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Cadence Tracking for Switched FES Cycling Combined with Voluntary Pedaling and Motor Resistance

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***Abstract*—A wide variation in muscle strength and range of motion exists in the movement disorder rehabilitation community. Functional Electrical Stimulation (FES) can be used to induce muscle contractions to assist a person who can contribute volitional coordinated torques. A motor can be used to both assist and resist a person's volitional and/or FES-induced pedaling. In this paper, a multi-level switched system is applied to a two-sided control objective to maintain a desired range of cadence using FES, motor assistance, motor resistance, and volitional pedaling. A system with assistive, passive, and resistive modes are developed based on cadence, each with a different combination of actuators. Lyapunov-based methods for switched systems are used to prove global exponential tracking to the desired cadence range for the combined FES-motor control system. Preliminary experiments show the feasibility and stability of the multi-level switched control system.**

I. Introduction

Functional Electrical Stimulation (FES) induced cycling is a common rehabilitation exercise for people with lower limb movement disorders [1]–[9], with FES imparting various notable health benefits. For instance, FES in general is known to improve muscle strength [10] and range of motion [11] and FES-cycling in particular was shown to improve bone mineral density [12], physiological motor control [13], and cardiovascular parameters [14]. Moreover, in the Spinal Cord Injured (SCI) population, [15] showed that FES increased muscle mass and decreased blood glucose and insulin levels, potentially lowering the risk for Type II diabetes. Compared to purely volitional pedaling for people with cerebral palsy, [16] concluded that also inducing muscle contractions through FES resulted in increased cadence, power output, and heart rate, and decreased variability in cycling performance. Therefore, clear motivation exists to engage in rehabilitative cycling with closed-loop FES methods; however, there are unanswered questions related to allowing a person's volitional contributions to interact with the closed-loop controller. For example, if a person can pedal faster than a cadence goal set by a physical therapist, should the closed-loop system fight against that input just to enforce the cadence goal? Motivated by such questions, we develop a new cycling strategy where closed-loop assistive measures are provided when the user is below a desired cadence goal but allows the user to pedal independently if they can exceed the cadence goal, up to an upper bound cadence where a closed-loop motor controller engages to limit the cadence for participant safety and provide resistance.

Among different populations that participate in physical therapy, there is a large variation of strength and abilities, giving motivation for an exercise protocol that accommodates each user by assisting when they do not meet minimum performance metrics and resisting when the person exceeds a desired target range. Motivated by these different cases, a novel control objective and closed-loop state dependent switched system strategy is developed in this paper with three modes: assistive, passive, and resistive. Switching between the three modes is based on feedback of the cadence state and the desired cadence range bounds, defined by a lower and upper threshold. When the individual is pedaling at speeds below the target cadence, FES is applied to assist the person. However, since FES (and even volitional pedaling) has inefficient kinematic regions (i.e., kinematic configurations that require large forces to produce comparatively low torque applied about the crank [8], [17]–[19]), switching also occurs within the assistive region between FES and an electric motor input that provides assistive torque contributions. Specifically, switching occurs between different combinations of muscle groups to coordinate the limb trajectories through FES and the motor during the inefficient kinematic regions. In the assistive mode, switching between the motor and FES control involves switching between controlled (stable) subsystems. If the person is able to voluntarily pedal above the cadence target then the system switches to an uncontrolled (but bounded) mode where the motor and FES inputs are turned off. This subsystem is denoted as a passive or uncontrolled subsystem in the sense that the controllers do not provide any assistance or resistance. Motivated by various safety reasons, an upper limit cadence is also defined. If the volitional efforts by the person attempt to exceed this upper bound, then switching will transition from the uncontrolled subsystem to a resistive subsystem where the motor will engage to provide resistive torques. A diagram of the combined switched system is depicted in Figure 1. A switched system Lyapunov-based analysis involving a common Lyapunov function candidate with a set valued

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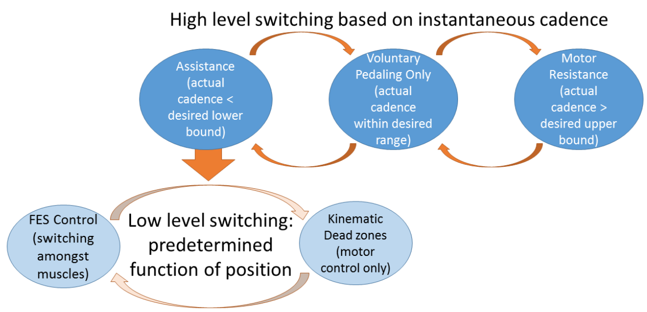

Figure 1. Diagram illustrating the combined two-level switched system.

generalized derivative is used to examine the stability of the family of sliding mode controllers, despite unknown bounded disturbances, provided sufficient gain conditions are satisfied. Specifically, global exponential stability of the controllers operating in the assistive and resistive modes, and the trajectories in the passive mode are bounded above and below by the controlled subsystems. Stability results are confirmed with experimental results that illustrate all three cycling modes.

II. Model

The combined cycle-rider dynamics are considered as[1]

$$\tau_e(t) = \tau_c(\dot{q}, \ddot{q}, t) + \tau_r(q, \dot{q}, \ddot{q}, t), \tag{1}$$

where $q : \mathbb{R}_{\geq 0} \to \mathcal{Q}$ denotes the measurable crank angle, $\mathcal{Q} \subseteq \mathbb{R}$ the set of all possible crank angles, and $\tau_c : \mathbb{R} \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ and $\tau_r : \mathcal{Q} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ the torques applied about the crank axis by the cycle. The torque applied about the crank axis by the electric motor, $\tau_e : \mathbb{R}_{\geq 0} \to \mathbb{R}$, can be written as

$$\tau_e(t) = B_e u_e(t), \tag{2}$$

where the motor control constant, $B_e \in \mathbb{R}_{>0}$, relates the motor's input current to output torque, and $u_e : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is the subsequently designed motor control current input. The cycle and rider torques, τ_c and τ_r, are defined as

$$\tau_c(\dot{q}, \ddot{q}, t) = J_c \ddot{q} + b_c \dot{q} + d_c, \tag{3}$$

$$\tau_r(q, \dot{q}, \ddot{q}, t) = \tau_p(q, \dot{q}, \ddot{q}) - \tau_M(q, \dot{q}, t) + d_r(t),$$

respectively, where $J_c \in \mathbb{R}_{>0}$, $b_c \in \mathbb{R}_{>0}$, and $d_c : \mathbb{R}_{\geq 0} \to \mathbb{R}$, denote inertial effects, viscous damping effects, and disturbances applied by the cycle, respectively. The torque applied about the crank by the rider can be separated into passive torques, $\tau_p : \mathcal{Q} \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, the volitional or FES induced muscle contribution, $\tau_M : \mathcal{Q} \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$, and the disturbances (e.g., spasticity or changes in load), $d_r : \mathbb{R}_{\geq 0} \to \mathbb{R}$. The passive torques applied by the rider are

$$\tau_p(q, \dot{q}, \ddot{q}) = M_p(q)\ddot{q} + V(q, \dot{q})\dot{q} + G(q) + P(q, \dot{q}), \tag{4}$$

where $M_p : \mathcal{Q} \to \mathbb{R}_{>0}$, $V : \mathcal{Q} \times \mathbb{R} \to \mathbb{R}$, $G : \mathcal{Q} \to \mathbb{R}$, and $P : \mathcal{Q} \times \mathbb{R} \to \mathbb{R}$, denote the inertial, centripetal-Coriolis, gravitational, and passive viscoelastic tissue forces, respectively. The torques applied by the muscles can be separated into volitional contributions and the sum of each muscle's individual contribution by FES as

$$\tau_M(q, \dot{q}, t) = \sum_{m \in \mathcal{M}} B_m(q, \dot{q}) u_m(t) + \tau_{vol}, \tag{5}$$

$\forall m \in \mathcal{M}$, where $u_m : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is the subsequently designed muscle control current input, and the subscript $m \in \mathcal{M} = \{RQ, RG, RH, LQ, LG, LH\}$ indicates the right (R) and left (L) quadriceps femoris (Q), gluteal (G), and hamstring (H) muscle groups, respectively. The rider's voluntary torque is denoted by $\tau_{vol} \in \mathbb{R}_{\geq 0}$. The uncertain muscle control effectiveness is denoted by $B_m : \mathcal{Q} \times \mathbb{R} \to \mathbb{R}_{>0}$, $\forall m \in \mathcal{M}$, and can be expanded as[2]

$$B_m = \lambda_m(q)\psi_m(q, \dot{q})\cos(\beta_m(q))T_m(q), \tag{6}$$

$\forall m \in \mathcal{M}$, where $\lambda_m : \mathcal{Q} \to \mathbb{R}_{>0}$ denotes the uncertain moment arm of each muscle group's force about its respective joint, $\psi_m : \mathcal{Q} \times \mathbb{R} \to \mathbb{R}_{>0}$ denotes the uncertain nonlinear function relating stimulation intensity to the force output by the muscle, and $\beta_m : \mathcal{Q} \to \mathbb{R}$ denotes the uncertain muscle fiber pennation angle. The function $T_m : \mathcal{Q} \to \mathbb{R}$ denotes the torque transfer ratio between each muscle group and the crank [8], [20]. Definitions for the subsequent stimulation regions and switching laws during the assistive mode are based on [8], where the portion of the crank cycle in which a particular muscle group is stimulated is denoted by $\mathcal{Q}_m \subset \mathcal{Q}$. In this manner, $\mathcal{Q}_m$ is defined for each muscle group as

$$\mathcal{Q}_m \triangleq \{q \in \mathcal{Q} \,|\, T_m(q) > \varepsilon_m\} \; \forall m \in \mathcal{M}, \tag{7}$$

where $\varepsilon_m \in \left(0, \max\left(T_m\right)\right]$, $\forall m \in \mathcal{M}$ is the lower threshold for each torque transfer ratio, which limits the stimulation regions for each muscle so that stimulation is only applied when the particular muscle group can contribute positive crank motion. Based on the defined stimulation regions defined by (7), let $\sigma_m(q) \in \{0, 1\}$ be a piecewise left-continuous switching signal for each muscle group such that $\sigma_m(q) = 1$ when $q \in \mathcal{Q}_m$ and $\sigma_m(q) = 0$ when $q(t) \notin \mathcal{Q}_m$, $\forall m \in \mathcal{M}$. The region of the crank cycle where FES produces efficient torques, $\mathcal{Q}_{FES}$, is defined as $\mathcal{Q}_{FES} \triangleq \bigcup_{m \in \mathcal{M}} \{\mathcal{Q}_m\}, \forall m \in \mathcal{M}$.

[1]For notational brevity, all explicit dependence on time, t, within the terms $q(t)$, $\dot{q}(t)$, $\ddot{q}(t)$ is suppressed.

[2]For notational brevity, all functional dependencies are hereafter suppressed unless required for clarity of exposition.

Within the assistive mode, position-based switching is used to switch between subsets of muscle groups and the motor. When switching between assistive, passive, and resistive modes, the switching velocity values $\left\{\dot{q}_{\underline{d}},\, \dot{q}_{\overline{d}}\right\}$ are known but the position values are not, where $\dot{q}_{\underline{d}} : \mathbb{R}_{>0} \to \mathbb{R}$ and $\dot{q}_{\overline{d}} : \mathbb{R}_{>0} \to \mathbb{R}$ are the minimum and maximum desired cadence values. To facilitate the analysis of a combination of position-based and velocity-based switching, switching times are denoted by $\left\{t_n^i\right\},\, i \in \{s,\, e,\, p\},\, n \in \{0, 1, 2, ...\}$, representing the times when the system switches to use stimulation, the electric motor (either assistive or resistive), or neither (i.e., passive mode). Also in (5), $u_m : \mathbb{R}_{>0} \to \mathbb{R}$ denotes the control input and the electrical stimulation intensity applied to each muscle, defined as

$$u_m \triangleq \sigma_m k_m u_s(t),\ \forall m \in \mathcal{M}, \tag{8}$$

where the subsequently designed FES control input is denoted by $u_s(t)$ and $k_m \in \mathbb{R}_{>0}$ is a control gain. Substituting (2)-(5) and (8) into (1) yields

$$\begin{aligned} B_M u_s + B_e u_e + \tau_{vol} &= M\ddot{q} + b_c\dot{q} + d_c \\ &\quad + V\dot{q} + G + P + d_r, \end{aligned} \tag{9}$$

where $B_M : \mathcal{Q} \times \mathbb{R} \to \mathbb{R}$ is the combined switched FES control effectiveness, defined as

$$B_M(q,\, \dot{q}) = \sum_{m \in \mathcal{M}} B_m \sigma_m k_m. \tag{10}$$

Note that $M : \mathcal{Q} \to \mathbb{R}$ is defined as the summation $M \triangleq J_c + M_p$. At times when the subject is cycling below the desired minimum cadence, $\dot{q}_{\underline{d}}$, FES is used to assist the subject similar to the protocol throughout [18]. In this paper, low-level switching occurs among the different muscle groups and motor assistance, while a high-level logical switching law selects between assistive, passive, and resistive modes, depending on the actual cycling cadence in relation to a desired range (although voluntary contribution is encouraged throughout). Both levels of switching are autonomous and state-dependent.

The switched system in (9) has the following properties and assumptions:

Property: 1 $c_m \le M \le c_M$, where $c_m,\, c_M \in \mathbb{R}_{>0}$ are known constants. **Property: 2** $|V| \le c_V|\dot{q}|$, where $c_V \in \mathbb{R}_{>0}$ is a known constant. **Property: 3** $|G| \le c_G$, where $c_G \in \mathbb{R}_{>0}$ is a known constant. **Property: 4** $|P| \le c_{P1} + c_{P2}|\dot{q}|$, where $c_{P1},\, c_{P2} \in \mathbb{R}_{>0}$ are known constants. **Property: 5** $|b_c| \le c_b|\dot{q}|$, where $c_b \in \mathbb{R}_{>0}$ is a known constant. **Property: 6** $|d_r + d_c| \le c_d$, where $c_d \in \mathbb{R}_{>0}$ is a known constant [17]. **Property: 7** The time derivative of the inertia matrix and the centripetal-Coriolis matrix are skew symmetric, $\frac{1}{2}\dot{M} = V$. **Property: 8** The unknown moment arm of each muscle group about their respective joint is non-zero, (i.e., $\lambda \ne 0$) [21]. **Property: 9** The auxiliary term ψ in (6) depends on the force-length and force-velocity relationships of the muscle being stimulated and is upper and lower bounded by known positive constants, $c_\psi,\, c_\Psi \in \mathbb{R}_{>0}$, respectively, provided the muscle is not fully extended [22] or contracting concentrically at its maximum shortening velocity [17]. **Property: 10** The function relating the unknown muscle fiber pennation angle to output torque is never zero, (i.e., $\cos(\beta_m) \ne 0$) [23]. **Property: 11** By properties 8-10, B_m is lower bounded $\forall m$, and thus, when $\sum_{m \in \mathcal{M}} \sigma_m > 0$, $c_{b_M} \le B_M$, where $c_{b_M} \in \mathbb{R}_{>0}$. **Property: 12** $c_{b_e} \le B_e \le c_{B_e}$, where $c_{b_e},\, c_{B_e} \in \mathbb{R}_{>0}$. **Assumption: 1** The volitional torque produced by the subject is bounded, due to human physical limitations, as $|\tau_{vol}| \le c_{vol}$, where $c_{vol} \in \mathbb{R}_{>0}$.

III. Control Development

The cadence tracking objective is quantified by the velocity error $e_1 : \mathbb{R}_{\ge 0} \to \mathbb{R}$ and auxiliary error $e_2 : \mathbb{R}_{\ge 0} \to \mathbb{R}$, defined as

$$e_1(t) \triangleq \dot{q}_{\underline{d}}(t) - \dot{q}(t), \tag{11}$$

$$e_2(t) \triangleq e_1(t) + (1 - \sigma_a(t))\,\Delta_d, \tag{12}$$

where $\dot{q}_{\underline{d}}$ was defined previously, along with $\dot{q}_{\overline{d}}$, which is now defined as $\dot{q}_{\overline{d}} \triangleq \dot{q}_{\underline{d}} + \Delta_d$, where $\Delta_d \in \mathbb{R}_{>0}$ is the range of desired cadence values. The switching signal designating the assistive mode $\sigma_a : \mathbb{R}_{\ge 0} \to \{0,\, 1\}$ is designed as

$$\sigma_a = \begin{cases} 1 & \text{if } \dot{q} < \dot{q}_{\underline{d}} \\ 0 & \text{if } \dot{q} \ge \dot{q}_{\underline{d}} \end{cases}. \tag{13}$$

Note that $e_1 = e_2$ when $\sigma_a = 1$. Taking the time derivative of (11), multiplying by M, and using (9) and (11) yields

$$M\dot{e}_1 = -B_e u_e - B_M u_s - \tau_{vol} - V e_1 + \chi, \tag{14}$$

where the auxiliary term $\chi : Q \times \mathbb{R} \times \mathbb{R}_{\ge 0} \to \mathbb{R}$ is defined as

$$\chi = b_c\dot{q} + d_c + G + P + d_r + V\dot{q}_{\underline{d}} + M\ddot{q}_{\underline{d}}.$$

From Properties 1-6, χ can be bounded as

$$\chi \le c_1 + c_2|e_1|, \tag{15}$$

where $c_1,\, c_2 \in \mathbb{R}_{>0}$ are known constants and $|\cdot|$ denotes absolute value. Based on (14), (15), and the subsequent stability analysis, the FES control input to the muscle is designed as

$$u_s = \sigma_a\left(k_{1s} + k_{2s}e_1\right), \tag{16}$$

where $k_{1s},\, k_{2s} \in \mathbb{R}_{>0}$ are constant control gains and σ_a is defined in (13). The switched control input to the motor is designed as

$$u_e = \sigma_e\left(k_{1e}\,\mathrm{sgn}(e_1) + k_{2e}e_2\right), \tag{17}$$

where $k_{1e}, k_{2e} \in \mathbb{R}_{>0}$ are constant control gains and $\sigma_e : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is the motor's switching signal, designed as

$$\sigma_e = \begin{cases} k_a & \text{if } \dot{q} < \dot{q}_{\underline{d}},\ q \notin Q_{FES} \\ 0 & \text{if } \dot{q} < \dot{q}_{\underline{d}},\ q \in Q_{FES} \\ 0 & \text{if } \dot{q}_{\underline{d}} \leq \dot{q} \leq \dot{q}_{\overline{d}} \\ k_r & \text{if } \dot{q} > \dot{q}_{\overline{d}} \end{cases}, \tag{18}$$

where $k_a, k_r \in \mathbb{R}_{>0}$ are constant control gains. Substituting (16) and (17) into (14) yields

$$\begin{aligned} M\dot{e}_1 &= -B_e \sigma_e \left(k_{1e} \text{sgn}\left(e_1\right) + k_{2e} e_2\right) \\ &\quad -B_M \sigma_a \left(k_{1s} + k_{2s} e_1\right) - \tau_{vol} - V e_1 + \chi. \end{aligned} \tag{19}$$

IV. Stability Analysis

Let $V_L : \mathbb{R} \to \mathbb{R}$ be a continuously differentiable, positive definite, common Lyapunov function candidate defined as

$$V_L = \frac{1}{2} M e_1^2, \tag{20}$$

which satisfies the following inequalities:

$$\frac{c_m}{2} e_1^2 \leq V_L \leq \frac{c_M}{2} e_1^2, \tag{21}$$

where c_m and c_M are introduced in Property 1.

Theorem 1. *When $\dot{q} < \dot{q}_{\underline{d}}$ and $q \in Q_{FES}$, the closed-loop error system in (19) is exponentially stable.*

Proof: When $\dot{q} < \dot{q}_{\underline{d}}$ and $q \in Q_{FES}$, $e_1 > 0$, $\sigma_a = 1$, and $\sigma_e = 0$ (i.e., the cycle-rider system is controlled by FES in the assistive mode). It can be demonstrated that, due to B_M discontinuously varying over time, the time derivative of (20) exists almost everywhere (a.e.), i.e., for almost all $t \in \left(t_n^s, t_{n+1}^i\right)$, $\forall i \in \{e, p\}$, and after substituting (19), the derivative of (20) can be upper bounded using Properties 7 and 11, Assumption II, and (15) as

$$\dot{V}_L \overset{\text{a.e.}}{\leq} -\left(c_{b_M} k_{1s} - c_{vol} - c_1\right) e_1 - \left(c_{b_M} k_{2s} - c_2\right) e_1^2, \tag{22}$$

which is negative definite since $e_1 > 0$, provided some gain conditions are satisfied. Furthermore, (21) can be used to upper bound (22) as

$$\dot{V}_L \leq -\lambda_s V_L, \tag{23}$$

where λ_s denotes a known positive bounding constant. The inequality in (23) can be solved to yield

$$V_L(t) \leq V_L\left(t_n^s\right) \exp\left[-\lambda_s \left(t - t_n^s\right)\right], \tag{24}$$

for all $t \in \left(t_n^s, t_{n+1}^i\right)$, $\forall i \in \{e, p\}$, $\forall n$. Rewriting (24) using (21) and performing some algebraic manipulation yields exponential convergence of $|e_1(t)|$ to zero.. ■

Theorem 2. *When $\dot{q} < \dot{q}_{\underline{d}}$ and $q \notin Q_{FES}$, the closed-loop error system in (19) results in exponential decay of the cadence errror*

Proof: When $\dot{q} < \dot{q}_{\underline{d}}$ and $q \notin Q_{FES}$, $e_1 > 0$, $\sigma_a = 1$, and $\sigma_e = k_a$, but $B_M = 0$ by its definition in (10) and the definition of σ_m. It can be demonstrated that, due to the signum function in (19), the time derivative of (20) exists a.e., i.e., for almost all $t \in \left(t_n^e, t_{n+1}^i\right)$, $\forall i \in \{s, p\}$, and, after substituting (12) and (19), can be upper bounded using Properties 7 and 12, Assumption II, and (15) as

$$\dot{V}_L \overset{\text{a.e.}}{\leq} -\left(c_{b_e} k_a k_{1e} - c_1\right) e_1 - \left(c_{b_e} k_a k_{2e} - c_2\right) e_1^2, \tag{25}$$

which is negative definite since $e_1 > 0$, provided some gain conditions are satisfied. Furthermore, (21) can be used to upper bound (25) as

$$\dot{V}_L \leq -\lambda_{e1} V_L, \tag{26}$$

where λ_{e1} denotes a known positive bounding constant. The inequality in (26) can be solved to yield

$$V_L(t) \leq V_L\left(t_n^e\right) \exp\left[-\lambda_{e1} \left(t - t_n^e\right)\right], \tag{27}$$

for all $t \in \left(t_n^e, t_{n+1}^i\right)$, $\forall i \in \{s, p\}$, $\forall n$. Rewriting (27) using (21), and performing some algebraic manipulation yields exponential convergence of $|e_1(t)|$ to zero. ■

Remark. Exponential convergence to $\dot{q}_{\underline{d}}$ throughout the assistive mode (Theorems 1 and 2) is guaranteed in the sense that

$$|e_1(t)| \leq \sqrt{\frac{c_M}{c_m}} |e_1\left(t_n^i\right)| \exp\left[-\frac{\lambda_a}{2}\left(t - t_n^i\right)\right], \tag{28}$$

for all $t \in \left(t_n^i, t_{n+1}^p\right) \forall i \in \{e, s\}$, $\forall n$, where $\lambda_a \in \mathbb{R}_{>0}$ is defined as

$$\lambda_a \triangleq \min\{\lambda_s, \lambda_{e1}\}.$$

Since (28) holds for all combinations of σ_e and σ_m while $\sigma_a = 1$, V_L is indeed a common Lyapunov function for switching during the assistive mode.

Theorem 3. *When $\dot{q} > \dot{q}_{\overline{d}}$, the closed-loop error system in (19) is exponentially stable.*

Proof: When $\dot{q} > \dot{q}_{\overline{d}}$, $\sigma_a = 0$, $e_2 < 0$, $e_1 < 0$, and $\sigma_e = k_r$ (i.e., the cycle-rider system is in the motor-resistance control mode). Due to the signum function in (19), the time derivative of (20) exists a.e., i.e., for almost all $t \in \left(t_n^e, t_{n+1}^p\right)$, and for all n, and, after substituting (12) and (19), can be upper bounded using Properties 7 and 12, Assumption II, and (15) as

$$\begin{aligned} \dot{V}_L \overset{\text{a.e.}}{\leq} & -\left(c_{b_e} k_r k_{1e} - c_{B_e} k_r k_{2e} \triangle_d - c_1 - c_{vol}\right) |e_1| \\ & -\left(c_{b_e} k_r k_{2e} - c_2\right) e_1^2, \end{aligned} \tag{29}$$

which is negative definite provided some gain conditions are satisfied. Furthermore, (29) can be upper bounded as

$$\dot{V}_L \leq -\lambda_{e2} V_L,$$

where λ_{e2} denotes a known positive bounding constant, and solved to yield

$$V_L(t) \leq V_L\left(t_n^e\right) \exp\left[-\lambda_{e2} \left(t - t_n^e\right)\right], \tag{30}$$

for all $t \in \left(t_n^e, t_{n+1}^i\right)$, $i = p$, $\forall n$. Rewriting (30) using (21), noting that $\left|e_1\left(t_n^e\right)\right| = \left|e_2\left(t_n^e\right) - \Delta_d\right| = \Delta_d$ when $\sigma_a = 0$, and performing algebraic manipulation yields exponential convergence of $\left|e_1(t)\right|$ to zero. ■

Remark 1. Since the passive mode is defined by $0 \leq e_1 \leq \Delta_d$, the error is always bounded in the passive mode. As described in Theorems 1-3, $|e_1|$ decays at an exponential rate in both the assistive and resistive modes. By the definition of e_2 in (12), $|e_2|$ also decays exponentially in the assistive and resistive modes. Therefore, sufficient conditions for overall stability of the two-sided system can be developed based on the exponential time constants λ_s, λ_{e1}, and λ_{e2}. When the system enters the resistive mode, the cadence will instantly exponentially decay back into the passive mode and when entering the assistive mode, the FES and motor controllers will ensure the cadence exponentially increases back into the voluntary range of desired cadence. For this particular application in FES cycling, where there is a desired cadence range, rather than a single desired trajectory, error convergence to a ball is desirable, rather than exponential error convergence to zero.

V. Experiment

To evaluate the performance of the FES and motor controllers in (16) and (17), respectively, experiments were conducted on an able-bodied subject of 23 years old after they gave written informed consent approved by the University of Florida Institutional Review Board. The subject was instructed to comfortably contribute to forward pedaling at various intensities to stay below, above, or within the desired region of cadence, showing the control system's three modes.

The experimental testbed and setup was done as in [8].It was desired to start from 0 RPM, smoothly approach 45 RPM with use of the motor, and then remain between 45 and 55 RPM with the switched control developed previously for the remainder of the experiment, which lasted 180s in total. Thus, the minimum desired crank velocity $\dot{q}_{\underline{d}}$ (rad/s) and velocity range Δ_d (rad/s) were designed as $\dot{q}_{\underline{d}} \triangleq \frac{3\pi}{2}\left\{1 - \exp\left[-\frac{2}{5}\left(t - t_0\right)\right]\right\}$, $\Delta_d \triangleq \frac{\pi}{3}$. The range of crank angles corresponding to the stimulation of each muscle group and activation of the motor within the assistive mode were determined based on the lower thresholds of the torque transfer ratios, which were calculated as $\varepsilon_{quad} = \varepsilon_{ham} = 0.42$, $\varepsilon_{glute} = 0.38$ for both the left and right legs. The gains, k_a and k_r were chosen as 0.8 and 1, respectively.

A. Results

Figure 2 depicts the activation of both the motor and FES as the cycle's cadence varies below, within, and above the set bounds during the experiment.

B. Discussion

The experiment was used to depict all three modes of the control system. After the first 10 seconds of the motor bringing the cadence up to 45 RPM, the participant was instructed to lightly pedal such that their voluntary efforts did not reach the minimum cadence threshold. As seen in Figure 2, control input was switched between FES and the motor during this time, often causing the cadence to cross above the lower threshold. From seconds 80-125, the participant was instructed to attempt to stay between the two cadence thresholds to demonstrate the passive mode. Figure 2 shows that there were few instances that input was sent to either FES or the motor, all of which corresponded to instances the cadence was above or below the desired region. From seconds 125-180, the participant was instructed to pedal much harder than necessary to stay within the threshold lines to demonstrate the resistive mode. During this time, no FES input was sent and input below 0.5 Amps was sent to the motor, often sending the cadence back into the desired range.

The goal of this experiment was to clearly depict the three modes of the control system separately in response to the cadence escaping the upper and lower bounds, which is expected to correspond to individuals at three different ability levels. However, it is possible that a person with a movement disorder or an able-bodied person pedaling at a higher cadence would switch modes more quickly and eventually fatigue such that assistance mode was utilized more, as in the first part of the current experiment.

VI. Conclusion

The combined motor and FES control system developed in this paper is designed to enable a cycle rider to maintain a cadence within a desired range with volitional pedaling. A Lyapunov-like analysis proved stability of the controllers for the multi-level switched system, despite unknown disturbances, showing exponential convergence to the desired cadence range (i.e., $e_1 \in (0, \Delta_d)$). Preliminary experiments validated the use of the control system in all three modes for an able-bodied person pedaling a custom tricycle within a range of 45-55 rpm.

With assistive, passive, and resistive modes, the developed control system has the potential to advance motorized FES-cycling as a rehabilitation exercise for people with movement disorders. Subjects with a wide range of volitional abilities could pedal within a desired cadence range, with FES and a motor assisting those with minimal leg strength or at the onset of fatigue, and with the motor providing resistance to someone who can easily pedal faster than a desired range. The authors also plan to expand the number of test subjects, including performing tests in individuals with neurological conditions.

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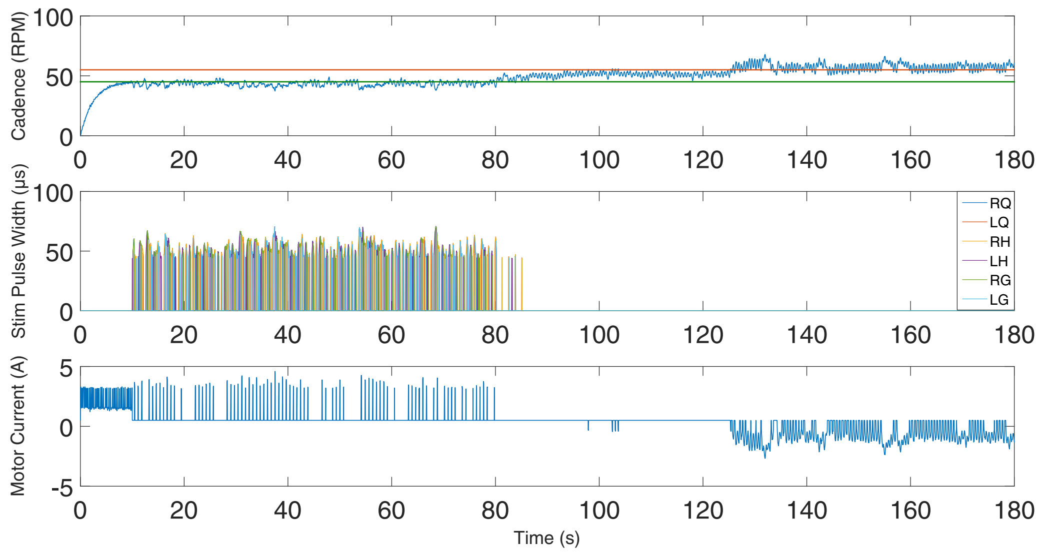

Figure 2. Cycle Cadence (top plot), stimulation pulse width (middle plot), and motor current (bottom plot). Motor current greater than 0.5A indicates assistance, motor current less than 0.5A indicates resistance, and an offset of 0.5 amps is used to combat friction within the motor. The solid green line at 45 RPM and red line at 55 RPM of the cadence plot indicate the chosen upper and lower bounds for the purely volitional pedaling mode. Seconds 10-80 depict the assistance mode (i.e., the subject does not maintain the minimum desired cadence on their own), the next 45 seconds depict the passive mode (i.e., the subject was able to maintain cadence within the desired range on their own), and the last 55 seconds depict the resistive mode (i.e., the subject fairly consistently voluntarily output a torque that resulted in a cadence above the maximum desired threshold).

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