Reliability of Geosynthetic Reinforced Soil Structure Design with Probabilistic and Finite Element Methods

Lei Wang[1]; Sandae Tait[2]; Ji Shin[3]; Sara Khoshnevisan[4]; Wenping Gong[5]

[1]Assistant Professor, Department of Civil Engineering, University of the District of Columbia, Washington, DC 20008. Email: lei.wang@udc.edu
[2]Research Assistant, Department of Civil Engineering, University of the District of Columbia, Washington, DC 20008. Email: sandae.tait@udc.edu
[3]Research Assistant, Department of Civil Engineering, University of the District of Columbia, Washington, DC 20008. Email: ji.shin@udc.edu
[4]Assistant Professor, Department of Civil and Environmental Engineering, Clarkson University, Potsdam, NY 13699. Email: sarak@clarkson.edu
[5]Professor, Faculty of Engineering, China University of Geosciences, Wuhan 430074, China. Email: wenpinggong@cug.edu.cn

Abstract: Geosynthetic reinforced soil structures are widely used for earth retention and stabilization in many geotechnical and transportation applications. In the traditional design of geosynthetic reinforced soil structures, factor of safety is used to address the uncertainties. However, this approach cannot systematically consider the uncertainties and usually result in over-conservativeness and inconsistence in the design practice. In this paper, a reliability assessment framework of geosynthetic reinforced soil structure design is developed using probabilistic and numerical methods. The geosynthetic reinforced soil structures are modeled using finite element method. In the finite element method, the soil behavior is modelled using the Mohr-Coulomb soil model and a strength reduction method is used to determine the factor of safety value for a given geosynthetic reinforced soil structure. Then the reliability method is combined with the finite element method to obtain the probability curves for the geosynthetic reinforced soil structure. A case study of geosynthetic reinforced soil wall design is used to illustrate the significance of the proposed framework. The reliability assessment framework provides a useful decision making tool for informed design of geosynthetic reinforced soil structures in the face of uncertainties.

Keywords: geosynthetic reinforced soil structure; finite element model; reliability assessment.

1 Introduction

The use of geosynthetics has gained popularity and been extensively used in structures such as roads, dams, levees, reservoirs, landfills, mines, erosion control, aquaculture, and agriculture. The "geosynthetics" is a generic term that encompasses most flexible polymeric materials in geotechnical engineering including geotextiles, geogrids, geomembranes, geofoam, and geocells (Elias et al. 2001). The geosynthetic reinforced soil structures are especially widely used in many transportation geotechnical applications for earth retention and stabilization such as highway mechanical stabilized earth walls, bridge abutments, ramps, overpasses, column-supported embankment, and roadway subgrade stabilization (Allen et al. 2002; Yang et al. 2010; Huang et al. 2011; Wu et al. 2013; Liu 2016). The geosynthetic reinforced soil structures have been identified by the U.S. Federal Highway Administration (FHWA) as a proven, market ready technology and are being promoted for various transportation geotechnical infrastructure (Wu and Ooi 2015).

While geosynthetic reinforced soil structures are extensively employed in engineering applications, the prediction of their performance can be highly uncertain because of geotechnical uncertainties involved in the design process. Under or over estimation of uncertain parameters could lead to either conservative and costly over-design or under-design that can cause the failure of the system or violation of serviceability requirements. Traditionally, the factor of safety-based design is adopted to cope with these uncertainties. However, the actual reliability level is unknown from the factor of safety approach, which can lead to inconsistency in the design practice. The design approaches for geosynthetic reinforced soil structures mainly include the empirical and analytical approaches, physical modeling, and numerical approaches (Karpurapu and Bathurst 1995; Leshchinsky and Han 2004; Xiao et al. 2016). Among these approaches, the finite element method (FEM) has been demonstrated as an effective way for predict the performance of geosynthetic reinforced soil structures under various loading conditions (Djabri and Benmebarek 2016). However, it is typically challenging to perform the reliability-based design combined with the finite element method due to its complexity in the implementation and computational cumbersomeness. Furthermore, most reliability evaluation of geosynthetic reinforced soil structures utilizes the empirical or analytical methods to evaluate the safety and performance of these structures and little studies have been focused on reliability assessment with finite element methods for such structures (Sayed et al. 2008; Basha and Babu 2012; Basha and Babu 2014). This paper aims to develop a reliability-based framework that can effectively perform the reliability assessment of the given design of geosynthetic reinforced soil structures that combines the probabilistic assessment with finite element modeling. An advanced point estimate method is adopted to estimate the moments of the performance function of geosynthetic reinforced soil wall design, and then the derived moments are used to derive the probability curves. A design example is used to illustrate the usefulness of the proposed approach as an effective design method for geosynthetic reinforced soil structures.

2 Finite Element Modeling with Strength Reduction Method

Finite element method is a validated way of evaluating large and complex geotechnical structures. The finite element method is suitable for modeling geosynthetic reinforced soil structures since the complexity of the structure including its various components and their geometry and material properties can be considered in the analysis. Finite element method models the entire structure body and divides it into smaller elements connected at nodes. In this study, the finite element program PLAXIS 2D is used to perform this analysis. PLAXIS is a user-friendly software and has widely recognized in the geotechnical engineering community. In this finite element program, a two-dimensional plane strain model is used to model the geosynthetic reinforced soil structure problem with six node triangular element through automatic generation of mesh. During the analysis, displacements are calculated at the nodes of the element and stresses are calculated at the stress points through numerical integration (Djabri and Benmebarek 2016; Juang et al. 2018).

In the finite element modeling, the Mohr-Coulomb model is adopted to model the soil behaviors in the model. The geogrid element is used to model the geosynthetics used and the interface elements are applied at top and bottom of the geogrid element that model the interaction between geosynthetics and the soil. The strength reduction method is adopted for determining the factor of safety for the finite element model of geosynthetic reinforced soil structures. In this method, the factor of safety is calculated by taking the strength parameters of the geotechnical materials and dividing it by the strength parameters at which the model will fail. The strength parameter at failure is determined by artificially reducing the strength parameters from original values in steps until the model fails. The finite element program runs a series of analyses to produce a factor of safety for the geosynthetic reinforced soil structure under its loading condition. The factor of safety is determined using the following equation (Cheng et al. 2007; Wang et al. 2018):

$$FS = \frac{\tan(\varphi')_{\text{input value}}}{\tan(\varphi')_{\text{value at falilure}}} \tag{1}$$

where $\tan(\varphi')_{\text{input value}}$ is the input effective friction angle of soil and $\tan(\varphi')_{\text{value at falilure}}$ is the effective friction angle of soil at failure after artificially reducing the strength parameter.

3 Reliability Assessment with Point Estimate Method

The point estimate method (PEM) formulated by Zhao and Ono (2000) is adopted herein to evaluate the moments of the performance function for design of geosynthetic reinforced soil structures. Many methods are available for the reliability assessment. However, the traditional methods such as First-order reliability method (FORM) requires complicated derivatives in determining the reliability index, which is difficult to be implemented combining with an implicit model based on the finite element method. This PEM only employs the weighted sum of the performance function evaluated at selected five sampling points of a probability distribution in the original space. The sampling points in the original space (x_j) are obtained using Rosenblatt transformation based on the corresponding sampling points (u_j) in the standard normal space as the following (Zhao and Ono 2000):

$$\begin{aligned} &u_1 = 0 \\ &P_1 = 8/15 \\ &u_2 = -u_3 = 1.3556262 \\ &P_2 = P_3 = 0.2220759 \\ &u_4 = -u_5 = 2.8569700 \\ &P_4 = P_5 = 1.12574 \times 10^{-2} \end{aligned} \tag{2}$$

where u_1, u_2, u_3, u_4, u_5 are the five sampling points, and P_1, P_2, P_3, P_4, P_5 are the corresponding weight for each of these points. The sampling points (u_j) in the standard normal space are transformed to the points in original space (x_j) and the kth central moment of the performance function $y = y(x)$ can be calculated using the following (Zhao and Ono 2000):

$$\mu_y = \sum_{j=1}^{m} P_j y[T^{-1}(u_j)] \tag{3}$$

$$M_{ky} = \sum_{j=1}^{m} P_j (y[T^{-1}(u_j)] - \mu_y)^k \tag{4}$$

where μ_y and M_{ky} are the mean and the kth central moment of the performance function; u_j and P_j are the sampling points and weighting from Eq. (2); $T^{-1}(u_j)$ is the inverse Rosenblatt transformation.

For the geosynthetics reinforced soil structure problem, the friction angle of the reinforced backfill, friction angle of foundation soil, the friction angle of the retained soil, Young's modulus of soils, stiffness of the geogrids and surcharge load are treaded as random variables in the probabilistic analyses. The performance function can be written as a function of these variables, and the performance function can be written in terms of either the ultimate limit state or the serviceability limit state for the wall design. The moments of performance function with multiple variables can be calculated using a set of equations developed by Zhao and Ono (2000) based on moment results from single variables using Eq. (3) and Eq. (4). Based on the obtained moments, various methods can be adopted to evaluate the reliability index and probability of exceedance. In this research, the first two moments are used to evaluate the probability of failure based on the principles of first-order second moment (Ang and Tang 2007).

4 Example Applications - Geosynthetic Reinforced Soil Wall

A geosynthetic reinforced soil wall as shown in Figure 1 with a height of 6 m is considered as a design example in this paper. The properties of the reinforced soil and foundation soil for the design (including cohesion, effective friction angle, unit weight and Young's modulus) are c' = 0, φ' = 34°, γ = 19 kN/m^2 and E = 140MPa. The properties of the backfill retained soil are c' = 0, φ' = 30°, γ = 19 kN/m^2 and E = 140MPa. It should be noted that the above mentioned effective friction angle and Young's modulus of the soils are mean values of these parameters. The interface friction angle (δ') between soil and geosynthetic reinforcement is assumed to be 2/3 of the effective friction angle (φ') based on the available literature (e.g., Das 2014). The surface of the backfill is assumed to be horizontal and carries a permanent uniform surcharge load of P = 11.5 kPa. 16 layers of geogrid are used to reinforce the soil, and the spacing between these layers of geogrid is 0.8 m. The length of the geogrid layers is 4.9 m. The stiffness of the geogrid are assumed to be random variables with the mean values shown in Figure 1. In the finite element model, the geogrid element is used to model geosynthetic layers, the plate element is used to model the wall facing and the line load is used to model the surchage load. The geosynthetic reinforced wall is constructed in stages by alternatively placing the compacted backfill soil with the geogrid layers. The computational time of a typical FEM run for this problem is approximately 4 min.

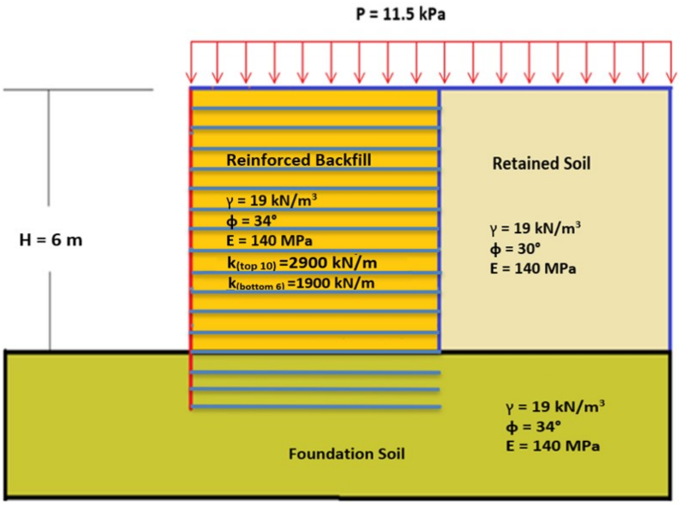

Figure 1. Schematic of geosynthetic reinforced soil wall

In this paper, a total of seven random variables are considered in the reliability analysis, which include friction angle of the reinforced backfill, friction angle of foundation soil, the friction angle of the retained soil, Young's modulus of soils, stiffness of the geogrids and surcharge load. The mean values, the coefficient of variations (COV), and the standard deviation of these parameters are summarized in Table 1. The COV of the friction angles are assumed as 10% (Phoon and Kulhawy 1999; Sert et al. 2016). The COV of Young's modulus is assumed to be 20% and the COV of the stiffness of the geogrids is assumed to be 10% (Sayed et al. 2008; Gong et al. 2018). The surcharge load is assumed with a COV of 15%. All of these random variables are assumed to follow normal distributions.

Table 1. Statistics of random variables used in the analyses

Uncertain Parameter	Mean Value	COV (%)	Std. Dev
Friction angle of reinforced backfill φ_1' (°)	34	10	3.4
Friction angle of foundation soil φ_2' (°)	30	10	3
Friction angle of retained soil φ_3' (°)	34	10	3.4
Young's modulus of soils, E (MPa)	140	20	28
k_1, stiffness of the upper geogrids (kN/m)	2900	10	290
k_2, stiffness of the lower geogrids (kN/m)	1900	10	190
P, surcharge load (kPa)	11.5	15	1.725

Firstly, the deterministic analyses are performed by taking mean values for all random variables. The deformation of the geosynthetic reinforced soil wall under the given load is shown in Figure 2. It is found that the maximum deformation of the geosynthetic reinforced soil wall occurs at the top of the wall with a maximum total displacement of 4.12 cm. Furthermore, the strength reduction method is used to determine the factor of safety of the wall by artificially weakening the soil strength parameters, and the resulting factor of safety determined from the finite element analyses is 1.477.

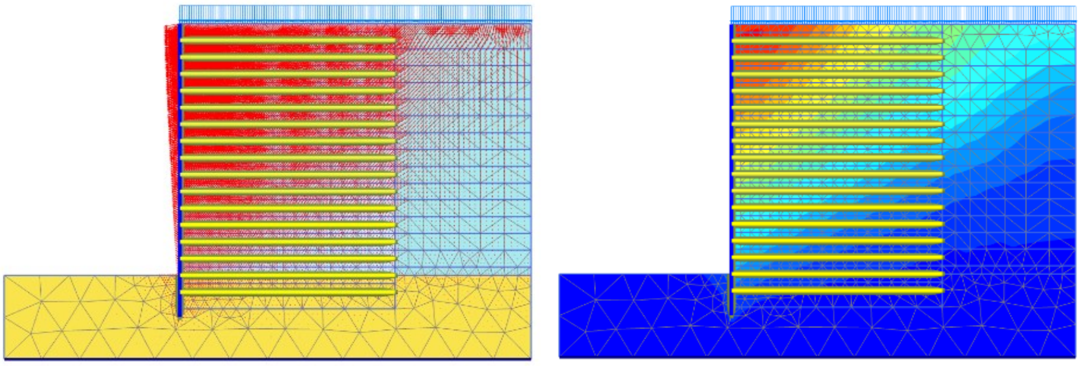

Figure 2. Displacement arrow and contour of geosynthetic reinforced soil wall based on deterministic analyses

Based on the statistics shown in Table 1, the probabilistic assessment is conducted by combining the reliability method described previously with the finite element methods. Here the limit state function for the probabilistic analyses is defined to determine the probability of exceedance of either a limiting factor of safety requirement or a limiting displacement requirement using the following equation:

$$g() = y_{\text{limiting}} - y_{FEM} \quad (5)$$

where y_{FEM} denotes the resulting factor of safety or displacement value determined by the PLAXIS, and $y_{limiting}$ represents the limiting threshold factor of safety value or displacement value as specified in the design. It should be noted that the resulting displacement from the FEM modeling in Eq. (5) is the maximum displacement of the geosynthetic reinforced soil wall (often located at the top left of the wall) determined by the PLAXIS. Following the above limit state function and reliability method, the probability curves can then be readily obtained for design of geosynthetic reinforced soil walls as shown in Figure 3 and Figure 4.

Figure 3 illustrates the results of the probability of exceedance for a series of limiting factor of safety values ranged from 1.0 to 2.0. For the limiting factor of safety of 1.0, the probability of exceedance is determined as 99.08%, which denotes the reliability of the geosynthetic reinforced soil wall under the current loading condition is 99.08%, and the actual probability of failure (the probability that the calculated factor of safety is less than 1) of the wall is 0.92%. As shown in Figure 3, with the increase of the limiting factor of safety requirement, the probability of exceeding such requirement is decreasing. For example, if the required factor of safety is 1.5, the probability of exceeding such requirement is about 33.5%.

Figure 4 shows the calculated probability of exceedance for various limiting displacement threshold values. As can be observed in Figure 4, the probability of exceedance depends significantly on the chosen limiting displacement threshold value specified by the designer. The greater the limiting displacement value, the lower probability of exceeding such limiting value. If the limiting displacement value is set as 3 cm, there is a 93.1% probability that the design will violate such displacement requirement. However, if the limiting displacement value is set as 7 cm, there is only a 0.11% probability that the design will violate such displacement requirement. The information from these probability curves (Figure 3 and Figure 4) provides valuable information for the designer to make a more informed design decision based on the target reliability requirements for geosynthetic reinforced soil structures.

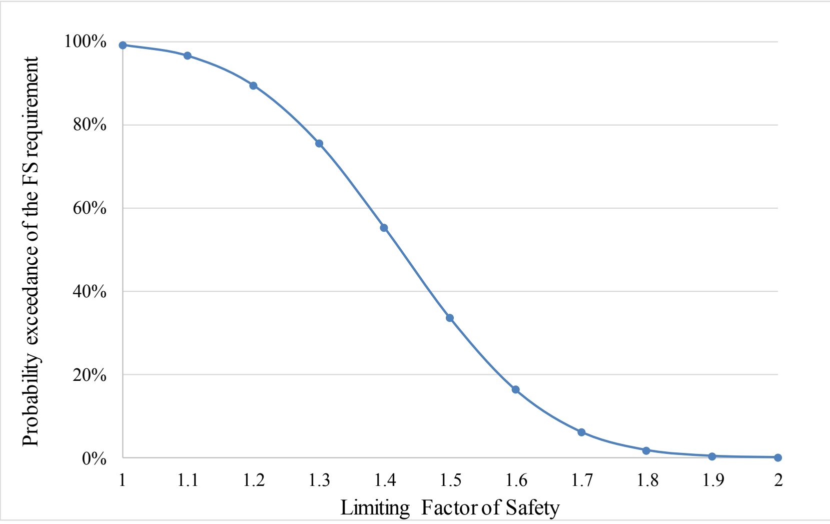

Figure 3. Computed probability of exceeding various limiting factor of safety threshold values

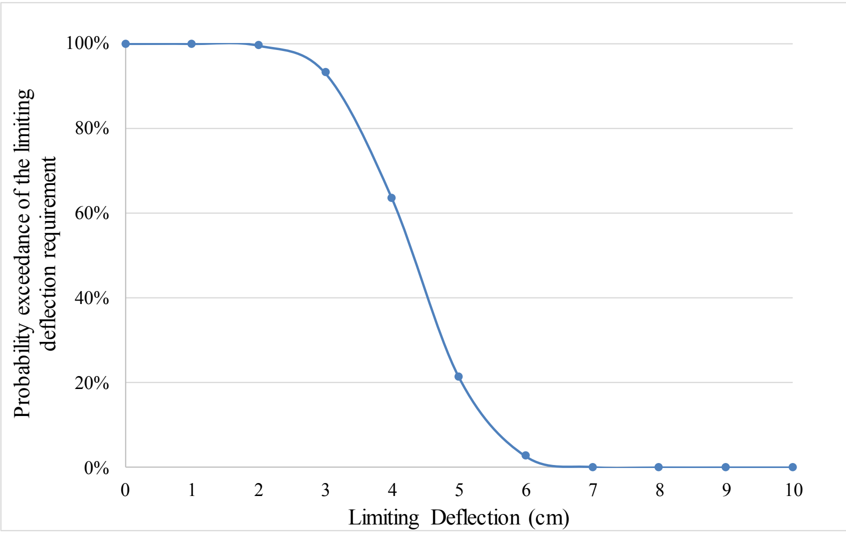

Figure 4. Computed probability of exceeding various limiting displacement threshold values

5 Conclusions

This paper presents a framework for conducting the reliability assessment of geosynthetic reinforced soil structure design based on the finite element method combined with probabilistic analyses. The deterministic analyses are performed using finite element analyses with the strength reduction method. Then a point estimate method is used to determine the moments for probability calculation. A case study of geosynthetic reinforced soil wall design is used to illustrate the significance of the proposed framework. Using the reliability method in combination with finite element method, the probability of exceedance of the limiting factor of safety and displacement requirements can be plotted against the limiting value. The resulting probability curves will provide information regarding the probability of failure of the given design as well as the probability of violating the factor of safety or serviceability (in terms of displacement) requirement. The proposed framework can improve the computational efficiency for performing reliability analyses with finite element method. The derived probability curves from the framework can provide useful reference for engineers to evaluate the probability and likelihood to meet the design requirement and assist them to make a more rational risk-informed decision.

Acknowledgments

The study on which this paper is based was supported in part by the National Science Foundation through Grants HRD-1818649 and 1900445. The results and opinions expressed in this paper do not necessarily reflect the views and policies of the National Science Foundation.

References

Allen, T. M., Bathurst, R. J., and Berg, R. R. (2002). "Global level of safety and performance of geosynthetic walls: an historical perspective." Geosynthetics International, 9(5-6), 395-450.

Ang, A. H. S., and Tang, W. H. (2007). *Probability Concepts in Engineering: Emphasis on Applications to Civil and Environmental Engineering*, 2nd edition, Wiley, New York.

Basha, B. M., & Babu, G. S. (2012). Target reliability-based optimisation for internal seismic stability of reinforced soil structures. Géotechnique, 62(1), 55-68.

Basha, B. M., and Babu, G.L.S. (2014). "Reliability-based load and resistance factor design approach for external seismic stability of reinforced soil walls." Soil Dynamics and Earthquake Engineering, 60: 8-21.

Cheng, Y. M., Lansivaara, T., & Wei, W. B. (2007). Two-dimensional slope stability analysis by limit equilibrium and strength reduction methods. Computers and Geotechnics, 34(3), 137-150.

Das, B. M. (2014). Principles of Foundation Engineering. 8th ed. Cengage Learning.

Djabri, M., and Benmebarek, S. (2016). FEM analysis of back-to-back geosynthetic-reinforced soil retaining walls. International Journal of Geosynthetics and Ground Engineericodung, 2(3), 26.

Elias, V., Christopher, B., and Berg, R. (2001). Mechanically stabilized earth walls and reinforced soil slopes design and construction guidelines. No. FHWA-NHI-10-024 Federal Highway Administration, Washington, D.C.

Gong, W., Juang, C. H., Martin, J. R., Tang, H., Wang, Q., and Huang, H. (2018). Probabilistic analysis of tunnel longitudinal performance based upon conditional random field simulation of soil properties. Tunnelling and Underground Space Technology, 73, 1-14.

Huang, J., Bhandari, A., and Yang, X. (2011). "Numerical modeling of geosynthetic-reinforced earth structures and geosynthetic-soil interactions." Geotechnical Engineering, 42(1), 43-56.

Juang, C. H., Gong, W., Martin, J. R., and Chen, Q. (2018). Model selection in geological and geotechnical engineering in the face of uncertainty-Does a complex model always outperform a simple model?. Engineering Geology.

Karpurapu, R., and Bathurst, R. J. (1995). Behaviour of geosynthetic reinforced soil retaining walls using the finite element method. Computers and Geotechnics, 17(3), 279-299.

Leshchinsky, D., and Han, J. (2004). Geosynthetic reinforced multitiered walls. Journal of Geotechnical and Geoenvironmental Engineering, 130(12), 1225-1235.

Liu, H. (2016). "Required reinforcement stiffness for vertical geosynthetic-reinforced-soil walls at strength limit state." Géotechnique, 66(5), 424-434.

Phoon, K. K., and Kulhawy, F. H. (1999). "Characterization of geotechnical variability." Canadian Geotechnical Journal, 36(4), 612-624.

Sayed, S., Dodagoudar, G. R., and Rajagopal, K. (2008). "Reliability analysis of reinforced soil walls." Indian Geotechnical Journal, 38(1), 47-65.

Sert, S., Luo, Z., Xiao, J., Gong, W., and Juang, C. H. (2016). Probabilistic analysis of responses of cantilever wall-supported excavations in sands considering vertical spatial variability. Computers and Geotechnics, 75, 182-191.

Wang, L., Powers, M., Studiner, M., Fallatah, M., and Gong, W. (2018). "Geotechnical Stability Analysis of Earthen Levees in the Face of Uncertainty." Proceedings of IFCEE 2018, ASCE Geotechnical Special Publication 297, pp. 247-256.

Wu, J.T., Pham, T.Q., and Adams, M.T. (2013). Composite behavior of geosynthetic reinforced soil mass. No. FHWA-HRT-10-077, Federal Highway Administration. Washington, D.C.

Wu, J.T., and Ooi, P.S. (2015). "Synthesis of geosynthetic reinforced soil design topics." No. FHWA-HRT-14-094, Federal Highway Administration, Washington DC.

Xiao, C., Han, J., and Zhang, Z. (2016). Experimental study on performance of geosynthetic-reinforced soil model walls on rigid foundations subjected to static footing loading. Geotextiles and Geomembranes, 44(1), 81-94.

Yang, K.H., Ching, J., and Zornberg, J.G. (2010). "Reliability-based design for external stability of narrow mechanically stabilized earth walls: calibration from centrifuge tests." Journal of Geotechnical and Geoenvironmental Engineering, 137(3), 239-253.

Zhao, Y.G., and Ono, T. (2000). New point-estimates for probability moments. Journal of Engineering Mechanics, 126(4), 433-436