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# Literature review and fundamental approaches for vehicle and tire state estimation*

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**ABSTRACT**

Most modern day automotive chassis control systems employ a feedback control structure. Therefore, real-time estimates of the vehicle dynamic states and tire-road contact parameters are invaluable for enhancing the performance of vehicle control systems, such as anti-lock brake system (ABS) and electronic stability program (ESP). Today's production vehicles are equipped with onboard sensors (e.g. a 3-axis accelerometer, 3-axis gyroscope, steering wheel angle sensor, and wheel speed sensors), which when used in conjunction with certain model-based or kinematics-based observers can be used to identify relevant tire and vehicle states for optimal control of comfort, stability and handling. Vehicle state estimation is becoming ever more relevant with the increased sophistication of chassis control systems. This paper presents a comprehensive overview of the state-of-the-art in the field of vehicle and tire state estimation. It is expected to serve as a resource for researchers interested in developing vehicle state estimation algorithms for usage in advanced vehicle control and safety systems.

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State estimation; vehicle dynamics; sliding mode observer; Kalman filter; recursive least squares

## Nomenclature

In this section, all symbols used in this work are listed.

| | |
|---|---|
| $r$ | yaw rate |
| $a_y$ | lateral body acceleration |
| $a_x$ | longitudinal body acceleration |
| $\delta$ | steering angle |
| $\delta_{\text{sus}}$ | suspension deflection |
| $v_x$ | vehicle velocity |
| $v_y$ | vehicle lateral velocity |
| $\lambda$ | wheel slip |
| $T_e$ | driveline torque |
| $T_b$ | brake torque |

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*The views and opinions expressed in this article are those of the authors and do not necessarily represent the views of The Goodyear Tire & Rubber Company.

| | |
|---|---|
| $\omega_w$ | wheel speed |
| $p$ | roll rate |
| $T_w$ | wheel torque |
| $\theta_w$ | wheel rotational speed |
| $F_y$ | lateral axle force |

## 1. Introduction

Accurate information about critical tire-vehicle dynamic states is crucial for the successful implementation of advanced chassis control systems, e.g. online computation of the optimised active longitudinal and lateral tire forces to be commanded by the electronic stability control module (Figure 1).

Key attributes such as the vehicle sideslip angle or absolute vehicle speed are difficult to measure within the desired accuracy level because of high costs and other associated impracticalities. Therefore, vehicle control systems currently available on production cars rely on available inexpensive measurements, such as wheel speeds, vehicle accelerations and yaw-rate. There is also unanimous agreement that knowledge about additional states of a vehicle (e.g. vehicle roll angle, tire-road grip level, etc.) can significantly reduce the risk of accidents through effective design and implementation of advanced chassis control systems. As a result, the problem of vehicle state estimation has attracted the considerable attention of many researchers (see Figure 2). Numerous studies have been conducted to estimate vehicle states using both model-based and kinematics-based estimation techniques.

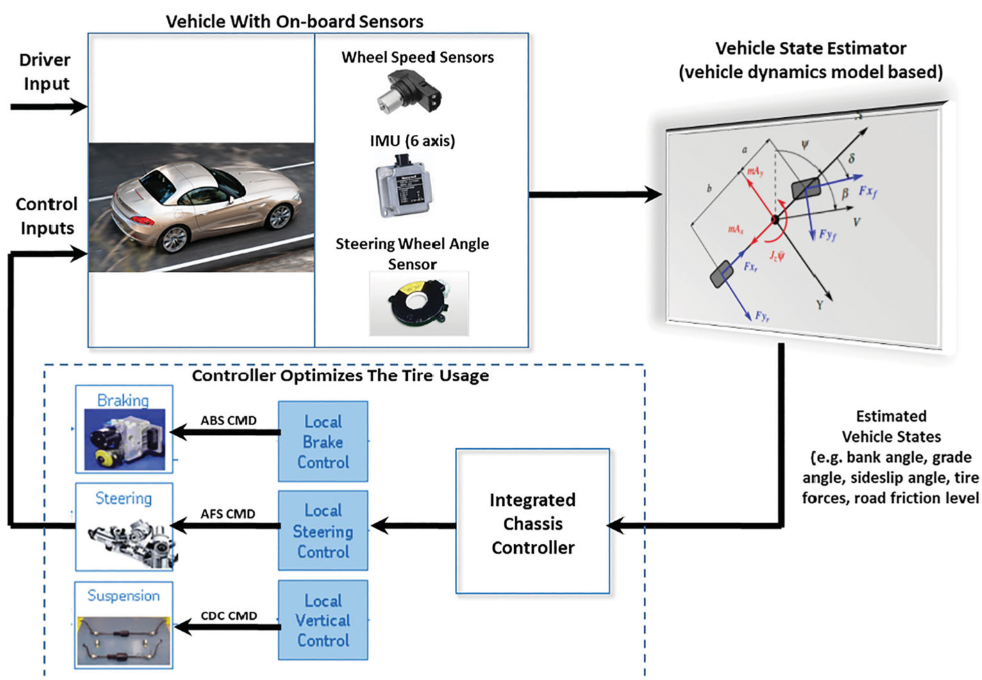



**Figure 1.** Block diagram representation of an integrated chassis control system.

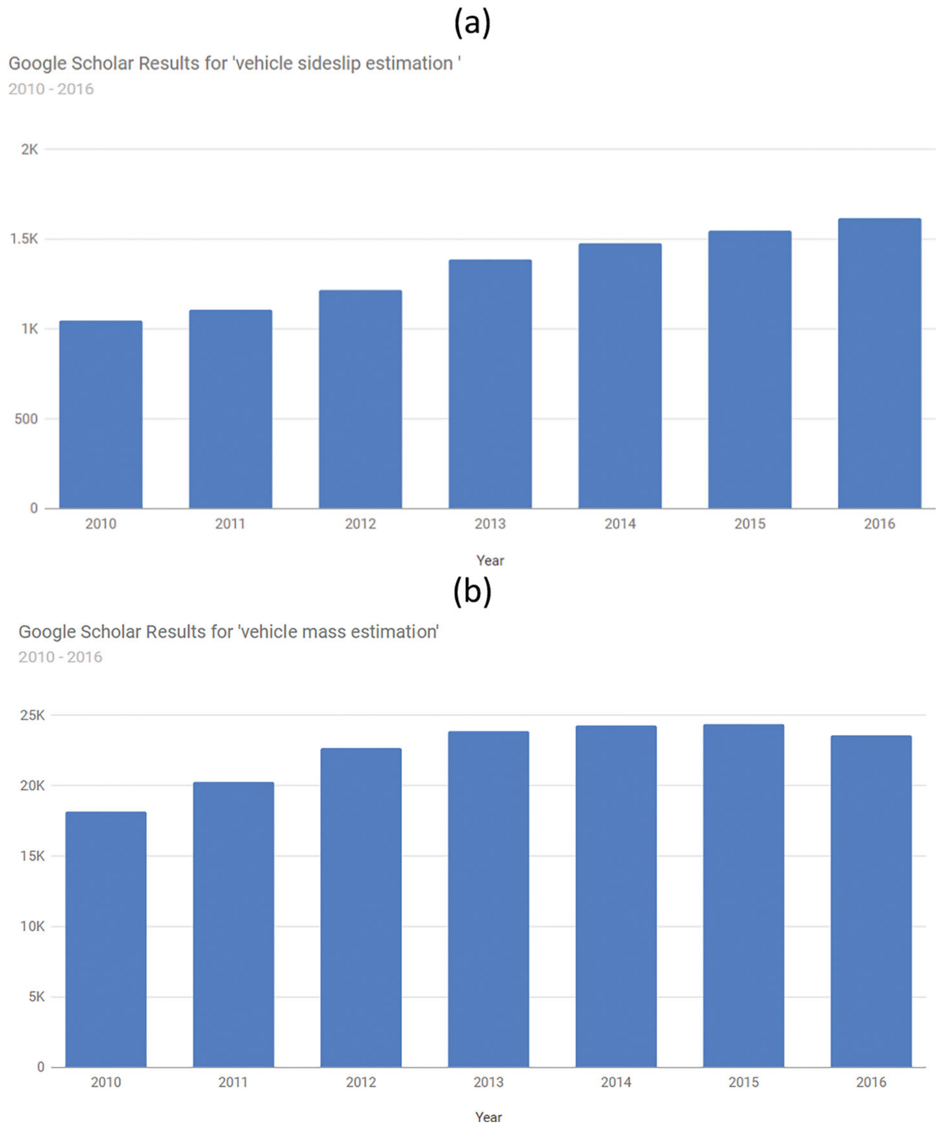



**Figure 2.** Publication trends – Google Scholar search result frequency by year: (a) keyword: vehicle sideslip estimation; (b) keyword: vehicle mass estimation.

This paper presents a state-of-the-art review of estimation techniques utilised in automotive applications. The basic organisation of this paper is as follows: Section 2.1 contains information about the different techniques proposed in the literature for estimating the tire forces. Section 2.2 summarises techniques for road profile estimation. Section 2.3 describes methods for estimating the vehicle lateral velocity, which subsequently is used for estimating the vehicle body sideslip angle. Section 2.4 focuses on the techniques proposed for vehicle roll estimation. Section 2.5 presents a summary of methods for estimating the tire cornering stiffness. Section 2.6 presents a summary of the different applications enabled thorough time and frequency domain analysis of wheel speed signals. Section 2.7 presents a state-of-the-art-review of the different strategies and algorithms used for real-time estimation of the tire-road friction coefficient. Section 2.8 summarises the advancement in the estimation vehicle motion states for active suspension control applications. Section 2.9 presents a comprehensive overview of the different signal processing techniques proposed by researchers to extract meaningful information from tire mounted accelerometers. Section 2.10 covers details about another novel sensor technology, the load sensing bearing, and conclusions are finally given in Section 3.

## 2. Literature review

A summary description of the state-of-the-art in the field of vehicle state estimation is given in Table 1.

### 2.1. Tire force estimation

Considering the cross coupling between different vehicle states, a cascaded observer structure has been employed by numerous researchers. One of the well-used approaches is presented in [1–3], which cascades two estimation blocks sequentially (see Figure 3). The first block provides as inputs the estimated tire forces to the second block, which then utilises these inputs to estimate the dynamic states of the vehicle motion, such as the vehicle sideslip angle. Both sliding mode observer (SMO) and Extended Kalman filter (EKF) methods are tested and evaluated within the cascaded blocks, and in-situ experiments indicate both methods converge close to laboratory measurements. Additionally, authors of [2] conducted a comparative study to investigate the use of different tire models to estimate the vehicle sideslip angle. They reported that an adaptive nonlinear tire model yields to considerably improved results. Adaptation is achieved by updating the tire cornering stiffness. These studies, however, do not assess the observer performance on low friction surfaces or on banked roads.

Road grade and bank angle pose a challenge in the estimation of vehicle states, primarily due to the bias they cause in conventional on-board sensors. In another series of studies, the same authors address the issue of road bank angle by investigating estimation of lateral load transfer and tire normal force [4–6] and methods of correction for such bias. Like their earlier results, their experimental evaluations emerge promisingly close to instrumented measurements. As for the road grade, most of the previous efforts have been based on vehicle longitudinal dynamics models due to the many common driving scenarios for which such models apply. The recursive least squares (RLS) algorithm with multiple forgetting factors has been the most cited method [7,8]. In more recent studies [52], an estimate of road grade is calculated by comparing the acceleration as measured by an on-board longitudinal accelerometer with that obtained by differentiation of the undriven wheel speeds. Knowledge of road grade also enables accurate real-time estimates of vehicle mass using the longitudinal vehicle model and a general recursive least squares (RLS) estimator [53].

There are numerous studies focused on the problem of tire force estimation. A comprehensive example is given in [9], which presents a scheme for simultaneous longitudinal and lateral tire-force estimation using a random-walk Kalman filter. From the simulation results, it is confirmed that the tire-force estimator performs well under various driving situations given that there is sufficient steering wheel excitation. Studies in [10–14]

**Table 1.** State-of-the-art literature review.

| Measurements used | Estimated states | Model used | Estimation methodology | Reference |
|---|---|---|---|---|
| $r, a_y, a_x, \delta$ | Tire forces and vehicle sideslip angle | Single-track model | SMO, EKF | [1–3] |
| $a_y, a_x, \delta_{sus}$ | Tire normal force | Vehicle roll dynamic model | EKF | [4–6] |
| $a_x, v_x, r, \lambda, T_e, T_b$ | Vehicle mass | Longitudinal dynamics | RLS | [7, 8] |
| $r, a_y, a_x, \delta, \omega_w, T_e, T_b$ | Tire forces | Wheel dynamics model, vehicle planar model | KF | [9] |
| $a_y, a_x, \delta_{sus}, r, p, \delta, \omega$ | Tire forces and vehicle sideslip angle | Four-wheel vehicle model | EKF,UKF | [10–14] |
| $a_y, a_x, \delta_{sus}, r, p, \delta, \omega$ | Tire-road friction coefficient and vehicle lateral skid indicator | Four-wheel vehicle model | EKF,UKF, NLLS | [15, 16] |
| $a_y, a_x, \delta_{sus}, r, p, \delta, \omega$ | LTR (Lateral load transfer) and LSI (Lateral skid indicator)- Accident risk prediction | Four-wheel vehicle model | EKF,UKF, NLLS | [17] |
| $a_y, \delta_{sus}$ | Road profile and wheel load | Quarter-car model | KF | [18] |
| $r, a_y, a_x$ | Vehicle sideslip angle | Kinematics model | Nonlinear observer | [19] |
| $r, a_y, a_x, \omega$ | Tire forces | Nonlinear vehicle model | EKF | [20, 21] |
| $T_w, v_x, \theta_w$ | Velocities and accelerations of the wheels, tire forces (vertical and longitudinal) and friction coefficient | Wheel dynamics model | Robust differentiator and sliding modes | [22, 23] |
| $T_w, \omega, r, a_x, a_y$ | Tire forces and vehicle parameter estimation | Wheel dynamics model, vehicle planar model, Friction ellipse | Model based | [24] |
| $r, a_x, a_y, \omega, \delta$ | Tire forces and road grade | Four-wheel vehicle model | EKF, Luenberger observer | [25, 26] |
| $a_x, a_y, \omega, \delta$ | Vehicle sideslip angle and yaw rate | Bicycle model | UKF | [27, 28] |
| $a_x, a_y, r$ | Vehicle sideslip angle | Kinematic model | EKF | [29] |
| $a_x, a_y, v_x, \delta, r$ | Vehicle sideslip angle, lateral tire road forces and tire road friction coefficient | Four-wheel vehicle model | UKF | [30] |
| $F_y$ | Vehicle sideslip angle | Yaw plane model | RLS | [31] |
| $r, a_x, a_y, \omega, \delta$ | Vehicle longitudinal and lateral velocity | Bicycle model | AKF, UKF | [32, 33] |
| $a_x, \omega$ | Vehicle longitudinal velocity | Kinematics-based | Rule Based | [34] |
| $a_x, \omega$ | Vehicle longitudinal velocity | Yaw plane model | KF, Fuzzy logic | [35] |
| $a_y, p$ | Roll angle | Vehicle roll dynamic model | KF | [36, 37] |
| $a_y, r, \delta$ | Roll angle and roll rate | Lateral-dynamics-model a four-degree-of-freedom half-car suspension model | KF | [38, 39] |
| $a_y, p$ | Roll angle | Vehicle roll dynamic model | Closed loop adaptive observer | [40] |
| $a_y, p, \phi_{tilt}$ angle sensor | Roll angle and centre of gravity height | Kinematic sensor fusion, Vehicle roll dynamic model | Sensor fusion | [41, 42] |
| $a_y, p$ | Load Transfer Ratio (LTR) and Predictive Load Transfer Ratio (PLTR) | Vehicle roll dynamic model | Model based | [43] |

(*continued*).

**Table 1.** Continued.

| Measurements used | Estimated states | Model used | Estimation methodology | Reference |
|---|---|---|---|---|
| $a_x, a_y, a_z, p, q, r$ | Roll and pitch angles, longitudinal, lateral, and vertical velocities | Kinematic and model-based (bicycle model) observer | Merging schemes | [44] |
| $a_y, r, \delta$ | Road bank angle | Bicycle model | Transfer function approach, superposition | [45] |
| $a_x, a_y, p, q, r, \omega$ | Vehicle roll and pitch angles | Kinematics-based observer | State observer | [46] |
| $r, a_x, a_y, \omega, \delta$ | Road bank and grade angles | Kinematic model | Observers using time-varying gains | [47] |
| $p$ | Roll angle | Vehicle roll dynamic model | Controlled integration | [48] |
| $a_y, r, v_x, p$ | Roll angle | Vehicle roll dynamic model, Kinematic model | Vehicle state index-based switching | [49] |
| $r, \omega, \delta, a_y, p$ | Vehicle roll angle and Sideslip angle | Kinematic model | Weighting function | [50] |
| $r, \delta, a_y, v_x$ | Tire Slip angle | Bicycle model | State observer | [51] |

Notes: SMO: Sliding mode observer, KF: Kalman Filter, EKF: Extended Kalman Filter, UKF: Unscented Kalman Filter, AKF: Adaptive Kalman Filter, RLS: Recursive least squares, NLLS: Nonlinear least squares. Please also see Nomenclature.

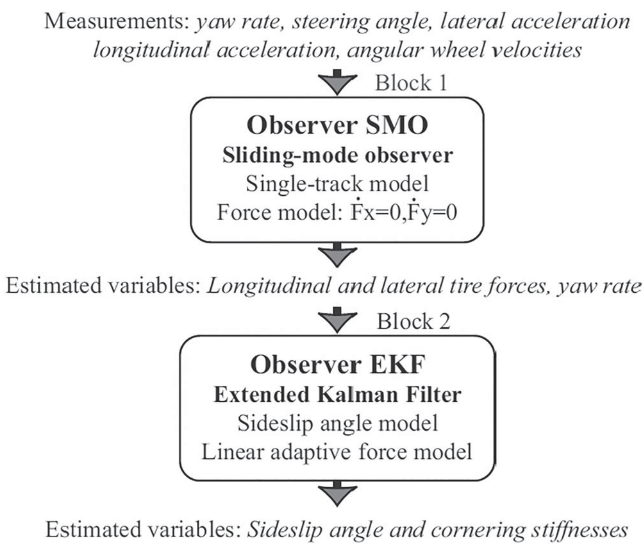



**Figure 3.** The cascade observer structure [1,2].

focus more on the lateral tire forces and examine two observers based on Extended [54] and Unscented [55] Kalman filtering techniques. The EKF is the nonlinear version of the Kalman filter. This non-linear filter linearises about the current mean and covariance using Jacobian matrices. Although EKF is straightforward and simple, it suffers from instability due to linearisation and erroneous parameters and the costly calculation of Jacobean matrices. Instead of linearising and using Jacobian matrices, the UKF uses a deterministic sampling approach to capture the mean and covariance estimates with a minimal set of sample points [56]. A simplified four-point contact vehicle model is utilised where the contact points are modelled by the Dugoff tire model. The Dugoff tire model is a simple analytical model that incorporates both longitudinal and lateral dynamics to calculate the tire-road force characteristics. It assumes a steady state tire behaviour. The effects of camber and turn slip are neglected. Furthermore, a uniform vertical pressure distribution is assumed in the tire. The basic principle of this model is that the linear tire force is calculated, which is then modified using the shape function [57]. The proposed scheme yields to the estimation of lateral forces on individual wheels along with lateral vehicle states such as vehicle sideslip angle. A comparison with experimental data demonstrates the potential of the developed process. In [20,21], an extended Kalman filter-based method is presented to estimate the dynamic states and tire-road forces for a nonlinear vehicle model. In [22,23], cascaded observer based on first and second order sliding modes are used to estimate the contact forces. The authors in [24] presents a tire force estimator, designed by accounting for the dependency between the longitudinal and lateral tire forces by introducing the friction ellipse into the estimation algorithm. In addition, the vehicle parameters are estimated online to alleviate the influence of variations in the model parameters on the lateral tire force estimation performance.

### 2.2. Road profile estimation

Road profile elevation is an essential input to vehicle dynamics models. Hence, an accurate knowledge of this data is essential for a better understanding of vehicle dynamics and control systems design. In [18], authors present a method to estimate the road profile elevation based on a classical Kalman filter. Once road profiles are estimated, it is possible to calculate the vertical forces on each wheel. In [25,26], a method for the estimation of vehicle states by an EKF, and to reconstitute the road slope using a Luenberger observer is presented. The estimation results have been compared to measurements collected with a prototype vehicle running on a test track. The authors in [15–17] describe a method for assessing the risk of vehicle skidding. The method is based on a nonlinear optimisation technique (Levemberg–Marquardt) applied to an error function between the forces estimated by observers installed in the vehicle and those calculated by a theoretical tire/road interaction model (Dugoff model).

### 2.3. Vehicle velocity estimation

The tire forces can be utilised in many different virtual sensing applications, for instance for the estimation of critical states such as the vehicle lateral velocity. The study in [31] provides a good example for the advantages of being able to compute real-time tire forces. The authors use lateral forces obtained from a multi-sensing hub unit to estimate vehicle lateral velocity and roll angle based on a recursive least square algorithm and a Kalman filter. Estimation performances and robustness of proposed estimators were discussed and evaluated by field tests on dry asphalt and a slippery road. The authors in [19] present a nonlinear vehicle sideslip observer (NVSO) with reduced computational complexity as compared to an extended Kalman filter. The observer is suitable for implementation in embedded hardware and has a reduced number of tuning parameters compared to the EKF. Nevertheless, because of the switching between operating regimes and the various thresholds involved in the switching logic, tuning the NVSO is nontrivial. The authors in [27–30] present the development of a nonlinear observer using unscented Kalman filter (UKF) to estimate sideslip angle. The authors in [32,33] present a vehicle lateral and longitudinal

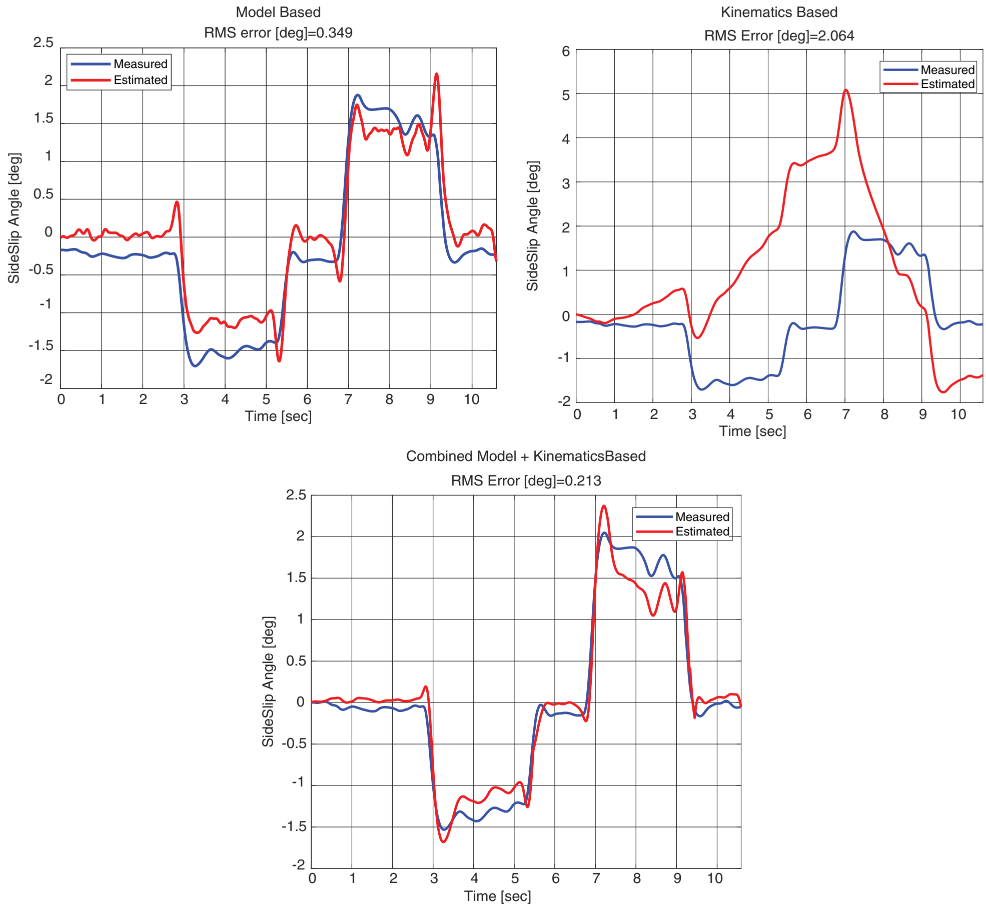



**Figure 4.** Sideslip estimation performance combining a model-based observer and a kinematics-based observers – experimental validation.

velocity estimation method using an adaptive/unscented Kalman filter. This method was evaluated under a variety of manoeuvres and road conditions (Figure 4).

From all these studies, it is fair to conclude that estimates from a pure model-based observer tend to deviate from the actual values because of mismatch between the vehicle actual parameters and those used by the model. Estimated from a pure kinematic based observer is prone to drift due to bias errors in the accelerometers and gyroscopes. In [58], a fusion-based method combing a model based observer and a kinematics-based observer are presented. The authors of this paper experimentally evaluated the performance of the approach presented in [58] and confirmed that the proposed algorithm can provide reasonably accurate estimates of the vehicle sideslip angle.

The authors in [34] present a new algorithm for the estimation of longitudinal vehicle speed, based on the measurements of the four-wheel rotational speeds and of the longitudinal vehicle acceleration. The main advantage of this approach is the low computational burden – which makes implementation on a commercial vehicle Electronic Control Unit (ECU) effective. The proposed algorithm was extensively tested on an instrumented test car in different driving and road conditions. In [35], a fuzzy logic is used to get an estimate of the vehicle longitudinal velocity; together with the estimated vehicle longitudinal acceleration, a Kalman filter is used to estimate the velocity of vehicle for use in ESC control applications. The fuzzy logic is used to get a first estimate of vehicle velocity. Subsequently, a Kalman filter is used to fuse this first estimate of the vehicle velocity with the vehicle longitudinal acceleration, to get a more precise estimate the vehicle velocity for use in an electronic stability controller (ESC).

### 2.4. Vehicle roll angle estimation

The studies summarised above generally refer to the planar motion of the vehicle and use the so-called two-track or bicycle model to represent vehicle dynamics within the observer structure. On the other hand, systems that are related to the body roll or pitch require inclusion of the vertical motion which than require an extended model so that additional states can be estimated. The authors in [36,37] present a Kalman filter-based approach to estimate roll angle and roll rate with either a three-degree-of freedom (3DOF), or 1DOF vehicle model. In [38], an estimator designed based on a 3DOF vehicle manoeuvring model and a 4DOF half-car suspension model is used to obtain estimates of the vehicle roll angle and roll rate in driving situations in which both manoeuvring and road disturbances affect the vehicle roll motions. In [40], an approach using a closed-loop adaptive observer for estimating roll angle and roll rate of vehicle body with respect to the road is proposed. The authors in [41,42] focuses on algorithms to estimate roll angle and centre of gravity (CoG) height. The algorithms investigated include a kinematic sensor fusion algorithm that utilizes a low-frequency tilt angle sensor and a gyroscope and a dynamic observer that utilizes only a lateral accelerometer and a gyroscope. The kinematic based sensor fusion algorithm combines the low-frequency content of the angle estimate from the tilt sensor with the high-frequency content of the gyroscope using a pseudo integration method. This helps eliminate the drift from integration of the gyroscope. In [43], two rollover indexes are proposed and analysed. The first rollover index estimates the actual Lateral Transfer Ratio (LTR) while the second, referred to as Predictive Lateral Transfer Ratio (PLTR), incorporates the predictive influence of the driver's steering input. The authors in [44] focuses on the accurate estimation of the vehicle states, including the longitudinal, lateral, and vertical velocities, as well as the roll and pitch angles, using merging schemes, that combine the kinematic and model-based observer outputs. The authors in [45–47] present methods for estimation of road inclination and bank angle. In [48], a scheme for the vehicle roll angle is derived based on the combination of sensors from vehicle dynamics control system and a rollover mitigation system. In [49,50], method for compensating the gravity components of the lateral acceleration is proposed. The authors in [51] presents a model-based estimation method that utilises pneumatic trail information in steering torque to identify a vehicle's lateral handling limits.

### 2.5. Tire cornering stiffness estimation

The techniques listed in Table 1 are based on a physical vehicle model, usually including a model of the tire-road friction forces. There is a main argument against using such a model due to its inherent uncertainty. Changes in the loading of the vehicle and the tire

**Table 2.** Factor affecting the tire cornering stiffness.

| Influencing factor | Effect on the tire cornering stiffness | Reasoning |
|---|---|---|
| Inflation pressure | Moderate | Caused by a variation in the carcass stiffness and tread stiffness (due to change in contact patch area) [65] |
| Tire wear | High | Caused by a variation in the tread stiffness [65] |
| Tire temperature | High | Caused by a variation in the rubber elasticity (modulus) [60] |
| Tire aging | High | Caused by stiffening on tread rubber [61] |

characteristics, for example, introduce unknown variations in the model. The cornering stiffness of a tire is known to be dependent on several factors (see Table 2), which are known to change as result of driving, operating and environmental conditions [59–61]. Consequently, estimation methods based on vehicle lateral/yaw dynamic equations (mostly based on the bicycle model) need to be made robust with respect to tire cornering stiffness changes.

In [62,63], algorithms for online estimation of the tire cornering stiffness during high excitation manoeuvres (steering frequency $> 0.5$ Hz) are presented. Among all the methods studied, the beta-less method has been found to have the highest potential for field implementation [64].

However, for most approaches presented in literature, the estimation accuracy of the algorithm is high only for driving manoeuvres that involve a high-frequency steering wheel excitation and not for manoeuvres with a low-frequency excitation. This is also corroborated in the benchmarking study conducted by the authors of this paper (Figure 5). The observer does not give satisfactory results in the steady state cornering manoeuvre. The estimates diverge from the actual measurements. In [66,67], an online cornering stiffness observer for low-frequency manoeuvres on public roads is presented. The estimation method presented shows good estimation results even under less extreme manoeuvres on public roads. The relative error of the cornering stiffness estimate was about 15%. Another set of methods without any need of vehicle parameters are using black-box regression models [68] which are non-linear models based in neural networks. For instance, after training the models with a measured sideslip angle and ESC sensor data and its derivatives of the order n, it is possible to estimate the sideslip angle. This method delivers very accurate estimation results, though only when working within the constraints of the trained operating conditions. For applying the estimator on different vehicle variants with different tire dimensions or vehicle mass etc., new data for training in every important operation point must be collected which results in huge time and costs efforts.

### 2.6. *Wheel speed signal analysis*

Wheel speed sensors are one of the most important sensors on a vehicle and used not only for ABS but also for various control systems. Researchers have extensively exploited wheel speed signals to estimate the tire and vehicle states indirectly. For instance, wheel speed signals are used in indirect tire pressure monitoring systems [69] to detect pressure loss (Figure 6).

Table 3 presents a summary of the different applications enabled through time and frequency domain analysis of wheel speed signals.

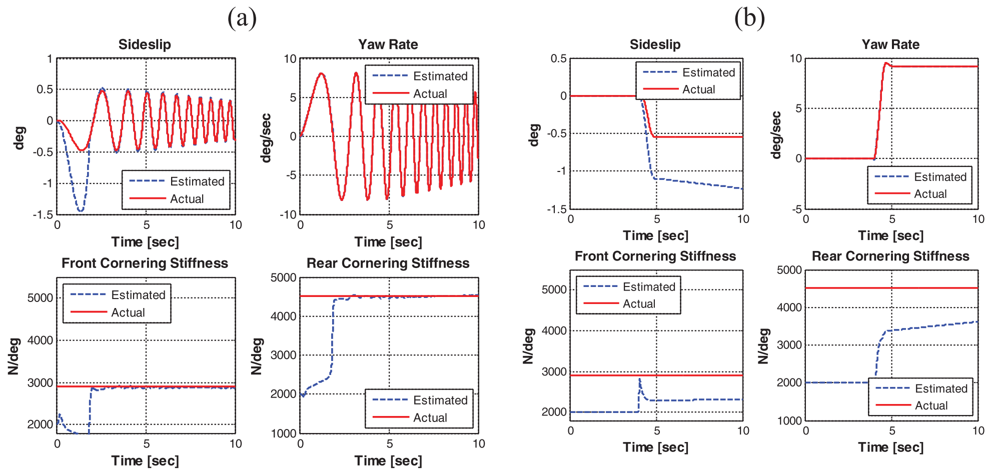



**Figure 5.** Cornering stiffness observer estimation performance: (a) Manoeuvre: Sine Sweep 0.25–4 Hz – good convergence seen (b) Manoeuvre: steady-state circular test – no convergence.

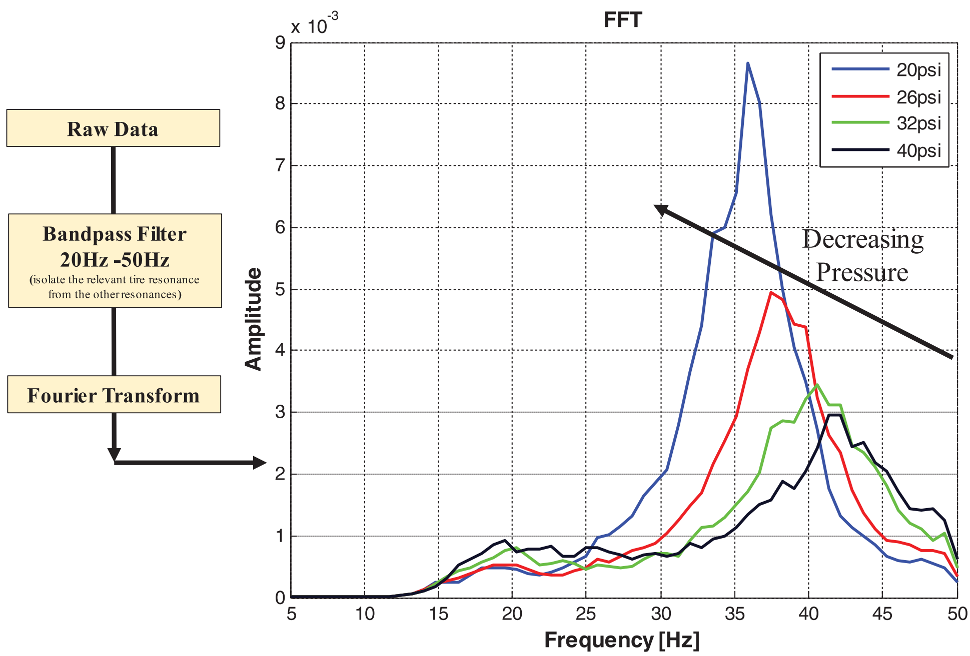



**Figure 6.** Monitoring change in tire torsional mode (30–50 Hz) with a change in the tire inflation pressure. Wheel speeds recorded at 100 Hz using a Racelogic VBOX 3i unit.

Most of these applications require high-resolution wheel speed signals, which poses some challenges. The wheel speed sensor unit consists of a toothed wheel that is subject to periodic pulse width errors. These pulse width errors occur at each edge of the sensor toothed wheel and are caused by mechanical tolerances during production. These pulse width errors reduce the quality of the wheel speed signal. Hence, is it important to pre-process the wheel speed signals [79].

**Table 3.** Usage of wheel speed signals.

| State estimated | Underlying physics | Reference |
|---|---|---|
| Tire wear state | Monitor shift in the 2nd torsional mode frequency (80–100 Hz)<br>Monitor tire slip behaviour during braking<br>Monitor change in the tire rolling radius | Singh et al.: US Patent 2016 [70]<br>Singh: US Patent 2017 [71]<br>Unterreiner: US Patent 2016 [72] |
| Vehicle loading state | Monitor increase in amplitude of the 1st torsional mode frequency (30–50 Hz) | Lee et al.: SAE 2017 [73] Kawasaki: US Patent 2015 [74] |
| Wheel imbalance state | Monitor wheel hop motion | Lu et al.: SAE 2011 [75] Dagh et al.:[76] |
| Absolute vehicle speed | Use time delay between the front wheel and rear wheel speed signals. | Gustavsson et al.: US Patent 2010 [77] |
| Road surface condition | Analyse wheel slip histogram | Engel et al.: US Patent 2007 [78] |

Numerous methods have been proposed in literature to estimate the absolute (i.e. reference) speed of a vehicle [80]. The commonly proposed method is to use wheel speed sensors, which accurately measure the angular velocity of the wheels. Wheel speed is proportional to the vehicle speed. However, the proportionality constant, the wheel rolling radius, is in general known to vary. The wheel rolling radius is known to change with tire inflation pressure, wheel loading condition, tire rolling speed and the remaining tread depth of the tire. Authors of this paper conducted a sensitivity study for tire rolling radius. Results are given in Figure 7. This makes vehicle speed estimates based on wheel speed sensor not so accurate in comparison to the actual speed over ground. Typically the error range is in $\pm 5\%$ [81]. Alternative solutions for everyday vehicle applications include GPS based velocity measurements or the usage of inertial measurement units. GPS access may be limited in tunnels, indoors environments and due to sensor imperfections, methods based on integration of accelerometer measurements yield large errors after short periods of time.

### *2.7. Tire-road friction estimation*

Apart from tire and vehicle dynamic states, instantaneous knowledge of the tire-road friction potential is expected to result in an improved performance of several of the active chassis control systems [82,83]. Examples of vehicle control systems that can benefit from the knowledge of tire-road friction include anti-lock braking systems (ABS), electronic stability control (ESC), adaptive cruise control, and collision warning or collision avoidance systems [84,85]. The quality of traffic management and road maintenance work (e.g. salt application and snow plowing) can also be improved if the estimated friction value is communicated to the traffic and highway authorities [86,87]. The importance of friction estimation is reflected by the considerable amount of work that has been done in the area (Table 4).

Lateral dynamics-based techniques can be utilised primarily while the vehicle is being steered.

Longitudinal dynamics-based techniques are in general applicable during vehicle acceleration and deceleration. Determination of friction coefficient is straightforward in cases where tire forces are saturated, such as under hard braking conditions. The difficulty lies in obtaining a friction estimate under more normal driving circumstances, in which the tire slip is smaller (lower utilisation conditions). The required utilisation of friction necessary

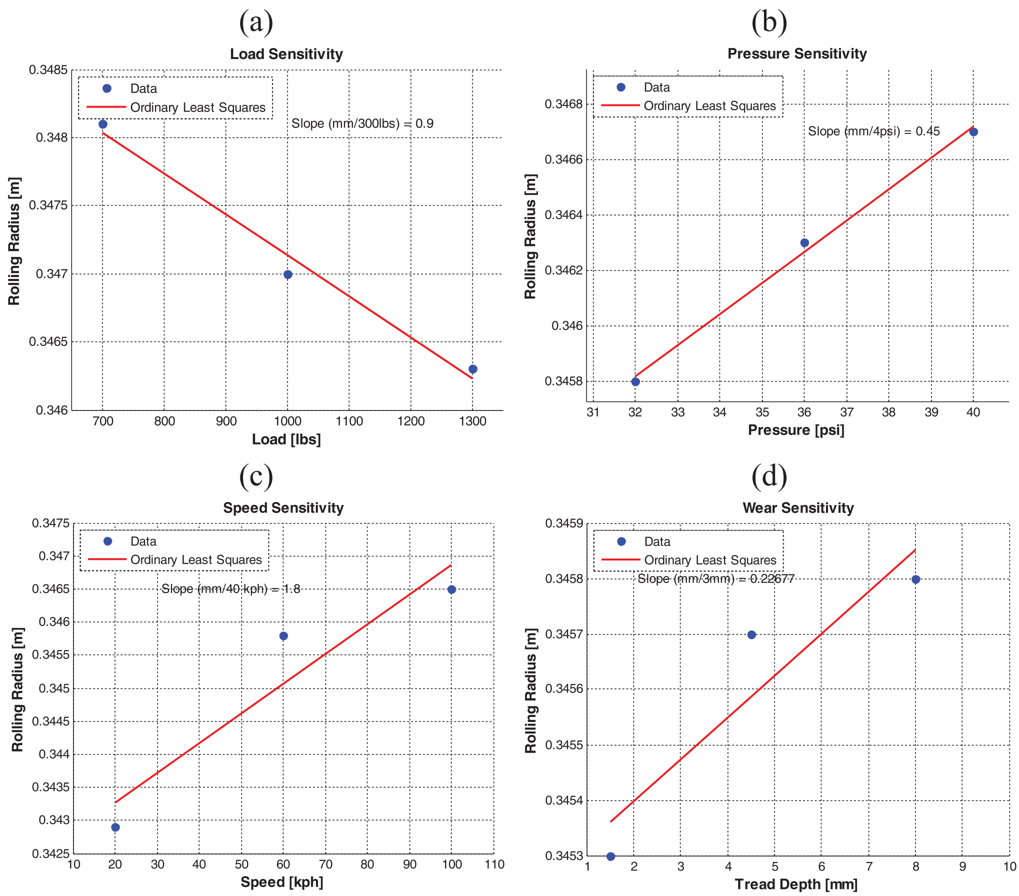



**Figure 7.** Rolling radius sensitivity to: (a) tire load, (b) inflation pressure, (c) rolling speed, and (d) tire tread depth. Measurements made indoors on a test drum.

to provide a road friction estimate within an accuracy range of ±10% is summarised in Figure 8. In the case of the “Force–Slip Regression Method” more than 75–80% of the available friction force must be generated before an accurate estimate can be derived. It is possible to estimate the tire-road friction coefficient for lower levels of utilisation (∼30–40%) if SAT (“Moment–Slip Regression Method”) is used as a basis for the estimator instead of the lateral force (“Lateral Force–Slip Regression Method”). One of the most promising approaches for friction estimation documented in literature is the longitudinal slip slope method, wherein the longitudinal stiffness of a tire is assumed to change in a near linear manner with the tire-road friction level. Another promising approach is to monitor the damping of the torsional frequency mode of a tire (typically between 35 and 45 Hz) under free rolling conditions.

## 2.8. Vertical state estimation

Another research topic related to state estimation that has garnered considerable attention is the concept of system state estimation for active suspension control. This is driven

**Table 4.** State-of-the-art literature review.

| Tire road friction estimation | | |
|---|---|---|
| **Lateral dynamics based** | **Longitudinal dynamics based** | **Torsional dynamics based** |
| Pasterkamp: JVSD 1997 [88]<br>Yasui: SAE 2004 [89]<br>Klomp: AVEC 2006 [90]<br>Hsu: PhD Thesis 2009 [91]<br>Erdogan:PhD Thesis 2009 [87]<br>Andersson: IVSS 2010 [92]<br>Ahn: PhD Thesis 2011 [93]<br>Nishihara: ASME 2011 [94]<br>Matilainen: IEEE 2011 [95]<br>Li: Elsevier 2014 [96]<br>Han: IEEE 2016 [97] | Germann: IEEE 1994 [98]<br>Gustafsson: Automatica 1997 [99]<br>Muller: ASME 2003 [100]<br>Lee: IEEE 2004 [101]<br>Li: IMechE 2007 [102]<br>Svendenius: PhD Thesis 2007 [103]<br>Rajamani: IEEE 2010 [104]<br>Andersson: IVSS 2010 [92]<br>Ahn: PhD Thesis 2011 [93]<br>Jonasson: US Patent 2016 [105]<br>Singh: US Patent 2017 [106]<br>Han: IEEE/ASME 2017 [107] | Umeno: SAE 2002 [108]<br>Pavkovi: SAE 2006 [109]<br>Schmeitz: 2014 [110]<br>Schmeitz: VSD 2016 [111] |

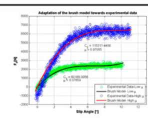
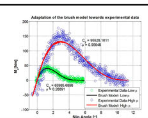
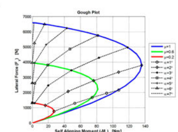
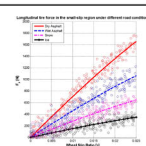
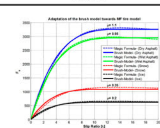
| Excitation Level | Estimator | Underlying Principle |
|---|---|---|
| Large Lateral Excitation (80 -100%) | **Tire Model Used: Brush Model**<br>$F_y = C_y\tan(\alpha) - \left(\frac{1}{3}\frac{C_y^2\lvert\tan(\alpha)\rvert\tan(\alpha)}{\mu F_z}\right) + \left(\frac{1}{27}\frac{C_y^3\tan^3(\alpha)}{(\mu F_z)^2}\right)$<br>**Estimation Algorithm: NLLS** | **"Force-Slip Method"**<br>**Fy v/s Slip angle**<br> |
| Medium Lateral Excitation (50 -80%) | **Tire Model Used : Brush Model**<br>$\tau_a = \frac{C_y\tan(\alpha)a}{3}\left(1-\left\lvert\frac{C_y\tan(\alpha)}{3\mu F_z}\right\rvert\right)^3$<br>**Estimation Algorithm: NLLS** | **"Moment-Slip Method"**<br>**Mz v/s Slip angle**<br> |
| Small Lateral Excitation (30 -50%) | **Tire Model Used : Brush Model**<br>$F_y = C_y\tan(\alpha) - \left(\frac{1}{3}\frac{C_y^2\lvert\tan(\alpha)\rvert\tan(\alpha)}{\mu F_z}\right) + \left(\frac{1}{27}\frac{C_y^3\tan^3(\alpha)}{(\mu F_z)^2}\right)$<br>$\tau_a = \frac{C_y\tan(\alpha)a}{3}\left(1-\left\lvert\frac{C_y\tan(\alpha)}{3\mu F_z}\right\rvert\right)^3$<br>**Estimation Algorithm: NLLS** | **"Force-Moment Method"**<br>**Fy v/s Mz**<br> |
| Small Longitudinal Excitation (0 -2%) | **Tire Model Used : Linear Model**<br>$F_x = C_x\lambda$<br>**Estimation Algorithm: RLS** | **"Force-Slip Method"**<br>**Fx v/s Slip ratio**<br> |
| Large Longitudinal Excitation (30 -100%) | **Tire Model Used : Brush Model**<br>$F_x = C_x\left(\frac{\lambda}{\lambda+1}\right) - \left(\frac{1}{3}\frac{C_x^2\left(\frac{\lambda}{\lambda+1}\right)\left(\frac{\lambda}{\lambda+1}\right)}{\mu F_z}\right) + \left(\frac{1}{27}\frac{C_x^3\left(\frac{\lambda}{\lambda+1}\right)^3}{(\mu F_z)^2}\right)$<br>**Estimation Algorithm: RLS** | **"Force-Slip Method"**<br>**Fx v/s Slip ratio**<br> |

**Figure 8.** Required utilisation of friction (in percent) to achieve a friction estimate within an accuracy of ±10%.

by the need to develop more advanced control systems for semi-active and fully active suspension systems are becoming more and more common on production vehicles. As other vehicle motion attributes, the nonlinearities become a significant challenge for such systems. To be able to overcome this, various linearisation and integration methods have been utilised. In [112], the authors refer to the concept of Takagi–Sugeno to establish an

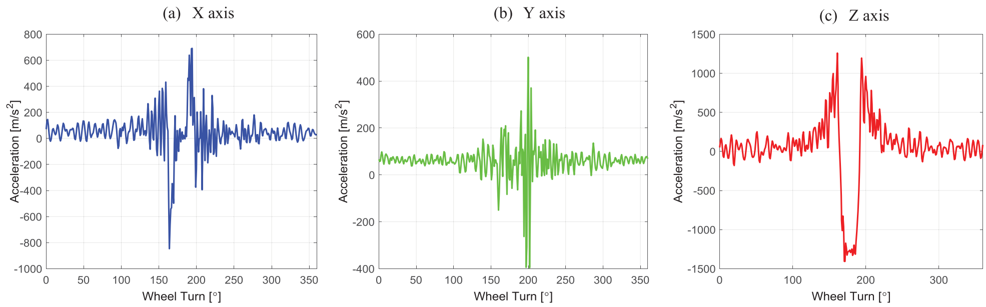



**Figure 9.** Signal for one-wheel turn from a tire attached tri-axial accelerometer. Sensor data sampled at 4000 Hz.

observer design for nonlinear state estimation in an actively controlled vehicle suspension application. Another challenge in observing vehicle vertical motion is measurement of the environmental inputs, such as the road profile or variations on mass. Authors of [113] utilise a linear quarter car model to design a virtual sensor estimating unmeasured state variables subject to unknown road inputs. Another approach as proposed in [114] is by using an adaptive super-twisting sliding mode observer (SMO) for state and unknown input estimation for the active suspension system. Super-twisting SMO belongs to the second order sliding mode approach that allows for finite-time convergence to zero of not only the sliding variable but its derivative as well, through a continuous control acting discontinuously on its second-time derivative [115].

For observing supplementary states such as sprung and unsprung mass motion, more conventional methods have been proposed. In [116], a Kalman Filter algorithm is constructed for bounce velocity estimation. [117] presents the design and development of a state estimator that accurately provides the information required by a sky-hook controller, using a minimum number of sensors. In [118], a road-frequency adaptive control for semi-active suspension systems is investigated. By using the data measured from a relative displacement sensor, a state estimator based on a Kalman filter for estimating the required state variables is designed. Road disturbance frequencies are estimated by using a first order zero-crossing algorithm. In [119], an estimator structure for active vehicle suspension control incorporating three parallel Kalman filters has been presented.

### 2.9. *Intelligent tire technology*

Although the methods based on chassis attached sensors present a relatively accurate solution, they rely heavily on tire and vehicle kinematic formulations and break down in case of abrupt changes in the measured quantities. To address this problem, researchers have been developing a certain sensor based advanced tire concepts for a direct measurement of critical tire states. The terms “Intelligent Tires” and “Smart Tires,” which mean online tire monitoring, are enjoying increasing popularity among researchers and automotive manufacturers. Most of the recent studies have evaluated the possibility of using a tire mounted accelerometer for tire state estimation (Figure 9).

Table 5 presents a summary of the different applications enabled through time and frequency domain analysis of tire accelerometer signals.

**Table 5.** State-of-the-art literature review – intelligent tires.

| State estimated | Underlying physics | Reference |
|---|---|---|
| Tire vertical force | Uses an empirical model to describe the shape of the radial acceleration signal - Vertical load is treated as an unknown parameter and is estimated used an EKF observer | Teerhuis et al. [120] |
| Tire longitudinal force | The observer contains a physical tire model. The Flexible Ring Tire model is adapted such that the tire belt deformation is calculated for prescribed contact patch boundary conditions. These boundary conditions are calculated using the vertical and longitudinal tire forces, which are the states of the model | Goos et al. [121] |
| Tire road friction propensity | Friction potential estimated through frequency domain analysis of the accelerometer signals | Niskanen [122]<br>Singh et al. [83] |
| Tire aquaplaning propensity | Remaining tire road contact length is determined based on the tangential acceleration signal | Niskanen [122] |
| Water depth | To detect the presence of water in the tire-road contact, the lateral acceleration signal is utilised. Since normal excitation from the road surface is lowest in the lateral direction, all external excitation produces rather noticeable difference. This is the case also with the turbulent water flow in the contact. | Niskanen [122] |

**Table 6.** State of the art for load sensing bearing technology.

| State estimated | Underlying physics | Reference |
|---|---|---|
| Wheel Forces | The strain on the outer ring of the bearing is measured and corrected based on the pass-by frequency of the rolling element | Nishikawa [123] |
| Wheel Slip and Forces | A decision-based hybrid algorithm yields to information regarding peak friction using the real-time feedback of the load-sensing bearing, which is primarily used in an ABS application. | Kerst et al. [124] |
| Tire road friction propensity | Friction potential estimated through the force feedback on the bearing sensor, combined slip and an SMO scheme. | Madhusudhanan et al. [125] |

### 2.10. *Load sensing bearing technology*

Another novel sensor technology has been developed for the mechanical bearings situated on the wheel hubs. In addition to the conventional speed encoders, the primary feedback on these types of sensors are related to the moments and forces, therefore these systems are generally called load-sensing bearings. The changes in the strain levels of the inner or outer rings of the bearing yields to an accurate estimate of the force and moment applied which then needs to be tuned for the specific construction using innovative methods. Table 6 summarises most relevant solutions and products introduced so far.

## 3. Discussion

Key vehicle state attributes for which numerous estimation algorithms have been proposed in literature include:

- Absolute vehicle speed

- Vehicle sideslip angle
- Tire road friction coefficient

The most commonly used state estimation algorithms include: SMC, KF, EKF, UKF.

- Both UKF and EKF have been found to be effective at identifying simple or complex vehicle models [81]. Although they use different methods for parameter error covariance estimation, both techniques have identical convergence characteristics and yield near-identical models. Unlike an EKF based observer, a UKF based observer avoids the need to calculate Jacobians and is computationally less expensive and easier to implement. It is also easily extended to system identification and dual estimation problems, in a similar fashion to the EKF. The convergence rate for these observers is primarily influenced by the quality of the excitation (i.e. in conditions of high excitation, a fast convergence of the estimation residuals is observed) and state covariance matrix tuning [96]. Hence, for real-world implementation, it is necessary to tune the Kalman filter covariance matrix Q and R. The disadvantage of Kalman filter-based estimators is that the optimality of the estimation algorithm depends on the quality of a priori knowledge of the process and measurement noise statistics. More recently, Artificial Intelligence (AI) algorithms have been proposed to eliminate some of its inadequacies. For instance, in [97, 98], AI-based algorithms are used to estimate the sideslip angle based an Adaptive Neuro-Fuzzy Inference System (ANFIS) [126]. ANFIS integrates both neural networks and fuzzy logic principles, hence has the potential to capture the benefits of both in a single framework.

Most model-based observers require adaptation of the model parameters.

- Different driving conditions, such as the number of passengers and seating arrangement, cause the inertial parameters to vary. However, most model-based observers proposed in literature assume fixed inertial parameters. Due to inherent interdependency of vehicle states and parameters, it is not realistic to separately solve the problems of vehicle state estimation and parameter identification. A variety of approaches have been proposed lately in literature [127,128] focused on combined estimation of vehicle states and identification of parameters. As these inertial parameters substantially influence the longitudinal and lateral dynamics of a vehicle, the availability of an accurate estimate could significantly improve the performance of the vehicle state observers.

Some of the shortcomings present in the state-of-the-art estimation techniques will be addressed in the future. General indication within most of the recent publications shows that more researchers will start using machine learning and data-driven modelling techniques for vehicle state estimation. Also expected is fusion of radar (or lidar) data, as these sensors are expected to become standard components on next-generation electric vehicle and autonomous vehicles [129–131].

## 4. Conclusion

This paper provides a comprehensive review of relevant works about virtual sensing and state estimation methods in vehicle dynamics and controls applications. Distinct approaches based on kinematics and model-based observers have been reviewed and the main advantages and drawbacks for these estimation methods have been discussed. As has been evidenced, model-based approaches require significant excitation levels to achieve an accurate estimation of the vehicle- and tire-states. In brief, additional investigations are needed to prove the efficacy of the proposed methods on production vehicles under daily drive cycles. Apart from model-based approaches, other non-model-based methods have also been proposed, namely, kinematic-based, black-box regression, neural networks, machine learning, etc., however, the effectiveness of these methods is also not proven on production vehicles. Therefore, the most suitable approach so far is a fusion of outputs from model-based and non-model-based methods.

Finally, a brief discussion has been presented concerning upcoming technologies related to intelligent tires and load sensing bearings. Even though these technologies are currently not matured for production vehicles, there is significant research and development work ongoing by major tire and bearing manufacturers. It is, therefore, reasonable to conclude that these technologies will make their way into series production vehicles in the next three to five years. The key value proposition for usage of virtual sensing techniques for state estimation in vehicles is the potential they offer in improving the performance of vehicle control systems. This would be of even more relevance in the case of autonomous vehicles, considering their high safety requirements.

## Disclosure statement

No potential conflict of interest was reported by the authors.

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