Transactions on Control Systems Technology

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Journal:	*IEEE Transactions on Control Systems Technology*
Manuscript ID	Draft
mstype:	Regular Paper
Date Submitted by the Author:	n/a

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Keywords:	Functional Electrical Stimulation (FES), FES- Cycling, Passivity, Iterative Learning Control (ILC), Switching Control

Passivity-Based Iterative Learning Control for Cycling Induced by Functional Electrical Stimulation with Electric Motor Assistance

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***Abstract*— This paper examines the use of a learning control method in a passivity-based framework to control a motorized cycle-rider system with functional electrical stimulation (FES) of the quadriceps muscle groups. FES-cycling with motorized assistance has been used in the rehabilitation of people with neurological conditions. The concepts of adaptation and passivity are explored to compensate for the uncertain nonlinear time-varying dynamics of the motorized FES cycle-rider system. The system is modeled as a closed-loop feedback, state-dependent switched system such that in each cycle, the quadriceps muscle groups produce functional torque and the electric motor provides assistance as needed. The output strictly passive (OSP) feature of the closed-loop system is proved by considering a learning control input. Then, an adaptive update law, based on iterative learning control (ILC), is developed to guarantee the convergence of the cadence tracking error. Experimental results from seven able-bodied participants are presented and discussed to demonstrate the effectiveness of this approach. The average cadence tracking error is 0.00±2.47 rpm for a desired trajectory of 50 rpm.**

I. Introduction

Functional electrical stimulation (FES) utilized in the lower body is a well-known rehabilitation technique, where muscle contractions are triggered due to the potential field applied across muscle groups to evoke functional tasks [1]. Specifically, FES-cycling is applied to people with neurological disorders such as spinal cord-injury, stroke, or traumatic brain injury [2]. FES has several therapeutic benefits resulting in the improvement of muscle strength [3]. The FES cycle-rider dynamic model is a complex nonlinear system due to the time-varying nature of the muscle dynamics, and the presence of disturbances, input delay, and muscle fatigue. Various control method approaches have been developed to address these challenges such as proportional-derivative (PD) and proportional-integral-derivative (PID) controllers [4], [5], [6], [7], [8]. FES-cycling studies have used fuzzy logic control scheme in [9], [10], and the comparison between fuzzy logic based control design and PD controller design is discussed. The results in [11], [12] investigate optimal stimulation patterns for FES-cycling to minimize the muscle stress time integral based on optimal control schemes.

During FES-cycling, muscle forces produce torque primarily about the knee joint, which is transferred to torque about the crank axis. However, there are regions of the crank cycle, where torque production is kinematically inefficient and thus, for efficient cycling the FES contribution is restricted to certain regions of the crank cycle. To maintain a constant cadence, an electric motor is used in the regions where it is inefficient to stimulate the muscle groups. The combination of FES and motor assist makes the overall system a state-dependent switched system. Recently, various studies have focused on motorized FES cycle-rider systems. FES and a motor assist based on fuzzy logic control methods are used to yield cadence tracking in impaired populations [13]. In [14], switching between muscle stimulation and a motor assist is studied to address muscle fatigue. Motorized FES cycling systems are studied in [15] to track a desired cadence and power output simultaneously. Since the automatic cycle-rider process is repetitive and possesses a number of uncertainties in its dynamics, the utilization of a learning control technique such as iterative learning control (ILC) scheme is very desirable.

ILC is a well established adaptive technique for repetitive tasks in which the control input is updated in each trial, based on the previous performance information [16]. For cyclic or repetitive nonlinear time-varying systems, ILC represents a promising learning control method to achieve asymptotic tracking. This paper employs ILC since the dynamics of the motorized FES cycle-rider system are repetitive. The purpose of ILC is to obtain asymptotic tracking and improve the performance of such system after a certain number of cycles/iterations. In, [17], [18], ILC was applied for robust tracking control of FES systems, and FES induced cycling based on repetitive learning control (RLC) is studied in [19]. FES of the upper limbs using ILC technique for rehabilitation purpose are studied in [20], [21]. All previous results are based on Lyapunov-based analysis and were able to show asymptotic tracking of the system. In this paper, the combination of adaptation and passivity are used to show $\mathcal{L}_2$ convergence of the closed-loop system's output.

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Systems that dissipate energy and do not generate their own energy are called dissipative systems. Passive systems are the most known classes of dissipative systems. An important point about passivity is its connection to the concept of stability. The fundamental concepts of stability and passivity are based on the proper choice of a positive definite function. In Lyapunov theory this function is known as the Lyapunov function and is used in stability analysis [22]. In passivity-based approaches this function is referred to as the storage function. More precisely, in passivity, the energy that is supplied into the system must be greater than or equal to the energy being stored by the system (the storage function), over a certain time interval [23], [24]. The rest of the energy is dissipated. This is simply referred to as the passivity inequality.

In this paper, the cycle-rider system is analyzed based on the concept of passivity. Specifically, the focus is on a particular subclass of passive systems: output strictly passive (OSP) systems. In [25], the relation between learnability, output-dissipativity, and strictly positive realness was studied. It was shown that based on an Arimoto-type ILC and the use of the passivity property, convergence of the error output is implied in the $\mathcal{L}_2$ norm sense [26]. In [27], [28] passivity-based control and ILC were utilized for robot manipulators with antagonistic bi-articular muscles. However, unlike in [28], this paper studies the passivity and adaptation of the nonlinear cycle-rider system with FES applied to lower limb muscles along with motorized assistance. The closed loop-system is viewed as a switched system, since it switches between muscle groups and the motor assist in each cycle. The learning control method which is build upon the passivity concept is employed to cope with system's repetitive nature and to guarantee the convergence of the output error trajectory to zero. To the best of our knowledge, this is the first time that the concept of passivity and learnability are applied to the motorized nonlinear cycle-rider system with switched control inputs.

This paper is organized as follows: in section II, preliminaries and background mathematics are introduced. In Section III the main results are presented, followed by experimental results in Section IV, and in Section V concluding remarks are discussed.

II. Background and Preliminaries

A. Dynamic Model

A motorized FES-cycling system can be modeled as [29], [30]

$$\tau_{cycle}(\dot{q}(t), \ddot{q}(t), t) + \tau_{rider}(q(t), \dot{q}(t), \ddot{q}(t), t) = \tau_{motor}(t), \quad (1)$$

where $q, \dot{q}, \ddot{q} : \mathbb{R}_{\geqslant 0} \to Q$ ($Q \subset \mathbb{R}$ denotes the set of crank angles) are the crank angle, velocity, and acceleration respectively (Fig. 1). In (1), $\tau_{cycle} : \mathbb{R} \times \mathbb{R} \times \mathbb{R}_{\geqslant 0} \to \mathbb{R}$ is the net cycle torque, $\tau_{rider} : Q \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}_{\geqslant 0} \to \mathbb{R}$ is the net rider torque, and $\tau_{motor} : \mathbb{R}_{\geqslant 0} \to \mathbb{R}$ is the torque about the crank axis provided by an electric motor, respectively. The torque applied by the motor is

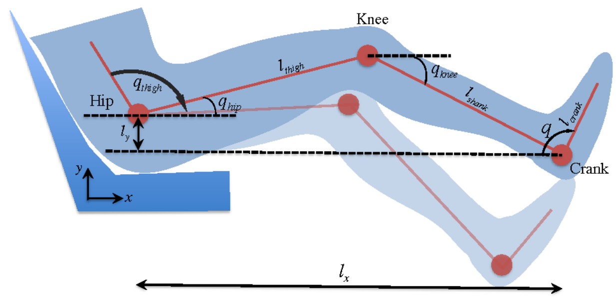

Figure 1. Schematic of the cycle and rider system.

$$\tau_{motor}(t) \triangleq B_e u_e(t), \quad (2)$$

where $B_e \in \mathbb{R}$ is a constant relating the current in the electric motor to the resulting torque about the crank axis, and $u_e : \mathbb{R}_{\geqslant 0} \to \mathbb{R}$ is the control current applied to the electric motor windings. The net cycle torque is defined as

$$\tau_{cycle}(\dot{q}(t), \ddot{q}(t), t) \triangleq J_{cycle}\ddot{q}(t) + b_{cycle}\dot{q}(t) + d_{cycle}(t), \quad (3)$$

where $J_{cycle} \in \mathbb{R}_{>0}$ is the unknown cycle inertia, $b_{cycle} \in \mathbb{R}_{>0}$ is the unknown coefficient of viscous damping in the cycle, and $d_{cycle} : \mathbb{R}_{\geqslant 0} \to \mathbb{R}$ denotes unknown disturbances such as load changes. The net rider torque is defined as

$$\begin{aligned}\tau_{rider}(q(t), \dot{q}(t), \ddot{q}(t), t) &\triangleq \tau_p(q(t), \dot{q}(t), \ddot{q}(t)) \\ &- \tau_a(q(t), \dot{q}(t), t) + d_{rider}(t),\end{aligned} \quad (4)$$

where $\tau_p : Q \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is the net torque produced by the rider's passive limb dynamics, $\tau_a : Q \times \mathbb{R} \times \mathbb{R}_{\geqslant 0} \to \mathbb{R}$ is the net torque produced by active contractions of the rider's muscles, and $d_{rider} : \mathbb{R}_{\geqslant 0} \to \mathbb{R}$ is the effects of unknown disturbances from the rider. The disturbances in (3) and (4) are combined as $d_{cycle}(t) + d_{rider}(t) = d_r(t)$, and the disturbance is assumed to have a known bound as $|d_r(t)| \leqslant c_{d_r}$, where $c_{d_r} \in \mathbb{R}_{>0}$ is a known constant.

The rider's passive limb dynamics are modeled as a single degree-of-freedom system [31] as

$$\begin{aligned}\tau_p(q(t), \dot{q}(t), \ddot{q}(t)) &\triangleq M(q(t))\ddot{q}(t) + V(q(t), \dot{q}(t))\dot{q}(t) \\ &+ G(q(t)) + P(q(t), \dot{q}(t)),\end{aligned} \quad (5)$$

where $M : Q \to \mathbb{R}_{>0}$ is the unknown rider inertia, $V : Q \times \mathbb{R} \to \mathbb{R}$ denotes centripetal and Coriolis forces, $G : Q \to \mathbb{R}$ represents gravitational effects, $P : Q \times \mathbb{R} \to \mathbb{R}$ accounts for the passive viscoelastic tissue forces in the knee joints. The passive viscoelastic tissue forces are defined as

$$P(q(t), \dot{q}(t)) \triangleq \sum_{j \in J} T_j(q(t)) p_j(q(t), \dot{q}(t)), \quad (6)$$

where $T_j : Q \to \mathbb{R}$ are the known joint torque transfer ratios [11] with subscript $j \in J \triangleq \{\text{RHip, RKnee, LHip, LKnee}\}$ indicating right and left hip and knee joints, and $p_j : Q \times \mathbb{R} \to \mathbb{R}$ is the resultant torque about the rider's joint from viscoelastic tissue forces.

The active torque resulting from the rider's muscles is defined as

$$\tau_a \triangleq \sum_{m \in \mathbb{M}} B_m(q(t), \dot{q}(t)) u_m(t) + \tau_r(t), \tag{7}$$

where $B_m : Q \times \mathbb{R} \to \mathbb{R}$ is the uncertain control effectiveness of a muscle group with subscript $m \in \mathbb{M} \triangleq \{\text{set of active muscles from right and left legs}\}$, $u_m : \mathbb{R}_{\geqslant 0} \to \mathbb{R}$ is the electrical stimulation intensity applied to each muscle group, and $\tau_r : \mathbb{R}_{\geqslant 0} \to \mathbb{R}$ is the torque applied about the crank axis by the rider's volitional cycling effort.

B. Switched System Model

This paper uses a motorized cycle combined with FES. Lower-limb muscles are stimulated to generate forward pedaling and an electric motor assists on the regions where FES induced torque is absent. The system is an arbitrary, state-dependent switched system since the system switches between two modes, the FES mode and motor mode in each cycle [32]. In other words, the muscle stimulation and the motor assist contribution are limited to certain portions of each cycle (Fig. 2). The electric motor region denoted by $Q_e \subset Q$ (where the electric motor is active) and the stimulation region denoted by $Q_m \subset Q$ for $m \in \mathbb{M}$ (where the muscle groups are stimulated) and for each muscle group, it is defined as [29], [30]

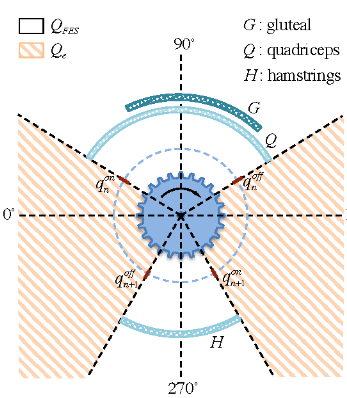

Figure 2. The switched system pattern shows the range of one cycle, in which the three groups of leg's muscle are stimulated (Q_{FES}), and the range where the electric motor generates the motion (Q_e).

$$Q_{*\text{Glute}} \triangleq \{q(t) | T_{*\text{Glute}}(q(t)) > \varepsilon_{*\text{Glute}}(t)\}, \tag{8}$$

$$Q_{*\text{Ham}} \triangleq \{q(t) | T_{*\text{Ham}}(q(t)) > \varepsilon_{*\text{Ham}}(t)\}, \tag{9}$$

$$Q_{*\text{Quad}} \triangleq \{q(t) | -T_{*\text{Quad}}(q(t)) > \varepsilon_{*\text{Quad}}(t)\}, \tag{10}$$

where $\varepsilon_m : \mathbb{R}_{\geqslant 0} \to [0, \ \max(T_m(q(t)))]$ is a time-varying signal defined for $m \in \mathbb{M}$.

The union of the stimulation regions is defined as $Q_{FES} \triangleq \bigcup_{m \in \mathbb{M}} Q_m$. Therefore, the region where the electric motor is active is defined as $Q_e \triangleq Q \backslash Q_{FES}$. Fig. 2 depicts how switching occurs among different modes (the FES region and the electric motor region) for one cycle. The known sequences of switching states are denoted by $\{q_0^{\text{on}}, \ q_0^{\text{off}} \in Q\}_{n=0}^{\infty}$, and the corresponding unknown switching times are denoted by $\{t_0^{\text{on}}, \ t_0^{\text{off}} \in \mathbb{R}_{\geqslant 0}\}_{n=0}^{\infty}$, where each on-time t_0^{on} and off-time t_0^{off} denotes the instant when q reaches the corresponding on-angle q_0^{on} and off-angle q_0^{off}, respectively [29]. The switching laws can be defined for each muscle group, $\sigma_m : \mathrm{Q} \to \{0, 1\}$, and for the electric motor, $\sigma_e : \mathrm{Q} \to \{0, 1\}$ as

$$\sigma_m \triangleq \begin{cases} 1 & \text{if } q(t) \in Q_m \\ 0 & \text{if } q(t) \notin Q_m \end{cases}, \ \sigma_e \triangleq \begin{cases} 1 & \text{if } q(t) \in Q_e \\ 0 & \text{if } q(t) \notin Q_e. \end{cases} \tag{11}$$

The muscle stimulation input, u_m, and the motor input, u_e, are defined as [29], [30]

$$\begin{aligned} u_m(t) &= k_m \sigma_m(q(t)) u_{FES}(t), \\ u_e(t) &= k_e \sigma_e(q(t)) u_{motor}(t), \end{aligned} \tag{12}$$

where $k_m, \ k_e \in \mathbb{R}_{>0}$ are positive constant gains, and

$$u(t) \triangleq u_{FES} = u_{motor}. \tag{13}$$

By substituting (2)-(5), (7), and (13) into (1), the nonlinear dynamics of motorized cycle-rider system with electrical stimulation can be modeled as [30]

$$\begin{aligned} &M(q(t))\ddot{q}(t) + V(q(t), \dot{q}(t))\dot{q}(t) + b_{cycle}\dot{q}(t) + G(q(t)) \\ &+ P(q(t), \dot{q}(t)) + d_r(t) = B_\sigma(q(t), \dot{q}(t)) u(t), \end{aligned} \tag{14}$$

where

$$B_\sigma \triangleq B_{FES} + B_{motor} \triangleq \sum_{m \in \mathbb{M}} B_m k_m \sigma_m + B_e k_e \sigma_e, \tag{15}$$

and $B_\sigma \in \mathbb{R}_{>0}$ is the switched control effectiveness, which accounts for the combination of the control effectiveness of each muscle group and the electric motor.

Note that in (14), the cycle and rider inertia are combined, in that, $M \triangleq (M + J_{cycle}) : Q \to \mathbb{R}_{>0}$, and τ_r is neglected since in the framework of this study it is assumed that the rider does not pedal voluntarily and the pedaling occurs due to the electrical muscle stimulation or the electric motor. The switched system represented by (14), has the following properties,

Property 1. $\underline{M} \leqslant M \leqslant \bar{M}$, where $\underline{M}, \ \bar{M} \in \mathbb{R}_{>0}$ are known constants.

Property 2. $\dot{M} \leqslant c_M |\dot{q}|$, where $c_M \in \mathbb{R}_{>0}$ is a known constant.

Property 3. $|V(q(t), \dot{q}(t))| \leqslant c_V |\dot{q}(t)|$, where $c_V \in \mathbb{R}_{>0}$ is a known constant.

Property 4. $0 < b_{cycle} < \bar{b}$, where $\bar{b} \in \mathbb{R}_{>0}$ is a known constant.

Property 5. $|G(q(t))| \leqslant c_G$, where $c_G \in \mathbb{R}_{>0}$ is a known constant.

Property 6. $|P(q(t), \dot{q}(t))| \leqslant c_{P_1} + c_{P_2} |\dot{q}(t)|$, where $c_{P_1},\ c_{P_2} \in \mathbb{R}_{>0}$ are known constants [29].

Property 7. $0 < c_b \leqslant B_\sigma \leqslant c_B,\ \forall \sigma \in p$, where $c_b,\ c_B \in \mathbb{R}_{>0}$ are known constants.

Property 8. $\dot{M}(q(t), \dot{q}(t)) - 2V(q(t), \dot{q}(t)) = 0$ by skew symmetry.

C. Dissipative and Passive Systems

Definition 1. [33] A dynamical system with supply rate $\omega(u, y)$ is dissipative if there exists a positive definite storage function $V(x(t))$ such that for all $T > 0$,

$$\int_0^T \omega(u, y) dt \geqslant V(x(T)) - V(x(0)), \tag{16}$$

where $u \in U \subset \mathbb{R}^m,\ y \in Y \subset \mathbb{R}^p$, and $x \in X \subset \mathbb{R}^n$, denote the input and its corresponding output, respectively.

Definition 2. [23] A system is output strictly passive (OSP) if it is dissipative with respect to the supply rate $\omega(u, y) = u^T y - \rho y^T y,$ for $\rho > 0$.

Definition 3. [22] The $\mathcal{L}_2$ norm of a signal is defined as

$$\|x(t)\|_2 \triangleq \sqrt{\int_0^\infty x(t)^T x(t) dt}. \tag{17}$$

III. Main Results

In this section, the control input error $u_{ce}(t)$ is defined such that the learning control input $u_l(t)$ converges to the desired control input $u_d(t)$, i.e., $u_l \to u_d$. Two theorems are represented in this section, one theorem shows that the closed-loop switched system of the motorized FES cycle-rider is output strictly passive, while the other one, based on a control update law, shows that the cadence error trajectory tends to zero.

A. Control Development

The control objective is to track a desired crank trajectory. The tracking error signals $e_1,\ e_2 : \mathbb{R}_{\geqslant 0} \to \mathbb{R}$ are defined as

$$e_1(t) = q(t) - q_d(t), \tag{18}$$

$$e_2(t) = \dot{e}_1(t) + \alpha e_1(t), \tag{19}$$

where $q_d : \mathbb{R}_{\geqslant 0} \to \mathbb{R}$ is the desired crank position such that its derivative exists and $|\dot{q}_d| \leqslant c_{d_1}$, $|\ddot{q}_d| \leqslant c_{d_2}$, where $c_{d_1},\ c_{d_2} \in \mathbb{R}_{>0}$ are known constants, and $\alpha \in \mathbb{R}_{>0}$ is a positive constant. Since the objective is to follow the desired crank trajectory, $e_2(t)$ in (19) is considered as the output of the system.

We propose the control input error as

$$u_{ce}(t) = u_l(t) - u_d(t), \tag{20}$$

where $u_l : \mathbb{R}_{\geqslant 0} \to \mathbb{R}$ is the learning control input and will be designed later based on iterative learning control techniques, and $u_d : \mathbb{R}_{\geqslant 0} \to \mathbb{R}$ is the ideal input. Based on the open-loop dynamics in (14) and the subsequent stability analysis the controller is designed as

$$\begin{aligned} u(t) = &-k_1 e_2 - k_2 |e_2| - k_3 |e_1| |e_2| \\ &- k_4 \operatorname{sgn}(e_2) + u_l, \end{aligned} \tag{21}$$

where $k_1,\ k_2,\ k_3,\ k_4 \in \mathbb{R}_{>0}$ are positive constants and $\operatorname{sgn} : \mathbb{R} \to [-1, 1]$ is the signum function. Substituting (21) into (14) yields

$$\begin{aligned} &M(q(t))\ddot{q}(t) + V(q(t), \dot{q}(t))\dot{q}(t) + b_{cycle}\dot{q}(t) \\ &+ G(q(t)) + P(q(t), \dot{q}(t)) + d_r(t) \\ &= B_\sigma(q(t), \dot{q}(t))(-k_1 e_2 - k_2 |e_2| \\ &- k_3 |e_1| |e_2| - k_4 \operatorname{sgn}(e_2)) + B_\sigma(q(t), \dot{q}(t)) u_l(t). \end{aligned} \tag{22}$$

After some algebraic manipulation, the closed-loop dynamics can be expressed as

$$\begin{aligned} &M(q)\ddot{e}_1 + V(q, \dot{q})\dot{e}_1 + b_{cycle}\dot{e}_1 + \chi \\ &= B_\sigma(q, \dot{q})(-k_1 e_2 - k_2 |e_2| - k_3 |e_1| |e_2| \\ &- k_4 \operatorname{sgn}(e_2)) + B_{\sigma e}(q, \dot{q}) u_{ce}, \end{aligned} \tag{23}$$

where

$$\begin{aligned} \chi = &(M(q) - M(q_d))\ddot{q}_d + (V(q, \dot{q}) - V(q_d, \dot{q}_d))\dot{q}_d \\ &+ (G(q) - G(q_d)) + P(q, \dot{q}) + d_r(t), \end{aligned} \tag{24}$$

and

$$B_{\sigma e}(q(t), \dot{q}(t)) = \min\{B_\sigma(q(t), \dot{q}(t)),\ B_\sigma(q_d(t), \dot{q}_d(t))\}, \tag{25}$$

$0 < c_{b_e} \leqslant B_{\sigma e} \leqslant c_{B_e},\ \forall \sigma e \in p$, where $c_{b_e},\ c_{B_e} \in \mathbb{R}_{>0}$ are known constants.

B. Passive Motorized FES Cycle and Rider Dynamics

To facilitate the subsequent analysis, positive constants $c_1,\ c_2,\ c_3,\ c_4,\ c_5 \in \mathbb{R}$ are defined as

$$\begin{cases} c_1 = \alpha \bar{M} c_{d_2} + \alpha c_G + \alpha c_V c_{d_1}^2, \\ c_2 = 1 + \alpha \bar{b} + \bar{M} c_{d_2} + c_G + c_V c_{d_1}^2 + \alpha c_V c_{d_1} \\ \qquad + \dfrac{1}{2} \alpha c_M c_{d_1} + \alpha c_{P_2}, \\ c_3 = \alpha \bar{M} + \bar{b} + c_V c_{d_1} + c_{P_2}, \\ c_4 = \frac{1}{2} \alpha c_M, \\ c_5 = c_{d_r} + c_{P_1} + c_{P_2} c_{d_1}. \end{cases} \tag{26}$$

Theorem 1. Consider the closed-loop system in (23), if the positive gains $k_1,\ k_2,\ k_3,\ k_4$ and the constant α are selected such that,

$$\bar{M} < \frac{1}{\alpha^2}, \tag{27}$$

$$k_1 > 0, \tag{28}$$

$$c_b^{-1} c_3 < k_2 < \frac{c_1 c_3 - \frac{1}{4} c_2^2}{\alpha^2 c_b c_3 + c_1 c_b - \alpha c_b c_2}, \tag{29}$$

$$k_3 > {c_b}^{-1} c_4, \tag{30}$$

$$k_4 > {c_b}^{-1} c_5, \tag{31}$$

where $\bar{M}$ is defined in the Property 1 then, the closed loop system (23) from the input u_{ce} to the output e_2 is output strictly passive (OSP).

Proof. Consider a storage function $V_s(t)$ as

$$V_s = \frac{1}{2} M \dot{e}_1^2 + \frac{1}{2} e_1^2 + \alpha M e_1 \dot{e}_1, \tag{32}$$

and can be expressed as

$$V_s = \frac{1}{2} M e_2^2 + \frac{1}{2} e_1^2 - \frac{1}{2} \alpha^2 M e_1^2, \tag{33}$$

which is positive provided (27) is satisfied.

Let $z(t) \triangleq \left[\begin{array}{cc} e_1(t) & \dot{e}_1(t) \end{array} \right]^T$ and $z(t)$ be a Filippov solution to the differential inclusion $\dot{z}(t) \in K[h](z(t))$, where $K[.]$ is defined in [34] and h is defined by (19) and (23) as [35]

$$h \triangleq \left[\begin{array}{c} h_1 \\ h_2 \end{array} \right] = \left[\begin{array}{c} \ddot{e}_1 \\ \dot{e}_1 \end{array} \right]. \tag{34}$$

Since (23) contains the sigmum function and the discontinuous control effectiveness B_σ and $B_{\sigma e}$, the time derivative of the storage function exists almost everywhere (a.e.), i.e., for almost all t. According to Lemma 1 in [36], the time derivative of the storage function can be obtained such that $\dot{V}_s \overset{a.e.}{\in} \dot{\tilde{V}}$, where $\dot{\tilde{V}}$ is the generalized time derivative of (32) along the Filippov trajectories of $\dot{z} \in h(z)$ and is defined as

$$\dot{\tilde{V}} \triangleq \bigcap_{\xi \in \partial V} \xi^T K \left[\begin{array}{c} \ddot{e}_1 \\ \dot{e}_1 \\ 1 \end{array} \right]. \tag{35}$$

The storage function is continuously differentiable in z, $\partial V = \{\nabla V\}$, thus,

$$\begin{aligned} \dot{\tilde{V}} \overset{a.e.}{=} & \dot{e}_1 e_1 + \alpha M \dot{e}_1^2 + \frac{1}{2} \alpha \dot{M} \dot{e}_1 e_1 - b_{cycle} \dot{e}_1^2 - \alpha b_{cycle} \dot{e}_1 e_1 \\ & - \chi(\dot{e}_1 + \alpha e_1) + B_{\sigma e}(q, \dot{q}) u_{ce} e_2 - k_1 B_\sigma(q, \dot{q}) e_2^2 \\ & - k_2 B_\sigma(q, \dot{q}) \left| e_2 \right| e_2 - k_3 B_\sigma(q, \dot{q}) \left| e_1 \right| \left| e_2 \right| e_2 \\ & - k_4 B_\sigma(q, \dot{q}) \left| e_2 \right|. \end{aligned} \tag{36}$$

After using Properties 1-7, and algebraic manipulation, (36) can be upper bounded by using the Mean Value Theorem as

$$\begin{aligned} \dot{\tilde{V}} \leqslant & -(\frac{1}{c_{b_e}}) \Gamma_1 \left| e_2 \right| - (\frac{1}{c_{b_e}}) \left[\begin{array}{c} |e_1| \\ |\dot{e}_1| \end{array} \right]^T \Gamma_2 \left[\begin{array}{c} |e_1| \\ |\dot{e}_1| \end{array} \right] \\ & - (\frac{1}{c_{b_e}}) \left[\begin{array}{c} |e_1|^2 \\ |\dot{e}_1|^2 \end{array} \right]^T \Gamma_3 \left[\begin{array}{c} |\dot{e}_1| \\ |e_1| \end{array} \right] - (\frac{c_b}{c_{b_e}}) k_1 e_2^2 + e_2 u_{ce}, \end{aligned} \tag{37}$$

where

$$\Gamma_1 = k_4 c_b - c_5, \tag{38}$$

$$\Gamma_2 = \left[\begin{array}{cc} \alpha^2 c_b k_2 - c_1 & \frac{1}{2}(2\alpha c_b k_2 - c_2) \\ \frac{1}{2}(2\alpha c_b k_2 - c_2) & c_b k_2 - c_3 \end{array} \right], \tag{39}$$

$$\Gamma_3 = \left[\begin{array}{cc} 2\alpha c_b k_3 & \alpha^2 c_b k_3 \\ 0 & c_b k_3 - c_4 \end{array} \right]. \tag{40}$$

Note that Γ_1, Γ_2, and Γ_3 are positive definite matrices provided (29), (30), and (31) are satisfied.

Integrating both sides of (37) and rearranging the terms yields

$$\begin{aligned} \int_0^T e_2 u_{ce} dt \geqslant & \tilde{V}(T) - \tilde{V}(0) + \int_0^T \Gamma_1 \left| e_2 \right| dt \\ & + \int_0^T \left[\begin{array}{c} |e_1| \\ |\dot{e}_1| \end{array} \right]^T \Gamma_2 \left[\begin{array}{c} |e_1| \\ |\dot{e}_1| \end{array} \right] dt \\ & + \int_0^T \left[\begin{array}{c} |e_1|^2 \\ |\dot{e}_1|^2 \end{array} \right]^T \Gamma_3 \left[\begin{array}{c} |\dot{e}_1| \\ |e_1| \end{array} \right] dt \\ & + k_1 (\frac{c_b}{c_{b_e}}) \int_0^T e_2 e_2 dt. \end{aligned} \tag{41}$$

The inequality in (41) can be further lower bounded as

$$\int_0^T e_2 u_{ce} dt \geqslant -\tilde{V}(0) + k_1 (\frac{c_b}{c_{b_e}}) \int_0^T e_2 e_2 dt. \tag{42}$$

The passivity inequality is satisfied through (42) and according to the Definition 2, the closed-loop system from the input u_{ce} to the output e_2 is output strictly passive (OSP). Note that $\tilde{V}(0)$ only depends on the initial conditions of e_1, $\dot{e}_1$ and it is a positive constant. □

Remark 1. In Theorem 1, the OSP property of the feedback closed-loop system (23) during the time interval $[0, T]$ is proven. In the passivity inequality of (42), the supply rate function is $\omega(u, y) = e_2 u_{ce} - k_1 (\frac{c_b}{c_{be}}) e_2 e_2$, according to Definition 2.

C. Iterative Learning Control for Automatic Cycle and Rider Dynamics

Now that the OSP property of the closed-loop system (23) is established, an ILC update law, inspired from [26], is added to the control framework. Based on the ILC, a learning update law $u_l^{k+1} = F(u_l^k, e_2^k)$ is defined to prove that the output of the system (e_2) converges to zero, in the sense that $e_2 \to 0$ as $k \to \infty$.

Theorem 2. Consider the learning control update law as

$$u_l^{k+1} = u_l^k - k_l e_2^k \tag{43}$$

where k_l is a positive gain. If the conditions of Theorem 1 ((27)-(31)) are satisfied and

$$0 < k_l < 2 k_1 (\frac{c_b}{c_{b_e}}), \tag{44}$$

then convergence of e_2^k to zero is guaranteed in the $\mathcal{L}_2$ norm sense.

Proof. From (20),

$$u_{ce}^{k+1} = u_l^{k+1} - u_d^{k+1} \tag{45}$$

$$u_{ce}^{k} = u_l^{k} - u_d^{k}. \tag{46}$$

Subtracting (46) from (45) and using (43) yields

$$u_{ce}^{k+1} = u_{ce}^{k} - k_l e_2^k. \tag{47}$$

Squaring (47) and multiplying by k_l^{-1} yields

$$k_l^{-1}(u_{ce}^{k+1})^2 = k_l^{-1}(u_{ce}^{k})^2 - 2u_{ce}^k e_2^k + k_l (e_2^k)^2. \tag{48}$$

Integrating (48) from 0 to T results in

$$\begin{aligned} k_l^{-1}\int_0^T (u_{ce}^{k+1})^2 dt &= k_l^{-1}\int_0^T (u_{ce}^{k})^2 dt \\ - 2\int_0^T u_{ce}^k e_2^k dt &+ k_l \int_0^T (e_2^k)^2 dt, \end{aligned} \tag{49}$$

which can be simplified as

$$k_l^{-1}\left\|u_{ce}^{k+1}\right\|_2 = k_l^{-1}\left\|u_{ce}^{k}\right\|_2 + k_l \left\|e_2^k\right\|_2 - 2\int_0^T u_{ce}^k e_2^k dt. \tag{50}$$

After substituting the passivity inequality from (42), the following inequality can be developed

$$\begin{aligned} &k_l^{-1}(\left\|u_{ce}^{k+1}\right\|_2 - \left\|u_{ce}^{k}\right\|_2) \leqslant k_l \left\|e_2^k\right\|_2 - 2(\tilde{V}^k(T) - \tilde{V}^k(0) \\ &+ \int_0^T \Gamma_1 \left|\dot{e}_1^k + \alpha e_1^k\right| dt + \int_0^T \begin{bmatrix} \left|e_1^k\right| \\ \left|\dot{e}_1^k\right| \end{bmatrix}^T \Gamma_2 \begin{bmatrix} \left|e_1^k\right| \\ \left|\dot{e}_1^k\right| \end{bmatrix} dt \\ &+ \int_0^T \begin{bmatrix} \left|e_1^k\right|^2 \\ \left|\dot{e}_1^k\right|^2 \end{bmatrix}^T \Gamma_3 \begin{bmatrix} \left|\dot{e}_1^k\right| \\ \left|e_1^k\right| \end{bmatrix} dt + k_1(\frac{c_b}{c_{b_e}})\left\|e_2^k\right\|_2). \end{aligned} \tag{51}$$

Since $\tilde{V}^k(0) \to 0$ as $k \to \infty$

$$\begin{aligned} k_l^{-1}(\left\|u_{ce}^{k+1}\right\|_2 - \left\|u_{ce}^{k}\right\|_2) &\leqslant -2\tilde{V}^k(T) \\ &- 2\int_0^T \Gamma_1 \left|\dot{e}_1^k + \alpha e_1^k\right| dt \\ &- 2\int_0^T \begin{bmatrix} \left|e_1^k\right| \\ \left|\dot{e}_1^k\right| \end{bmatrix}^T \Gamma_2 \begin{bmatrix} \left|e_1^k\right| \\ \left|\dot{e}_1^k\right| \end{bmatrix} dt \\ &- 2\int_0^T \begin{bmatrix} \left|e_1^k\right|^2 \\ \left|\dot{e}_1^k\right|^2 \end{bmatrix}^T \Gamma_3 \begin{bmatrix} \left|\dot{e}_1^k\right| \\ \left|e_1^k\right| \end{bmatrix} dt \\ &- (2k_1(\frac{c_b}{c_{b_e}}) - k_l)\left\|e_2^k\right\|_2. \end{aligned} \tag{52}$$

It can be seen that $\left\|u_{ce}^k\right\|_2$ in (52) is monotonically decreasing if $0 < k_l < 2k_1(\frac{c_b}{c_{b_e}})$ and the term $(2k_1(\frac{c_b}{c_{b_e}}) - k_l)\left\|e_2^k\right\|_2$ goes to zero as $k \to \infty$ meaning that $e_2^k \to 0$ in the $\mathcal{L}_2$ norm sense. From (18)-(19), $e_1 \to 0$ when $e_2 \to 0$, which means that the crank trajectory will follow the desired path as $k \to \infty$. □

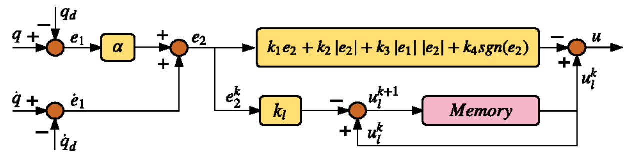

Figure 3. Block diagram of iterative learning controller.

Remark 2. Theorem 2 states that the output of the closed-loop system, (i.e.,(23)), which satisfies the OSP property based on Theorem 1, converges to zero in the $\mathcal{L}_2$ norm sense as $k \to \infty$. According to (19), the crank velocity $\dot{q}$ will follow the desired velocity $\dot{q}_d$ as $k \to \infty$. Moreover, $\left\|u_{ce}^k\right\|_2$ is a monotonically decreasing function and eventually goes to zero as $k \to \infty$. The block diagram of the iterative learning controller is shown in Fig. 3.

IV. Experiments

FES-cycling experiments were performed to demonstrate the tracking performance of the designed controller with iterative learning in (12), (13), (21), and (43). Seven able-bodied subjects (two female, five male) participated in the experiments. Each subject gave written informed consent approved by the University of Florida Institutional Review Board. All participants were asked to relax and make no volitional effort to assist the cycling and were not informed of the desired trajectory, and could not see the desired or actual trajectory. In these experiments only the quadriceps femoris muscle groups were stimulated. For safety considerations, the subject could stop the experiment at any time by using an emergency stop switch.

A. Instrumentation

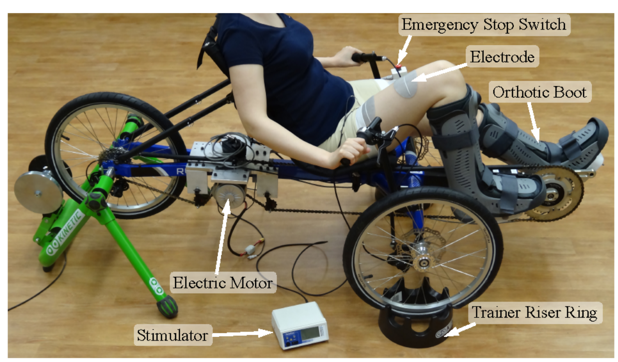

Figure 4. The motorized FES-cycling test bed used for the experiment.

The motorized FES-cycling test bed used in the study (Fig. 4), was instrumented like in [29], [30]. The stimulation pulse width for each muscle group was determined by u_m and commanded to the stimulator by the control software. Self-adhesive PALS electrodes (3 in. × 5 in.)

[1] were placed on each muscle group. The stimulation amplitudes and frequency for the quadriceps muscle groups were fixed at 90 mA and 60 Hz, respectively.

B. Experimental Setup

Electrodes were placed over the subjects' quadriceps femoris muscle groups according to Axelgaard's electrode placement manual.[2] Seat position adjustments were performed at the beginning of each experiment to ensure a proper interaction between the subject and tricycle. The distance from the cycle crank to the subject's right greater trochanter was measured according to [29], preventing full knee extension. The torque transfer ratio for subjects' muscle groups were calculated based on geometric measurements of each individual. Each experiment lasted between 180-300 seconds, depending on the subject. Two primary factors determined if an experiment was terminated before 300 seconds, mainly if the subject's sensitivity to stimulation produced an uncomfortable sensation or if the cadence tracking error grow above a range of (± 5 rpm) at steady state. The desired cadence was designed to smoothly reach 50 rpm, remaining at this value for the duration of the experiment. The desired crank velocity $\dot{q}_d$ was defined in radians per second [29] as

$$\dot{q}_d \triangleq \frac{5\pi}{3}(1 - e^{-\frac{2}{5}(t)}). \quad (53)$$

To avoid exerting large muscle forces at the beginning of the experiment, the motor was initially activated and the muscle stimulation intensities were progressively incorporated. The motor was active for the first 16 seconds of the experiment until the cadence reached 50 rpm, then the muscle intensities were gradually increased during a transition period of 10 seconds until the desired steady state stimulation pattern was achieved. The control gains and learning control gain in (12), (21), and (43) and the constant α in (19) were tuned prior to each experiment to achieve a proper tacking performance and they were selected as $\alpha = 2$, $k_m = 0.5$, $k_e = 1$, $k_l \in [1.5, 3.5]$, $k_{m_1} \in [65, 111]$, $k_{m_2} = 0.25$, $k_{m_3} \in [5, 7.5]$, $k_{m_4} \in [0.5, 1]$, $k_{e_1} = 13.5$, $k_{e_2} = 0.09$, $k_{e_3} = 4.5$, $k_{e_4} = 0.09$, where subscript m refers to the muscle controller gains and subscript e refers to the motor controller gains.

C. Results

Tracking performance for Subject 1, quantified by the cadence tracking error $\dot{e}_1$ and the root mean square (RMS), is depicted in Fig. 5. Fig. 6 shows the stimulation intensity input to each muscle group u_m and the electric motor current input u_e. In Fig. 7 the distribution of the control input between FES and the motor across one crank cycle for Subject 1 is represented. Fig. 8 illustrates a closer look at the stimulation intensity input to each muscle group u_m, the electric motor current input u_e, the learning control input u_l, and the cadence tracking error $\dot{e}_1$ over four revolutions for Subject 1. Table I summarizes the transitory and steady state of the RMS, the cadence tracking error $\dot{e}_1$, and the percentage of error for Subjects 1-7.

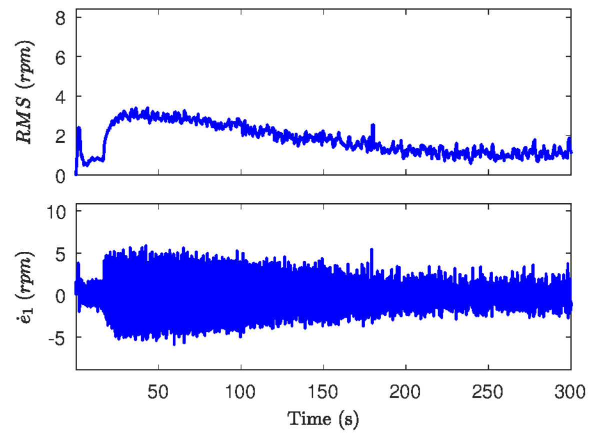

Figure 5. Tracking performance for Subject 1 characterized by the cadence tracking error $\dot{e}_1$ and its RMS over $t \in [0, 300]$.

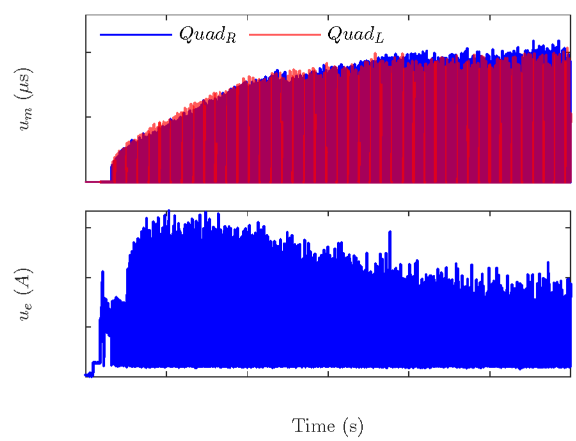

Figure 6. FES control input to quadriceps femoris muscle groups u_m and the electric motor current input u_e for Subject 1 over $t \in [0, 300]$.

D. Discussions

The control strategy was developed based on the passivity concept and the switched input was properly distributed between the FES control input u_m and the electric motor u_e. The results show that the learning controller was able to successfully regulate the cadence tracking error $\dot{e}_1$ close to zero. Note that, the exact zero convergence could not be achieved due to unknown disturbances such as electromechanical delay between muscle activation and force production [37] and muscle fatigue. The root mean

[1]Surface electrodes for the study were provided compliments of Axelgaard Manufacturing Co., Ltd.

[2]http://www.palsclinicalsupport.com/videoElements/videoPage.php

Table I
SUMMARY OF AUTOMATIC CYCLE-RIDER SYSTEM PERFORMANCE FOR SEVEN SUBJECTS. THE TRANSITORY AND STEADY STATE OF THE RMS, THE CADENCE TRACKING ERROR $\dot{e}_1$, AND THE PERCENTAGE OF ERROR FOR EACH SUBJECT ARE PROVIDED.

	RMS		$\dot{e}_1$		% Error	
	Transitory	Steady State	Transitory	Steady State	Transitory	Steady State
Subject 1	2.20±0.64	1.83±0.73	0.07±2.98	-0.01±1.92	0.42±4.79	-0.02±3.93
Subject 2	2.32±0.60	2.72±0.29	0.04±2.93	0.00±2.73	0.45±4.97	-0.00±5.48
Subject 3	1.58±0.46	1.86±0.53	0.04±2.55	-0.01±1.91	0.29±3.40	-0.03±3.88
Subject 4	2.11±0.65	1.97±0.40	-0.03±2.55	-0.00±1.99	0.16±4.59	-0.01±4.03
Subject 5	2.09±0.61	2.82±0.20	0.02±2.82	-0.01±2.83	0.26±4.61	-0.03±5.66
Subject 6	2.22±0.69	2.88±0.28	-0.10±2.98	-0.00±2.89	0.54±4.88	-0.02±5.79
Subject 7	2.49±0.61	3.00±0.17	0.02±2.96	-0.00±3.00	0.23±5.29	-0.00±6.01
Mean	2.15	2.44	0.01	-0.00	0.34	-0.01
STD	0.28	0.52	0.02	0.00	0.13	0.01

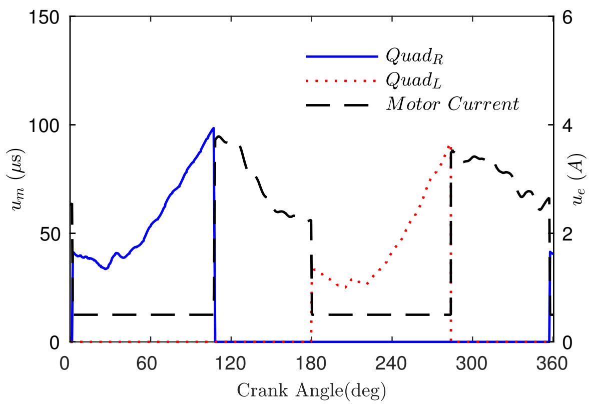

Figure 7. Switched control input among FES quadriceps femoris muscle groups and electric motor over a single crank cycle for Subject 1.

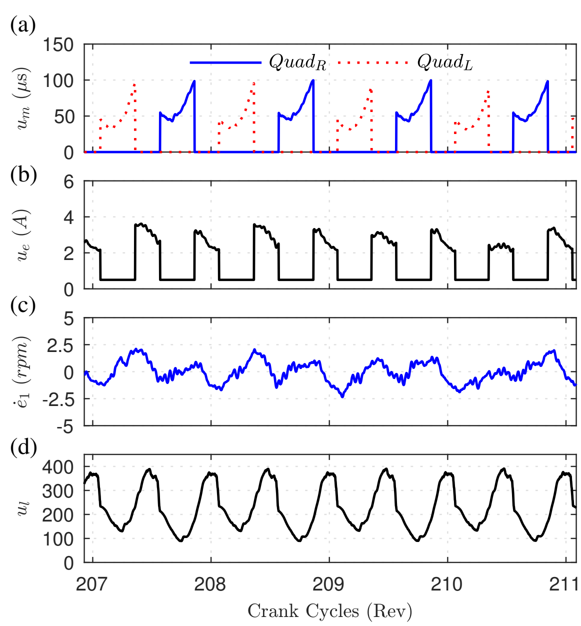

Figure 8. A zoom-in representation of the FES control input u_m, the electric motor control input u_e, the learning control input u_l, and the cadence tracking error $\dot{e}_1$ over four revolutions for Subject 1.

square (RMS) of the cadence tracking error in Fig. 5 clearly illustrates the transitory and steady state behavior of cadence tracking error $\dot{e}_1$, showing the convergence of the cadence tracking error for Subject 1. As evident from Fig. 8c, $\dot{e}_1$ has a steady state error of ±2.47 rpms. In addition, Fig. 8d shows the contribution of the learning controller u_l over the same revolutions.

V. CONCLUSION

The passivity property of an automatic stationary cycle where cycling is either produced by motorized assistance or induced through muscle stimulation was studied. Due to the uncertain nonlinear dynamics of the switched closed-loop system, and the repetitive nature of the cycling task, ILC was used to achieve the desired output trajectory. The developed method ensured the $\mathcal{L}_2$ norm of the output error trajectory converges to zero. The OSP property of the system was proven and the ILC scheme based on the Arimoto learning control update law was developed. Results obtained from experiments on a recumbent stationary bicycle for seven able-bodied participants, where the average cadence tracking error was 0.00±2.47 rpm (0.01±4.97% error) for 50 rpm at steady state. Future studies could apply the same approach to the control of upper body limbs such as hand-cycling. Moreover, for more general cases, the learning control update law could be developed based on the output dissipativity property of the closed-loop system in which, QSR-dissipativity, a generalization of passivity, will be studied.

REFERENCES

[1] A. R. Kralj and T. Bajd, *Functional electrical stimulation: standing and walking after spinal cord injury*. CRC press, 1989.
[2] C. A. Phillips, J. S. Petrofsky, D. M. Hendershot, and D. Stafford, "Functional electrical exercise: a comprehensive approach for physical conditioning of the spinal cord injured patient," *Orthopedics*, vol. 7, no. 7, pp. 1112–1123, 1984.
[3] G. M. Davis, N. A. Hamzaid, and C. Fornusek, "Cardiorespiratory, metabolic, and biomechanical responses during functional electrical stimulation leg exercise: health and fitness benefits," *Artificial organs*, vol. 32, no. 8, pp. 625–629, 2008.

[4] R. Jaeger, “Design and simulation of closed-loop electrical stimulation orthoses for restoration of quiet standing in paraplegia,” *Journal of biomechanics*, vol. 19, no. 10, pp. 825–835, 1986.

[5] A. H. Vette, K. Masani, and M. R. Popovic, “Implementation of a physiologically identified pd feedback controller for regulating the active ankle torque during quiet stance,” *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, vol. 15, no. 2, pp. 235–243, 2007.

[6] K. Masani, A. H. Vette, and M. R. Popovic, “Controlling balance during quiet standing: proportional and derivative controller generates preceding motor command to body sway position observed in experiments,” *Gait & posture*, vol. 23, no. 2, pp. 164–172, 2006.

[7] M. Ferrarin, E. D’acquisto, A. Mingrino, and A. Pedotti, “An experimental pid controller for knee movement restoration with closed loop fes system,” in *Engineering in Medicine and Biology Society, 1996. Bridging Disciplines for Biomedicine. Proceedings of the 18th Annual International Conference of the IEEE*, vol. 1. IEEE, 1996, pp. 453–454.

[8] A. H. Vette, K. Masani, and M. R. Popovic, “Neural-mechanical feedback control scheme can generate physiological ankle torque fluctuation during quiet standing: A comparative analysis of contributing torque components,” in *Control Applications, 2008. CCA 2008. IEEE International Conference on*. IEEE, 2008, pp. 660–665.

[9] J. Chen, N.-Y. Yu, D.-G. Huang, B.-T. Ann, and G.-C. Chang, “Applying fuzzy logic to control cycling movement induced by functional electrical stimulation,” *IEEE transactions on rehabilitation engineering*, vol. 5, no. 2, pp. 158–169, 1997.

[10] A. Farhoud and A. Erfanian, “Fully automatic control of paraplegic fes pedaling using higher-order sliding mode and fuzzy logic control,” *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, vol. 22, no. 3, pp. 533–542, 2014.

[11] E. Idsø, T. Johansen, and K. Hunt, “Finding the metabolically optimal stimulation pattern for fes-cycling,” *Power [W]*, vol. 10, no. 12, p. 14, 2004.

[12] N. A. Hakansson and M. L. Hull, “Can the efficacy of electrically stimulated pedaling using a commercially available ergometer be improved by minimizing the muscle stress–time integral?” *Muscle & nerve*, vol. 45, no. 3, pp. 393–402, 2012.

[13] Y. Ogawa, T. Inoue, T. Inada, Y. Tagawa, K. Yoshimitsu, and N. Shiba, “Locomotion assistance for the person with mobility impairment: Fuzzy control of cycling movement by means of surface electrical-stimulation,” in *Engineering in Medicine and Biology Society, 2007. EMBS 2007. 29th Annual International Conference of the IEEE*. IEEE, 2007, pp. 2420–2423.

[14] N. Kirsch, N. Alibeji, B. E. Dicianno, and N. Sharma, “Switching control of functional electrical stimulation and motor assist for muscle fatigue compensation,” in *American Control Conference (ACC), 2016*. IEEE, 2016, pp. 4865–4870.

[15] K. J. Hunt, B. Stone, N.-O. Negard, T. Schauer, M. H. Fraser, A. J. Cathcart, C. Ferrario, S. A. Ward, and S. Grant, “Control strategies for integration of electric motor assist and functional electrical stimulation in paraplegic cycling: utility for exercise testing and mobile cycling,” *IEEE Transactions on Neural systems and rehabilitation engineering*, vol. 12, no. 1, pp. 89–101, 2004.

[16] S. Arimoto, S. Kawamura, and F. Miyazaki, “Bettering operation of dynamic systems by learning: A new control theory for servomechanism or mechatronics systems,” in *Decision and Control, 1984. The 23rd IEEE Conference on*, vol. 23. IEEE, 1984, pp. 1064–1069.

[17] H. Dou, K. K. Tan, T. H. Lee, and Z. Zhou, “Iterative learning feedback control of human limbs via functional electrical stimulation,” *Control Engineering Practice*, vol. 7, no. 3, pp. 315–325, 1999.

[18] K. Wang, T. Schauer, H. Nahrstaedt, and J. Raisch, “Iterative learning control of cadence for functional electrical stimulation induced cycling in paraplegia,” in *14th Annual Conference of the International FES Society*, 2009, pp. 71–73.

[19] V. H. Duenas, C. A. Cousin, A. Parikh, and W. E. Dixon, “Functional electrical stimulation induced cycling using repetitive learning control,” in *Decision and Control (CDC), 2016 IEEE 55th Conference on*. IEEE, 2016, pp. 2190–2195.

[20] C. Freeman, D. Tong, K. Meadmore, A.-M. Hughes, E. Rogers, and J. Burridge, “Fes based rehabilitation of the upper limb using input/output linearization and ilc,” in *American Control Conference (ACC), 2012*. IEEE, 2012, pp. 4825–4830.

[21] P. Sampson, C. Freeman, S. Coote, S. Demain, P. Feys, K. Meadmore, and A.-M. Hughes, “Using functional electrical stimulation mediated by iterative learning control and robotics to improve arm movement for people with multiple sclerosis,” *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, vol. 24, no. 2, pp. 235–248, 2016.

[22] H. K. Khalil and J. Grizzle, *Nonlinear systems*. Prentice hall New Jersey, 1996, vol. 3.

[23] J. Bao and P. Lee, *Process control : the passive systems approach*. London: Springer-Verlag, 2007.

[24] A. Van der Schaft, *L2-gain and passivity techniques in nonlinear control*. Springer Science & Business Media, 2012.

[25] S. Arimoto and T. Naniwa, “Equivalence relations between learnability, output-dissipativity and strict positive realness,” *International Journal of Control*, vol. 73, no. 10, pp. 824–831, 2000.

[26] ——, “Learnability and adaptability from the viewpoint of passivity analysis,” *Intelligent Automation & Soft Computing*, vol. 8, no. 2, pp. 71–94, 2002.

[27] H. Kawai, T. Murao, R. Sato, and M. Fujita, “Passivity-based control for 2dof robot manipulators with antagonistic bi-articular muscles,” in *Control Applications (CCA), 2011 IEEE International Conference on*. IEEE, 2011, pp. 1451–1456.

[28] Y. Ichijo, T. Murao, H. Kawai, and M. Fujita, “Passivity-based iterative learning control for 2dof robot manipulators with antagonistic bi-articular muscles,” in *Control Applications (CCA), 2014 IEEE Conference on*. IEEE, 2014, pp. 234–239.

[29] M. J. Bellman, R. J. Downey, A. Parikh, and W. E. Dixon, “Automatic control of cycling induced by functional electrical stimulation with electric motor assistance,” *IEEE Transactions on Automation Science and Engineering*, vol. 14, no. 2, pp. 1225–1234, April 2017.

[30] M. J. Bellman, T.-H. Cheng, R. J. Downey, C. J. Hass, and W. E. Dixon, “Switched control of cadence during stationary cycling induced by functional electrical stimulation,” *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, vol. 24, no. 12, pp. 1373–1383, 2016.

[31] E. S. Idsø, “Development of a mathematical model of a rider-tricycle system,” *Dept. of Engineering Cybernetics, NTNU, Tech. Rep*, 2002.

[32] D. Liberzon, *Switching in systems and control*. Springer Science & Business Media, 2012.

[33] J. C. Willems, “Dissipative dynamical systems part i: General theory,” *Archive for Rational Mechanics and Analysis*, vol. 45, no. 5, pp. 321–351, 1972.

[34] A. F. Filippov, *Differential equations with discontinuous righthand sides: control systems*. Springer Science & Business Media, 2013, vol. 18.

[35] B. Paden and S. Sastry, “A calculus for computing filippov’s differential inclusion with application to the variable structure control of robot manipulators,” *IEEE transactions on circuits and systems*, vol. 34, no. 1, pp. 73–82, 1987.

[36] N. Fischer, R. Kamalapurkar, and W. E. Dixon, “Lasalle-yoshizawa corollaries for nonsmooth systems,” *IEEE Transactions on Automatic Control*, vol. 9, no. 58, pp. 2333–2338, 2013.

[37] N. Sharma, C. M. Gregory, and W. E. Dixon, “Predictor-based compensation for electromechanical delay during neuromuscular electrical stimulation,” *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, vol. 19, no. 6, pp. 601–611, 2011.