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Precision Test of the Limits to Universality in Few-Body Physics

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(Received 21 June 2019; revised manuscript received 13 October 2019; published 2 December 2019)

We perform precise studies of two- and three-body interactions near an intermediate-strength Feshbach resonance in ^{39}K at 33.5820(14) G. Precise measurement of dimer binding energies, spanning three orders of magnitude, enables the construction of a complete two-body coupled-channel model for determination of the scattering lengths with an unprecedented low uncertainty. Utilizing an accurate scattering length map, we measure the precise location of the Efimov ground state to test van der Waals universality. Precise control of the sample's temperature and density ensures that systematic effects on the Efimov trimer state are well understood. We measure the ground Efimov resonance location to be at $-14.05(17)$ times the van der Waals length r_{vdW}, significantly deviating from the value of $-9.7 r_{\mathrm{vdW}}$ predicted by van der Waals universality. We find that a refined multichannel three-body model, built on our measurement of two-body physics, can account for this difference and even successfully predict the Efimov inelasticity parameter η.

DOI: 10.1103/PhysRevLett.123.233402

The few- and many-body physics of an interacting gas are intractable when treated in full microscopic detail. However, the problem can be greatly simplified in a dilute ultracold atomic gas with near-resonant interactions, where the two-body scattering length a greatly exceeds the van der Waals length r_{vdW} characterizing the range of the interacting potential. In such a scenario, all physical observables can be parametrized by only two dimensionless quantities describing the strength of interactions and the level of quantum degeneracy [1]: na^3 and $n\lambda^3$, where n is the atomic density and λ is the thermal wavelength. Then, continuous scaling transformations such as $n \to \zeta^{-3} n$, $a \to \zeta a$, and $\lambda \to \zeta \lambda$ will leave all observables and their dynamics invariant when measured in rescaled units. Such behavior is regarded as universal, insensitive to microscopic details in the problem and the chosen atomic species.

Nevertheless, the principle of universality has its limitations. For example, unless all length scales in the problem ($|a|$, λ, $n^{-1/3}$, etc.) greatly exceed r_{vdW}, nonuniversal corrections due to short-ranged physics must be implemented. Even when these conditions are well satisfied, a more fundamental effect concerning few-body interactions can break universality: the Efimov effect [2]. Within this phenomenon, short-ranged near-resonant two-body interactions give rise to a three-body attraction that hosts an infinite series of Efimov trimer states. Each consecutive state meets the three-body continuum at a particular value of scattering length that is 22.7 times larger than the previous state, with a_- defining the ground state location [1]. Although these fixed length scales break the continuous aspect of universality, there remains a discrete version of scale transformations, with ζ values restricted to 22.7^j, where j is an integer.

The value of a_- was originally thought to be set by the details of the short-range interaction, and therefore to be thoroughly nonuniversal. However, it was noted that, across many atomic species and different Feshbach resonances, the measured a_- value was within 20% of $-9 r_{\mathrm{vdW}}$ [3–6]. This suggested that a_- depends only on the longest-range part of the short-range physical interaction. Theory indeed predicts a similar value of $a_- = -9.7 r_{\mathrm{vdW}}$ [4–10]. This "van der Waals universality," together with the Efimov scaling, allows one to predict the full Efimov structure to arbitrarily large length scales.

Our experimental goal is to definitively challenge the robustness of this van der Waals universality. It has been speculated [11–18] that universality of the Efimov structure depends on the breadth of the Feshbach resonance quantified by the dimensionless parameter s_{res}. Very roughly, s_{res} may be understood as the parameter that characterizes the range of the scattering length, $|a| \gtrsim 4 r_{\mathrm{vdW}}/s_{\mathrm{res}}$, over which the two-body Feshbach resonance has universal structure, meaning, e.g., that the two-body binding energy is $E_b = \hbar^2/(ma^2)$ [19]. One might expect the three-body Efimov resonances to be more precisely universal when they fall more deeply into that range of a for which the two-body Feshbach resonant structure is universal. In previous experiments on homonuclear Efimov states [3,17,37–45], there is some support for the notion that, as s_{res} gets smaller,

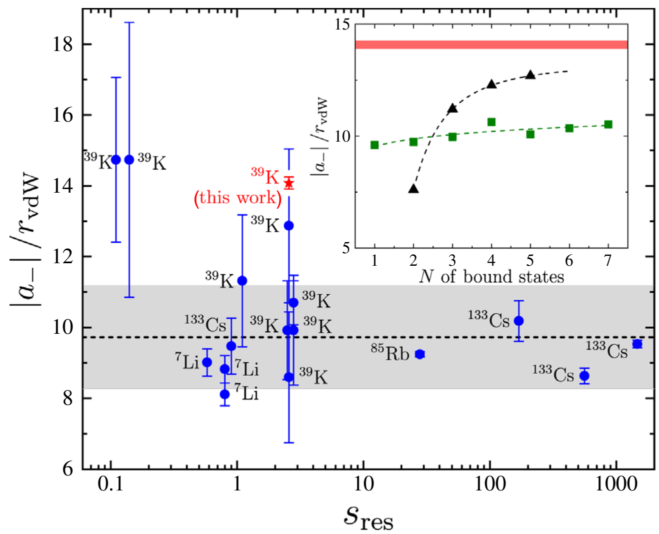

FIG. 1. Survey of experimental a_- values in homonuclear systems, inspired by [15]. Previous results (blue circles) [3,17,37,41–45] show a tentative dependence of the a_- value on the Feshbach resonance strength parameter $s_{\rm res}$. Our measurement (red star; red band in the inset) is the strongest evidence of departure from the $-9.7 \pm 15\% r_{\rm vdW}$ value (dashed line and gray area) predicted by van der Waals universality [4,5,7,9]. The inset shows calculations for a_- based on a single van der Waals potential [7] with $N = 1-7$ s-wave two-body bound states (green squares) and results from our multichannel model [19] with $N = 2-5$ (black triangles).

the measured a_- values should begin to deviate from the universal $a_- = -9.7r_{\rm vdW}$ value; see Fig. 1. However, this conclusion is only tentative due to large experimental uncertainties in the measured a_- [17], unexpected temperature dependence [45], and large systematic uncertainties in the parameters of the underlying two-body Feshbach resonance [17,37,38,43]. Although there are intriguing experimental [15,46–55] and theoretical [52,56–58] results for the heteronuclear cases, the possible influence of many additional parameters (mass ratio, quantum statistics, and inter- and intraspecies scattering lengths) makes the question of universality in those systems a topic for an entirely separate investigation.

In this Letter, we present a precise test of van der Waals universality near a Feshbach resonance with $s_{\rm res} = 2.57$ [19], which is intermediate between the narrow ($s_{\rm res} \ll 1$) and broad ($s_{\rm res} \gg 1$) regimes. Specifically, we accurately determine the value of a_- by having precise control of critical experimental parameters such as temperature, density, and scattering length. Because of our tight control of both systematic and statistical errors, ours is the first measurement of a compelling nonuniversal a_- value in a homonuclear Efimov resonance.

A thorough characterization of the Feshbach resonance and an accurate map of the scattering length are required for precise determination of the a_- value. Accordingly, we perform high-precision spectroscopy on a pure gas of Feshbach dimers and accurately determine their binding energies. This measurement enables us to refine our two-body model and accurately predict the scattering length in our Efimov measurements [19]. In other Feshbach resonance studies, methods based on number loss or thermalization rate have occasionally given inconsistent results. By contrast, dissociation spectroscopy of Feshbach dimers isolates two-body physics and accurately determines resonance properties [59–62].

Precision molecular spectroscopy requires long interrogation times under unperturbed conditions. We stabilize the magnetic field to the milligauss-level and eject all unpaired atoms, whose presence affects dimer lifetimes and complicates the spectroscopy. A pure molecular sample is prepared by starting with $\sim 10^5$ atoms confined in an optical dipole trap and a temperature of $\sim$300 nK. We transfer a fraction of atoms in the $|F = 1, m_F = -1\rangle$ hyperfine state to the dimer state by magnetoassociation [63]. Subsequently, all residual unpaired atoms are blasted away by multiple radio-frequency (rf) and optical pulses, leaving a pure sample of $\sim 10^4$ molecules. Lastly, the magnetic field B is ramped to various values, corresponding to different binding energies, where we perform rf spectroscopy.

We dissociate molecules by transferring one atom of the pair from the $|F = 1, m_F = -1\rangle$ interacting state to the $|F = 1, m_F = 0\rangle$ imaging state. The final state being nearly noninteracting enables us to directly probe the dimer binding energy. Additionally, the transition being magnetically less sensitive near B values of interest allows long molecular interrogation times, limited only by dimer lifetimes, to achieve high spectral resolution. We scan the rf frequency and measure the transferred fraction, keeping the pulse energy low to limit saturation effects and dissociate a maximum 50% of molecules. We fit the measured spectrum to a functional form given by the Franck-Condon factor of the bound-free transition [59], and we extract the molecular binding energy E_b [19,64]. We repeat this procedure to determine E_b at different magnetic field values, as depicted in Fig. 2.

The universal expression $E_b = \hbar^2/(ma^2)$ is always accurate for large enough a. A more refined expression $E_b = \hbar^2/[m(a - \bar{a})^2]$, which introduces the mean scattering length $\bar{a} \approx 0.956 r_{\rm vdW}$ [66], is valid at smaller values of a as long as $a \gg r_{\rm vdW}/s_{\rm res}$ [65]. However, such treatments are inadequate for narrow and intermediate resonances. To better compare to our experimental data, we developed a coupled-channel model [19] capable of describing our high-precision E_b data. We fine-tune the model's parameters, the singlet and triplet scattering potentials, to accurately match most of our measurements to within 1%, as depicted in Fig. 2's inset. As a result, we determine a particular linear combination of the singlet and triplet scattering lengths of $0.2470a_S + 0.9690a_T = 1.926(2)a_0$ [19], further constraining the previously reported values of $a_S = 138.49(12)a_0$ and $a_T = -33.48(18)a_0$ [67,68].

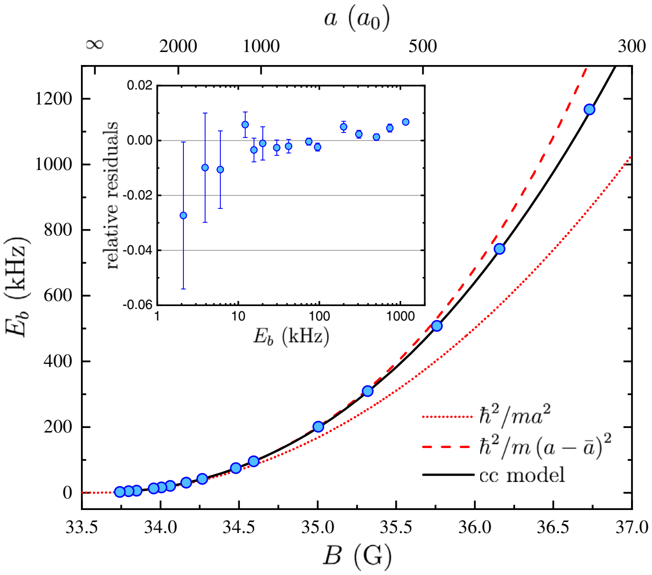

FIG. 2. Precise measurement of Feshbach dimer binding energies E_b as a function of magnetic field B. Small experimental uncertainties on E_b, spanning from 56 Hz at $E_b/h = 2.103$ kHz to 1.0 kHz at $E_b/h = 1167.2$ kHz, are not resolvable in the figure. A coupled-channel (cc) model is required to describe our data [19]. The solid curve shows the resulting fit, and the inset shows remarkably small fractional residuals. Contrary to applicability near broad Feshbach resonances [65], universal expressions (dashed and dotted curves) are insufficient for describing E_b near our intermediate strength resonance.

Furthermore, we constrain the Feshbach resonance location to within 33.5820(14) G [19], which is a two-order-of-magnitude improvement over the previous measurement [17] and an unprecedented [60] accuracy better than 3×10^{-5} of the resonance width.

With a good grasp on two-body physics, we seek to test the validity of van der Waals universality near our Feshbach resonance. We perform precision atom-loss spectroscopy to obtain the Efimov ground state location a_- [69]. Specifically, we measure the inelastic three-body recombination coefficient L_3 in the vicinity of a_-, where the presence of the nearby Efimov state leads to a resonant enhancement of the three-body loss: an Efimov resonance. A zero-temperature zero-range expression [1] relates L_3 features to a_- for $a < 0$:

$$L_3^{T=0}(a) \approx \frac{3\hbar a^4}{m}\frac{4590\sinh(2\eta)}{\sin^2[s_0\ln(a/a_-)] + \sinh^2(\eta)}, \quad (1)$$

where the dimensionless inelasticity parameter η characterizes the Efimov resonance width, and the constant $s_0 \approx 1.00624$ fixes Efimov series spacing $e^{\pi/s_0} \approx 22.7$. Although Eq. (1) adequately describes L_3 in the limit of $\lambda \gg |a|$, for increasing temperatures, it becomes less valid; and a finite-temperature zero-range model [70,71] is required to describe the three-body loss for $a < 0$:

$$L_3(a,T) = A\frac{72\sqrt{3}\pi^2\hbar(1-e^{-4\eta})}{mk_{\mathrm{th}}^6 a^2} \times \int_0^\infty \frac{(1-|s_{11}(x)|^2)e^{-x^2/(k_{\mathrm{th}}a)^2}x}{|1+s_{11}(x)(\frac{-xa_-}{1.017|a|})^{-2is_0}e^{-2\eta}|^2}dx, \quad (2)$$

where $k_{\mathrm{th}} = \sqrt{mk_B T}/\hbar$, $x = k|a|$, A is a numerical factor that improves the fit quality by allowing for uncertainty in the absolute density, and the complex function $s_{11}(x)$ is an S-matrix element from Refs. [70] and [72].

We perform $L_3(a)$ measurements at different temperatures and extract a_- using the zero-range model [Eq. (2)]. We begin with dilute thermal samples at $a = -100a_0$. We ensure our gas is fully thermalized and make trapping potentials sufficiently deep to be certain that evaporative losses have a negligible effect on our measurements. Then, we ramp a to a value of interest and let three-body loss occur for a varied amount of time, allowing up to 30% decay of the initial atom number. Subsequently, we ramp a to a value of $-200a_0$, transfer the remaining atoms to the $|F = 2, m_F = -2\rangle$ state, and perform time-of-flight imaging. We determine the time-dependent density n from the measured temperatures and atom numbers [19]. For each scattering length, we extract the L_3 value by numerically solving the expression [73]:

$$\frac{1}{N}\frac{dN}{dt} = -L_3\langle n^2\rangle - \alpha, \quad (3)$$

where $\langle n^2\rangle = (1/N)\int n^3(\vec{x})d^3x$, and the constant $1/\alpha > 40$ s is the a-independent one-body decay time measured at $a = -100a_0$, which is negligible as compared to the three-body loss timescales of 50–170 ms for our n near a_-. Additionally, we check that the two-body loss contribution $-L_2\langle n\rangle$ to Eq. (3), with L_2 predicted by our two-body model, is also negligible.

Accurate calibration of a and density (and not just relative changes) enables accurate comparison of the measured $L_3(a)$ values at different temperatures, as depicted in Fig. 3. We fit the data collected at each temperature to Eq. (2) with three parameters: a_- (see inset of Fig. 3), η, and A. We take the weighted mean across all temperatures to extract single values $a_- = -908(11)a_0 = -14.05(17)r_{\mathrm{vdW}}$ and $\eta = 0.25(1)$ [19]. Equation (2) will eventually become inaccurate at large a, if only because the functional form requires the first two resonances be a factor of 22.7 apart. We vary the fit range from all a to only $|a| < \lambda/10$ and take the maximal spread of all fit errors as the uncertainty on a_- and η.

In addition to finite-temperature effects, we check the effect of high density on L_3 measurements. We prepare samples with varied densities yet similar temperatures of ~200 nK. Although measurements with the two lowest densities (where initial $|na^3| = 1.3 \times 10^{-5}$ and 2.4×10^{-5} at a_-) are consistent, we observe a suppression and shift of the Efimov resonance for our highest-density gas

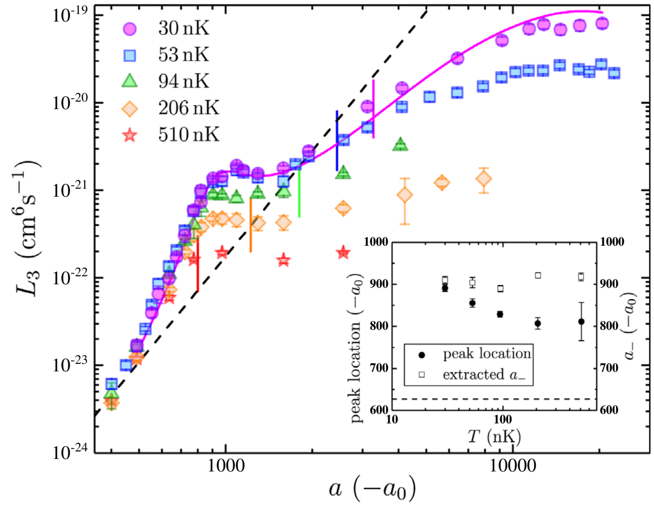

FIG. 3. Temperature dependence of the three-body loss coefficient L_3, scaling as a^4 scaling (dashed) [73–75], enhanced near an Efimov ground state located at a_-. For each temperature, we fit our data using a zero-temperature zero-range model [Eq. (1)], limiting fits to data points for which $|a| < \lambda/10$ (short vertical lines), to extract the L_3/a^4 peak location and a finite-temperature zero-range model [Eq. (2), solid] to extract the true a_- value. The inset shows the extracted peak locations (circles) and a_- values (squares), where both coincide at the lowest temperature. The observed a_- value significantly deviates from the $a_- = -630a_0$ value (inset dashed line) predicted by van der Waals universality [7].

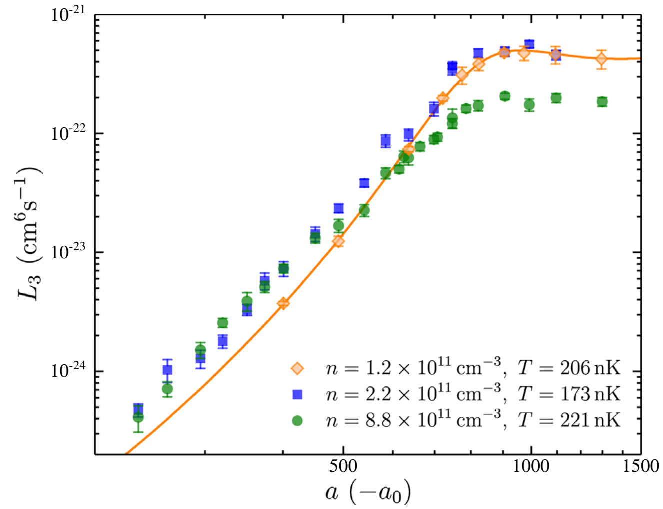

FIG. 4. Suppression of the Efimov resonance in a high-density gas. Measurements of high- and intermediate-density samples performed with the same experimental conditions, contrasting only in the initial atom number. As a result, differential comparison of L_3 values between those two measurements is of greatest interest. Small L_3 deviations at low $|a|$ between the lowest-density data and other data are attributed to differing trap conditions that result in evaporation. However, for our highest-density data, we observed a strong suppression of L_3 near $a = a_-$.

(see Fig. 4), where $|na^3| = 9.7 \times 10^{-5}$ at a_- and the collision rate is no longer small as compared to the trapping frequency. The latter condition in particular can lead to systematic errors. A recently published study [45] on the same resonance as we discuss here reported counterintuitive temperature-induced shifts in the Efimov peak at high values of $|na^3|$, the collision rate, and $n\lambda^3$. We see no such effects in the data (shown in Fig. 3) that we use to determine a_-; for those fits, we use only $|na^3| < 4 \times 10^{-5}$ and $n\lambda^3 < 0.2$ [19]. The data shown in Fig. 3 agree well with the prediction of Eq. (2): not just in the shape of $L_3(a, T)$ but in its overall amplitude A. The fact that, for all values of T, our fit A is within 43% of 1.0 (which is consistent with small discrepancies in the density calibration) is further evidence that our results are not contaminated by high degeneracy, many-body effects, collisional avalanche, or misassignment of resonance peaks.

Our final value for $a_- = -908(11)a_0$, plotted as a red star in Fig. 1, differs from the range of theoretical predictions [4–7,9] for the universal result of $a_- = -630 \pm 15\%a_0$ by many times our estimated error. How does this firmly established discrepancy compare to theoretical efforts to model the "edges of universality"?

The range $a_- = [-11.2, -8.3]r_{\mathrm{vdW}}$ of theoretical predictions for the universal value arises because the calculated value of the ostensibly universal a_- depends, even if only modestly, on the details of short-range treatment [7]. It seems likely this variability will be only more pronounced for a regime where universality is already beginning to fail on its own. A key qualitative lesson from Ref. [16] is the prediction of a nonuniversal value of $a_- \approx -12r_{\mathrm{vdW}}$ for $s_{\mathrm{res}} = 2.57$ and $a_{\mathrm{bg}} = -19.6a_0$. However, going beyond the results from Ref. [16], we find that a_- also depends on the number of bound states in the model for small s_{res} and a_{bg}. In our theoretical effort to accurately describe three-body physics [19], we constructed a more realistic multichannel model using a realistic hyperfine and Zeeman spin structure, with triplet and singlet scattering lengths constrained to equal our empirically determined values. The adjustable parameters are the inner walls of the van der Waals potentials tuned to give the desired number of bound states. The results are shown as black triangles in the inset of Fig. 1. We see that the predicted a_- result more closely approximates our distinctly nonuniversal measurement as we go to a larger number of bound states. An empirical attempt to extrapolate to a very large number of bound states yields $a_-^{\mathrm{lim}} = -13.1r_{\mathrm{vdW}}$ and $\eta^{\mathrm{lim}} = 0.21$. This is the first attempt to get a quantitatively accurate calculation for η close to our measured value of 0.25(1). The reasonable agreement with the experimental value shows the importance of properly modeling the diatomic molecular spectra and its hyperfine structure [19].

To conclude, we precisely measure dimer binding energies, the Feshbach resonance location, and the Efimov ground location. Our results (in particular, the observation of a definitively nonuniversal Efimov state location and its corresponding inelasticity parameter) suggest that more realistic models, like the one we used, can be necessary to fully understand and accurately describe few-body physics in ultracold atomic systems.

The authors thank Jeremy Hutson, John Bohn, Wilhelm Zwerger, and Richard Schmidt for useful discussions. This work is supported by NSF Physics Frontier Center Grant No. 1734006, NASA, the Marsico Research Chair, and the NIST.

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