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Passivity-Based Learning Control for Torque and Cadence Tracking in Functional Electrical Stimulation (FES) Induced Cycling

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Abstract—**This paper examines torque tracking accomplished by the activation of lower-limb muscles via Functional Electrical Stimulation (FES) and cadence regulation by an electric motor. Challenges arise from the fact that skeletal muscles evoke torque via FES in a time-varying, nonlinear, and delayed manner. A desired torque trajectory is constructed based on the crank position and determined by the knee joint torque transfer ratio (i.e., kinematic efficiency of the knee), which varies as a periodic function of the crank angle. To cope with this periodicity, a repetitive learning controller is developed to track the desired periodic torque trajectory by stimulating the muscle groups. Concurrently, a sliding-mode controller is designed for the electric motor to maintain cadence tracking throughout the entire crank cycle. A passivity-based analysis is developed to ensure stability of the torque and cadence closed-loop systems.**

Index Terms—**Functional Electrical Stimulation (FES), FES-Cycling, Repetitive Learning Control (RLC), Passivity-Based Control**

I. Introduction

Functional Electrical Stimulation (FES) is a rehabilitative strategy that applies electrical current to a neuromuscular system to enable function by assisting a person's motor output [1]. Motorized FES-cycling is recommended as an effective exercise to activate lower-limb muscles, thus exploiting the physiological benefits of electrical stimulation [2], [3]. Additionally, the exercise duration can be prolonged due to the assistance provided by the electric motor. Closed-loop controllers have been developed for cadence tracking in FES-cycling while including a motor in the loop [4]–[6]. In [4], [5], switching control was used to activate lower-limb muscles and an electric motor to cooperatively track cadence. However, to maximize FES-cycling benefits, it is recommended to maximize the torque output produced by the activation of lower-limb muscles for strength and mass building [7].

Several objectives have been identified for the design of assistive devices, such as strength training (control of resistive torque to enhance power output) and cardiovascular workout (focused on lower resistive torque but longer exercise duration) [8]–[10]. A combined goal of torque tracking with position/speed regulation is desired for the control of an assistive device that interacts with people. This motivates the use of robotic assistance to achieve a desired motion, while the patient voluntarily or via FES exerts a torque output. This shared control task between the human and the machine exploits the functional benefits of the repetitive exercise and avoids, or to a lesser degree delays, the issues related to muscle fatigue and lack of controllability. However, skeletal muscles evoke torque via FES in a time-varying, nonlinear, and delayed manner. In addition, torque feedback is challenging due to the sensing limitations to measure the isolated muscle torque contribution in a system. To improve cycling performance during power tracking protocols, there is motivation to develop adaptive control algorithms with proof of stability of the human-machine closed-loop system [9], [11].

Torque tracking trajectories for human-robot applications have been based on time, joint angles, mechanical phase-variables, and electromyographic measurements [12]–[14]. The relationship between lower-limb joint angles and joint torques has been utilized to regulate net joint power, especially in lower limb exoskeletons [12], [15]. In [16] a transfemoral amputee subject pedaled a bicycle with asistance from a powered prosthetic device, where the joint torque references were generated using the knee joint power distribution (as a function of crank angle) presented in [17]. The purpose of the aforementioned results is to develop position-dependent stiffness or torque trajectories to avoid unsafe interactions between people and the robotic devices. Similarly, speed regulation is beneficial to avoid unstable motion while evoking a torque output.

Passivity-based control has been applied in human-robot interaction applications and series elastic actuation to ensure safety of operation [9], [10], [18]. The use of passivity properties has been instrumental for control design and to prove stability of nonlinear systems [19], [20]. Adaptive controllers have been developed for trajectory tracking of fully actuated robot manipulators by taking advantage of preserved passivity under parallel and feedback connections of subsystems [20], [21]. Moreover, passivity-based control has played a dominant role in designing and analyzing the stability of feedback interconnections of complex systems, such as hybrid and switched systems [22], [23].

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Learning control techniques, such as iterative learning control (ILC) and repetitive learning control (RLC), have been developed to improve tracking performance for repetitive or periodic (time or state) systems by using control inputs from previous trials, iterations, cycles, or periods [24]–[27]. However in applications such as the control of the motorized FES-cycling system, the associated dynamics and/or control design involve a switched systems analysis.

In this paper, a switched FES repetitive learning control is designed to track a periodic torque trajectory by stimulating lower-limb muscles on a stationary recumbent cycle. The periodic desired torque trajectory is designed based on the knee kinematic effectiveness of the rider, which varies as a function of the crank angle. In parallel, a robust sliding-mode controller is designed for the electric motor to achieve cadence tracking.The cycle-rider model includes the switching effects of activating multiple muscle groups based on a state-dependent activation pattern that exploits the kinematic effectiveness of the rider. A passivity-based analysis is developed to ensure stability of the torque (muscle control) and cadence (motor control) closed-loop systems.

II. Cycle-Rider Dynamic Model

The stationary cycle-rider system is modeled as a single degree-of-freedom system with the following dynamics [28]

$$\begin{aligned} M(q)\ddot{q} + V(q,\dot{q})\dot{q} + G(q) & \\ +P(q,\dot{q}) + c_d\dot{q} + d(t) &= \tau_e(t) + \tau_a(q,\dot{q},t), \quad (1) \end{aligned}$$

where $q : \mathbb{R}_{\geq t_0} \to \mathcal{Q}$ denotes the positive clockwise measurable crank angle, $\mathcal{Q} \subseteq \mathbb{R}$ denotes the set of crank angles contained between $[0, 2\pi)$, and $t_0 \in \mathbb{R}$ is the initial time; $M : \mathcal{Q} \to \mathbb{R}_{>0}$, denotes the combined inertial effects of the rider and cycle; $V : \mathcal{Q} \times \mathbb{R} \to \mathbb{R}$ and $G : \mathcal{Q} \to \mathbb{R}$ denote the centripetal-Coriolis, and gravitational effects, respectively; $P : \mathcal{Q} \times \mathbb{R} \to \mathbb{R}$ denotes the effects of passive viscoelastic tissue forces in the rider's joints; $c_d \in \mathbb{R}_{>0}$ denotes the unknown coefficient of viscous damping in the cycle; $d : \mathbb{R}_{\geq t_0} \to \mathbb{R}$ denotes the disturbances applied by the rider and unmodeled effects in the system; $\tau_a : \mathcal{Q} \times \mathbb{R} \times \mathbb{R}_{\geq t_0} \to \mathbb{R}$ denotes the net active torque produced by the rider's lower limb muscles, and $\tau_e : \mathbb{R}_{\geq t_0} \to \mathbb{R}$ denotes the torque applied about the cycle crank axis by the electric motor. The model in (1) can be generalized as

$$\tau_c(\dot{q},\ddot{q}) + \tau_r(q,\dot{q},\ddot{q},t) = \tau_e(t), \quad (2)$$

where $\tau_c : \mathbb{R}^2 \to \mathbb{R}$ denotes the torque applied about the crank by the cycle and is given by $\tau_c(\dot{q},\ddot{q}) = J\ddot{q} + c_d\dot{q}$, and $\tau_r : \mathcal{Q} \times \mathbb{R}^2 \times \mathbb{R}_{\geq t_0} \to \mathbb{R}$ denotes the rider torque applied about the crank as

$$\tau_r(q,\dot{q},\ddot{q},t) = \tau_p(q,\dot{q},\ddot{q},t) - \tau_a(q,\dot{q},t), \quad (3)$$

where $\tau_p : \mathcal{Q} \times \mathbb{R}^2 \times \mathbb{R}_{\geq t_0} \to \mathbb{R}$ denotes the passive torque denoted as $\tau_p = M_p(q)\ddot{q} + V(q,\dot{q})\dot{q} + G(q) + P(q,\dot{q}) + d(t)$. The torque applied by the electric motor about the crank axis is defined as

$$\tau_e(t) \triangleq B_e u_e(t), \quad (4)$$

where $B_e \in \mathbb{R}_{>0}$ is a positive torque constant, and satisfies $B_e \geq c_e$, where $c_e \in \mathbb{R}_{>0}$ is a positive known constant, and $u_e : \mathbb{R}_{\geq t_0} \to \mathbb{R}$ is the motor current control input. The net active torque produced by muscle contractions is

$$\tau_a(q,\dot{q},t) \triangleq \sum_{m\in\mathcal{M}} B_m(q,\dot{q})u_m(t), \quad (5)$$

where $B_m : \mathcal{Q} \times \mathbb{R} \to \mathbb{R}_{>0}$ represents the uncertain control effectiveness of the muscle groups with subscript m indicating an element in the muscle set $\mathcal{M} \triangleq \{RQuad, RHam, RGlute, LQuad, LHam, LGlute\}$ that contains the right (R) and left (L) quadriceps femoris $(Quad)$, hamstrings (Ham), and gluteal $(Glute)$ muscle groups respectively, and $u_m : \mathbb{R}_{\geq t_0} \to \mathbb{R}$ denotes the stimulation intensity applied to each muscle group. The control effectiveness for the muscle groups is nonzero and denoted as [28]

$$B_m(q,\dot{q}) \triangleq \Omega_m(q,\dot{q})T_m(q), \forall m \in \mathcal{M}, \quad (6)$$

where $\Omega_m : \mathcal{Q} \times \mathbb{R} \to \mathbb{R}$ denotes the uncertain relationship between stimulation intensity and the muscle group's evoked force which produces a resultant torque about the joint it spans, and $T_m : \mathcal{Q} \to \mathbb{R}$ denotes the relationship between a muscle's resultant torque about a joint to torque about the crank axis.

The stimulation intensities $u_m, \forall m \in \mathcal{M}$ are applied to the muscle groups in regions of the crank cycle where the torque transfer ratios T_m are above a predefined threshold $\varepsilon_m \triangleq \Lambda_m max(T_m), \forall m \in \mathcal{M}$, where $\Lambda_m \in [0,1]$ is a selectable value. The muscle switching control design yields an autonomous, state-dependent, switched control system. The portion of the crank cycle over which a particular muscle group is stimulated is denoted by $\mathcal{Q}_m \subset \mathcal{Q}, \forall m \in \mathcal{M}$, where the muscle groups are activated as described in [28] so that $\mathcal{Q}_M \triangleq \underset{m\in\mathcal{M}}{\cup} \mathcal{Q}_m$. A piecewise constant switching signal can be developed for each muscle group, $\sigma_m \in \{0,\ 1\}, \forall m \in \mathcal{M}$ as

$$\sigma_m(q) \triangleq \begin{cases} 1 & \text{if } q \in \mathcal{Q}_m \\ 0 & \text{if } q \notin \mathcal{Q}_m \end{cases}. \quad (7)$$

Using (7), the stimulation intensity to the muscle groups is defined as

$$u_m(t) \triangleq k_m\sigma_m u_{FES}, \quad (8)$$

where $k_m \in \mathbb{R}_{>0}, \forall m \in \mathcal{M}$ are selectable positive control gains, and $u_{FES} : \mathbb{R}_{\geq t_0} \to \mathbb{R}$ is a subsequently designed muscle input. Substituting (4), (5), and (8) into (1) and rearranging terms yields

$$M(q)\ddot{q} + V(q,\dot{q})\dot{q} + G(q)$$

$$+P(q,\dot{q})+c_d\dot{q}+d \quad = \quad B_\sigma u_{FES}+B_e u_e, \quad (9)$$

where $B_\sigma \in \mathbb{R}_{\geq 0}$ is a lumped, switched control effectiveness term defined as

$$B_\sigma(q,\dot{q}) \triangleq \sum_{m\in\mathcal{M}} B_m(q,\dot{q})k_m\sigma_m(q). \quad (10)$$

The subscript $\sigma \in \mathcal{P} \triangleq \{1,2,3,...n\}$, $\mathcal{P} \subset \mathbb{N}$, $n \in \mathbb{N}$ indicates the index of B_σ, which switches according to the crank position and $\mathcal{P}$. The known sequence of switching states, which are the limit points of $\mathcal{Q}_m$, $\forall m \in \mathcal{M}$, is defined as $\{q_n\}$, and the corresponding sequence of unknown switching times $\{t_n\}$ is defined such that each t_n denotes the instant when q reaches the corresponding switching state q_n. The switching signal σ_m is assumed to be continuous from the right (i.e., $\sigma_m(q) = \lim_{q\to q_n^+}\sigma_m(q)$) and designed to produce forward pedaling only. The following assumption and properties of the switched system in (9) will be exploited in the subsequent control design and stability analysis:

Assumption 1. The disturbance term d is bounded as $|d| \leq \xi_d$, where $\xi_d \in \mathbb{R}_{>0}$ is a known constant.

Property 1. $c_m \leq M \leq c_M$, where $c_m,\ c_M \in \mathbb{R}_{>0}$ are known constants.

Property 2. $|V| \leq c_V\,|\dot{q}|$, where $c_V \in \mathbb{R}_{>0}$ is a known constant.

Property 3. $|G| \leq c_G$, where $c_G \in \mathbb{R}_{>0}$ is a known constant.

Property 4. $|P| \leq c_{P1}+c_{P2}\,|\dot{q}|$, where $c_{P1},\ c_{P2} \in \mathbb{R}_{>0}$ are known constants [28].

Property 5. $\frac{1}{2}\dot{M} - V = 0$ by skew symmetry [29].

Property 6. The lumped switching control effectiveness is bounded as $c_b \leq B_\sigma \leq c_B$, $\forall\sigma_m \in \mathcal{P}$, where $c_b, c_B \in \mathbb{R}_{>0}$ are known constants.

III. Control Development

A. Cadence Control

The first objective is to design a motor controller that tracks a target cadence. The measurable crank position trajectory tracking error $e : \mathbb{R}_{\geq t_0} \to \mathbb{R}$ is defined as[1]

$$e(t) \triangleq q(t) - q_d(t), \quad (11)$$

where $q_d : \mathbb{R}_{\geq t_0} \to \mathbb{R}$ denotes the desired crank position and its first two time derivatives are bounded such that $|\dot{q}_d(t)| \leq \xi_1$ and $|\ddot{q}_d(t)| \leq \xi_2$, where $\xi_1,\ \xi_2 \in \mathbb{R}_{>0}$ are known positive constants. To facilitate the subsequent control development, an auxiliary tracking error $r : \mathbb{R}_{\geq t_0} \to \mathbb{R}$ is defined as[2]

$$r \triangleq \dot{e} + \alpha e, \quad (12)$$

[1]The control objective is quantified using the first time derivative of $e(t)$.

[2]Functional dependencies are removed henceforth unless they add clarity to the exposition.

where $\alpha \in \mathbb{R}_{>0}$ is a constant control gain. After taking the time derivative of (12) and premultiplying by M, substituting for (9) and (11), and then performing some algebraic manipulation yields

$$M\dot{r} = -Vr + \chi + \tilde{N} + B_\sigma u_{FES} + B_e u_e - e, \quad (13)$$

where the auxiliary signals $\chi : \mathbb{R}_{\geq t_0} \to \mathbb{R}$ and $\tilde{N} : \mathbb{R}_{\geq t_0} \to \mathbb{R}$ are defined as

$$\begin{aligned} \chi &= W_d - M(q)(\ddot{q}_d - \alpha\dot{e}) - V(q,\dot{q})(\dot{q}_d - \alpha e) - G(q) \\ &\quad -P(q,\dot{q}) - c_d\dot{q} + N_d + e, \quad (14) \\ \tilde{N} &\triangleq -(W_d + N_d + d), \quad (15) \end{aligned}$$

and the signals $W_d : \mathbb{R}_{\geq t_0} \to \mathbb{R}$ and $N_d : \mathbb{R}_{\geq t_0} \to \mathbb{R}_{>0}$ are defined as

$$\begin{aligned} W_d &\triangleq M(q_d)\ddot{q}_d + V(q_d,\dot{q}_d)\dot{q}_d + G(q_d) + c_d\dot{q}_d, \quad (16) \\ N_d &\triangleq c_{P1} + c_{P2}\dot{q}_d. \quad (17) \end{aligned}$$

The auxiliary signal in (15) can be upper bounded as

$$|\tilde{N}| \leq \Theta_1, \quad (18)$$

where $\Theta_1 \in \mathbb{R}_{>0}$ is a known positive constant. By using Properties 1-5, (11) and (12), the Mean Value Theorem can be used to develop an upper bound for (14) as

$$\chi \leq \rho(\|z\|)\|z\|, \quad (19)$$

where $z : \mathbb{R}_{\geq t_0} \to \mathbb{R}^2$ is a composite vector of error signals defined as

$$z \triangleq [e\ r]^T, \quad (20)$$

and $\rho(\cdot) \in \mathbb{R}$ is a known positive, radially unbounded, nondecreasing function. Given the cadence open-loop error system in (13), the control input to the motor is designed as

$$u_e = -k_1 r - (k_2 + k_3\rho(\|z\|)\|z\|)\,sgn(r) + \nu_p, \quad (21)$$

where $k_1, k_2, k_3 \in \mathbb{R}_{>0}$ are selectable positive gain constants, $sgn(\cdot) : \mathbb{R} \to [-1,1]$ is the signum function, and ν_p is a subsequently designed control input. The closed-loop error system is obtained by substituting (21) into (13)

$$\begin{aligned} M\dot{r} \quad &= \quad -Vr + \chi + \tilde{N} + B_\sigma u_{FES} - e - B_e(k_1 r - \nu_p \\ &\quad + (k_2 + k_3\rho(\|z\|)\|z\|)\,sgn(r)). \quad (22) \end{aligned}$$

B. Torque Control

The second objective is to track a desired torque trajectory in the muscle stimulation regions (i.e., $q \in \mathcal{Q}_M$). The torque tracking error signal is designed based on the difference between desired torque and the torque produced by the muscle contractions defined in (5). Wireless torque sensors are commonly included on rehabilitation cycles which provide a measurement of the net torque contributions about the crank. Direct measurement of muscle force requires real-time invasive sensing which is not practical as discussed in [15]. Similar to previous FES experiments (cf. [15], [30]), a baseline measurement of the required torque to drive the cycle-rider system at a desired speed is obtained a priori where no electrical stimulation is applied to the lower-limb muscles (i.e., $\tau_a = 0$ such that $\tau_r = \tau_p$) and under the assumption that the disturbances d by the rider and the cycle are sufficiently small. Invariance of the cycle-rider system dynamics is assumed along the system's trajectory in the absence of FES. Setting $\tau_a = 0$ in (3), a nominal torque measurement $\tau_n : \mathbb{R}_{\geq t_0} \to \mathbb{R}$ of (2) can be obtained as

$$\tau_n = \tau_e = \tau_c + \tau_p. \tag{23}$$

An estimate of the nominal torque measurement $\hat{\tau}_n : \mathbb{R}_{\geq t_0} \to \mathbb{R}$ (i.e., $\hat{\tau}_n = \hat{\tau}_c + \hat{\tau}_p$) can be obtained by using fitting techniques such as Fourier series using torque measurements [30]. The mismatch between the nominal torque and the nominal torque estimate $\tilde{\tau}_n : \mathbb{R}_{\geq t_0} \to \mathbb{R}$ is obtained as

$$\tilde{\tau}_n = \tau_n - \hat{\tau}_n \leq \epsilon_n, \tag{24}$$

where $\epsilon_n \in \mathbb{R}_{>0}$ is a known upper bound in the estimation error. The measurable net active muscle torque τ_a is obtained by subtracting the nominal torque estimate $\hat{\tau}_n$ from the continuous time torque measurement $\tau_M : \mathbb{R}_{\geq t_0} \to \mathbb{R}$ (i.e., the net torque contributions about the crank) such that

$$\tau_a = \tau_M - \hat{\tau}_n. \tag{25}$$

To quantify the torque control objective, a torque tracking error-like term (integral of the actual torque tracking objective) $e_\tau : \mathbb{R}_{\geq t_0} \to \mathbb{R}$ is defined as [29]

$$e_\tau = \int_{t_0}^{t} (\tau_d(\varphi) - \tau_a(\varphi))\, d\varphi, \tag{26}$$

where $\tau_d : \mathbb{R}_{\geq t_0} \to \mathbb{R}$ denotes the periodic desired torque trajectory.

Remark 1. The desired torque trajectory τ_d is periodic with known period T in the sense that $\tau_d(t) = \tau_d(t - T)$.

The torque open-loop error system is obtained by taking the time derivative of (26) and using (5), (8), and (10) yields

$$\dot{e}_\tau = \tau_d - B_\sigma u_{FES}. \tag{27}$$

Given the open-loop error system in (27), the muscle control input is designed as

$$u_{FES} = \hat{W}_d + k_4 e_\tau - \nu_{FES}, \tag{28}$$

where $k_4 \in \mathbb{R}_{>0}$ is a constant control gain, $\nu_{FES} : \mathbb{R}_{\geq t_0} \to \mathbb{R}$ is a control term to be designed, and $\hat{W}_d : \mathbb{R}_{\geq t_0} \to \mathbb{R}$ is the repetitive control law designed as

$$\hat{W}_d(t) = sat_{\beta_r}(\hat{W}_d(t - T)) + k_L e_\tau, \tag{29}$$

where $k_L \in \mathbb{R}_{>0}$ is a positive constant learning control gain, and $sat_{\beta_r}(\cdot)$ is defined as $sat_{\beta_r}(\Xi) \triangleq \begin{cases} \Xi & \text{for } |\Xi| \leq \beta_r \\ sgn(\Xi)\beta_r & \text{for } |\Xi| > \beta_r \end{cases}$, $\forall \Xi \in \mathbb{R}$. The closed-loop error system is obtained by substituting (28) into (27) as

$$\dot{e}_\tau = \tilde{W}_d + \hat{W}_d - B_\sigma(\hat{W}_d + k_4 e_\tau - \nu_{FES}), \tag{30}$$

where $\tilde{W}_d : \mathbb{R}_{\geq t_0} \to \mathbb{R}$ is the learning estimation error defined as $\tilde{W}_d \triangleq \tau_d - \hat{W}_d$. Based on the periodicity and boundedness of τ_d, $\tau_d(t) = sat_{\beta_r}(\tau_d(t)) = sat_{\beta_r}(\tau_d(t-T))$. Hence, by exploiting (29), the following expression can be developed for $\tilde{W}_d$

$$\tilde{W}_d = sat_{\beta_r}(\tau_d(t-T)) - sat_{\beta_r}(\hat{W}_d(t-T)) - k_L e_\tau(t). \tag{31}$$

IV. Stability Analysis

Theorem 1. *Given the closed loop error system in (30), the system is output strictly passive (OSP) from input $v_1 = \gamma_1 \hat{W}_d + c_b \nu_{FES}$ to output e_τ in $q \in \mathcal{Q}_M$ and the controller designed in (28) and repetitive learning law in (29) ensures asymptotic tracking in the sense that $\lim_{t\to\infty} e_\tau(t) = 0$.*

Proof: Let $V_1 : \mathbb{R}^2 \times \mathbb{R}_{\geq t_0} \to \mathbb{R}$ be a nonnegative, continuously differentiable, storage function defined as

$$V_1 \triangleq \frac{1}{2}e_\tau^2 + \frac{1}{2k_L}\int_{t-T}^{t} (sat_{\beta_r}(\tau_d(\varphi)) - sat_{\beta_r}(\hat{W}_d(\varphi)))^2 d\varphi. \tag{32}$$

The storage function in (32) satisfies the following inequalities:

$$\lambda_1 \|w\|^2 \leq V_1(w,t) \leq \lambda_2 \|w\|^2,$$

where $\lambda_1 \triangleq min(\frac{1}{2}, \frac{1}{2k_L})$, $\lambda_2 \triangleq max(\frac{1}{2}, \frac{1}{2k_L})$ and $w \triangleq [e_\tau\ \sqrt{Q_L}]^T$ where $Q_L \triangleq \int_{t-T}^{t}(sat_{\beta_r}(\tau_d(\varphi)) - sat_{\beta r}(\hat{W}_d(\varphi)))^2 d\varphi$. Let $w(t)$ be a Filippov solution to the differential inclusion $\dot{w} \in K[h](w)$, where $K[\cdot]$ is defined as [31] and h is defined by using (30) and the first time derivative of Q_L as $h \triangleq [h_1 \quad h_2]$, where $h_1 \triangleq \tilde{W}_d + \hat{W}_d - B_\sigma(\hat{W}_d + k_4 e_\tau - \nu_{FES})$, $h_2 \triangleq \frac{1}{2\sqrt{Q_L}}\{(sat_{\beta_r}(\tau_d(t)) - sat_{\beta_r}(\hat{W}_d(t)))^2 - (sat_{\beta_r}(\tau_d(t-T)) - sat_{\beta_r}(\hat{W}_d(t-T)))^2\}$. The control input in (27) has the discontinuous lumped control effectiveness B_σ, hence the time derivative of (32) exists almost everywhere (a.e.), i.e., for almost all t. Based on [32, Lemma 1], the time derivative of (32), $\dot{V}_1(w(t),t) \overset{a.e.}{\in} \dot{\tilde{V}}_1(w(t),t)$, where $\dot{\tilde{V}}_1$ is the generalized time derivative of (32) along the Filippov trajectories of $\dot{w} = h(w)$ and is defined as in [32] as $\dot{\tilde{V}}_1 \triangleq \bigcap_{\xi \in \partial V_1} \xi^T K \begin{bmatrix} \dot{e}_\tau & \frac{\dot{Q}_L}{2\sqrt{Q_L}} & 1 \end{bmatrix}^T (e_\tau,\ 2\sqrt{Q_L},\ t)$. Since $V_1(w,t)$ is continuously differentiable in w, $\partial V_1 = \{\nabla V_1\}$, thus

$$\dot{\tilde{V}}_1 \overset{a.e.}{\subset} [e_\tau,\ \left(\frac{1}{2k_L}\right) 2\sqrt{Q_L}] K \begin{bmatrix} \dot{e}_\tau & \frac{\dot{Q}_L}{2\sqrt{Q_L}} \end{bmatrix}^T. \tag{33}$$

Therefore, after substituting for (30), the generalized time derivative of (32) can be expressed as

$$\begin{aligned}\dot{\tilde{V}}_1 \overset{a.e.}{\subset} & \; e_\tau \left(\tilde{W}_d + \hat{W}_d - K[B_\sigma](k_4 e_\tau + \hat{W}_d - \nu_{FES})\right) \\ & -\frac{1}{2k_L}(sat_{\beta_r}(\tau_d(t-T)) - sat_{\beta_r}(\hat{W}_d(t-T)))^2 \\ & +\frac{1}{2k_L}(sat_{\beta_r}(\tau_d(t)) - sat_{\beta_r}(\hat{W}_d(t)))^2. \end{aligned} \quad (34)$$

By employing the following property

$$\left(\tau_d(t) - \hat{W}_d(t)\right)^2 \geq \left(sat_{\beta_r}(\tau_d(t)) - sat_{\beta_r}(\hat{W}_d(t))\right)^2,$$

using a similar proof as developed in [33, Appendix I], and using Property 6 to lower bound $K[B_\sigma]$, and canceling terms, an upper bound for (34) can be developed as

$$\dot{\tilde{V}}_1 \overset{a.e.}{\leq} -\delta_1 e_\tau^2 + v_1 e_\tau, \quad (35)$$

where $v_1 = \gamma_1 \hat{W}_d + c_b \nu_{FES}$, $\gamma_1 = 1 + c_B$, and $\delta_1 \triangleq c_b k_4 + \frac{k_L}{2}$, $\delta_1 > 0$. Integrating (35) yields $\int_{t_0}^{t} v_1(\varphi) e_\tau(\varphi) d\varphi \overset{a.e.}{\geq} \left(\tilde{V}_1(t) - \tilde{V}_1(t_0) + \int_{t_0}^{t} \delta_1 e_\tau^2(\varphi) d\varphi\right)$. Hence the system is output strictly passive (OSP) from the input v_1 to the output e_τ. Therefore, the closed-loop system in (30) is passive with a radially unbounded positive definite storage function. From [34, Theorem 2.28], to prove asymptotic tracking, the zero-state observability condition has to be satisfied[3]. Thus by designing ν_{FES} in (28) as $\nu_{FES} \triangleq -k_5 \hat{W}_d$, where $k_5 \triangleq \frac{\gamma_1}{c_b}$, and substituting it in (35), $\dot{\tilde{V}}_1 \overset{a.e.}{\leq} -\delta_1 e_\tau^2 \leq 0$. By invoking [32, Corollary 2] and since $\dot{\tilde{V}}_1(w,t) \overset{a.e.}{\leq} -W(w)$, W is a continuous positive semi-definite function, $|e_\tau| \to 0$ as $t \to \infty$. Since $V_1 \geq 0$ and $\dot{V}_1 \overset{a.e.}{\leq} 0$, $V_1 \in \mathcal{L}_\infty$, hence, $e_\tau, Q_L \in \mathcal{L}_\infty$. From (29), $\hat{W}_d \in \mathcal{L}_\infty$, which along with the fact that $\tau_d \in \mathcal{L}_\infty$ implies that $\tilde{W}_d \in \mathcal{L}_\infty$. Then from (28), $u_{FES} \in \mathcal{L}_\infty$, and from (8), $u_m \in \mathcal{L}_\infty$. Hence the closed-loop system in (30) is passive and asymptotic tracking is achieved. ■

Remark 2. The actual torque tracking error $\dot{e}_\tau$ is uniformly bounded from (30). Based on Theorem 1, $e_\tau, \hat{W}_d, \tilde{W}_d \in \mathcal{L}_\infty$, hence $\dot{e}_\tau \in \mathcal{L}_\infty$.

Theorem 2. *Given the closed loop error system in (22), the system is output strictly passive (OSP) from input $v_2 = B_\sigma u_{FES} + c_e \nu_p$ to output r and achieves exponential tracking when $u_{FES} = 0$.*

Proof: Let $V_2 : \mathbb{R}^2 \times \mathbb{R}_{\geq t_0} \to \mathbb{R}$ be a nonnegative, continuously differentiable, storage function defined as

$$V_2 = \frac{1}{2}e^2 + \frac{1}{2}Mr^2. \quad (36)$$

The storage function in (36) satisfies the following inequalities:

[3]In [35], the definition of zero-state observability is described for Filippov solutions.

$$\lambda_3 \|z\|^2 \leq V_2(z,t) \leq \lambda_4 \|z\|^2,$$

where $\lambda_3 \triangleq min(\frac{1}{2}, \frac{c_m}{2})$, $\lambda_4 \triangleq max(\frac{1}{2}, \frac{c_M}{2})$ and z was defined in (20). Let $z(t)$ be a Filippov solution to the differential inclusion $\dot{z} \in K[h](z)$, where $K[\cdot]$ is defined as in [32], and h is defined by using (11) and (12) as $h \triangleq [h_3 \; h_4]$, where $h_3 \triangleq r - \alpha e$ and $h_4 \triangleq M^{-1}\{-Vr + \chi + \tilde{N} + B_\sigma u_{FES} - e - B_e(k_1 r - \nu_p + (k_2 + k_3\rho(\|z\|)\|z\|)\, sgn(r))\}$.The control input in (21) includes the discontinuous signum function; hence, the time derivative of (36) exists almost everywhere (a.e.), i.e., for almost all t. Based on [32, Lemma 1], the time derivative of (36), $\dot{V}_2(z(t),t) \overset{a.e.}{\in} \dot{\tilde{V}}_2(z(t),t)$, where $\dot{\tilde{V}}_2$ is the generalized time derivative of (36) along the Filippov trajectories of $\dot{z} = h(z)$ and is defined as $\dot{\tilde{V}}_2 \triangleq \bigcap_{\xi \in \partial V_2} \xi^T K \begin{bmatrix} \dot{e} & \dot{r} & 1 \end{bmatrix}^T (e, r, t)$. Since $V_2(z,t)$ is continuously differentiable in z, $\partial V_2 = \{\nabla V_2\}$, thus $\dot{\tilde{V}}_2 \overset{a.e.}{\subset} [e, \; Mr, \; \frac{1}{2}\dot{M}r^2] K[\dot{e} \; \dot{r} \; 1]^T$. Using (22), Property 5, and canceling common terms, the generalized time derivative of (36) can be expressed as

$$\begin{aligned}\dot{\tilde{V}}_2 \overset{a.e.}{\subset} & -\alpha e^2 + r\left(\chi + \tilde{N}\right) + rK[B_\sigma]u_{FES} \\ & -B_e k_1 r^2 - B_e K[sgn(r)]k_2 r \\ & -B_e K[sgn(r)]k_3 \rho(\|z\|)\|z\|r + B_e \nu_p r, \end{aligned} \quad (37)$$

where, $K[sgn(r)] = SGN(r)$ such that $SGN(r) = \{1\}$ if $r > 0$, $[-1, 1]$ if $r = 0$, and $\{-1\}$ if $r < 0$. Using (18), (19), and using Property 6, the expression in (37) can be upper bounded as

$$\begin{aligned}\dot{\tilde{V}}_2 \overset{a.e.}{\leq} & -\alpha e^2 - k_1 c_e r^2 - (k_2 c_e - \Theta_1)|r| \\ & - (k_3 c_e - 1)\rho(\|z\|)\|z\||r| \\ & + (B_\sigma u_{FES} + c_e \nu_p) r. \end{aligned} \quad (38)$$

Integrating (38) yields $\int_{t_0}^{t} v_2(\varphi) r(\varphi) d\varphi \overset{a.e.}{\geq} (\tilde{V}_2(t) - \tilde{V}_2(t_0) + \int_{t_0}^{t} \delta_2 \|z(\varphi)\|^2 d\varphi)$, where $\delta_2 = min\{\alpha, k_1 c_e\}$, and $v_2 = B_\sigma u_{FES} + c_e \nu_p$, which can be used to prove that the closed-loop system in (22) is output strictly passive (OSP) from input v_2 to output r, provided the following sufficient gain conditions are satisfied

$$\delta_2 > 0, \;\; k_2 > \frac{\Theta_1}{c_e}, \;\; k_3 > \frac{1}{c_e}. \quad (39)$$

In fact, the system is strictly passive [19]. Moreover, set $\nu_p \triangleq -k_p r$, $k_p \in \mathbb{R}_{>0}$ in (38); hence, the system achieves exponential tracking, $\dot{V}_2 \overset{a.e.}{\leq} -\delta_3 V_2$ where $\delta_3 = \frac{min\{\delta_2, k_p\}}{\lambda_4}$, during $q \notin \mathcal{Q}_M$ since $u_{FES} = 0$, provided the gain conditions in (39) are satisfied. ■

V. Conclusion

A cadence controller that commanded current to the electric motor and a torque controller that commanded stimulation intensities to six muscle groups were implemented in this paper to achieve cadence and torque tracking in FES-cycling. The switched muscle torque controller included a feedforward learning input that compensated for the periodic dynamics of the desired torque trajectory. A passivity-based analysis was developed to ensure stability of the torque and cadence closed-loop systems. Future work includes the implementation of the control technique with people with neurological conditions to study the long-term rehabilitative benefits of FES-cycling using learning control methods.

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