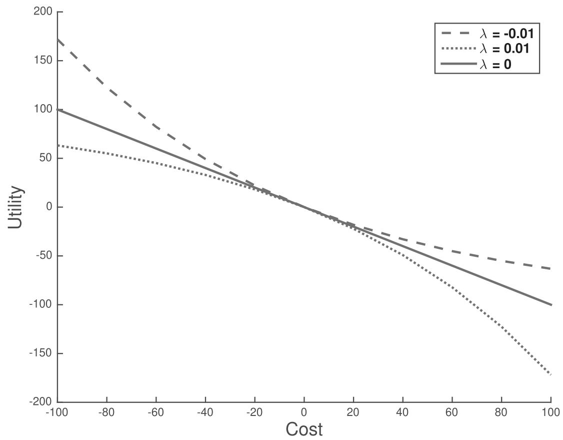


Fig. 1. Exponential utility function.

while that of a risk-aversion user ($\lambda = 0.01$) is concave. Note that the utility function is ordinal. Although the curve with $\lambda = -0.01$ lies above the other two at all possible travel costs, it does not imply that the user with risk attitude $\lambda = -0.01$ always gains more benefits than the other two users ($\lambda = 0, 0.01$) for the same travel cost. The value of utilities can only be compared within the same utility curve (namely for the same user). This paper works with a general utility function $U(c)$, but will adopt the exponential utility $U^e(c, \lambda)$ when providing specific examples.

Our proposed raffle scheme is for general users with any forms of utility functions that follows a consistent order of degree of risk aversion. In the remaining part while discussing the properties of the equilibrium for our raffle schemes, we will assume general utility functions when possible. While solving the equilibrium and conducting numerical experiments, we adopt the exponential utility functions as a demonstrative example.

Studies have argued that many people tend to be more conservative dealing with decisions regarding a large amount of money, and more adventurous with little money. In other words, λ may be dependent on the cost c for any particular user in the real world. Here, we assume λ is independent of c for three reasons: (1) it allows deriving analytical results for policy insights; (2) in our case, given the total commuting demand is fixed and the same risk aversion order across all the users is preserved regardless of stakes, the resultant users' choices would not deviate too much from the case with cost-varying risk attitudes; and (3) under long-term system equilibrium when the cost c becomes relatively stable to users, one's expected utility could only vary in a small range. Therefore, it may be reasonable to assume an independent λ within a small range of c.

2.3. The morning commute model under a general utility function

In the classic bottleneck model, since all users are considered to be homogeneous in terms of risks and Eq. (3) is adopted, the equilibrium is achieved when all users experience the same generalized travel cost. This may not hold in general. Provided that each user has his/her own general utility function, we first define an equilibrium of all users' choices based on their utilities.

Definition 1 (*Utility-based User Equilibrium***).** An equilibrium is defined as a state that, given the respective departure time choices of all users, no single user can increase the (expected) utility by unilaterally changing his/her departure time choice.

Since the capacity of the bottleneck is assumed to be fixed and deterministic, once every user determines their departure times, the cumulative arrival and departure curves of the bottleneck are known. The two cumulative curves are also known as a "profile" of the morning commute travel pattern. We have the following proposition.

Proposition 1. *Given the respective utility function for each user, any travel profile under the cost-based user equilibrium is still a utility-based user equilibrium, if there is no uncertainty associated with users' generalized travel cost and if both Assumptions 1 and 2 hold.*

Proof. Under the equilibrium based on the generalized travel cost, any user cannot reduce his/her cost by unilaterally changing his/her departure time choice. A general utility function is a monotonically decreasing function of the generalized travel cost that is known, fixed and deterministic. Hence under the profile of cost-based user equilibrium, no user can increase his/her utility by unilaterally changing his departure choice. The utility-based equilibrium still holds. □

Recall that under the cost-based user equilibrium profile, every user experiences the identical cost c. While under the utility-based user equilibrium, users' utilities are not identical across users, unless for users who have identical risk attitude (e.g., *ARA*). The absolute value of the utility cannot be compared among different users in general.

As a result, the equilibrium profile derived in Tian et al. (2013), Yang and Huang (1997), and Arnott et al. (1993) and many others can be directly generalized to a utility-based equilibrium. However, this is not the case when there exists uncertainty associated with the utility or cost of users' choices. We will show that Proposition 1 does not hold when we take into account uncertain outcomes in users' choices.

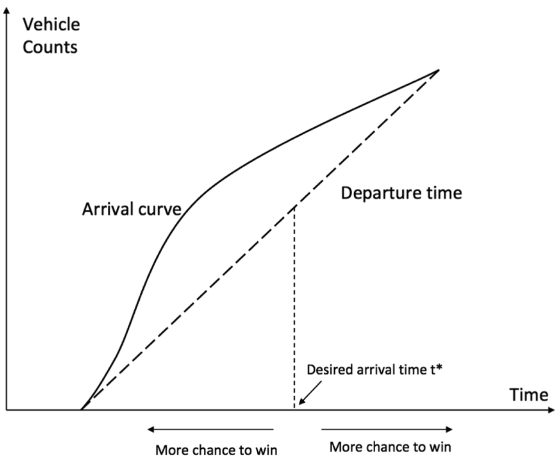


Fig. 2. A general raffle scheme.

Note that we work with the utility-based equilibrium in the reminder of this paper. For notation simplicity, an "equilibrium" refers to a utility-based equilibrium unless specified otherwise.

2.4. A class of raffle-based schemes

In this section, we propose a class of raffle-based schemes. The transportation system manager will hold a daily lottery to choose one winner from a subset of commuters to share a prize R. These commuters are those who voluntarily participate to travel within a desired time window that is set as part of the lottery system. The chance that a user can win the prize depends on his/her departure time choice, and in general we use $P(t)$ to represent the probability of winning the prize at a departure time t. Upon the lottery participation, the expected utility of a user departing at time t is,

$$\mathbb{E}(U(c(t))) = (1-P(t))U(c(t)) + P(t)U(c(t)-R) \tag{7}$$

Intuitively, if the system goal is to alleviate morning commute traffic congestion, we would incentivize some users to shift their departure time from peak hours to off-peak hours. Thus, a higher probability could be assigned to users traveling farther away from the desired work starting time t^* than those close to t^*, as described as in Fig. 2. $P(t)$ is expected to be positively proportional to the marginal cost of a user to the system, just like the optimal congestion pricing.

For instance, if we adopt the exponential utility function, the expected utility of a user with risk λ is,

$$\mathbb{E}(U(c(t))) = \begin{cases} \frac{1}{\lambda} - \frac{\exp\{\lambda c(t)\}}{\lambda}[1-P(t)+P(t)\exp\{-\lambda R\}] & \lambda \neq 0 \\ -c(t) + P(t)R & \lambda = 0 \end{cases} \tag{8}$$

Proposition 2. *If all users are risk neutral, the following raffle scheme completely eliminates all queue delay for the morning commute.*

$$P(t) = \begin{cases} -\frac{\beta}{R}t + c_1 & t \leqslant t^* \\ \frac{\gamma}{R}t + c_2 & t > t^* \end{cases} \tag{9}$$

where

$$c_1 = \frac{-N^2\beta\gamma/2 + N\beta^2st^* + N\beta\gamma st^* + R\beta s + R\gamma s}{NRs(\beta+\gamma)} = -\frac{N\beta\gamma}{2Rs(\beta+\gamma)} + \frac{\beta t^*}{R} + \frac{1}{N} \tag{10}$$

$$c_2 = \frac{-N^2\beta\gamma/2 - N\beta\gamma st^* - N\gamma^2st^* + R\beta s + R\gamma s}{NRs(\beta+\gamma)} = -\frac{N\beta\gamma}{2Rs(\beta+\gamma)} - \frac{\gamma t^*}{R} + \frac{1}{N} \tag{11}$$

Proof. When all the queue was eliminated, the travel time $w(t)$ will remain 0 for all t. For a user with early arrival, the utility is $u(t) = -\beta(t^*-t) + P(t)R$. For a user with late arrival, the utility is $u(t) = -\gamma(t-t^*) + P(t)R$. Since all users are risk neutral and have identical values of time, under equilibrium they will experience the same utility. Thus we have

$$u'(t) = \begin{cases} \beta + P'(t)R & , t \leqslant t^* \\ -\gamma + P'(t)R & , t > t^* \end{cases} = 0 \tag{12}$$

If the queue was totally eliminated, the incoming flow rate of the bottleneck will be equal to the capacity s. Since $P(t)$ is a probability we will have

$$\int_{t_0}^{t^e} sP(t) = 1 \tag{13}$$

From Eq. (12) we have

$$P(t) = \begin{cases} -\frac{\beta}{R}t + c_1 & t \leqslant t^* \\ \frac{\gamma}{R}t + c_2 & t > t^* \end{cases} \quad \square \tag{14}$$

Remark: there exists a lower bound of R such that the probability $P(t)$ may be negative when the prize is set to lower than this bound. This indicates that there is a minimum prize R in order for the time-varying raffle scheme to work.

Arnott et al. (1990) and Xiao et al. (2013) have showed that both the congestion toll and tradable credits can achieve the system optimum (SO) for the morning commute. They do not consider the risk attitudes of users. Here we show that if all users are risk neutral (essentially an underlying assumption in those studies) we can also find a raffle scheme leading to SO. The winning probability over time is analogous to the SO credit/toll, and is related to the amount of prize R.

In reality, users have heterogeneous risk attitudes. Assuming homogeneity of risk neutral would undermine the promises of raffles in congestion mitigation, since raffles would otherwise incentivize those who are relatively risk-seeking to effectively change their travel choices. For example, if we assume all users' utilities follow the exponential function with homogeneously $\lambda = 1$ (namely we assume all users are risk aversion rather than risk neutral), we could solve a differentiable equation based on the first order condition of utility-based user equilibrium,

$$\beta e^{\beta(t^*-t)} - \beta e^{\beta(t^*-t)}P(t) + e^{\beta(t^*-t)}P'(T) + \beta e^{\beta(t^*-t)}P(t)e^{-R} - e^{\beta(t^*-t)}P'(t)e^{-R} = 0 \tag{15}$$

In practice, it can be challenging to implement the winning probabilities for the lottery (or congestion tolls, credits) that continuously change over time of day, and a simple structure of lottery would be much preferred and acceptable to users. More importantly, we would like to start with a simply structured lottery scheme to derive analytical results for policy insights. In addition, we will work with a population of commuters with heterogeneous risk attitudes, modeled by a probability distribution of λ.

In the congestion pricing literature, a coarse pricing scheme was proposed instead of continuous time-varying scheme (Arnott et al., 1990). In the tradable credit literature, past studies usually fix the price of the credit as a constant within a time window, instead of a time-varying credit price (Xiao et al., 2013). Following a similar fashion, this paper proposes a class of raffle schemes where the winning chance within a predetermined time window is constant, called the Flat Raffle scheme in the reminder of this paper.

The Flat Raffle scheme is designed as follows: the transportation system manager chooses a time interval $[t^+, t^-]$ considered as the peak time window, and provides a raffle prize R. All users who arrive at the bottleneck outside the peak time ($t + w(t) \notin [t^+, t^-]$) will receive an entry with a small probability to win the prize. This probability is identical for all entries and is dependent on the total number of entries given out on a particular day. All users who pass the bottleneck during the peak time window will not enter the lottery. The overall idea is represented as in Fig. 3.

We use N_{np} to denote the number of users who pass the bottleneck in non-peak time intervals. The probability function $P(t)$ can be written as

$$P(t) = \begin{cases} \frac{1}{N_{np}} & t + w(t) \notin [t^+, t^-] \\ 0 & t + w(t) \in [t^+, t^-] \end{cases} \tag{16}$$

A user's expected utility is then,

$$\mathbb{E}(U(c(t))) = \begin{cases} U(c(t)) & t \in [t^+, t^-] \\ (1 - P(t))U(c(t)) + P(t)U(c(t) - R) & t \notin [t^+, t^-] \end{cases} \tag{17}$$

Using the exponential utility function as an example, the expected utility of a user with risk λ is,

$$\mathbb{E}(U(c(t))) = \begin{cases} -\alpha w(t) - \max\{\beta(t^* - t - w(t)), \gamma(t + w(t) - t^*)\} & t + w(t) \in [t^-, t^+], \lambda = 0 \\ -\alpha w(t) - \max\{\beta(t^* - t - w(t)), \gamma(t + w(t) - t^*)\} + \frac{R}{N_{np}} & t + w(t) \notin [t^-, t^+], \lambda = 0 \\ \frac{1}{\lambda} - \frac{\exp\{\lambda(\alpha w(t) + \max\{\beta(t^* - t - w(t)), \gamma(t + w(t) - t^*)\})\}}{\lambda} & t + w(t) \in [t^-, t^+], \lambda \neq 0 \\ \frac{1}{\lambda} - \frac{\exp\{\lambda(\alpha w(t) + \max\{\beta(t^* - t - w(t)), \gamma(t + w(t) - t^*)\})\}}{\lambda}\left(1 - \frac{1}{N_{np}} + \frac{1}{N_{np}}e^{-\lambda R}\right) & t + w(t) \notin [t^-, t^+], \lambda \neq 0 \end{cases} \tag{18}$$

The flat raffle scheme is easy to implement. There exist inexpensive technologies that allow the system managers to record the commuters arriving at the bottleneck during a desired time window. Comparing to congestion pricing or tradable credit scheme, it requires similar technologies, but may be less controversial, since the participation is voluntary. Unlike the tradable credit scheme where a platform for credit trading is required, the implementation of a raffle scheme requires a centralized lottery system to collect commuters' information and draw the lottery on a regular basis. The cost of such a lottery system is likely cheaper than a platform that requires frequent trading among many entities.

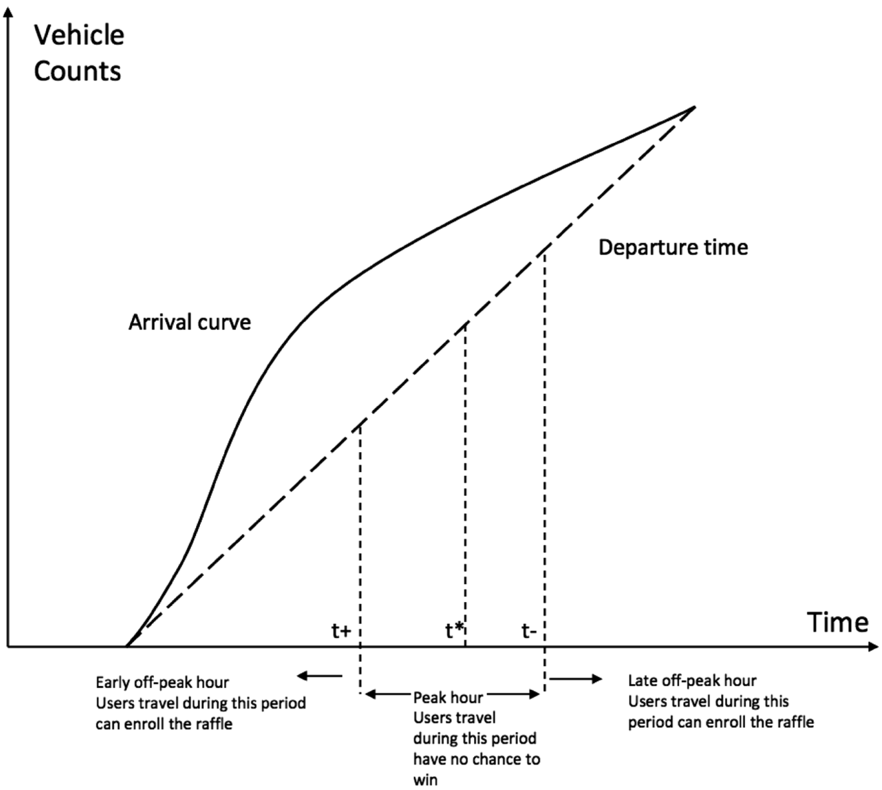


Fig. 3. The flat raffle scheme.

3. Equilibrium under the Flat Raffle scheme

In this section, we will derive and discuss the properties of equilibrium under the Flat Raffle scheme using a general utility function in general, followed by an example in the exponential utility function. We will define what makes a good raffle scheme.

3.1. Notations

First of all, we introduce some notations that will be used in the reminder of this paper in Table 1.

3.2. Properties of the Flat Raffle scheme

Proposition 3. *Assume an equilibrium exists under the flat raffle scheme. Under equilibrium, a raffle (non-raffle) user must experience identical expected utility by unilaterally changing his/her departure time within the raffle (non-raffle) time window. Denote the raffle users' generalized cost as c_r and the non-raffle users' generalized cost as c_{nr}, then $c_r \geqslant c_{nr} \geqslant c_r - R$.*

Proof of Proposition 3. First work with the non-raffle users. Denote $t' \in [t^-, t^+]$ be the departure time of a non-raffle user who experiences the most generalized cost under equilibrium among all non-raffle users. Hence for any time $t \in [t^-, t^+]$ we have $c(t') \geqslant c(t)$. By the definition of an equilibrium, we have $U(c(t')) \geqslant U(c(t))$ for this user departing at time t'.

On the other hand, by Assumption 1 where $\frac{\partial U}{\partial c} < 0$, if $c(t') \leqslant c(t)$, we must have $U(c(t')) \geqslant U(c(t))$. Hence, $c(t) = c(t')$ hold for any $t \in [t^-, t^+]$.

The proof is similar to raffle users due to the fact that the winning probability is constant throughout the raffle time period.

Assume $u_1(c)$ is the utility function of a non-raffle user. Under equilibrium, he/she would not voluntarily participate in the raffle, and in a view of $\partial U/\partial c < 0$ therefore,

$$u_1(c_{nr}) \geqslant (1 - P)u_1(c_r) + Pu_1(c_r - R) \geqslant u_1(c_r)$$

Thus we have $c_{nr} \leqslant c_r$. Similarly we can find $c_{nr} \geqslant c_r - R$. □

Remark: The definition of equilibrium states that a user cannot increase his/her utility by changing his/her departure time choice. Proposition 3 further shows that changing his/her departure time within either raffle or non-raffle time period will not change his/her utility. Given Assumption 1, the generalized travel cost is the same for any user within the raffle (or non-raffle) time period

Table 1
Table of notation.

α	Cost per unit travel time
β	Cost per unit early arrival time
γ	Cost per unit delay arrival time
R	The total amount of raffle prize, determined by the policy maker
t^+	The starting time of peak hours (with a raffle), determined by the policy maker
t^-	The ending time of peak hours (with a raffle), determined by the policy maker
t	Time, usually means the user departure time if not specified
$\mathbf{t}$	The profile of all departure times for all commuters
t^*	The desired arrival time of all morning commuters
$w(\mathbf{t}, t)$	Given $\mathbf{t}$, the travel time of a user departing at time t, written as $w(t)$ for simplicity
$P(\mathbf{t}, t)$	Given $\mathbf{t}$, the probability of winning the prize if arriving the bottleneck at time t, written as $P(t)$ for simplicity
N	Total number of daily users of the bottleneck
N_{np}	Number of users departing from the bottleneck during the non-peak time window, aka the number of raffle users
t_0	The departure time (from home) of the first morning commuter
t_e	The departure time (from home) of the last morning commuter
t_1	The departure time (from home) of the last early-arrival, raffle user
t_2	The departure time (from home) of the first early-arrival, non-raffle user
t_3	The departure time (from home) of the user who departs from the bottleneck at t^*
t_4	The departure time (from home) of the first braking user, also of the last non-braking user
t_b	The arrival time (to the bottleneck) of the first braking user, also of the last non-braking user
$U(c)$, $u(c)$	A general utility function
$U^e(c, \lambda)$, $u^e(c, \lambda)$	The exponential utility function
λ	The parameter in the exponential function, which is the degree of risk aversion
λ_{cr}	The degree of risk aversion of the marginal user who is indifferent participating the lottery or not
c_r (c_{nr})	The generalized travel cost of all raffle (non raffle) users under equilibrium

according to Proposition 3. In other words, all raffle (non-raffle) users will experience identical generalized cost $c(t)$ under equilibrium. Then part of the results from the classic morning commute model that the cumulative arrival curve to the bottleneck has a slope of $\frac{\alpha s}{\alpha-\beta}$ before t_3 and $\frac{\alpha s}{\alpha+\gamma}$ after t_3 still hold here.

Proposition 3 helps determine the shape of both arrival curve and departure curve within the raffle and non-raffle time period, respectively. Next we explore the travel profile at the two boundary time (t^+ and t^-) when travelers switches from raffle to non-raffle (or the other way around).

Proposition 4. *Under the equilibrium with the flat raffle scheme, any raffle user is more risk seeking than any non-raffle user.*

Proof. According to Proposition 3, denote that under equilibrium all raffle users' generalized cost is c_r and all non-raffle users' generalized cost is c_{nr}. All users have the two choices: either 100% chance to cost c_{nr}, or with a probability of P to cost c_r and a probability of $1-P$ to cost $c_r - R$. Apparently, those who prefer choice 1 are more risk averse than those who prefer choice 2. □

The intuition is that participating in the raffle will increase the uncertainty of a user's utility. As a result, only those users who are more risk-seeking would see the uncertainty as a potential "benefit". In other words, the raffle scheme naturally order the users' departure time by their risk attitudes (e.g., λ in the case of exponential utility functions).

Next we give the definition of a marginal user which is critical to solve for equilibrium profiles. We argue that we can always find such a user that preserves the order of degree of risk aversion.

Definition 2 (*Marginal user***).** Under the equilibrium with the flat raffle scheme, a marginal user is indifferent between participating in the raffle or not.

Proposition 5 (*Existence of a marginal user***).** *Given a Flat Raffle scheme and any arbitrary utility functions for all users such that the order of degree of risk aversion is preserved at any cost, we can always find a utility function that (1) adding a user associated with this utility function will still preserve the order of degree of risk aversion across all users; and (2) this new user is indifferent between participating in the raffle or not, under equilibrium.*

Proof of Proposition 5. by construction.

Since all users follow the same order of degree of risk aversion at any cost, we are able to find the most risk seeking non-raffle user, denoted as user 1 with his/her own utility function $u_1(c)$, and the most risk averse raffle user, denoted as user 2 with his/her own utility function $u_2(c)$. According to Proposition 4 we must have user 2 being more risk seeking than user 1.

We first show that we can find a user with a utility function $u(c, \lambda)$ such that it is continuous on λ this user is indifferent between participating in the raffle or not. Suppose the largest degree of risk aversion of user 1 can have given the cost interval $[c_r - R, c_r]$ is λ_1. We can construct a virtual user 1 with the utility function $u(c, \lambda_1)$. Under the equilibrium, this virtual user 1 would not participate in the raffle, therefore $u(c_{nr}, \lambda_1) - Pu(c_r - R, \lambda_1) - (1-P)u(c_r, \lambda_1) \geqslant 0$. Similarly suppose the lowest degree of risk aversion of user 2 can have $[c_r - R, c_r]$ is λ_2. We then construct a virtual user 2 with the utility function $u(c, \lambda_2)$. Under the equilibrium, this virtual user would participate in the raffle, therefore $u(c_{nr}, \lambda_2) - Pu(c_r - R, \lambda_2) - (1-P)u(c_r, \lambda_2) \leq 0$.

Now define a function of λ:

$$F(\lambda) = u(c_{nr}, \lambda) - Pu(c_r - R, \lambda) - (1 - P)u(c_r, \lambda) \tag{19}$$

Since the exponential utility function $u^e(c, \lambda)$ is a continuous function with respect to λ, $F(\lambda)$ is also a continuous function with respect to λ. $F(\lambda_1) \geqslant 0$ and $F(\lambda_2) \leqslant 0$, then there must exist a λ' between λ_1 and λ_2 such that $F(\lambda') = 0$. This indicates that this user is indifferent between participating in the raffle or not, at the degree of risk aversion λ' and at some cost $c \in [c_r - R, c_r]$. Furthermore, we can always find a utility function for this new user $u(c)$ such that the degree of risk aversion of this new user is between user 1 an user 2 for $\forall\ c \in [c_r - R, c_r]$, and meanwhile his/her degree of risk aversion is λ' and at some cost $c \in [c_r - R, c_r]$. This guarantees the consistent order of risk aversion for this particular user among all the existing users. □

3.3. The Flat Raffle scheme with exponential utility functions

We have two corollaries for the case when the utility function of all users takes the form of an exponential utility function.

Corollary of Proposition 5 (*Existence of a marginal user when all users follow exponential utility functions***).** Suppose the utility functions of all users take the form of exponential utility functions. Given a Flat Raffle scheme and the probability distribution of risk attitudes λ over a population, there must exist some value $\lambda_{cr} \in [\min\{\lambda\}, \max\{\lambda\}]$ such that a user with degree of risk aversion λ_{cr} is indifferent between participating in the raffle or not under equilibrium.

Corollary of Proposition 4 (*Existence of a critical risk attitude that separates users from participating in the lottery or not***).** When all users follow exponential utility functions, all raffle users' degrees of risk aversion are no greater than λ_{cr}, and all non-raffle users' degrees of risk aversion are no less than λ_{cr}.

Starting from this subsection, we will particularly investigate the case of exponential utility function, and discuss how we can find the travel profiles under equilibrium, given a flat raffle scheme (represented by a tuple of (t^+, t^-, R)). A similar method could be developed for any other utility functions.

The number of travelers participating in the raffle is N_{np}. When all users' utility functions take the form of exponential utility functions we have,

$$\lambda_{cr} = \lambda^{(N_{np}/N)} \tag{20}$$

where λ^a is the a-th quantile of the λ in the population.

Let t_1 denote the departure time from home of the last user who experiences early schedule delay and enrolls the raffle, and t_2 the departure time from home of the first user who experiences early schedule delay but does not take part in the raffle. We know the slope of the cumulative curves before t_1 and after t_2 under equilibrium. We present the following claims for the two specific users (t_1 user and t_2 user for short),

Claim 1. Under equilibrium, the t_1 user must pass the bottleneck exactly at t^+. In other words, $t_1 + w(t_1) = t^+$.

Proof of Claim 1. Suppose $t_1 + w(t_1) < t^+$. The first early-arrival and non-raffle user arrives no earlier than t^+, since t^+ is the starting time of the raffle. In this case, the t_1 user is able to switch his/her departure time to a time between $t_1 + w(t_1)$ and t^+, staying in the raffle and experiencing less travel time and early schedule delay cost. This would not be an equilibrium if $t_1 + w(t_1) < t^+$. □

Claim 2. Under equilibrium, for the t_2 user, if he leaves home before t^+, then we must have $t_2 + w(t_2) = t^+$ and if he leaves after t^+, we must have $w(t_2) = 0$.

Proof of Claim 2. We know that $t_2 + w(t_2) \geqslant t^+$.

If $t_2 \leqslant t^+$, according to first-in-first-out (FIFO) principle, the t_2 user can always choose $t_2 + w(t_2) = t^+$ unilaterally. Comparing to any other case that $t_2 + w(t_2) > t^+$, the t_2 user simply get less travel time by experiencing more early delay. Since we assume $\beta < \alpha$, the user is willing to do so.

If $t_2 > t^+$, similarly since he wants to minimize his travel time, he will choose to set $w(t_2) = 0$. □

Claims 1 and 2 reveal how the cumulative curves work near t^+. Next we discuss the curves near t^-. Nie (2015) showed that it is challenging to determine the profiles at a 'discontinuous' point in the late arrival part of the bottleneck model under equilibrium defined in Definition 1.

As summarized in Nie (2015), there are three main ways to deal with this part, Mass Arrival (MA) (Vickrey, 1969), Separated Waiting (SW) (Laih, 1994) and Braking-Induced (BI) (Lindsey et al., 2010). The MA assumption used a mixed strategy Nash Equilibrium. All users under the MA will experience the same expected generalized cost. This is based on the cost-based equilibrium. However, this would not be the case under utility-based equilibrium with uncertainties. Hence the MA is not suitable for the raffles. The SW solves the equilibrium at the late arrival by assuming that several independent lanes (or queues) exist and users can choose to pass or wait before the bottleneck on a particular lane (or queue). Although it solves the discontinuity problem, the SW assumption can violate FIFO in the first place. In this paper, the BI assumption will be used to build the utility-based equilibrium which meets FIFO and does not require mixed strategy under equilibrium. The basic idea is that near the end of the peak hour $[t^+, t^-]$ (either for a coarse toll or a flat raffle), a user who prefers not to pass the bottleneck during the peak hour (but arrives to the bottleneck near t^-) can slow down or brake and wait until he/she ensures to pass the bottleneck immediately after the peak hours. In Lindsey et al. (2010), the user slows down because he/she wants to avoid the congestion pricing, just so he/she can pass the bottleneck without paying immediately after the toll period. In our case, a user may want to enroll the raffle to increase his/her expected utility by doing so.

We use t_4 to denote the departure time from home of the first braking user, namely the first late arrival user who enrolls the raffle. t_4 is also the departure time from home of the last non-braking user, namely the last late arrival user who does not enroll the raffle. We use t_b to denote the arrival time to the bottleneck of the last non-braking user. For these two users, we have following claims,

Claim 3. Under equilibrium, the last non-braking user passes the bottleneck between t^* and t^-.

Proof of Claim 3. If $t_b < t^*$, then this user can change his/her departure time such that he/she passes the bottleneck right at t^*, which does not change the order of the departure time of any other users and he/she can reduce his generalized travel cost by reducing the schedule delay. This contradicts the equilibrium definition. If $t_b > t^-$, this directly contradicts the BI assumption. □

Claim 4. The first braking user passes the bottleneck exactly at t^-.

Proof of Claim 4. According to Claim 3, this user is able to pass the bottleneck at any time after t_b, including t^-. If the user passes the bottleneck before t^-, then he/she cannot enroll the raffle, which defeats the goal for braking (namely he/she can reduce the generalized cost by not doing so). If the user passes the bottleneck after t^-, as $\alpha < \gamma$, he/she can unilaterally change his/her arrival time to t^- and increase the utility. □

These above two claims show that the braking user can neither leave home too early nor pass the bottleneck too late. We have one more claim for the first and last commuters:

Claim 5. Under equilibrium, the travel time of first user and last user must be 0.

Proof of Claim 5. For the last user, if he/she experience a non-zero travel time, since there is no user traveling after him/her, he/she can unilaterally change the arrival time to t_e, so that he/she departs the bottleneck as the same time as before, but experience less travel time. □

Given all the claims above, we can plot the possible shapes of the profile under equilibrium. Fig. 4 shows two possible cases of the travel profiles under equilibrium, due to Claim 2 near t^+. Fig. 4a corresponds to the case when $w(t_2) = 0$, and Fig. 4b corresponds to the case when $t_2 + w(t_2) = t^+$. Fig. 4c is a special case of both Fig. 4b and a where the conditions of the two cases are met simultaneously.

Next we analytically derive the travel profiles based on the claims and graphs. We will show that solving the equilibrium is equivalent to solving a system of equations, given a flat raffle scheme uniquely defined by t^+, t^- and R. We will represent t_1, t_2 and $w(t_2)$ in three equations, so as to solve for the equilibrium profiles.

Claim 1 leads to,

$$w(t_1) = t^+ - t_1 \tag{21}$$

According to slopes of the arrival and departure curves and FIFO, we have

$$(t_1 + w(t_1) - t_0)s - (t_1 - t_0)\frac{\alpha s}{\alpha - \beta} = 0 \tag{22}$$

$$(t_3 - t_2)\frac{\alpha s}{\alpha - \beta} - (t^* - t_2 - w(t_2))s = 0 \tag{23}$$

$$(t_e - (t^- - t_b) - (t_2 + w(t_2) - t_1 - w(t_1)) - t_0)s = N \tag{24}$$

$$(t_b - (t_2 + w(t_2)) + (t_1 + w(t_1) - t0))s = (t_1 - t_0 + t_3 - t_2)\frac{\alpha s}{\alpha - \beta} + (t_4 - t_3)\frac{\alpha s}{\alpha + \gamma} \tag{25}$$

Every raffle user experiences the same generalized cost by Proposition 3,

$$\beta(t^* - t_0) = \gamma(t_e - t^*) \tag{26}$$

From Eqs. (21)–(26), we can represent the values of t_0, $w(t_1)$, t_3, t_4, t_b, t_e in terms of t_1, t_2 and $w(t_2)$ (given t^+, t^- and R as known). The number of raffle users N_{np} is the total number of users who pass the bottleneck outside of the time window $[t^+, t^-]$. Hence,

$$N_{np} = (t_1 + w(t_1) - t_0)c + (t_e - t^-)s \tag{27}$$

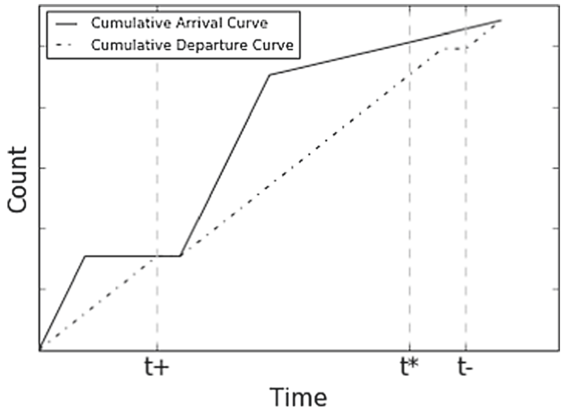
So N_{np} can be represented in terms of t_1, t_2 and $w(t_2)$. According to Eq. (20), λ_{cr} can also be represented in terms of t_1, t_2 and $w(t_2)$. All the travelers must add up to the total demand N,

$$\frac{\alpha s}{\alpha - \beta}(t_3 - t_2 + t_1 - t_0) + \frac{\alpha s}{\alpha + \gamma}(t_e - t_3) = N \tag{28}$$

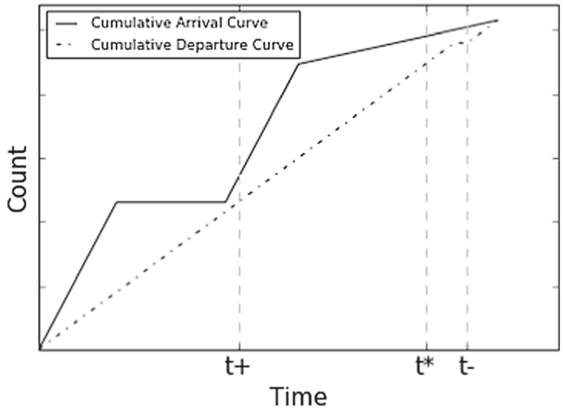
which can be used as the third equation for the equilibrium. Substitute all variables in terms of t_1, t_2 and $w(t_2)$ in Eq. (28),

$$\frac{\beta + \gamma}{\gamma}t^* - \frac{\alpha^2\beta + \alpha^2\gamma + \alpha\gamma^2}{\gamma(\alpha + \gamma)\beta}t_1 + \frac{\alpha^2\beta - \alpha\beta^2 + \alpha^2\gamma + \alpha\gamma^2}{\beta\gamma(\alpha + \gamma)}t^+ - \frac{\beta + \gamma}{\alpha + \gamma}w(t_2) - \frac{\beta + \alpha + \gamma}{\alpha + \gamma}t_2 = \frac{N}{s} \tag{29}$$

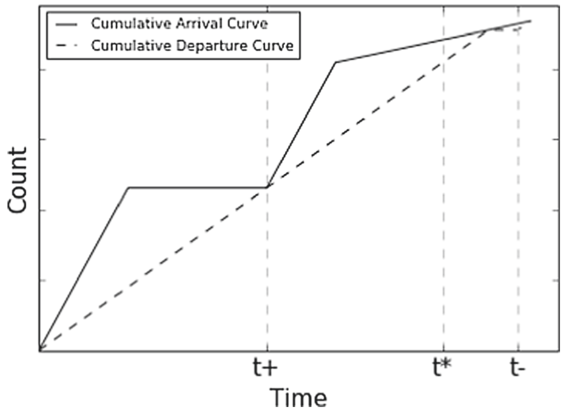
Hence we can have a system of three equations that solves the equilibrium



(a) $t_2 > t^+, w(t_2) = 0$



(b) $t_2 < t^+, t_2 + w(t_2) = t^+$



(c) $t_2 = t^+,\ w(t_2) = 0,\ w(t_4) = 0$

Fig. 4. Three possible shapes of the equilibrium profiles.

$$\left(1 - \frac{1}{N_{np}} + \frac{1}{N_{np}} e^{-\lambda_{cr} R}\right) = \exp\{\lambda_{cr}(\alpha w(t_2) + \beta(t^* - t_2 - w(t_2)) - \alpha w(t_1) - \beta(t^* - t_1 - w(t_1))\} \tag{30a}$$

$$(t_e - t_3)\frac{\alpha s}{\alpha + \gamma} + (t_3 - t_2 + t_1 - t_0)\frac{\alpha s}{\alpha - \beta} = N \tag{30b}$$

$$w(t_2) = 0 \tag{30c}$$

with $t_2 > t^+$, which gives a profile in Fig. 4a, and

$$(1 - \frac{1}{N_{np}} + \frac{1}{N_{np}}e^{-\lambda_{cr}R}) = \exp\{\lambda_{cr}(\alpha w(t_2) + \beta(t^* - t_2 - w(t_2)) - \alpha w(t_1) - \beta(t^* - t_1 - w(t_1))\} \tag{31a}$$

$$(t_e - t_3)\frac{\alpha s}{\alpha + \gamma} + (t_3 - t_2 + t_1 - t_0)\frac{\alpha s}{\alpha - \beta} = N \tag{31b}$$

$$t_2 + w(t_2) = t^+ \tag{31c}$$

with $t_2 < t^+$, which gives a profile in Fig. 4b.

3.4. Optimize the Flat Raffle schemes

We have derived the equilibrium for any given Flat Raffle scheme t^+, t^-, R. The more important problem is how to choose a 'best' scheme that 'benefits' the society and the population of travelers most. A demand management strategy is hardly one-size-fit-all. In reality, policymakers may have different focuses to tailor the policies/strategies for specific communities. Therefore, we define four possible objectives for improving social welfare under raffles. We will show later how those objectives vary by different raffle schemes. Those objectives will also be compared among various incentives, such as Flat Raffles and coarse tolls.

1. Minimizing the length of morning commuting time period

$$\min\{t_e - t_0\} \tag{32}$$

2. Maximizing the total social utility;

$$\min \int_t U(c(t)) \tag{33}$$

3. Minimizing the total queuing time;

$$\min \int_t w(t) \tag{34}$$

4. Minimizing the total generalized travel cost.

$$\min \int_t c(t) \tag{35}$$

Minimizing the length of morning commuting time period means we would like to shorten the tine period of morning commute as much as we can. However, the length of morning commuting time period is dependent on the number of commuters and the capacity of bottleneck. As both the demand and capacity are fixed, the theoretically minimal value is $\frac{N}{s}$ when no road capacity is wasted. When travelers tend to wait at the bottleneck to enter raffles (due to the BI assumption), it may lead to capacity waste and hence stretching the commuting period. We would like to minimize such a capacity waste.

Maximizing the total social utility is the most reasonable in cases where the individual utility function is linear and utilities can be compared among users. But it is not the case when we adopt a general utility function that only carries ordinal information of utilities. In this sense, utilities are not additive among different users. In particular, for the commonly used exponential utility function, different values of risk measures λ will produce a wide spectrum of magnitudes of utilities. For example, with a same travel cost, the utility for a extremely risk-seeking user can be much higher than an extremely risk-averse user. If we solely use this objective for management strategies, risk-seeking users are placed more importantly than risk-averse users, leading to possible controversy.

Minimizing the total queuing time has been used in past studies as the queuing time is considered as deadweight loss. It can be a reasonable objective. However, as we will show later, in some cases the minimum of queuing time may lead to undesirable results.

Minimizing the total generalized travel cost was also used in past studies. As discussed before, it implicitly assumes that all users are risk-neutral. Although in reality users have different degrees of risk-aversion, a system manager may want to view users all equally to best serve the interest of social equity. In this case, the system would not prefer any user due to their degrees of risk-aversion. This paper would prefer using this objective when identifying the optimal raffle schemes, but also considers other objectives. Minimizing the total generalized travel cost results in an equilibrium profile with the following property,

Theorem 1. *The raffle scheme with the minimal value of total generalized travel cost (TTC) must have $t_2 = t^+$, $w(t_2) = 0$, $w(t_4) = 0$ under equilibrium, as depicted in Fig. 4c.*

Proof of Theorem 1. First we argue that, given a profile like in Fig. 4b, we can always change it to the one in Fig. 4c with holding the same values of t^+ and t^- and solely changing the value of reward R. Recall that we have two different cases near t^+ where the marginal user would have, either by shorter queuing time or less early schedule delay, to compensate his/her loss of not enrolling the raffle. Hence we can change the value of R in order to change t_2 and $w(t_2)$. We can always choose a value of R such that $t_2 = t^+$. With this new R and holding t^+ and t^- the same, the cost of raffle users do not change. However, the cost of non-raffle users will decrease as their queuing time decreases. Thus, a profile in Fig. 4b cannot result in the minimal TTC.

Second, we argue that, given a profile like in Fig. 4a, we can always change the value of R and t^- in order to change t_1 and hold $w(t_2) = 0$ the same, in such a way that there would be no capacity waste around time t^+ as in Fig. 4c. In this case, all the early arrival raffle users keep the same total queuing time but experience less early delay. Hence the TTC will decrease by applying this change. Hence profile like in Fig. 4a cannot give the minimal TTC.

Last but not least, we show that $w(t_4) = 0$ for the minimum TTC. If $w(t_4) \neq 0$, then one can always postpone t^- and hold t^+ and R the same in such a way that $w(t_4) = 0$. In this case, all late arrival raffle users are subject to less late schedule delay and less queuing delay, which clearly results a less TTC than $w(t_4) \neq 0$. □

Theorem 1 shows that the equilibrium that gives the minimal TTC must have a travel profile shown in Fig. 4c, namely $t_2 = t^+$, $w(t_2) = 0$ and $w(t_4) = 0$, despite the unavoidable waste of capacity during the BI time period. The first (last) non-raffle user can depart exactly at the starting (close to ending, subject to flow idling) time of the raffle, and experience no queuing delay. On the other hand, Theorem 1 helps us find the best Flat Raffle scheme. If minimizing TTC is the objective, we will solve Eq. (30) or Eq. (31) simultaneously to obtain the best raffle scheme.

4. Numerical experiments and a comparative study

In this section we show the results of numerical experiments to illustrate how we could alleviate commuting congestion and improve social welfare using a Flat Raffle Scheme. We also compare and contrast the raffle scheme with other strategies to provide insights when and why raffles may be preferred than other strategies.

4.1. Solution algorithm

Solving the optimization formulas is not trivial. Eqs. (30a) and (31a) are non-linear and complex. The probability distribution of λ can also be very complex. For most probability distributions, their quantile functions do not have a closed form. Thanks to the simplified network, fortunately, our solution space is not very large which makes it possible to do a grid search. We use the following method to find an equilibrium solution,

```
Define minC as the minimum value of the objective. Initialization minC = ∞;
while sparse grid search t1,t2 in solution space do
    find all other parameters for the model by optimizing the objective functions;
    if A feasible points is found then
        thisC = local minimum of the feasible point;
        minC = (thisC<minC)?thisC:minC;
    else
        continue;
    end
end
```

In short we first do a grid search for feasible points of t_1. t_2. Then for each feasible points t_1. t_2 found, we use an optimization method, e.g. the Levenberg-Marquardt Algorithm (LMA), to find a local minimum for Eq. (30) and (31) simultaneously given t_1. t_2. Finally we take the minimum of all local minimums as the global minimum.

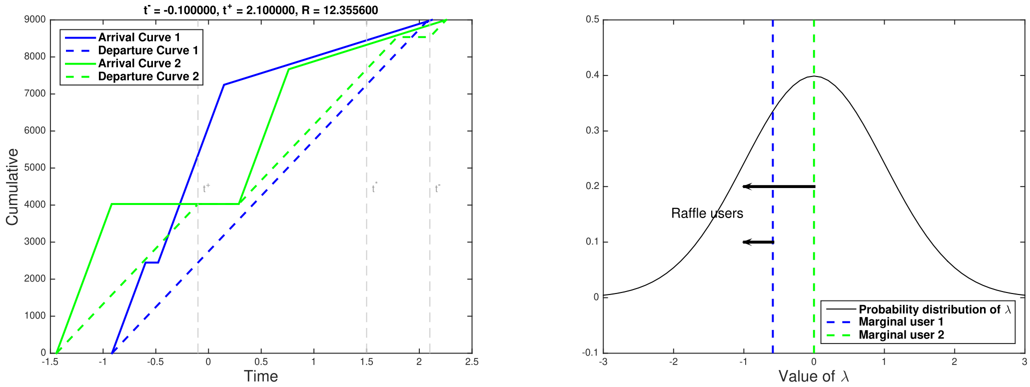
4.2. Numerical settings

Unless otherwise specified, in following experiments we choose α = \$6.4/hour, β = \$3.9/hour, γ = \$15.21/hour based on Small (1982). We also set $t^* = 1.5$ h, $D = 9000$ vehicles and $s = 3000$ vehicles/hour. Note that, the value of t^* can be arbitrary, and is only used for references. The roadway capacity s is set to roughly a little less than the maximum flow rate of two highway lanes. Based on the classical bottleneck model (Vickrey, 1969), the total social cost under equilibrium when no management strategy is applied is \$83,776.

4.3. Examples of Flat Raffle schemes: possibly undesirable cases

Given any set of t^+, t^- and R, the system would reach an equilibrium. We here provide an example to show that there exist two possible equilibrium profiles, both of which are not so desirable or not necessarily helping reduce congestion as much. Fig. 5a shows the two equilibrium states with a flat raffle scheme $[t^+, t^-, R] = [-0.1, 2.1, 12.3556]$. We assume that the degree of risk aversion follows a standard normal distribution $\mathbb{N}(0, 1)$ as shown in Fig. 5b. The Equilibrium 1 (in blue color) is the case of Fig. 4b and the Equilibrium 2 (in green color) is the case of Fig. 4a.

We see that Equilibrium 1 has an overall shorter commuting time period than Equilibrium 2, while the total queuing delay under Equilibrium 2 is substantially less. The system TTC under Equilibrium 1 is \$79,836, 8.7% greater than that under Equilibrium 2, \$



(a) An example with multiple equilibrium solutions (b) The marginal user under the two equilibra

Fig. 5. Multiple equilibria can exist under a Flat Raffle scheme.

72,890. Given the same traveler population, more travelers would participate in the raffle under Equilibrium 2 than under Equilibrium 1, as shown in Fig. 5b. We show that there may exist two equilibrium profiles, and either one may be preferred to the other under a particular objective. However, both equilibrium profiles are not so desirable. Equilibrium 1 is almost the same as the case without any demand management strategies, while Equilibrium 2 has a substantial capacity waste before work starting time. We expect to improve the system by proposing an optimal Flat Raffle scheme, namely choosing the optimal t^+, t^- and R.

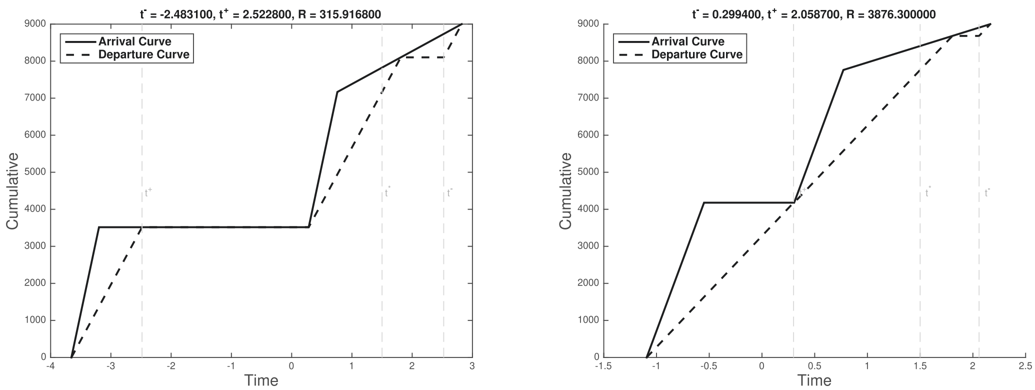
4.4. Minimizing TTC or total queuing delay

Fig. 6 shows the optimal equilibrium profiles by minimizing the total queuing delay and minimizing the TTC, respectively. For solving for the best scheme, we set the scheme parameters t^+, t^- and R as decision variables, Eq. (30) and (31) as constraints, one of the two criteria as the objective function.

As we can see, if queuing delay is the sole objective, the equilibrium under the best raffle scheme would have a very long time period (nearly 6 h) with a period of almost 3 h when no one uses the bottleneck. This would not be desirable for the morning commute. In this case, queuing delay can be the secondary consideration, while the primary objective is to minimize the TTC.

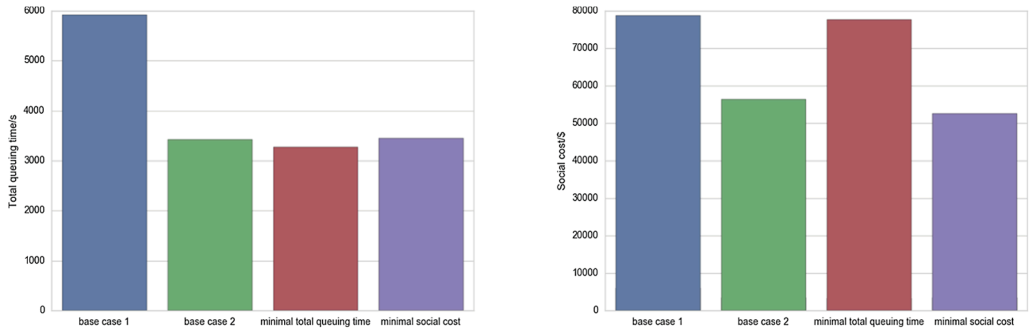
In contrary, the optimal travel profile minimizing the TTC is more reasonable as shown in Fig. 6b. The bottleneck is effectively used throughout the entire morning commute time period, except the unavoidable braking idle period, about 10 min long. The total queuing time also declines comparing to the case without any raffles. This confirms our speculation that minimizing TTC would be a good objective in most cases.

We provide the values of total queuing time and TTC in Fig. 7 under four cases: the two optimal solutions as well as the two solutions in Fig. 5. One can see that for the profile minimizing the TTC, both the TTC and queuing delay are low among all four cases, while the optimal profile minimizing the queuing delay leads to relatively high TTC even comparing to a base case.



(a) Best scheme in terms of social queuing time (b) Best scheme in terms of social generalized cost

Fig. 6. The equilibrium profiles under the best scheme.



(a) The total queuing time (b) The total generalized travel cost (TTC)

Fig. 7. The total queuing delay and total generalized travel cost for four cases (Base case 1: Equilibrium 1 in Fig. 5; Base case 2: Equilibrium 2 in Fig. 5).

4.5. Sensitivity analysis of Flat Raffle schemes

In this section we conduct a sensitivity analysis of Flat Raffle schemes. We focus on the parameters t^+ and t^- as both influence the equilibrium profiles more substantially than the reward R.

In Fig. 8, we plot the change in the morning commuting duration, braking period duration, the number of users enrolled in the raffle and the TTC, respectively, with respect to the raffle starting time t^+. Those plots are for three possible values of t^-. From Fig. 8d,

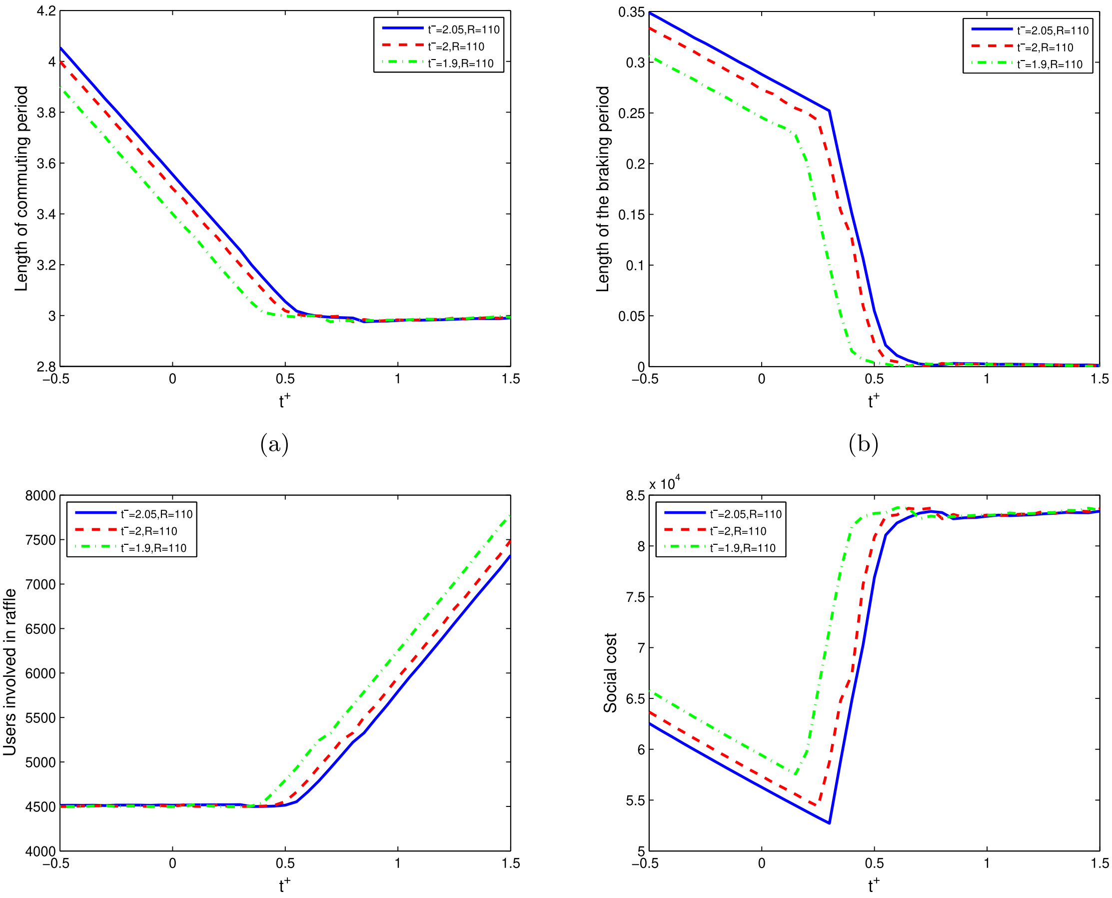


Fig. 8. Fix $R = \$110$ and $t^- = [1.9, 2, 2.05]$ and change the value of t^+, the plots show the change in: (a) duration of the commuting period, (b) duration of the braking period, (c) the number of users participating the raffle, (d) the total generalized travel cost.

we see that, given t^- and R, there exists a t^+ that minimizes the TTC. When the raffle starting time is set later than 0.5 h ($t^+ > 0.5$), the morning commute duration almost converges to 3 h. This is the minimum possible duration for the morning commute, but not necessarily the optimum for the system. In the case of $t^+ > 0.5$, with the increase in t^+ (namely the raffle duration reduces), the braking period of capacity waste becomes very close to zero. Also, it leads to substantial increase in the raffle enrollments, lowering the chance of winning the raffle for everyone. In addition, when $t^+ > 0.5$, the TTC is among the greatest, implying the raffle becomes inefficient in improving the system. It does not change the system as compared to the case without raffles, other than the system manager has to pay the raffle for someone. On the other hand, when $t^+ < 0.5$, the morning commute duration increases as the raffle starts earlier, while the raffle enrollments stay constant.

An interesting observation is that although the raffle duration being too long or too short can both result in a relatively high TTC, a slightly longer raffle duration (which means a slightly smaller t^+ than 0.5 h) is often a safe choice, along with a stabilized and highest raffle winning chance.

4.6. The effect of population distribution of risk attitudes λ

It has been found that many factors such as gender (Jianakoplos and Bernasek, 1998), age (Pålsson, 1996), and income (Shaw, 1996) are correlated with users' degree of risk aversion. In this subsection, we examine how the probability distribution of the degree of risk aversion λ affects the performance of the raffle scheme. Three classes of probability distributions are used: Gaussian distribution, Uniform distribution, and Log-normal distribution.[1]

Fig. 9 shows how the raffle scheme performs under different population distribution of λ and with different population means. The performance at each point is obtained from the best scheme that minimizes the TTC. As we can see, the performance of the raffle schemes under different distributions look similar in general. The performance under the Log-Normal distribution is more volatile than the other two.

From Fig. 9a, we can see that the TTC reaches the minimum when the mean of population λ is slightly less than 0. When the population has slightly more risk-seeking users than risk-averse users under the raffle award (in this case $R = \$110$), the raffle works the best, even better than the optimal one-step coarse toll that minimizes the TTC. When the mean of λ is positive and large, a considerable number of risk-averse travelers would enroll the raffle in order to receive a higher utility, which is clearly not desirable. On the other hand, then the mean of λ is negative and too small implying most users are highly risk-seeking, there are so many of them who would participate in the raffle, which lowers the winning chance and in turn would reduce the raffle performance. Clearly, a Flat Raffle scheme works the best for a slightly risk-seeking population. Note that we discussed before that users' risk attitudes can change with respect to the award and cost. When the raffle reward R is significant comparing with a not-so-significant cost increase, we may be dealing with a slightly risk-seeking population and it is likely the raffle can work well.

Fig. 9b shows the duration of the braking period. Since we show that under the optimal raffle scheme that minimizes the TTC, we have $w(t_2) = 0$, thus, the duration of the braking period is equivalent to the extra time required for the morning commuting. We find that the curves in Fig. 9b have the reverse trend as in Fig. 9a. This implies that we are making a trade-off between the TTC and the morning commute duration. Fig. 9c and d show the performance of raffle schemes with respect to the total number of raffle users. It further highlights the trade-off between the morning commute duration and TTC.

4.7. A comparative study

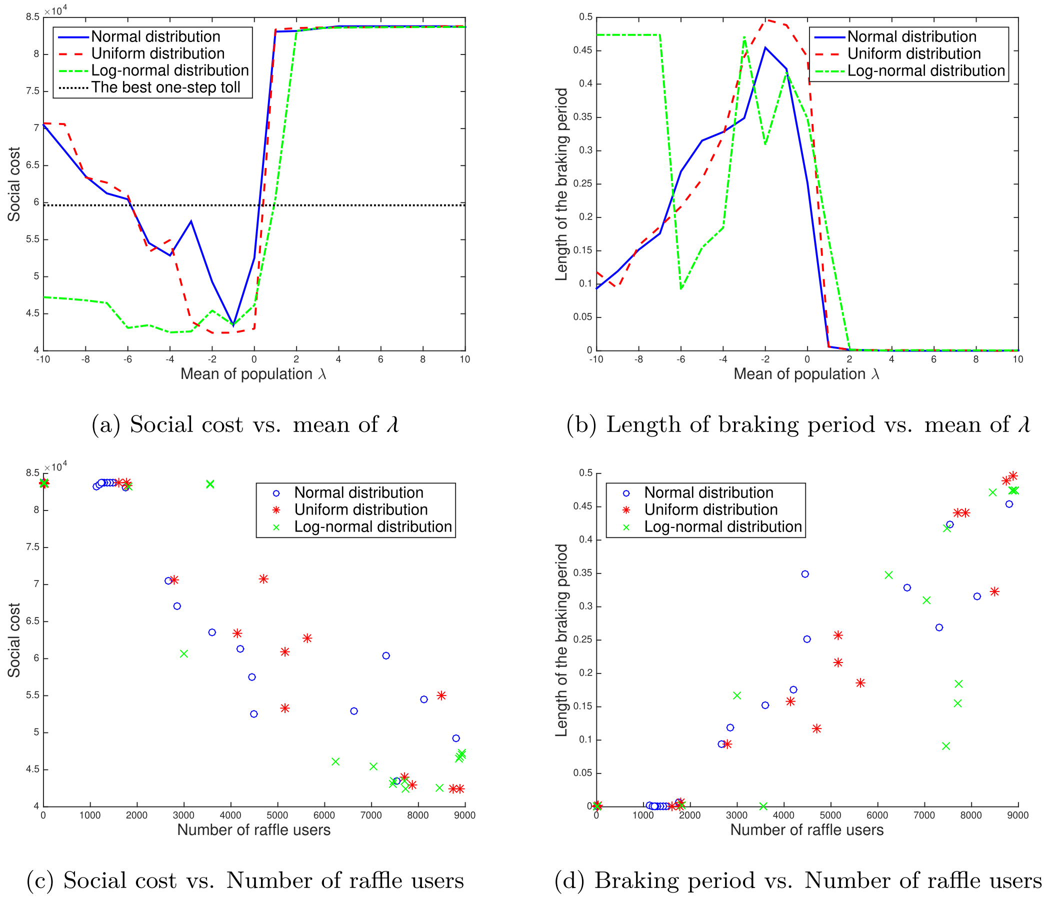
We conduct a comparative study between the raffle-based scheme and the one-step congestion pricing scheme. Note that although the tradable credit scheme has many benefits over the congestion pricing, they are equivalent from the modeling perspective in this morning commute settings. The comparison between one-step toll and the raffle scheme is under the same settings of users' choice models, meaning that the commuters' utility function and their desired arrival time are the same. Under these two cases, commuters are facing deterministic and stochastic results, and will have to make their decisions according to their utility functions.

To make a fair comparison, we solve the optimal one-step toll under the bottleneck model settings, as suggested by Lindsey (2004). We obtain that $[t^+, t^-, \rho] = [0.155, 2.066, 4.723]$.

We plot the TTC of the optimal one-step toll in Fig. 9a. Note that, as a deterministic policy, the TTC of the one-step toll does not change with respect to the probability distribution or the mean of risks λ. When most users are risk averse (meaning the mean of λ is positive) or extremely risk seeking (the mean of λ is extremely small and negative), the optimal one-step toll is better than the flat raffle scheme, in terms of TTC. However, when the mean of λ is from -5 to 0, the raffle scheme performs substantially better than the optimal one-step toll. It can offer up to 30% less TTC than the optimal step toll. More interestingly, when the raffle-based scheme offers a lower TTC, every single user will experience a higher utility than under the optimal one-step toll.

Again, users' risk attitudes are highly dependent on the stakes and potential reward. Recall that users are more risk seeking when stakes are low. In our setting, the travelers only need to shift their departure times a bit (e.g., 10 min earlier than what they used to) in order to participate in the raffle, it may be that the stakes are very low here for many commuters. As a result, it is possible that the mean of λ may be slightly negative and a raffle scheme may outperform the one-step toll.

[1] Instead of the common Log-normal distribution that is derived directly by taking logarithm of a Gaussian distribution, we shift the mean of the Log-normal distribution to have same mean (0) and variance (1) of the normal distribution we use, so that it can be comparable with two other distributions.



(a) Social cost vs. mean of λ (b) Length of braking period vs. mean of λ

(c) Social cost vs. Number of raffle users (d) Braking period vs. Number of raffle users

Fig. 9. The performance of optimal raffle schemes that minimizes the TTC.

Another aspect of the raffle scheme worth mentioning is the source of the funding for both lottery configurations and lottery prize. Note that from the stand point of transportation authorities, a congestion toll scheme will lead to some revenue that can be re-distributed to subsidize public projects. A tradable credit scheme in the long term can be revenue neutral and thus self-sustainable. In the raffle scheme, the transportation authorities have to fund the lottery prize. We argue that a lottery would be funded through current transportation demand management (TDM) or Transportation Improvement Projects (TIP) funding. A lottery could be possible alternative projects to substitute current TDM or infrastructure projects. In addition, because the required funding to set up a lottery prize (e.g., at the level of tens of thousands dollars) is not as substantial as those TDM or infrastructure projects, it is possible to run it in the long term if it is effective. In fact, the fact that having the transportation authorities to run the lottery with public funding and thus only requires voluntary participation of the public is what make it less controversial than congestion pricing and tradable credits. On the other hand, as we have shown in the numerical examples, it is possible that the raffle scheme will gain more social welfare for the entire system, we would expect that this could make other arguments why a lottery is preferred. For example, in the long term, since the highway is less congested, facilities in the networks are more accessible, leading to possible more economics prosperity. It is reasonable to expect that the long-term social welfare gained from the raffle scheme will be greater than what the government needs to pay for the prize.

Note that the total social cost with no management strategy is \$83,776 and the optimal value of the reward R is around \$110. When users are in general slightly risk seeking such that the mean of population λ is a bit smaller than 0, the order of magnitude of the social welfare gain (namely the reduction of social costs) from the raffle scheme is 10^4, which is much greater than the value of R. Comparing to the system with the one-step toll, the social welfare gain from the raffle scheme is still way much greater than R under this “ideal” commuter population for raffles. If this is the case for raffles, the transportation authorities can easily justify the funding allocated for raffles to improve the transportation system performance as an alternative to other TDM or infrastructure projects.

5. Conclusion

Raffle based incentives have been widely used in commercial market to influence consumers’ behaviors. In this paper, we examine the effect of raffle or lotteries for travel demand management, particularly the morning commute. Instead of charging users a fee or issuing a credit to users as incentives, the raffle-based scheme incentivizes users to change their choices by giving credits that are used to enter a lottery. The promises of a raffle-based scheme is that users are more risk-seeking, thus willing to participate in the lottery, when stakes are low. Raffles may be less controversial comparing to tolls and tradable credits, thanks to its voluntary nature. This would particularly fit the traffic management where the cost for a user to change his/her choices may be relatively insignificant comparing to the reward of the raffle.

To evaluate the performance of raffle-based schemes, we first introduce users' utility functions into the classic Vickrey's bottleneck model, and establish the utility-based user equilibrium. We show the classic equilibrium based on the users' generalized travel cost is equivalent to the utility-based equilibrium, only if the outcomes of users' choices are certain. As the raffle scheme introduces uncertain reward, using utility-based equilibrium would be needed to understand users' departure time choices under their respective risk attitudes.

We show that the raffle-based scheme can achieve the system optimum(defined as all queuing delay is eliminated) when all users are risk neutral as other policies or strategies would, such as congestion pricing or tradable credits by setting a continuously changing probabilities for winning the prize. However, since in practice simpler pricing structures are more likely to be acceptable by users, we study Flat Raffle schemes where a constant lottery winning chance is set during the off-peak of morning commute. The Flat Raffle is particularly compared to the one-step coarse toll in the same spirit of practical simplicity. We show several characteristics of the utility-based equilibrium for arbitrary utility functions, as long as they follow two general assumptions. We also show that a marginal user who is indifferent from participating in the lottery or not would exist for any utility function. Furthermore, we analytically derive and solve for the equilibrium using exponential utility functions, and we solve for the optimal travel profiles under four potential system management objectives. When minimizing the total generalized travel cost (TTC), we show that there will be no queuing delay for the user who is indifferent from participating in the lottery or not.

Through both the analytics and numerical experiments, we find that minimizing the TTC seems a reasonable goal for an optimal Flat Raffle scheme possibly resulting in approximately minimum commuting duration and minimum total queuing delay at the same time. Most interestingly, a Flat Raffle scheme works the best and outperforms the optimal one-step toll, when the population is overall slightly risk-seeking. This is particularly appealing since the raffle reward is relatively significant comparing with a not-so-significant cost increase associated with users' behavioral change.

Implementing lottery or raffles in traffic demand management can be fully supported by emerging sensing and communication technologies in general. In-vehicle sensors or smart phones could be used to identify the actual choices of individuals (Merugu et al., 2009; Yue et al., 2015). Those data are then recorded and entered to a centralized platform where a lottery is drawn on a regular basis. In the era of ubiquitous sensing and vehicle technologies, collecting data of individual vehicles or passengers with required privacy protection would be possible and increasingly inexpensive. Yue et al. (2015) also noted that marketing and social networks may help enlarge the number of lottery participants, partially due to peer pressure (Feygin and Pozdnoukhov, 2018).

The research shows great promises of raffle applications in managing transportation systems, but is merely a beginning step before refined analysis. It can be further extended in many directions. We would like to consider its effectiveness on travelers' route choices in addition to departure time choices. We would also like to examine its effectiveness with a more refined behavioral choice model. At this step, we assume that users are only heterogeneous in terms of risk attitudes. It would be interesting to examine users' heterogeneity in other aspects and how this would further impact our understanding of raffles' effectiveness. We would be interested in experimenting raffles in real world through a pilot study and surveys, and in large-scale networks.

Acknowledgements

This research is funded in part by National Science Foundation Award CMMI-1751448 and Carnegie Mellon University's Mobility21, a National University Transportation Center for Mobility sponsored by the US Department of Transportation. The contents of this report reflect the views of the authors, who are responsible for the facts and the accuracy of the information presented herein.

References

Akamatsu, T., Wada, K., 2017. Tradable network permits: a new scheme for the most efficient use of network capacity. Transp. Res. Part C: Emerging Technol. 79, 178–195.
Angelopoulos, A., Gavalas, D., Konstantopoulos, C., Kypriadis, D., Pantziou, G., 2018. Incentivized vehicle relocation in vehicle sharing systems. Transp. Res. Part C: Emerging Technol. 97, 175–193.
Arnott, R., De Palma, A., Lindsey, R., 1990. Economics of a bottleneck. J. Urban Econ. 27 (1), 111–130.
Arnott, R., De Palma, A., Lindsey, R., 1993. A structural model of peak-period congestion: a traffic bottleneck with elastic demand. Am. Econ. Rev. 161–179.
Arnott, R., De Palma, A., Lindsey, R., 1999. Information and time-of-usage decisions in the bottleneck model with stochastic capacity and demand. Eur. Econ. Rev. 43 (3), 525–548.
Arrow, K.J., 1965. Aspects of the theory of risk-bearing. Yrjö Jahnssonin Säätiö.
Arrow, K.J., 1971. The theory of risk aversion. Essays in the theory of risk-bearing 90–120.
Becker, G.S., 1962. Irrational behavior and economic theory. J. Polit. Econ. 1–13.
Feygin, S., Pozdnoukhov, A., 2018. Peer pressure enables actuation of mobility lifestyles. Transp. Res. Part C: Emerging Technol. 87, 26–45.
Haisley, E., Mostafa, R., Loewenstein, G., 2008. Myopic risk-seeking: the impact of narrow decision bracketing on lottery play. J. Risk Uncertainty 37 (1), 57–75.
Harsanyi, J.C., 1953. Cardinal utility in welfare economics and in the theory of risk-taking. J. Polit. Econ. 61 (5), 434–435.
High, J., Bloch, H., 1989. On the history of ordinal utility theory: 1900–1932. Hist. Polit. Econ. 21 (2), 351–365.
Hillson, D., Murray-Webster, R., 2004. Understanding and managing risk attitude. In: 'Proceedings of 7th Annual Risk Conference, held in London, UK', vol. 26.
Hirshleifer, D., Teoh, S.H., 2003. Limited attention, information disclosure, and financial reporting. J. Acc. Econ. 36 (1), 337–386.
Holguín-Veras, J., Wang, X.C., Sánchez-Díaz, I., Campbell, S., Hodge, S.D., Jaller, M., Wojtowicz, J., 2017. Fostering unassisted off-hour deliveries: The role of incentives. Transp. Res. Part A: Policy Pract. 102, 172–187.

Holt, C.A., Laury, S.K., et al., 2002. Risk aversion and incentive effects. Am. Econ. Rev. 92 (5), 1644–1655.
Jianakoplos, N.A., Bernasek, A., 1998. Are women more risk averse? Econ. Inquiry 36 (4), 620–630.
Kahneman, D., 1994. New challenges to the rationality assumption. J. Inst. Theoret. Econ. (JITE)/Zeitschrift für die gesamte Staatswissenschaft 18–36.
Koopmans, T.C., 1960. Stationary ordinal utility and impatience. Econometrica: J. Econometric Soc. 287–309.
Laih, C.-H., 1994. Queueing at a bottleneck with single-and multi-step tolls. Transp. Res. Part A: Policy Pract. 28 (3), 197–208.
Li, H., Bovy, P., Bliemer, M., 2008. Departure time distribution in the stochastic bottleneck model. Int. J. ITS Res. 6 (2).
Lindsey, R., 2004. Existence, uniqueness, and trip cost function properties of user equilibrium in the bottleneck model with multiple user classes. Transp. Sci. 38 (3), 293–314.
Lindsey, R., Van den Berg, V.A., Verhoef, E.T., 2010. 'Step by step: revisiting step tolling in the bottleneck model', Tinbergen Institute Discussion Paper 10-118/3.
Loiseau, P., Schwartz, G., Musacchio, J., Amin, S., 2011. Incentive schemes for internet congestion management: Raffles versus time-of-day pricing. In: 2011 49th Annual Allerton Conference on Communication, Control, and Computing (Allerton). IEEE, pp. 103–110.
Loiseau, P., Schwartz, G., Musacchio, J., Amin, S., Sastry, S.S., 2011. Congestion pricing using a raffle-based scheme. In: 2011 5th International Conference on Network Games, Control and Optimization (NetGCooP). IEEE, pp. 1–8.
Merugu, D., Prabhakar, B.S., Rama, N., 2009. An incentive mechanism for decongesting the roads: a pilot program in bangalore. In: Proc. of ACM NetEcon Workshop, Citeseer.
Morgan, J., 2000. Financing public goods by means of lotteries. Rev. Econ. Stud. 67 (4), 761–784.
Nie, Y.M., 2015. A new tradable credit scheme for the morning commute problem. Networks Spatial Econ. 15 (3), 719–741.
Pålsson, A.-M., 1996. Does the degree of relative risk aversion vary with household characteristics? J. Econ. Psychol. 17 (6), 771–787.
Pratt, J.W., 1992. Risk aversion in the small and in the large. In: Foundations of Insurance Economics. Springer, pp. 83–98.
Qian, Z., Rajagopal, R., 2015. Optimal dynamic pricing for morning commute parking. Transportmetrica A: Transport Sci. 11 (4), 291–316.
Qian, Z.S., Xiao, F.E., Zhang, H., 2012. Managing morning commute traffic with parking. Transp. Res. Part B: Methodol. 46 (7), 894–916.
Qian, Z.S., Xiao, F.E., Zhang, H., et al., 2011. The economics of parking provision for the morning commute. Transp. Res. Part A: Policy Pract. 45 (9), 861–879.
Rey, D., Dixit, V.V., Ygnace, J.-L., Waller, S.T., 2016. An endogenous lottery-based incentive mechanism to promote off-peak usage in congested transit systems. Transp. Policy 46, 46–55.
Shaw, K.L., 1996. An empirical analysis of risk aversion and income growth. J. Lab. Econ. 626–653.
Small, K.A., 1982. The scheduling of consumer activities: work trips. Am. Econ. Rev. 72 (3), 467–479.
Tian, L.-J., Yang, H., Huang, H.-J., 2013. Tradable credit schemes for managing bottleneck congestion and modal split with heterogeneous users. Transp. Res. Part E: Logist. Transport. Rev. 54, 1–13.
Tscharaktschiew, S., Evangelinos, C., 2019. Pigouvian road congestion pricing under autonomous driving mode choice. Transp. Res. Part C: Emerging Technol. 101, 79–95.
van den Berg, V., Verhoef, E.T., 2011. Congestion tolling in the bottleneck model with heterogeneous values of time. Transp. Res. Part B: Methodol. 45 (1), 60–78.
Verhoef, E., Nijkamp, P., Rietveld, P., 1997. Tradeable permits: their potential in the regulation of road transport externalities. Environ. Plann. B: Plann. Des. 24 (4), 527–548.
Vickrey, W.S., 1969. 'Congestion theory and transport investment. Am. Econ. Rev. 251–260.
Xiao, F., Qian, Z.S., Zhang, H.M., 2013. Managing bottleneck congestion with tradable credits. Transp. Res. Part B: Methodol. 56, 1–14.
Xiong, C., Shahabi, M., Zhao, J., Yin, Y., Zhou, X., Zhang, L., 2019. An integrated and personalized traveler information and incentive scheme for energy efficient mobility systems. Transp. Res. Part C: Emerging Technol.
Yang, H., Huang, H.-J., 1997. Analysis of the time-varying pricing of a bottleneck with elastic demand using optimal control theory. Transp. Res. Part B: Methodol. 31 (6), 425–440.
Yang, H., Wang, X., 2011. Managing network mobility with tradable credits. Transp. Res. Part B: Methodol. 45 (3), 580–594.
Yue, J.S., Mandayam, C.V., Merugu, D., Abadi, H.K., Prabhakar, B., 2015. Reducing road congestion through incentives: a case study, Transportation Research Board 94th Annual Meeting Compendium of Papers.
Zhu, S., Jiang, G., Lo, H.K., 2018. Capturing value of reliability through road pricing in congested traffic under uncertainty. Transp. Res. Part C: Emerging Technol. 94, 236–249.
Zhu, X., Wang, F., Chen, C., Reed, D.D., 2019. Personalized incentives for promoting sustainable travel behaviors. Transp. Res. Part C: Emerging Technol.