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Admittance Trajectory Tracking using a Challenge-Based Rehabilitation Robot with Functional Electrical Stimulation

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Abstract—**In an effort to combine two rehabilitation strategies, Functional Electrical Stimulation (FES) and robotic therapy, a rehabilitation robot was developed to challenge an arm during bicep curls elicited by closed-loop control of FES. The robot is designed to act as an admittance and its robust, sliding mode controller is proven to be passive with respect to the human. The FES controller utilizes a robust, sliding mode control design to then dominate the robot effects and obtain global exponential stability as demonstrated by a Lyapunov-based stability analysis. The two interacting controllers yield arm position and velocity regulation, where the robot challenges this movement with a desired admittance.**

Index Terms—**Functional Electrical Stimulation (FES), Rehabilitation Robot, Lyapunov, Admittance, Passivity**

I. Introduction

Functional Electrical Stimulation (FES) has been used in the field of rehabilitation because of its numerous benefits such as increased muscular strength [1], improved motor control [2], and others [3]–[5]. However, some inherent challenges to FES control include the nonlinear dynamics exhibited by muscles and dynamically changing muscle characteristics such as fatigue [6]. Hence, closed-loop control of FES is motivated to produce accurate regulation of generated movements [7].

An additional option for rehabilitation is robotic therapy [8], which has shown promise in promoting somatosensory stimulation [9], [10] and motor function [11]–[15]. However, like FES, robotic therapy possesses challenges such as selecting the appropriate control scheme [16] and ensuring human safety [17]. Admittance control, pioneered in [18], has been shown to be an intuitive solution to many of the challenges rising from human-robot interaction [6], [19]–[22] because it modulates behavior instead of explicit force or position trajectories [23], [24].

Furthermore, when controlling rehabilitation robots using admittance, they can be assistive or resistive (challenge-based) [25]. Assistive robots aid rehabilitative movements [26]–[28], while resistive robots challenge rehabilitative movements [16]. Because muscle effort is essential for eliciting motor plasticity [29], [30], challenge based robots are most often used to build muscle mass and strength. Additionally, it has been shown that specific, goal-oriented, repetitive tasks can be effective in reducing motor impairments [31], which makes a case for further integration of robotics into rehabilitative therapies. Based on recent trials, the American Heart Association, Veterans Administration, and Department of Defense have all endorsed upper extremity robotic therapy [32].

In this paper, a single degree of freedom rehabilitation robot is considered that is attached to a person's forearm and pivots at the elbow joint. Closed-loop control of FES is employed on the biceps brachii muscle group to actuate the forearm and perform biceps curls (tracking position), while an admittance controller is employed on the robot to regulate behavior and challenge the user. Although the user and robot are interacting at all times, by treating them as two isolated subsystems linked only by common positions, velocities, and interaction torques, FES can be employed on the user to regulate position and velocity while a motor is employed on the robot to regulate behavior. For position tracking to occur, the muscle must work to overcome the challenge posed by the robot, while the robot maintains its compliant, safe operation. A Lyapunov stability analysis is conducted on the muscle subsystem and is proven to be globally exponentially stable. The robot subsystem requires a passivity-based analysis and is shown to be strictly passive for all time, and globally exponentially stable when in isolation (i.e., when a person is not coupled to the robot). The robot's admittance parameters are selected so that the robot will resist the user's movement during the biceps curl with a constant desired interaction torque. A saturation function is also employed on the admitted trajectory to ensure boundedness of the signal for use in feedback. The goal of such a design is to allow disabled individuals possessing partial or total loss of upper extremity control to perform intensive, repetitive exercises to regain muscle mass, motor control, and perform a range of motion exercise simultaneously.

II. Dynamics

A. Human Subsystem

The human subsystem dynamics, which consist of the forearm and muscle, are considered as

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$$M_m\ddot{q}(t) + G_m(q(t)) + \tau_p(q(t), \dot{q}(t)) + d_m(t) = \tau_{int}(q(t), \dot{q}(t), \ddot{q}(t), t) + \tau_m(q(t), \dot{q}(t), t), \tag{1}$$

where $q : \mathbb{R}_{\geq 0} \to \mathcal{Q}$ and $\dot{q} : \mathbb{R}_{\geq 0} \to \mathbb{R}$ denote the measurable elbow joint axis angle and computable velocity, respectively, and $\mathcal{Q} \subseteq \mathbb{R}$ denotes the set of safely achievable angles (i.e., the range of motion) by the elbow joint. The inertial and gravitational effects of the forearm are denoted by $M_m \in \mathbb{R}_{>0}$ and $G_m : \mathcal{Q} \to \mathbb{R}$, respectively. The torques applied about the elbow joint by passive viscoelastic tissue forces and unknown disturbances (e.g., spasticity or changes in load) are denoted by $\tau_p : \mathcal{Q} \times \mathbb{R} \to \mathbb{R}$ and $d_m : \mathbb{R}_{\geq 0} \to \mathbb{R}$, respectively. The torque applied on the forearm about the joint axis by the robot's interaction is denoted by $\tau_{int} : \mathcal{Q} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ and the torque applied about the joint axis by the biceps brachii muscle group is denoted by $\tau_m : \mathcal{Q} \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$. The torque applied by the muscle can be expressed as [1]

$$\tau_m = B_m(q(t), \dot{q}(t))\, u_m(q(t), \dot{q}(t), t), \tag{2}$$

where $B_m : \mathcal{Q} \times \mathbb{R} \to \mathbb{R}_{>0}$ is the function relating stimulation current to torque, and is represented by

$$B_m = \lambda(q(t))\, \psi(q(t), \dot{q}(t)) \cos(\beta(q(t))), \tag{3}$$

where $\lambda : \mathcal{Q} \to \mathbb{R}_{>0}$, $\psi : \mathcal{Q} \times \mathbb{R} \to \mathbb{R}_{>0}$, and $\beta : \mathcal{Q} \to \mathbb{R}$ denote the uncertain moment arm of the bicep brachii's output torque about the elbow joint, the uncertain nonlinear function relating stimulation current to muscle fiber force, and the uncertain pennation angle of the muscle fibers, respectively. The subsequently designed muscle stimulation current input is denoted by $u_m : \mathcal{Q} \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$.

B. Robot Subsystem

The robot subsystem dynamics, which consist of a single degree of freedom rotating arm, are considered as

$$M_r\ddot{q}(t) + G_r(q(t)) + \tau_b(\dot{q}(t)) + d_r(t) = \tau_{int}(q(t), \dot{q}(t), \ddot{q}(t), t) + \tau_r(q(t), \dot{q}(t), t), \tag{4}$$

where the robot's inertial and gravitational effects are denoted by $M_r \in \mathbb{R}_{>0}$ and $G_r : \mathcal{Q} \to \mathbb{R}$, respectively. The torques resulting from viscous friction and unknown disturbances are denoted by $\tau_b : \mathbb{R} \to \mathbb{R}$ and $d_r : \mathbb{R}_{\geq 0} \to \mathbb{R}$, respectively. The torque applied about the joint axis by the robot's motor is denoted by $\tau_r : \mathcal{Q} \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ and can be expressed as

$$\tau_r = B_r u_r(q(t), \dot{q}(t), t), \tag{5}$$

[1]For notational brevity, all functional dependencies are hereafter suppressed unless required for clarity of exposition.

where $B_r \in \mathbb{R}_{>0}$ is the electric motor control constant relating input current to output torque, and $u_r : \mathcal{Q} \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ is the subsequently designed motor control current input. The following properties and assumptions are used in the subsequent development.

Property 1. The unknown inertia terms in (1) and (4) can be bounded by $c_{\underline{Mm}} \leq M_m \leq c_{\overline{Mm}}$, and $c_{\underline{Mr}} \leq M_r \leq c_{\overline{Mr}}$, where $c_{\underline{Mm}}$, $c_{\overline{Mm}}$, $c_{\underline{Mr}}$, and $c_{\overline{Mr}} \in \mathbb{R}_{>0}$ are known constants [33].

Property 2. The unknown gravitational terms in (1) and (4) can be bounded by $|G_m| \leq c_{\overline{Gm}}$, and $|G_r| \leq c_{\overline{Gr}}$, where $c_{\overline{Gm}}$ and $c_{\overline{Gr}} \in \mathbb{R}_{>0}$ are known constants [33].

Property 3. The unknown passive viscoelastic tissue torques in (1) can be bounded as $|\tau_p| \leq c_{\overline{P1}} + c_{\overline{P2}}|\dot{q}|$, where $c_{\overline{P1}}$ and $c_{\overline{P2}} \in \mathbb{R}_{>0}$ are known constants [34].

Property 4. The unknown viscous torques in (4) can be bounded as $|\tau_b| \leq c_{\overline{b}}|\dot{q}|$, where $c_{\overline{b}} \in \mathbb{R}_{>0}$ is a known constant [34].

Property 5. The unknown motor control constant in (5) is bounded below by $B_{\underline{r}} \leq B_r$, where $B_{\underline{r}} \in \mathbb{R}_{>0}$ is a known constant.

Assumption 1. The unknown disturbances in (1) and (4) can be bounded as $|d_m| \leq c_{\overline{dm}}$, and $|d_r| \leq c_{\overline{dr}}$, where $c_{\overline{dm}}$ and $c_{\overline{dr}} \in \mathbb{R}_{>0}$ are known constants.

Remark 1. Because the unknown moment arm of the biceps brachii muscle group in (3) about the elbow joint is non-zero, (i.e. $\lambda \neq 0$) [35]; because the unknown nonlinear function relating stimulation current to muscle fiber force can be bounded below as $c_\psi \leq \psi$, where $c_\psi \in \mathbb{R}_{>0}$ is a known constant, provided the muscle is not fully extended [36] or contracting concentrically at its maximum shortening velocity [37]; and because the unknown muscle fiber pennation angle for the biceps brachii muscle group is approximately zero (i.e., $\cos(\beta) \approx 1$) due to parallel muscle architecture [38]; (3) can be bounded below by $B_{\underline{m}} \leq B_m$, where $B_{\underline{m}} \in \mathbb{R}_{>0}$ is a known positive constant.

C. Admittance Model

An additional admittance model is used to describe the human-robot interaction. The admittance model is

$$\tau_{int} = M_d(\ddot{q}_\alpha - \ddot{q}_o) + D_d(\dot{q}_\alpha - \dot{q}_o) + K_d(q_\alpha - q_o) \tag{6}$$

where $q_o : \mathbb{R}_{\geq 0} \to \mathcal{Q}$ is the robot's nominal non-contact trajectory designed such that $q_o(t),\ \dot{q}_o(t),\ \ddot{q}_o(t) \in \mathcal{L}_\infty$ (i.e., bounded). The designed inertial, damping, and spring effects are denoted by $M_d \in \mathbb{R}_{>0}$, $D_d :\in \mathbb{R}_{>0}$ and $K_d :\in \mathbb{R}_{>0}$, respectively. The admitted (or contact) trajectory is denoted by $q_\alpha : \mathbb{R}_{\geq 0} \to \mathcal{Q}$ and generated by the above model using the interaction torque and nominal trajectory. The admittance model in (6) is designed to be passive (i.e., $\tau_{int} \in \mathcal{L}_\infty \Rightarrow q_\alpha \in \mathcal{L}_\infty$), by selecting the admittance parameters such that the transfer function is positive real [39] and critically damped if $D_d = \sqrt{4M_dK_d}$ [40].

A projection algorithm is employed so that a modified (saturated) admitted trajectory, $q_a : \mathbb{R}_{\geq 0} \to \mathcal{Q}$, can be used in feedback that not only preserves $q_\alpha(t)$ when $|q_\alpha(t)| \leq 2q_o(t)$, but also remains bounded. The saturation function also preserves smoothness because if $q_\alpha(t)$ is continuously differentiable, $q_a(t)$ is also continuously differentiable. The saturation algorithm is given by

$$q_a(t) \triangleq q_o\left[2.13\mathrm{erf}\left(\frac{\sqrt{\pi}}{4}\left(\frac{q_\alpha}{q_o} - 1\right)\right) + 1\right], \quad (7)$$

where the Gauss error function is given by $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$ [41]. Note, this saturation function is passive because it is a memoryless system and $q_a(t)q_\alpha(t) \geq 0$ by definition [39, Definition 6.1].

After $q_\alpha(t)$ is passed through the saturation function in (7), and a continuously differentiable $q_a(t)$ is generated, derivatives may be taken to generate the saturated velocity and acceleration, $\dot{q}_a(t)$ and $\ddot{q}_a(t)$ to be used in the subsequently designed controller.

III. Control Development

A. Muscle Position Controller

The muscle's position tracking objective is quantified by $e_m : \mathbb{R}_{\geq 0} \to \mathbb{R}$, defined as

$$e_m(t) \triangleq q_m(t) - q(t), \quad (8)$$

where $q_m : \mathbb{R}_{\geq 0} \to \mathbb{R}$ denotes the desired angular trajectory which is sufficiently smooth (i.e., $\dot{q}_m(t),\ \ddot{q}_m(t) \in \mathcal{L}_\infty$). To facilitate the control development and stability analysis, an auxiliary tracking error $r_m : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is defined as

$$r_m(t) \triangleq \dot{e}_m + \alpha_m e_m, \quad (9)$$

where $\alpha_m \in \mathbb{R}_{>0}$ is a constant control gain. Taking the time derivative of (9), premultiplying by M_m, then substituting (1), (2), and (8) yields the open-loop error system

$$M_m\dot{r}_m = \chi_m - \tau_m - \tau_{int} - e_m. \quad (10)$$

From Properties 1-3, and Assumption 1, $\chi_m : \mathbb{R}^2 \to \mathbb{R}$ is bounded as

$$|\chi_m| \leq c_1 + c_2\|z_m\|,$$

where $c_1,\ c_2 \in \mathbb{R}_{>0}$ are known constants, $\|\cdot\|$ denotes the standard Euclidean norm, and the error vector $z_m \in \mathbb{R}^2$ is defined as $z_m \triangleq [e_m \quad r_m]^T$. Based on (10) and the subsequent stability analysis, the muscle controller is designed as

$$u_m = \frac{1}{B_{\underline{m}}}\left(k_1r_m + k_2 + k_3\|z_m\| + k_4|\tau_{int}|\right)\mathrm{sgn}(r_m), \quad (11)$$

where $k_1,\ k_2,\ k_3,\ k_4 \in \mathbb{R}_{>0}$ denote constant control gains, and $\mathrm{sgn}(\cdot)$ denotes the signum function. Substituting (11) into (10) yields the closed-loop error system

$$M\dot{r}_m = \chi_m - \tau_{int} - e_m - \frac{B_m}{B_{\underline{m}}}\left(k_1r_m + k_2 + k_3\|z_m\| + k_4|\tau_{int}|\right)\mathrm{sgn}(r_m). \quad (12)$$

B. Robot Admittance Controller

The robot's admittance behavior is accomplished through the generation of an admitted trajectory based on the interaction torque. However, the robot still requires an inner loop position controller to track this admitted trajectory. This position objective is quantified by $e_r : \mathbb{R}_{\geq 0} \to \mathbb{R}$, defined as

$$e_r(t) \triangleq q_a - q, \quad (13)$$

where q_a is defined in (7). To facilitate the control development and stability analysis, the auxiliary tracking error $r_r : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is defined as

$$r_r(t) \triangleq \dot{e}_r + \alpha_r e_r, \quad (14)$$

where $\alpha_r \in \mathbb{R}_{>0}$ is a constant control gain. Taking the time derivative of (14), premultiplying by M_r and then substituting (4), (5), and (13) yields

$$M_r\dot{r}_r = \chi_r - \tau_{int} - \tau_r - e_r. \quad (15)$$

By Properties 1-2 and 4-5, and Assumption 1, $\chi_r : \mathbb{R}^2 \to \mathbb{R}$ is bounded as

$$|\chi_r| \leq c_3 + c_4\|z_r\| + c_5|\ddot{q}_a| + c_6|\dot{q}_a|,$$

where $c_3,\ c_4,\ c_5 \in \mathbb{R}_{>0}$ are known constants, and the error vector $z_r \in \mathbb{R}^2$ is defined as $z_r \triangleq [e_r \quad r_r]^T$. Based on (15) and the subsequent stability analysis, a sliding mode controller is designed for the motor input as

$$u_r \triangleq \frac{1}{B_{\underline{r}}}\Big(k_5r_r + k_6 + k_7\|z_r\| + k_8|\dot{q}_a| + k_9|\ddot{q}_a|\Big)\mathrm{sgn}(r_r). \quad (16)$$

Substituting (16) into (15) yields the closed-loop error system

$$M_r\dot{r}_r = \chi_r - \tau_{int} - e_r - \frac{B_r}{B_{\underline{r}}}\Big(k_5r_r + k_6 + k_7\|z_r\| + k_8|\dot{q}_a| + k_9|\ddot{q}_a|\Big)\mathrm{sgn}(r_r). \quad (17)$$

IV. Stability Analysis

Theorem 1. *Given the open-loop error system in (15), the controller in (16), and the admittance relation in (6), the robot is passive with output $y \triangleq r_r$ and input $v \triangleq \tau_{int}$, $\forall t \in [t_0, \infty)$, where $t_0 \in \mathbb{R}_{\geq 0}$ is the initial time and the robot error system is globally exponentially stable for $q \in \mathcal{Q}$ when $v = 0$ in the sense that*

$$\|z_r\| \leq \sqrt{\frac{\overline{\Lambda_r}}{\underline{\Lambda_r}}}\, \|z_r(t_0)\| \exp\left[-\frac{1}{2}\lambda_r(t-t_0)\right], \tag{18}$$

$\forall t \in [t_0, \infty)$, where $\underline{\Lambda_r}, \overline{\Lambda_r} \in \mathbb{R}_{>0}$ are known constants defined as $\underline{\Lambda_r} \triangleq min\left(\frac{\underline{c_{Mr}}}{2}, \frac{1}{2}\right)$, $\overline{\Lambda_r} \triangleq max\left(\frac{\overline{c_{Mr}}}{2}, \frac{1}{2}\right)$, and $\lambda_r \in \mathbb{R}_{>0}$ is defined as

$$\lambda_r = \frac{1}{\overline{\Lambda_r}} min\left(k_5, \alpha_r\right), \tag{19}$$

provided the following constant gain conditions are satisfied:

$$k_6 \geq c_3,\ k_7 \geq c_4,\ k_8 \geq c_5,\ k_9 \geq c_6. \tag{20}$$

Proof: Let $V_r : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ denote a continuously differentiable, positive definite storage function defined as

$$V_r\left(r_r, e_r\right) \triangleq \frac{1}{2}M_r r_r^2 + \frac{1}{2}e_r^2. \tag{21}$$

The storage function satisfies the following inequalities:

$$\underline{\Lambda_r}\left\|z_r\right\|^2 \leq V_r \leq \overline{\Lambda_r}\left\|z_r\right\|^2. \tag{22}$$

Due to the signum function in the control input (16), the time derivative of (21) exists almost everywhere (a.e.) and can be expressed as

$$\dot{V}_r \overset{a.e.}{=} r_r\left(\chi_r - \tau_{int} - \tau_r\right) - \alpha_r e_r^2. \tag{23}$$

Utilizing (16), (23) can be upper bounded as

$$\begin{aligned}\dot{V}_r \overset{\text{a.e.}}{\leq} & \|z_r\|\left|\tau_{int}\right| - k_5 r_r^2 - \alpha_r e_r^2 - \lambda_1\left|r_r\right| - \lambda_2\left|r_r\right|\|z_r\| \\ & - \lambda_3\left|r_r\right|\left|\dot{q}_a\right| - \lambda_4\left|r_r\right|\left|\ddot{q}_a\right|,\end{aligned} \tag{24}$$

where $\lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4 \in \mathbb{R}$ are defined as $\lambda_1 \triangleq k_6 - c_3$, $\lambda_2 \triangleq k_7 - c_4$, $\lambda_3 \triangleq k_8 - c_5$, and $\lambda_4 \triangleq k_9 - c_6$. Provided the gain conditions in (20) are satisfied, $\lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4 \geq 0$; thus, (24) can be upper bounded as

$$\dot{V}_r \overset{\text{a.e.}}{\leq} \|z_r\|\left|\tau_{int}\right| - \delta z_r^2, \tag{25}$$

where $\delta \triangleq \min(k_5, \alpha_r)$. Hence, by [39, Definition 6.3] the robot system is output strictly passive with input $|\tau_{int}|$, output $\|z_r\|$, and storage function V_r. By [39, Lemma 6.5] it follows that the system is finite-gain $\mathcal{L}_2$ stable and its $\mathcal{L}_2$ gain is than than or equal to $\frac{1}{\delta}$. Supposing the robot acts in isolation (i.e., the human is decoupled from the robot), $\tau_{int} = 0$, and (25) can be rewritten as

$$\dot{V}_r \overset{\text{a.e.}}{\leq} -\lambda_r V_r, \tag{26}$$

where λ_r was defined in (19). Hence, the system is zero-state observable by [39, Definition 6.5] and the storage function qualifies as a radially unbounded positive definite Lyapunov function, resulting in global exponential stability in isolation. Because $\tau_{int} = 0$, $q_a = q_\alpha$ and (22) can be utilized in (25) to obtain the result in (18). From the closed-loop error system in (17), the admittance relation in (6), and the saturation function in (7), the robot controller is bounded. ■

Theorem 2. *Given the open-loop error system in (10), the muscle controller in (11) yields global exponential tracking for $q \in \mathcal{Q}$ in the sense that*

$$\|z_m\| \leq \sqrt{\frac{\overline{\Lambda_m}}{\underline{\Lambda_m}}}\, \|z_m(t_0)\| \exp\left[-\frac{1}{2}\lambda_m(t-t_0)\right], \tag{27}$$

$\forall t \in [t_0, \infty)$, where $\underline{\Lambda_m}, \overline{\Lambda_m} \in \mathbb{R}_{>0}$ are known constants defined as $\underline{\Lambda_m} \triangleq min\left(\frac{\underline{c_{Mm}}}{2}, \frac{1}{2}\right)$, $\overline{\Lambda_m} \triangleq max\left(\frac{\overline{c_{Mm}}}{2}, \frac{1}{2}\right)$, and where $\lambda_m \in \mathbb{R}_{>0}$ is defined as

$$\lambda_m = \frac{1}{\overline{\Lambda_m}} min\left(k_1, \alpha_m\right), \tag{28}$$

provided the following constant gain conditions are satisfied:

$$k_2 \geq c_1,\ k_3 \geq c_2,\ k_4 \geq 1. \tag{29}$$

Proof: Let $V_m : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ denote a continuously differentiable, positive definite Lyapunov function candidate defined as

$$V_m\left(r_m, e_m\right) \triangleq \frac{1}{2}M_m r_m^2 + \frac{1}{2}e_m^2. \tag{30}$$

The Lyapunov function candidate satisfies the following inequalities:

$$\underline{\Lambda_m}\left\|z_m\right\|^2 \leq V \leq \overline{\Lambda_m}\left\|z_m\right\|^2. \tag{31}$$

Due to the signum function in the control input (11), the time derivative of (30) exists almost everywhere (a.e.) and can be expressed as

$$\dot{V}_m \overset{a.e.}{=} r_m\left(\chi_m - \tau_m - \tau_{int}\right) - \alpha_m e_m^2. \tag{32}$$

Utilizing (11), (32) can be upper bounded as

$$\begin{aligned}\dot{V}_m \overset{\text{a.e.}}{\leq} & -k_1 r_m^2 - \alpha_m e_m^2 - \lambda_5\left|r_m\right| \\ & - \lambda_6\left|r_m\right|\|z_m\| - \lambda_7\left|r_m\right|\left|\tau_{int}\right|,\end{aligned} \tag{33}$$

where $\lambda_5, \lambda_6, \lambda_7 \in \mathbb{R}$ are defined as $\lambda_5 \triangleq k_2 - c_1,\ \lambda_6 \triangleq k_3 - c_2,$ and $\lambda_7 \triangleq k_4 - 1$. Provided the gain conditions in (29) are satisfied, $\lambda_5, \lambda_6, \lambda_7 \geq 0$; thus, (33) can be upper bounded as

$$\dot{V}_m \overset{\text{a.e.}}{\leq} -\lambda_m V_m, \tag{34}$$

where λ_m was defined in (28). Based on (31) and (34) the result in (27) can be obtained, and from the closed-loop error systems, the muscle controller is bounded. ■

V. Conclusion

In this paper, a rehabilitative robot was successfully designed and controlled to challenge an FES position controller for the biceps brachii muscle group. With this method, the two controllers operate alongside each other, challenging each other, while ensuring safety for the user arm coupled to the robot. A Lyapunov stability analysis was conducted on the muscle subsystem and was shown to be globally exponentially stable. A passivity based analysis was conducted on the robot subsystem, and was shown to be strictly passive with respect to the interaction torque.

By appropriately designing the admittance, the nominal robot trajectory, the saturation function, and the desired interaction torque, the saturated admitted robot trajectory overlapped the desired muscle trajectory. This ensured that as long as the desired interaction torque existed, both controllers could accomplish their tracking objective simultaneously yet challenge each other. Controllers like these are extremely promising for the field of rehabilitation because of their safe performance during exercises that focus on both range of motion and producing useful work from the muscle due to FES. The admitting robot also ensures that it can be utilized by a variety of people with movement conditions or neurological disorders without the need for excessive gain tuning.

Future work will involve making the admittance controller on the robot adaptive in nature to improve overall system performance. Because the robot admits to the interaction torque which arises from the user and the FES controller, the robot will adapt to the user and their performance, leading to customized online rehabilitative therapy. Experiments will be conducted with a focus on individuals with neurological conditions.

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