

Mode Clustering for Markov Jump Systems

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Abstract—In this work, we consider the problem of mode clustering in Markov jump models. This model class consists of multiple dynamical modes with a switching sequence that determines how the system switches between them over time. Under different active modes, the observations can have different characteristics. Given the observations only and without knowing the mode sequence, the goal is to cluster the modes based on their transition distributions in the Markov chain to find a reduced-rank Markov matrix that is embedded in the original Markov chain. Our approach involves mode sequence estimation, mode clustering and reduced-rank model estimation, where mode clustering is achieved by applying the singular value decomposition and k-means. We show that, under certain conditions, the clustering error can be bounded, and the reduced-rank Markov chain is a good approximation to the original Markov chain. Through simulations, we show the efficacy of our approach and the application of our approach to real world scenarios.

Index Terms—Switched model, Markov chain, clustering

I. INTRODUCTION

Modeling dynamic systems has been a problem of great interest in the signal processing and control communities for decades. Many real-world phenomena cannot be described with one dynamical model, and so switched models wherein the dynamics transition between different system models have been studied and applied widely. In human-made systems, for example, a robot may have different dynamics under different battery levels or when different modules within the robot fail. In nature, the temperature and humidity level will have different fluctuations under different weather conditions; brain electricity signals will behave differently under different emotions of the test subject. Note that in all these examples, the modes can switch over time. To model this switching, one systematic and probabilistic way is to assume the mode switching follows a Markov chain where future modes do not depend on past modes given the most recent mode. This Markov jump model [2], [3] has been used in power systems, air traffic management, economics, and communication systems [4]–[9].

A key challenge for such models is the model compactness – how does one represent such a complicated dynamical system with as simple a model as possible? For example, modes like weather conditions and human emotions have extremely complex underlying dynamics with strong correlations over

time. To satisfy the Markov property, one may concatenate underlying modes into a single Markov state, and Markov chains built in this way will have a state space that grows exponentially with the number of modes concatenated in the sequence. The same exponential growth rate applies when one models human-made systems with multiple sub-modules that each have multiple behavior modes (normal/abnormal). Allowing the Markov model to get extremely large is computationally inefficient for analysis and control.

In the context of model reduction for Markov jump systems (MJS), prior work mainly focuses on the reduction of continuous state-space (or observation space): [10] studies the $\mathcal{H}_\infty$ model reduction and derives conditions under which a reduced order system can be obtained via linear matrix inequalities; [11] reduces the model order with the help of generalized dissipation inequalities and storage functions; [12] proposes a balanced truncation algorithm to reduce model order and gives upper bound on approximation error. To the best of our knowledge, the reduction of discrete state-space (number of modes) for Markov jump systems has not been considered before. And prior work in state space reduction of Markov chains does not further consider Markov jump models (i.e., models are fully discrete). There have been several works studying the aggregation of states for Markov chains, which mainly relies on assumptions such as strong/weak lumpability, or aggregatibility properties of a Markov chain [13]–[16]. There is therefore significant potential in applying the abundant algorithms and theory in Markov chain aggregation to Markov jump systems. This can achieve model reduction from a new perspective and will benefit the analysis and control of systems with large discrete state-spaces.

The work presented here addresses this gap. We observe that often times certain modes have similar transition behaviors, and these correlations between the modes can be exploited to construct a reduced-order model. By doing so, one may gain more insight into the nature of the complex model. Moreover, we will have fewer parameters to estimate when learning the model or monitoring the system using the constructed model, thus this may significantly reduce the computation burden. We are interested in situations where the bottleneck is due to a large discrete state-space (i.e., large number of modes) and aim to cluster and aggregate the modes for reduction. We achieve this model aggregation by clustering the modes with similar transition distributions together. We assume the dynamics for each mode are known, but we have no knowledge

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The complete version and proofs are in [1]

of the true mode sequence. In our approach, we cluster the modes based on a reduced-dimensional representation of the empirical Markov transition matrix. We then re-estimate the empirical Markov matrix using this cluster information, giving us a final low-rank estimate. We discuss our method's computational advantage, and show its performance is guaranteed in the sense that the clustering error and difference between reduced model and the true model can be upper bounded. Experiments show the efficacy of our approach as well as how the performance scales with the problem complexity.

II. PROBLEM FORMULATION

A. Notation

In this paper, boldface and uppercase (lowercase) letters denote matrices (vectors); plain letters denote scalars. If $\mathbf{A}$ is a matrix, then $\mathbf{A}(i, j)$ indexes the (i, j) th element in $\mathbf{A}$ and $\mathbf{A}(i, j:k)$ indexes the row vector corresponding to the i th row and column j through k . $\mathbf{A}(i, :)$ indexes the i th row of $\mathbf{A}$. Norms without subscript, i.e. $\|\cdot\|$, all denote the ℓ_2 -norm. We let $[n] := \{1, 2, \dots, n\}$ and $X_{0:N} := \{X_i\}_{i=0}^N$.

For Markov chain with state space $[n]$ and row stochastic transition matrix $\mathbf{P} \in \mathbb{R}^{n \times n}$, we let $\boldsymbol{\pi} \in \mathbb{R}^n$ denote the stationary distribution vector of $\mathbf{P}$, i.e. $\boldsymbol{\pi}^\top \mathbf{P} = \boldsymbol{\pi}^\top$. Furthermore, we let $\pi_{\max} := \max_i \pi_i, \pi_{\min} := \min_i \pi_i$. If $\mathbf{P}$ is ergodic, then $\boldsymbol{\pi}$ is unique and $\pi_{\min} > 0$. Let $\boldsymbol{\pi}_t \in \mathbb{R}^n$ denote the transient state distribution of $\mathbf{P}$ and $\boldsymbol{\pi}_t^\top = \boldsymbol{\pi}_{t-1}^\top \mathbf{P}$. We denote with $\{\Omega_1, \dots, \Omega_r\}$ a partition of the state space $[n]$, where each Ω_k denotes a cluster of states. We let $\Omega_{(i)}$ denote the cluster with i th largest cardinality.

B. Preliminaries

The Markovian jump model we consider is the following:

$$y_t = \sum_{i=1}^{n_a} a_i(X_t) y_{t-i} + \sum_{j=1}^{n_c} c_j(X_t) u_{t-j} + n_t, \quad (1)$$

$$X_{0:N} \in [n]^{N+1} \sim \text{Markov chain}(\mathbf{P}), \quad (2)$$

where y_t, u_t, n_t are scalars and represent the model output, input and noise at time t respectively. And y_t depends on $\{y_{t-i}\}_{i=1}^{n_a}, \{u_{t-j}\}_{j=1}^{n_c}$ linearly through the parameters $\{a_i(X_t)\}_{i=1}^{n_a}, \{c_j(X_t)\}_{j=1}^{n_c}$ from mode X_t at time t . There are n modes in total and the mode sequence $X_{0:N}$ is assumed to follow a Markov chain with row stochastic Markov matrix $\mathbf{P} \in \mathbb{R}^{n \times n}$. The initial state distribution $\boldsymbol{\pi}_0$ can be arbitrary. Note that one can omit input u_t by taking $n_c = 0$, which corresponds to an autonomous model. If we let

$$\mathbf{w}_{X_t} := [a_1(X_t), \dots, a_{n_a}(X_t), c_1(X_t), \dots, c_{n_c}(X_t)]^\top, \quad (3)$$

$$\boldsymbol{\phi}_t := [y_{t-1}, \dots, y_{t-n_a}, u_{t-1}, \dots, u_{t-n_c}]^\top, \quad (4)$$

then we obtain a simpler representation of the model:

$$y_t = \mathbf{w}_{X_t}^\top \boldsymbol{\phi}_t + n_t, \quad (5)$$

where the pair $\{y_t, \boldsymbol{\phi}_t\}$ can be viewed as the observation/data.

Furthermore, we assume the Markov matrix $\mathbf{P}$ has the following structure:

$$\mathbf{P} = \bar{\mathbf{P}} + \boldsymbol{\Delta}, \quad (6)$$

where $\bar{\mathbf{P}}$ is a Markov matrix that is r -aggregatable, i.e. there exists an r -cluster partition $\{\Omega_1, \Omega_2, \dots, \Omega_r\}$ on the state space $[n]$ such that

$$\forall k \in [r], \forall i, j \in \Omega_k, \bar{\mathbf{P}}(i, :) = \bar{\mathbf{P}}(j, :). \quad (7)$$

We assume $\text{rank}(\bar{\mathbf{P}}) = r$, which guarantees there are only r unique rows in $\bar{\mathbf{P}}$. Matrix $\boldsymbol{\Delta}$ is the perturbation that accounts for the difference of the true Markov matrix $\mathbf{P}$ and the r -aggregatable Markov matrix $\bar{\mathbf{P}}$. We assume modes are clustered only based on the transition distributions in current work and leave the consideration of mode dynamics and group connectivity for future work.

C. Problem Formulation

Assuming parameters for all the modes $\{\mathbf{w}_k\}_{k=1}^n$ are known, given observation trajectory $\{y_t, u_t\}_{t=0}^N$ with length N , we want to find an r -aggregatable approximation $\tilde{\mathbf{P}}$ of $\mathbf{P}$ such that the partition information in $\tilde{\mathbf{P}}$ could recover $\{\Omega_1, \Omega_2, \dots, \Omega_r\}$ in $\bar{\mathbf{P}}$.

We seek an r -aggregatable approximation of the original Markov matrix while preserving the clustering information in the underlying aggregatable Markov matrix. Given a Markov chain, one could use the power method [17] to iteratively simulate the evolution of the state distribution or compute the stationary distribution. So, one motivation to solve the aforementioned problem is that, during the power method, it requires $O(n^2)$ scalar multiplications in one iteration for $\mathbf{P}$ but only $O(rn)$ for the r -aggregatable $\tilde{\mathbf{P}}$. Meanwhile, the compromise in accuracy brought by the reduction of computation can be upper bounded with the following theorem.

Theorem 1: The differences between two Markov matrices $\mathbf{P}$ and $\tilde{\mathbf{P}}$ in terms of stationary distribution satisfy

$$\|\boldsymbol{\pi} - \tilde{\boldsymbol{\pi}}\|_1 \leq \sum_{i=2}^n \frac{1}{1 - \lambda_i(\mathbf{P})} \|\mathbf{P} - \tilde{\mathbf{P}}\|_\infty. \quad (8)$$

Furthermore, if $\mathbf{P}$ and $\tilde{\mathbf{P}}$ are both ergodic, their transient distributions and satisfy

$$\|\boldsymbol{\pi}_t - \tilde{\boldsymbol{\pi}}_t\|_1 \leq C\rho^t + \|\boldsymbol{\pi} - \tilde{\boldsymbol{\pi}}\|_1 \quad (9)$$

for some $C > 0$ and $0 < \rho < 1$.

We can see that as long as the approximation error $\|\mathbf{P} - \tilde{\mathbf{P}}\|_\infty$ is upper bounded, the stationary and transient behavior differences between the true Markov matrix $\mathbf{P}$ and the r -aggregatable approximation $\tilde{\mathbf{P}}$ can be bounded. This gives the justification for using $\tilde{\mathbf{P}}$ as a surrogate for $\mathbf{P}$ in the power method. The distance $\|\mathbf{P} - \tilde{\mathbf{P}}\|_\infty$ with $\tilde{\mathbf{P}}$ obtained from our approach is bounded in Theorem 4.

III. OUR APPROACH

Our approach to solve the problem mentioned above is given in Algorithm 1. In Line 3, we estimate the active mode at time t by picking the mode whose dynamics gives the smallest residual error $|y_t - \mathbf{w}_k^\top \boldsymbol{\phi}_t|$. Then, in Line 5, based on the estimated mode sequence, we estimate $\mathbf{P}$ with the empirical Markov matrix $\hat{\mathbf{P}}$. The SVD and truncation in Line 7 is essentially a denoising step that reduces the

Algorithm 1: Mode Clustering for MJS

Input: Observation $\{y_t, u_t\}_{t=0}^N$, dynamics $\{\mathbf{w}_k\}_{k=1}^n$

1 **for** $t = 0, \dots, N$ **do**

2 $\Phi_t := [y_{t-1}, \dots, y_{t-n_a}, u_{t-1}, \dots, u_{t-n_c}]^\top$

3 $\hat{X}_t = \arg \min_{k \in [n]} |y_t - \mathbf{w}_k^\top \Phi_t|$

4 **end**

5 Compute empirical Markov matrix:

$$\hat{\mathbf{P}}(i, j) = \frac{\sum_{t=1}^N \mathbb{1}\{\hat{X}_{t-1} = i, \hat{X}_t = j\}}{\sum_{t=1}^N \mathbb{1}\{\hat{X}_{t-1} = i\}} \quad (10)$$

6 SVD decomposition: $\hat{\mathbf{P}} = \mathbf{U}\mathbf{S}\mathbf{V}^\top$

7 $\mathbf{U}_r = \mathbf{U}(:, 1:r)$

8 Solve the following k-means problem:

$$\hat{\Omega}_{1:r}, \hat{\mathbf{c}}_{1:r} = \arg \min_{\substack{\hat{\Omega}_1, \dots, \hat{\Omega}_r \\ \hat{\mathbf{c}}_1, \dots, \hat{\mathbf{c}}_r}} \sum_{k=1}^r \sum_{i \in \hat{\Omega}_k} \|\mathbf{U}_r(i, :) - \hat{\mathbf{c}}_k\|^2 \quad (11)$$

9 Aggregatable approximation: assume $i \in \hat{\Omega}_s$

$$\tilde{\mathbf{P}}(i, j) = \frac{\sum_{k \in \hat{\Omega}_s} \sum_{t=1}^N \mathbb{1}\{\hat{X}_{t-1} = k, \hat{X}_t = j\}}{\sum_{k \in \hat{\Omega}_s} \sum_{t=1}^N \mathbb{1}\{\hat{X}_{t-1} = k\}} \quad (12)$$

Output: Partition $\{\hat{\Omega}_1, \dots, \hat{\Omega}_r\}$ and matrix $\tilde{\mathbf{P}}$

influence of perturbation Δ and estimation error in $\hat{\mathbf{P}}$. Then, we use k-means to estimate the clustering information in $\tilde{\mathbf{P}}$. Finally, in Line 9, we compute $\tilde{\mathbf{P}}$ by taking modes within the same estimated cluster as a single mode and re-computing the empirical Markov matrix.

Note that if a certain mode does not show up at all in the trajectory, i.e. the denominators in Line 5 and Line 9 might be 0, then we simply assign uniform distribution to that mode, i.e. $\hat{\mathbf{P}}(i, j) = 1/n$. We show in the proof that when the trajectory is long enough, every mode will show up with high probability.

IV. THEORETICAL GUARANTEES

A. Relevant Definitions

Before discussing theoretical guarantees of the proposed approach, we introduce some definitions that will be used later.

Definition 1 (Mixing Time of MC): Let $\mathbf{P} \in \mathbb{R}^{n \times n}$ be a row stochastic Markov transition matrix with stationary distribution $\boldsymbol{\pi}$. Then for any $\epsilon > 0$, the ϵ -mixing time is defined as

$$\tau(\epsilon) = \min \left\{ k : \max_{i \in [n]} \left\| (\mathbf{P}^k)(i, :)^\top - \boldsymbol{\pi} \right\|_1 \leq \epsilon \right\}. \quad (13)$$

Moreover, we let $\tau_* = \tau(\frac{1}{4})$.

Since k-means is used in Algorithm 1, we assume a $(1 + \epsilon)$ solution to the k-means problem can be obtained and later show how ϵ affects the overall clustering error.

Definition 2 (Approximate Solution to k-means Clustering Problem): For problem in (11), we say $\hat{\Omega}_1, \dots, \hat{\Omega}_r, \hat{\mathbf{c}}_1, \dots, \hat{\mathbf{c}}_r$ is a $(1 + \epsilon)$ solution [18] if

$$\sum_{s=1}^r \sum_{i \in \hat{\Omega}_s} \|\mathbf{U}_r(i, :) - \hat{\mathbf{c}}_s\|^2 \leq (1 + \epsilon) \min_{\substack{\Omega_1, \dots, \Omega_r \\ \mathbf{c}_1, \dots, \mathbf{c}_r}} \sum_{s=1}^r \sum_{i \in \Omega_s} \|\mathbf{U}_r(i, :) - \mathbf{c}_s\|^2. \quad (14)$$

Definition 3 (Misclustering Rate): Let $\{\Omega_1, \Omega_2, \dots, \Omega_r\}$ be the underlying true clustering partition of $[n]$ and $\{\hat{\Omega}_1, \hat{\Omega}_2, \dots, \hat{\Omega}_r\}$ be an estimate of the true partition. We define *misclustering rate* of $\{\hat{\Omega}_1, \hat{\Omega}_2, \dots, \hat{\Omega}_r\}$ as

$$MR(\hat{\Omega}_1, \hat{\Omega}_2, \dots, \hat{\Omega}_r) = \min_{k \in \mathcal{K}} \sum_{j=1}^r \frac{|\{i : i \in \Omega_j; i \notin \hat{\Omega}_{k(j)}\}|}{|\Omega_j|}, \quad (15)$$

where $\mathcal{K}$ is the set of all bijections from $[r]$ to $[r]$.

Since the partition is invariant to the labels of clusters, when we evaluate the misclustering rate, we compute the error under the best label matching, which is the reason we need $\mathcal{K}$.

B. Main Results

Let $N' := \sum_{t=0}^{N-1} \mathbb{1}\{\hat{X}_t \neq X_t\}$ denote the number of mistakes in the estimated mode sequence and $\eta := \frac{N'}{N}$ denote the mistake rate. In the following analyses, Lemma 2 gives conditions under which $N' = 0$. Theorem 3 and 4 give the upper bounds on misclustering rate and approximation error.

Lemma 2: Assume for any $t, |n_t| < n_{\max}$,

$$|\Phi_t^\top (\mathbf{w}_{X_t} - \mathbf{w}_j)| > 2n_{\max} \quad \forall j \in [n] \setminus X_t, \quad (16)$$

then the sequence estimated in Line 3 of Algorithm 1 is correct, i.e. $N' = 0$.

When $n_t = 0$, the dynamics given in (5) defines a hyperplane plus noise. Data points at the intersection of these hyperplanes (a set of measure zero in the noiseless case) are not useful in distinguishing the mode. Equation (16) essentially means that such data points do not exist.

Theorem 3: Assume: (i) $\mathbf{P}$ is ergodic; (ii) $\{\hat{\Omega}_1, \dots, \hat{\Omega}_r\}$ is a $(1 + \epsilon_1)$ solution to the k-means problem in Algorithm 1; (iii) $\|\Delta\| \leq \frac{\sigma_r(\tilde{\mathbf{P}})}{8\sqrt{(2 + \epsilon_1)r}} \sqrt{\frac{|\Omega_{(r)}|}{|\Omega_{(1)}|}} + 1$; (iv) mistake rate $\eta < \frac{\pi_{\min}}{2}$. Then, for any $\epsilon_2 > 0$ small enough, when $N \geq 400\tau_*\pi_{\max} \log(\log(\epsilon_2^{-1})) \log(\epsilon_2^{-1})\epsilon_2^{-2}$, with probability no less than

$$1 - \exp\left(-\frac{N}{200\tau_*\pi_{\max} \log(\epsilon_2^{-1})\epsilon_2^{-2}}\right), \quad (17)$$

we have

$$\begin{aligned} &MR(\hat{\Omega}_1, \hat{\Omega}_2, \dots, \hat{\Omega}_r) \\ &\leq 64(2 + \epsilon_1)r \left(\frac{\|\Delta\|}{\sigma_r(\tilde{\mathbf{P}})} + \frac{4(\epsilon_2 + 1.5\eta)(\|\Delta\| + \|\tilde{\mathbf{P}}\|)}{\pi_{\min}\sigma_r(\tilde{\mathbf{P}})} \right)^2. \end{aligned} \quad (18)$$

Theorem 4: Under the same conditions as Theorem 3, if $MR=0$, then with the same probability lower bound we have

$$\|\mathbf{P} - \tilde{\mathbf{P}}\|_\infty \leq 12\sqrt{n}\pi_{\min}^{-1}\sigma_1(\mathbf{P})(\epsilon_2 + 1.5\eta) + 2\|\Delta\|_\infty. \quad (19)$$

From Theorem 3 and 4, the bounds improve as any of the following decreases: number of modes n , number of

clusters r , perturbation $\|\Delta\|$, mixing time τ_* , condition number $\sigma_1(\bar{\mathbf{P}})/\sigma_r(\bar{\mathbf{P}})$, and disparities in stationary distribution π and cluster population, namely $\pi_{\max}/\pi_{\min}$ and $|\Omega_{(1)}|/|\Omega_{(r)}|$. Assumption (iii) restricts how the true Markov matrix is different from the underlying aggregatable matrix and assumption (iv) guarantees the Markov matrix can be learned accurately enough from the trajectory.

V. EXPERIMENTS

A. Synthetic Data

We first study the performance of our approach with synthetic data. In the Markov jump model, we let $n_a=3, n_c=2$ and number of modes $n = 50$. For each mode, the dynamics are generated by uniformly sampling its poles on $(-1, 1)$. We let input $u_t \sim \mathcal{N}(0, 1)$ and noise $n_t \sim \text{Unif}(-n_{\max}, n_{\max})$. The state space $[n]$ is partitioned into r clusters $\Omega_{1:r}$ randomly such that every possible partition is sampled with equal probability. The mode transition probabilities $\bar{\mathbf{P}}(\Omega_k, \cdot)$ for every k and initial mode distribution π_0 are sampled from uniform Dirichlet distribution.

The error metrics we evaluate are: (i) clustering error $\text{CE} = n^{-1} \min_{k \in \mathcal{K}} \sum_{j=1}^r |\{i : i \in \Omega_j; i \notin \hat{\Omega}_{k(j)}\}|$ where $\mathcal{K}$ is given in Definition 3; (ii) $\|\tilde{\pi} - \pi\|_1$, i.e. the stationary distribution difference between $\bar{\mathbf{P}}$ and $\mathbf{P}$. For each parameter setup, we record the average of these two metrics over 100 experiments.

1) *Without Perturbation* ($\Delta = 0$): We first evaluate how the performance depend on number of clusters r and noise magnitude $n_{\max}$. We set perturbation $\Delta = 0$ for these test cases. The experiment results are given in Fig.(1a-1d). We set $n_{\max} = 0.1$ in Fig.(1a-1b) and $r = 6$ in Fig.(1c-1d).

2) *With Perturbation* ($\Delta \neq 0$): In this test case, we fix $n = 50, r = 6, n_{\max} = 0.05, N = 10^5$. The space of Δ is a polytope which makes it difficult to sample uniformly, so instead for $i \in \Omega_k$, we sample $\mathbf{P}(i, \cdot)$ from Dirichlet distribution with parameters $\alpha \mathbf{P}(\Omega_k, \cdot)$ and record $\Delta = \mathbf{P} - \bar{\mathbf{P}}$. In this case, $\mathbb{E}[\mathbf{P}(i, \cdot)] = \mathbf{P}(\Omega_k, \cdot)$ and α controls how much $\mathbf{P}(i, \cdot)$ deviates from $\mathbf{P}(\Omega_k, \cdot)$. We sweep α and use scatter plots Fig.(1e-1f) to show how the error metrics vary with $\|\Delta\|$.

B. Practically Motivated Example—Patrol Robot

Now we consider a more realistic case involving Markov jump system that can possibly benefit from our approach. Assume in a region, we have n stations each with position $p_i \in \mathbb{R}$ and at time t there is only one active station s_t that generates requests; the sequence of active stations $s_{0:t}$ follows a Markov chain $\mathbf{P}$. There is a robot with position $x_t \in \mathbb{R}$ at time t aiming to reach the active station as fast and close as possible. Assuming the dynamics and control law of the robot are given by

$$\begin{aligned} x_{t+1} &= x_t + u_t + n_t, \\ u_t &= K(p_{s_t} - x_t), \end{aligned} \quad (20)$$

the closed-loop dynamics take the form

$$x_{t+1} = (1 - K)x_t + Kp_{s_t} + n_t, \quad (21)$$

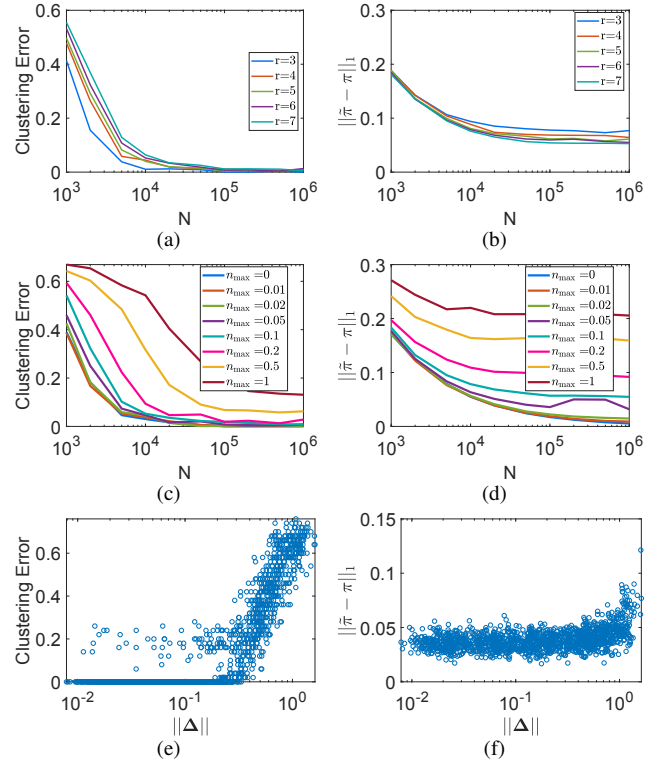


Fig. 1: Performance vs: (a,b) N and r ; (c,d) N and $n_{\max}$; (e,f) $\|\Delta\|$

which is a Markov jump model. In this setting, if the underlying Markov chain bears aggregatability property to some extent, we could use our approach to uncover the corresponding partition of modes as well as find an approximation of Markov transition matrix with stationary distribution that is easier to compute. Understanding the similarities between the stations' activation schedule can be useful to design improved control strategies for the robot.

In the experiment, we set $n = 50, p_i = i, K = 0.7, n_t \sim \mathcal{N}(0, 0.1), N = 10^6$ and sample clusters, $\bar{\mathbf{P}}$, and $\mathbf{P}$ same as V-A1. Over the average of 100 runs, clustering error $\text{CE} = 0.04$ and $\|\tilde{\pi} - \pi\|_1 = 0.07$.

VI. CONCLUSIONS & FUTURE WORK

In this paper, we consider the problem of model reduction for Markov jump systems by clustering the modes based on their transition distributions. The proposed approach has guaranteed clustering error upper bound and exhibits decent performance in the experiments.

One direction for future for is to consider lumpable Markov chains as an alternative criterion for reduction. We will also study clustering of the continuous-dynamics in each mode based on their similarities and quantify the model mismatch between the actual and reduced order models in this setting.

REFERENCES

- [1] Z. Du, N. Ozay, and L. Balzano, "Mode clustering for markov jump systems," arXiv:1910.02193 [eess.SY], 2019.

- [2] V. Gupta, R. M. Murray, L. Shi, and B. Sinopoli, "Networked sensing, estimation and control systems," *California Institute of Technology Report*, 2009.
- [3] P. Shi and F. Li, "A survey on markovian jump systems: modeling and design," *International Journal of Control, Automation and Systems*, vol. 13, no. 1, pp. 1–16, 2015.
- [4] V. Ugrinovskii* and H. R. Pota, "Decentralized control of power systems via robust control of uncertain markov jump parameter systems," *International Journal of Control*, vol. 78, no. 9, pp. 662–677, 2005.
- [5] K. Loparo and F. Abdel-Malek, "A probabilistic approach to dynamic power system security," *IEEE transactions on circuits and systems*, vol. 37, no. 6, pp. 787–798, 1990.
- [6] W. Liu and I. Hwang, "Probabilistic trajectory prediction and conflict detection for air traffic control," *Journal of Guidance, Control, and Dynamics*, vol. 34, no. 6, pp. 1779–1789, 2011.
- [7] K. Gopalakrishnan and H. Balakrishnan, "A comparative analysis of models for predicting delays in air traffic networks." ATM Seminar, 2017.
- [8] L. E. Svensson, N. Williams *et al.*, "Optimal monetary policy under uncertainty: A markov jump-linear-quadratic approach," *Federal Reserve Bank of St. Louis Review*, vol. 90, no. 4, pp. 275–293, 2008.
- [9] Y. Zhu, Z. Zhong, W. X. Zheng, and D. Zhou, "Hmm-based $\mathcal{H}_\infty$ filtering for discrete-time markov jump lpv systems over unreliable communication channels," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 48, no. 12, pp. 2035–2046, 2017.
- [10] L. Zhang, B. Huang, and J. Lam, " $\mathcal{H}_\infty$ model reduction of markovian jump linear systems," *Systems & Control Letters*, vol. 50, no. 2, pp. 103–118, 2003.
- [11] G. Kotsalis, A. Megretski, and M. A. Dahleh, "Model reduction of discrete-time markov jump linear systems," in *2006 American Control Conference*. IEEE, 2006, pp. 6–pp.
- [12] G. Kotsalis and A. Rantzer, "Balanced truncation for discrete time markov jump linear systems," *IEEE Transactions on Automatic Control*, vol. 55, no. 11, pp. 2606–2611, 2010.
- [13] J. Sanders, A. Proutière, and S.-Y. Yun, "Clustering in block markov chains," *arXiv preprint arXiv:1712.09232*, 2017.
- [14] A. Ganguly, T. Petrov, and H. Koepl, "Markov chain aggregation and its applications to combinatorial reaction networks," *Journal of mathematical biology*, vol. 69, no. 3, pp. 767–797, 2014.
- [15] K. Deng, Y. Sun, P. G. Mehta, and S. P. Meyn, "An information-theoretic framework to aggregate a markov chain," in *2009 American Control Conference*. IEEE, 2009, pp. 731–736.
- [16] A. Zhang and M. Wang, "State compression of markov processes via empirical low-rank estimation," *arXiv preprint arXiv:1802.02920*, 2018.
- [17] W. J. Stewart, "A comparison of numerical techniques in markov modeling," *Commun. ACM*, vol. 21, no. 2, pp. 144–152, Feb. 1978. [Online]. Available: <http://doi.acm.org/10.1145/359340.359350>
- [18] J. Lei, A. Rinaldo *et al.*, "Consistency of spectral clustering in stochastic block models," *The Annals of Statistics*, vol. 43, no. 1, pp. 215–237, 2015.