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Transport of Topological Semimetals

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Keywords

[**Topological semimetals, Dirac semimetals, Weyl semimetals, Transport, Quantum Oscillation**]

Abstract

[**AU: Abstracts should be no more than 150 words, per house style, so all the abstract text can appear on the first page of the typeset article. Please trim as appropriate.**] Three-

dimensional (3D) topological semimetals represent a new class of topological matters. The study of this family of materials has been at the frontiers of condensed matter physics, and many breakthroughs have been made. Several topological semimetal phases, including Dirac semimetals (DSMs), Weyl semimetals (WSMs), nodal-line semimetals (NLSMs), and triple-point semimetals, have been theoretically predicted and experimentally demonstrated. The <mark>low-energy excitation</mark> around the Dirac/Weyl nodal points, nodal line, or triply degenerated nodal point can be viewed as emergent relativistic fermions. [**←AU: Could the highlighted be hyphenated to "low-energy excitation"? JH: OK**] Experimental studies have shown relativistic fermions can result in a rich variety of exotic transport properties, e.g., extremely large magnetoresistance, the <mark>chiral anomaly</mark>, intrinsic anomalous Hall effect (AHE), etc. [**←AU: Below, sometimes "the chiral anomaly" appears and sometimes simply "chiral anomaly" (with no definite object "the"). For consistency, please indicate whether this term should have "the" before it; changes will be made to text according to your response**] In this review, we first briefly introduce band structural characteristics of each topological semimetal phase and then review the current studies on quantum oscillations and exotic transport properties of various topological semimetals, and finally provide a perspective of this area.

1. INTRODUCTION

The rich cross-pollination between high-energy physics and condensed matter physics has led to deeper knowledge of important topics in physics such as spontaneous symmetry breaking, phase transitions, and renormalization (1, 2). Such knowledge has, in turn, greatly helped physicists and materials scientists to better understand magnets, superconductors, and other novel materials, leading to practical device applications (1). In the past decade, there has been significant interest in realizing high-energy particles in solid-state systems. The theoretical attempts to explain graphene's properties (3) by using solid-state physics led to an equation similar to one otherwise seen in cosmology and colliders: the Dirac equation. Following graphene's discovery, many materials with nodal band crossings, known as topological insulators and semimetals (4–11), were discovered, generating significant research excitement. The topological Dirac semimetals (DSMs) (12–14) and Weyl semimetals (WSMs) (2, 15–23) are crystalline solids whose low-energy electronic excitations resemble the Dirac (24) and Weyl (15) fermions in high-energy particle physics, respectively. In particular, although the Weyl fermion played a crucial rule in

the Standard Model ([15]), it has never been observed as a fundamental particle. The realization of the topological WSM state ([22], [23], [25]–[27]) enables the observation of this elusive particle in physics. Topological semimetals further allow for band crossings beyond high-energy classifications. Primary examples include the type II WSMs ([28]) and DSMs ([29]), the nodal-line semimetals (NLSMs) ([30]), and the unconventional fermion semimetals ([31]–[36]). Due to the rich variety of crystalline and magnetic symmetry properties of condensed matter systems ([37]), it is likely that such breakthroughs are only the tip of an iceberg and that there are ample new topological semimetals awaiting discovery. [**⬅****AU: OK? – JH: OK**] These topological semimetals provide platforms for studying a number of important concepts in high-energy physics (e.g., the chiral anomaly) in tabletop experiments. Moreover, such materials extend the classification of topological phases from gapped matter (e.g., insulators) to gapless systems (e.g., metals).

Topological semimetals enable a kaleidoscope of novel electronic properties. They support exotic, topologically protected boundary modes such as the topological Fermi arcs and drumhead surface states. These surface states have been directly observed in spectroscopic measurements ([19], [25], [27], [38]–[42]). The Fermi arcs also lead to unusual quantum cyclotron orbits (the Weyl orbits) as observed in quantum oscillation measurements ([43], [44]). Because of linear dispersion and spin (pseudospin) momentum locking, low-energy electrons in topological semimetals are highly robust against crystalline disorder and imperfections, leading to very high electron mobilities ([45], [46]). The compensating electron and hole carriers further cause nonsaturating magnetoresistance (MR) ([46]–[48]) and magnetothermopower ([49]–[51]). The application of parallel electric and magnetic fields can break the apparent conservation of the chiral charge ([10], [11], [52], [53]). Such chiral anomaly leads to enhanced conductivity with an increasing magnetic field. The diverging Berry curvatures near the nodal points support distinct anomalous transport phenomena, including intrinsic AHEs ([54]–[56]) and anomalous Nernst effects ([57], [58]). Such curvatures also support significantly enhanced optical and optoelectronic phenomena, including large (even quantized) photocurrents ([59]–[64]), second-harmonic generation ([65], [66]), optical activity and gyrotopy ([67]–[69]), and Kerr rotation ([70], [71]). Furthermore, thinning down a 3D topological semimetal into 2D may give rise to new 2D topology, including the quantum spin Hall insulator (QSHI) and the quantum anomalous Hall insulator (QAHI) ([14], [20], [21], [72]–[76]). These unconventional transport and optical properties of topological semimetals can pave the

way for the realization of dissipationless electronic and spintronic devices as well as efficient photodetectors and energy harvesters.

The area of 3D topological semimetals is fast growing; many papers have been published on theoretical predictions and experimental studies. There have been many reviews that introduce progress in theoretical and experimental studies on topological semimetals ([8](#)–[11](#), [76](#)–[85](#)). In this review, we focus on electronic transport and quantum oscillation studies on topological semimetals; these two topics have not been reviewed comprehensively in previous reviews. Before we discuss these topics in detail, we first briefly introduce each prototype topological semimetal phase and discuss their band structure characteristics, topological <mark>invariants</mark>, [**←****AU: invariants? **JH: yes****] and symmetry protections.

2. CATEGORIES OF TOPOLOGICAL SEMIMETALS

In this section, we discuss various 3D topological semimetal phases of matter, including WSMs, DSMs, NLSMs, and unconventional fermion semimetals beyond the Dirac and Weyl paradigm. For each kind of topological semimetal, we focus on three aspects: the characteristic band structure (the number of bands that cross, the dimensionality of the band crossing in k space, and the typical energy-momentum dispersion), the topological invariant and the symmetry protections, and representative materials.

2.1. Weyl Semimetals

WSMs are a class of topological semimetals that host Weyl fermions as low-energy quasiparticle excitations ([2](#), [6](#)–[11](#), [15](#)–[21](#)). In a WSM, two singly degenerate bands cross at discrete points, i.e., Weyl nodes, and disperse linearly in all three momentum space directions away from each Weyl node (**Figure 1***a*). Weyl fermions have distinct chiralities that are [**←****AU: OK? **JH: OK****] either left handed or right handed. The chiralities of the Weyl nodes give rise to chiral charges, which can be understood as monopoles and antimonopoles of Berry flux in momentum space. The separation of the opposite chiral charges in momentum space leads to surface Fermi arcs, whose constant energy contours are open arcs that connect the Weyl nodes of opposite chiralities on the surface.

<COMP: PLEASE INSERT FIGURE 1 HERE>

Figure 1 Schematic band structure of different types of topological semimetals. (*a*) Type I Weyl/Dirac semimetal. The degeneracy of a Weyl point is half of that of a Dirac point. On a 2D closed surface (the *green* surface) that encloses the Weyl node in *k* space, the band structure is fully gapped and therefore allows a topological invariant to be defined (19). [⬅**AU: Meaning that panel a is reproduced from Ref 19? (Are any of the other panels from Ref 19, or are they from other references?) Please clarify JH: none of the panels from Ref 19. This reference can be deleted**]** Specifically, the topological invariant for a Weyl node is a chiral charge, which corresponds to the Chern number associated with the 2D closed surface. (*b*) Type II Weyl/Dirac semimetal. At the energy of the type II Weyl/Dirac node, the constant energy contour consists of an electron pocket and a hole pocket touching at the node. (*c*) Nodal-line semimetal. The conduction and valence bands are degenerate on a 1D closed loop, shown as the green circle in the Brillouin zone. The topological invariant of the nodal line is a winding number *w*, which is defined as the line integral of the Berry connection along a closed loop, shown as the green circle that interlinks the nodal line. (*d*) Triple-point semimetal. Three singly degenerate bands cross at discrete points, the triple points. The triple point can also be viewed as the meeting point between two nodal lines along the k_y axis.

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Because of the existence of Weyl nodes, WSMs lack a global band gap. The absence of a global band gap prevents the definition of a topological invariant for the entire 3D bulk Brillouin zone (BZ). In contrast, on a 2D closed surface that encloses the Weyl node in *k* space (**Figure 1a**), the band structure is fully gapped and therefore allows a topological invariant to be defined (19). Specifically, the Chern number associated with the 2D closed surface directly corresponds to the topological invariant of a Weyl node (i.e., the chiral charge). Mathematically, the chiral charge *C* can be calculated by the integral of the Berry curvature (the Berry flux) as shown below:

$$C = \int_S \mathbf{\Omega} \cdot d\mathbf{S}, \qquad 1.$$

where *S* is the 2D closed surface in *k* space that encloses the Weyl node and $\mathbf{\Omega}$ is the Berry curvature. Due to the chiral charge, Weyl nodes can appear at generic *k* points of the BZ. In the presence of translational symmetry, these Weyl nodes are topologically stable and cannot be removed without pair annihilation. The existence of Weyl nodes does not rely on any additional crystalline point group symmetries.

Real materials that host the WSM state are usually further classified into either inversion-

symmetry–breaking WSMs or time-reversal symmetry–breaking WSMs. Representative inversion-symmetry–breaking WSMs include the TaAs family of noncentrosymmetric crystals (22, 23, 25, 27, 39, 86–92). Representative time-reversal symmetry–breaking WSMs can be realized in naturally occurring ferromagnetic (FM) semimetals such as pyrochlore iridate (19), $HgCr_2Se_4$ (21), $Co_3Sn_2S_2$ (93, 94), Heuslers (95–99), and the noncollinear antiferromagnets Mn_3Sn and Mn_3Ge (57, 100–103) or by applying an external magnetic field to a nonmagnetic or antiferromagnetic (AFM) semimetal, as demonstrated in the magnetotransport experiments (104) on Na_3Bi (105), Cd_3As_2 (45, 106), $ZrTe_5$ (107), and half-Heuslers (108–110). From a different angle, WSMs can also be classified by the energy-momentum dispersions near the Weyl nodes. Type I WSMs have untilted or weakly tilted Weyl cones with a point-like Fermi surface when the chemical potential is placed at the Weyl node. By contrast, type II WSMs have strongly tilted Weyl cones (**Figure 1*b***) (28). Their Fermi surface consists of electron and hole pockets that touch at the type II Weyl nodes. Representative type II WSMs include WTe_2 (28, 111–113), $MoTe_2$ (114–122), $TaIrTe_4$ (123, 124), and $(W/Mo)P_2$ (125). These different classifications are not mutually exclusive. For instance, $MoTe_2$ is not only an inversion-breaking WSM but also a type II WSM.

2.2. Dirac Semimetals

DSMs host Dirac fermions as low-energy quasiparticle excitations (12–14, 38, 126–131). In a DSM, two doubly degenerate bands cross to form a Dirac node and disperse linearly in all three momentum directions away from the node. Each Dirac node can be viewed as a pair of degenerate Weyl nodes of opposite chiralities. Since a pair of degenerate Weyl nodes of opposite chiralities is in general unstable and may annihilate, additional crystalline point group symmetries are needed to realize a stable DSM phase (131). One route is to rely on uniaxial rotational symmetries (131). Specifically, a band inversion can create a pair of 3D Dirac nodes on the opposite sites of the time-reversal invariant momenta. Representative DSMs of this kind include Na_3Bi (13, 38, 126) and Cd_3As_2 (14, 127–130) (type I) as well as VAl_3 (29) (type II). Another route is to rely on nonsymmorphic symmetries, i.e., glide reflections and screw rotations. It has been theoretically shown that nonsymmorphic symmetries can lead to nontrivial band connectivity at the BZ boundaries, giving rise to filling-enforced DSMs or NLSMs, depending on the specific space groups (12, 132–135). Representative filling-enforced DSM candidates include β-BiO_2 (12) and distorted spinels (132). Furthermore, a DSM can be realized

as the critical point of the topological phase transition between a trivial insulator and a topological insulator. This is achieved in the $BiTl(S_{1-x}Se_x)_2$ (12, 136), $Bi_{2-x}In_xSe_3$ (137), and $Pb_{1-x}Sn_xTe$ (138) systems by fine-tuning the chemical doping concentration. Alternatively, compounds like ==ZrTe₅ (107, 139, 140) and those in the SrMnSb₂ family (141–143)== naturally sit near the critical point of such a topological phase transition and therefore approximate a DSM state. [**←**AU: By** ZrTe₅ **do you mean "ZrTe₅ family"? If so, the highlighted will be edited to "those in the** ZrTe₅ **(107, 139, 140) and SrMnSb₂ (141–143) families"** – if not, please clarify JH: ZrTe5 is one material, not family. For SrMnSb2, there are some others materials with same SrMnSb2-type structure, so it is a "SrMnSb2 family". ****]** According to current theoretical understanding, Dirac nodes are not associated with any nontrivial topological invariant (i.e., they have zero chiral charge) (144).

2.3. Nodal-Line Semimetals

In NLSMs, conduction and valence bands cross at 1D lines in k space (**Figure 1a**) (30, 40, 78, 85, 133, 134, 145–161). Compared to DSMs/WSMs, the electronic structure of NLSMs is distinct in three aspects: (*a*) The bulk Fermi surface consists of 1D lines in NLSMs but of 0D points in WSMs; (*b*) the density of states (DOS) is proportional to $(E - E_F)^2$ in NLSMs but to [**←**AU: should "is" or "to" be inserted here? JH: "to" is better ****] $|E - E_F|$ in WSMs; and (*c*) on the surface, nodal lines are accompanied by drumhead-like surface states, whereas Weyl nodes are connected by 1D Fermi arc surface states.

We now discuss the topological invariant of NLSMs. We consider a 1D closed loop that interlinks the nodal line in k space (**Figure 1c**). The band structure is fully gapped and therefore allows for the definition of a topological invariant, i.e., the winding number (150). Mathematically, the winding number w is defined as the integral of the Berry connection along the 1D closed loop that links the nodal line as shown below:

$$w = \int_l \mathbf{A} \cdot d\mathbf{l}, \qquad 2.$$

where l is the 1D closed loop that links the nodal line and $\mathbf{A}$ is the Berry connection.

NLSMs also come in a variety of forms, depending on the characteristic band structure and the symmetry protection. First, nodal lines can be closed loops (also termed nodal circles) inside the 3D BZ. [**←**AU: OK? Please clarify as necessary JH: OK****] Such nodal circles are

naturally formed by a band inversion. The nodal circles are further classified on the basis of the symmetry protection. There are nodal circles that are strictly gapless only in the absence of spin-orbit coupling (SOC) ([78], [146], [149], [150]). They are usually protected by the combination of time-reversal and time-inversion symmetries ([78], [146], [150]). Representative materials include Cu_3N ([149]), Ca_3P_2 ([147]), Cu_3PdN ([148]), and those in the ZrSiS family ([154]–[158]). Alternatively, nodal circles can be formed in noncentrosymmetric crystals protected by a mirror plane. These nodal circles are stable even upon the inclusion of SOC. Representative materials include $PbTaSe_2$, $TlTaSe_2$, and CaAgAs ([40], [145], [159], [160]). Second, nodal lines can also be a straight line that span across the BZ. Representative materials include those in the $BaNbS_3$ family ([161]). Third, nodal circles can interlink with each other in k space, forming Hopf links and nodal chains ([162]–[167]). These Hopf links and nodal chains [**←****AU: please clarify. **JH:** Hopf links and nodal chains **] may be protected by the presence of multiple perpendicular mirror planes ([167]) or by nonsymmorphic symmetries ([162], [163]).

2.4. Unconventional Fermion Semimetals

In contrast to [**←****AU: insert "the case for"? **JH: not necessary****] high-energy physics, solid-state crystals can support band crossings beyond the Dirac/Weyl paradigm ([31]–[36]). These band crossings, broadly referred as unconventional fermions, include three-, four-, six-, and eightfold degeneracies ([31]). [****AU: Quotation marks used for purposes other than direct quotation have been removed throughout, per house style. JH: OK****]

Here we take a particular type of three-band crossing as an example ([33]–[36], [168]–[170]). In such a triple-point semimetal, three singly degenerate bands cross at discrete points, the triple points (**Figure 1d**). Moving away from one triple point along k_x or k_z, all three bands become nondegenerate. By contrast, moving away along k_y, bands 1 and 2 remain degenerate for $-k_y$, whereas bands 2 and 3 remain degenerate for $+k_y$. Therefore, the triple point can also be viewed as the meeting point between two nodal lines along the k_y axis. These triple points are protected by the combination of a uniaxial rotational axis, mirror planes, and time-reversal symmetry. These triple points are not associated with any topological invariant due to the lack of a global band gap on any 2D closed surface that encloses the triple point. Representative materials include MoC, WC, MoP, and ZrTe ([33]–[36], [169], [170]).

3. TRANSPORT SIGNATURES OF TOPOLOGICAL SEMIMETALS

The relativistic nature of the Dirac and Weyl fermions in topological semimetals manifests in many distinct transport properties, including extremely large MR, high mobility, light effective mass, nontrivial Berry phase, chiral anomaly, and the AHE. These relativistic fermion properties have great potential for future electronic and spintronic applications. Characterization of relativistic fermions through transport measurements provides a convenient approach to verify a nontrivial topological state, complementary to the direct observation of nontrivial band topology by ARPES experiments. In this section, we summarize these transport signatures of topological DSMs and WSMs.

3.1. Magnetoresistance

Electron transport in topological semimetals is usually strongly affected by external magnetic field. Large MR is a common signature often seen in most DSMs and WSMs. MR is usually expressed as the change in resistance (resistivity) under field normalized by the zero-field resistance (resistivity), i.e., $[R(B) - R(B = 0)]/R(B = 0)$ or $[\rho(B) - \rho(B = 0)]/\rho(B = 0)$. The transverse MR, measured with the field perpendicular to the current direction, can reach up to 0.1–1 million percent at low temperatures (0.5–5 K) and a field of 9 T (see **Table 1**), without any sign of saturation up to 30–100 T in WSMs/DSMs such as Cd_3As_2, $PtBi_2$, WTe_2, and NbP ([46], [48], [171], [172]). A power law field dependence (MR $\propto B^n$) is usually seen in various topological semimetals, with the exponent n ranging from 1 to 2 ([45], [46], [48], [107], [171]–[187]).

<COMP: PLEASE INSERT TABLE 1 HERE>

In a simple metal, a positive transverse MR with quadratic field dependence is generally expected due to the Lorentz effect ([47]). Such Lorentz effect–induced orbital MR is usually weak and saturates for systems with a closed Fermi surface, contrasted with the giant, nonsaturating MR seen in topological semimetals. The origin of the unusually large MR of topological semimetals has been intensively studied. Electron-hole compensation has been proposed to be a possible mechanism ([46], [48], [171]). However, reports also indicate that carrier compensation is not achieved in some topological semimetals ([188], [189]). An alternative explanation is that the backscattering at zero field is strongly suppressed by some protection mechanisms associated with nontrivial band topology but is [←**AU: OK? JH: also change "and" to "but"**] significantly enhanced by magnetic fields ([45]).

The strong coupling between MR, high mobility, and linearly dispersed Dirac/Weyl cones may provide some clues for further understanding of the large MR. High mobility is another signature accompanied with large MR in topological semimetals. Mobility (μ) is related to conductivity σ via $\sigma = nq\mu$, where n and q are the carrier density and charge, respectively. For a single-band system, the Hall coefficient $R_H = 1/nq$, and thus $\mu = \sigma \cdot R_H$. However, in multiple-band systems, the field dependence of Hall resistivity ρ_{xy} deviates from linearity. **Figure 2a** shows one example. In this case, the Hall coefficient, defined as $d\rho_{xy}/dB$, becomes field dependent, and both mobility and carrier density cannot be directly derived as for a single-band system. A commonly used approach for analyzing the transport properties of multiband systems is the multiple-band model, i.e., assuming that the contributions of various bands to the conductivity are additive. In practice, for a system with more than two bands, a further simplified model, which considers only one electron band and one hole band, is widely used to describe the longitudinal resistivity (ρ_{xx}) and transverse resistivity (ρ_{xy}, i.e., the Hall resistivity), as shown by Equations 3 and 4 below ([190]):

$$\rho_{xx} = \frac{(n_e\mu_e + n_h\mu_h) + (n_e\mu_e\mu_h^2 + n_h\mu_h\mu_e^2)B^2}{(n_e\mu_e + n_h\mu_h)^2 + \mu_e^2\mu_h^2(n_h-n_e)^2 B^2} \cdot \frac{1}{e}, \quad 3.$$

$$\rho_{xy} = \frac{(n_h\mu_h^2 - n_e\mu_e^2) + \mu_h^2\mu_e^2(n_h-n_e)B^2}{(n_e\mu_e + n_h\mu_h)^2 + \mu_h^2\mu_e^2(n_h-n_e)^2 B^2} \cdot \frac{B}{e}, \quad 4.$$

where n_e (n_h) and μ_e (μ_h) are the density and mobility of the electron (hole) band, respectively. From the simultaneous fitting for $\rho_{xx}(B)$ and $\rho_{xy}(B)$ by using such a two-band model, both the densities and mobilities of the electron bands and hole bands can be obtained. Clearly, for a real system with more than one electron or hole band, this oversimplified model averages electron and hole bands and neglects any interband interactions. Although adding more bands to the above model is possible in principle, more accurate results may not be obtained with an overparameterized model. In fact, the two-band model already yields reasonable results for a variety of material systems, so it is reasonable to extend its application to topological semimetals.

<COMP: PLEASE INSERT FIGURE 2 HERE>

Figure 2 Magnetoresistance (MR). (*a*) Magnetic field dependence of the longitudinal (ρ_{xx}) and

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transverse (Hall) (ρ_{xy}) resistivity for Cd_3As_2 <mark>(45)</mark>. (*b*) MR normalized by the zero-field resistivity for WTe_2 at 2 K and 10 K. Shubnikov–de Haas (SdH) oscillation is seen for the $T = 2$ K data. (*Upper inset*) MR at higher temperatures. (*Lower inset*) Oscillatory component of the resistivity oscillation, obtained by subtracting the smooth MR background <mark>(48)</mark>. (*c*) MR normalized by the zero-field resistivity for NbP at various temperatures. SdH oscillation is seen at $T < 10$ K. (*Inset*) MR at higher temperatures <mark>(46)</mark>. **[←**AU: Do the highlights in this caption indicate that panel a is reproduced from Ref 45, panel b is reproduced from Ref 48, and panel c is reproduced from Ref 46? Please clarify if not** JH: yes there are from those references]**

Equation 3 indicates that ρ_{xx} tends to saturate at high fields where the B^2 terms dominate. Only when $n_e = n_h$, i.e., the case of electron-hole compensation, **[←**AU: OK? JH: not good**]** $\rho_{xx} \propto B^2$ without saturation. Under such a circumstance, large MR is expected when mobility is high. **Table 1** shows the mobilities of some representative topological semimetals acquired from two-band model analysis; the mobilities are indeed high, in the range of 10^3–10^6 cm^2/(V·s). Such high transport mobility is consistent with the ultralow residual resistivity at the zero-temperature limit ($\sim n\Omega$ to a few $\mu\Omega$; see **Table 1**) as well as with the high quantum mobility revealed by quantum oscillation studies (discussed in Section 3.2.2).

The two-band model, while widely used, provides only an approximate description for the magnetotransport properties of multiple-band materials. First, Equations 3 and 4 are not applicable if there are open orbits, which occur when the Fermi surface is not closed in the momentum space (190). Second, the negligence of interband interaction leads to an apparent contradiction: The carrier compensation appears to be necessary for the nonsaturated MR according to Equation 3, but the Hall resistivity expressed by Equation 4 must be linearly dependent on the field when $n_e = n_h$, which is not true for most topological semimetals (e.g., see **Figure 2***a*). Third, according to Equation 3, even approximate electron-hole compensation should be able to lead to a quadratic or nearly-quadratic field dependence for ρ_{xx}. Such a dependence **[←**AU: OK?**]** has indeed been observed in a number of topological semimetals (48, 183, 191–193), but linear or even sublinear MR has also been observed in a variety of samples (107, 171, 172, 174–180, 182, 183, 191, 194). The linear MR may be a classical effect due to strong current inhomogeneity (172) or may have a quantum mechanical interpretation (195) (see Section 3.2.8), while the sublinear MR may be attributed to the weak antilocalization caused by strong SOC (196). With these considerations, the two-band model appears to be applicable only for a limited field range or at higher temperatures at which quantum effects are

not significant.

Although obtaining the precise value of carrier mobility for individual bands might be challenging, the two-band model still provides an effective approach for the approximate description of magnetotransport properties of multiband materials. This model successfully explains the extremely large MR arising from high mobility and approximate carrier compensation. Then, a key question for topological semimetals is why Dirac/Weyl fermions have high mobility. This question [←**AU: OK? Or "The answer"?** JH: OK] can be understood in terms of the energy band characteristics of topological semimetals. Given that the carrier mobility is determined by relaxation time τ and effective mass m^*,[←**AU: insert "is determined by"? **].i.e. $\mu = e\tau/m^*$, greater relaxation time and smaller effective mass favor higher mobility. As shown in Section 3.2.2, the cyclotron effective masses derived from quantum oscillations are indeed small for many topological semimetals, reaching as low as $0.02m_e$ (where m_e is the free electron mass) for some materials. Such massless behavior is naturally expected for ideal topological fermions since they are hosted by linearly dispersed bands crossing near the Fermi level, which requires zero mass in the Hamiltonian ([11]).

Greater relaxation time in topological materials may be associated with symmetry protection in many cases. For topological insulators, it has been well established that backscattering is forbidden by time-reversal symmetry, even though nonmagnetic defects exist, thus resulting in longer relaxation time ([197]–[201]). In some topological semimetals, a strong suppression of backscattering due to nontrivial band topology has also been proposed ([45]); such suppression would lead to enhanced transport relaxation time. [←**AU: OK? JH: OK**] This idea is partially supported by the quantum oscillation studies that reveal a long quantum relaxation time in topological semimetals, as shown in Section 3.2.2.

3.2. Landau Quantization and Quantum Oscillations

In addition to the extremely large MR, another important phenomenon in the magnetotransport of topological semimetals is quantum oscillation (**Figure 2b,c**), i.e., the Shubnikov–de Haas (SdH) effect. Quantum oscillations can also be probed in other measurements such as magnetization/magnetic torque [i.e., the de Haas–van Alphen (dHvA) effect], thermoelectric power, and ultrasonic absorption. Quantum oscillations have been widely used for the study of the 3D topological insulators ([202]) and topological semimetals, and [←**AU: please clarify JH: change to "and"**] reveal key parameters for Dirac/Weyl fermions such as effective mass,

quantum mobility, and (most [←**AU: OK? JH: OK**] importantly) the Berry phase. In this section, we review quantum oscillation studies of topological semimetals.

3.2.1. The zeroth Landau level for relativistic fermions.

Quantum oscillation theory for nonrelativistic electrons has been well established and documented in earlier textbooks and reviews ([203](#), [204](#)). Here we briefly recall the fundamental theory and put major emphasis on its extension to relativistic fermions. Quantum oscillation originates from the quantized cyclotron motion of charge carriers under magnetic fields, i.e., the Landau quantization of the energy states. With the conduction band splitting to LLs, the DOS at the Fermi level, $DOS(E_F)$, becomes periodically modulated by magnetic field (more precisely, periodic in $1/B$), leading to periodic oscillations of physical quantities.

Panels *a* and *b* of **Figure 3** show the textbook drawings of the Landau quantization for spinless (i.e., ignoring the Zeeman splitting) nonrelativistic electrons with parabolic dispersion. The quantized LL energy is $\varepsilon_n = (n + 1/2)\hbar\omega_c$, where $\omega_c = eB/m$ is the cyclotron motion frequency and the LL index $n = 0, 1, \ldots$ The energies of all LLs are field dependent and evenly spaced by $\hbar\omega_c$, as shown in **Figure 3b**. For the lowest LL, a finite zero-point energy $\hbar\omega_c/2$ exists, which is in analogy to the zero-point energy of a harmonic oscillator. To distinguish the lowest LL for the nonrelativistic fermions [←**AU: please clarify JH: change to "the lowest LL for the nonrelativistic fermions"**] from the exotic zeroth LL with field-independent zero energy for the relativistic fermions shown below, we rewrite the LL energy of nonrelativistic electrons as $\varepsilon_n = (n - 1/2)\hbar\omega_c$, where n becomes a nonzero integer (1, 2, …).

<COMP: PLEASE INSERT FIGURE 3 HERE>

Figure 3 Landau quantization. (*a,c*) Schematics for energy-momentum dispersions of the (*a*) normal (nonrelativistic) and (*c*) relativistic electrons. (*b,d*) Landau spectra for the 2D spinless (*b*) nonrelativistic and (*d*) relativistic electrons. (*e,f*) Landau spectra for the 3D spinless (*e*) nonrelativistic and (*f*) relativistic electrons with the magnetic field along the k_z direction ($B//k_z$). [←**AU: OK that the slashes go to the right? Should they be vertical? JH: can be vertical or forward slash**] (*g*) Landau tubes intersecting a 3D spherical Fermi surface. (*h*) Landau rings within the 2D Fermi surface (ring). Panels *g* and *h* show the scenario for nonrelativistic electrons without the zeroth Landau level.

The LL quantization is completely different for the relativistic fermions with linear dispersion (**Figure 3c**). Earlier studies on graphene ([205](#), [206](#)) [←**AU: OK? JH: OK**]

established that the quantized energies of LLs for spinless 2D Dirac fermions are

$$\varepsilon_n = \upsilon_F \, \text{sgn}(n)\sqrt{2e\hbar\,|B|\,|n|} \quad (n = 0, \pm1, \pm2\ldots) \qquad \text{5.}$$

where $\text{sgn}(n)$ is the sign function and υ_F is the Fermi velocity. [⬅**AU: (1) The highlighted symbol appears a bit different from the one shown in Equation 5 – please clarify JH: it is the symbol for velocity, so it should be letter "v". (2) In Equation 5, the "e" in $2e\hbar$ is not italicized – should it be italicized (if it is, e.g., a variable, then it should be italicized)? Please also check all equations and indicate any necessary formatting changes to "e" (i.e., italics to roman font or vice versa) JH: e means the electron charge, italicized **]** As illustrated in **Figure 3***d*, LLs are no longer equally spaced for relativistic fermions given $\varepsilon_n \propto \sqrt{|n|}$. Most strikingly, a field-independent zeroth ($n = 0$) LL locked at the band crossing point ($\varepsilon_0 = 0$) appears, which is a signature unique to 2D relativistic electron systems. Such a zero energy can be understood in terms of the Berry phase arising from the cyclotron motion of carriers in momentum space (206). The detailed theoretical background of the Berry phase and its manifestation in transport measurements have been well understood (202, 207–209). In short, the Berry phase describes a geometrical phase factor of a quantum mechanical system acquired in the adiabatic evolution along a closed trajectory in the parameter space. Such a phase factor does not depend on the details of the temporal evolution and thus differs from the dynamical phase. A nonzero Berry phase ϕ_B originates from the band touching point, such as Dirac nodes. [⬅**AU: For consistency, OK to define "Berry phase" as ϕ_B and then use ϕ_B subsequently in text? JH: under some situations it is better to use the words rather the symbol**]** Under magnetic fields, the cyclotron motion of Dirac fermions, i.e., the [⬅**AU: OK?JH: OK**]** closed trajectory in momentum space, induces a Berry phase that changes the phase of quantum oscillations. Ideally, $\phi_B = \pi$ for an exact linear energy-momentum dispersion, and this value shifts when the bands deviate from linear dispersion and/or the Zeeman effect is strong (209, 210).

Before formulizing the quantum oscillation for relativistic fermions by incorporating the Berry phase–induced phase shift, we should pay attention to the dimensionality of the investigated material systems. The Landau quantization of the 2D surface state of topological insulators is very different from that of the Dirac or Weyl fermions in 3D topological

semimetals. Most topological semimetals reported so far are 3D in nature [such as Cd_3As_2 ([14], [127]–[130]), Na_3Bi ([13], [126]), and the TaAs family ([22], [23], [25], [27], [39], [86], [87], [211])], and 3D is necessarily required for a Weyl state ([10]). For nonrelativistic electrons in 3D, the motion along the magnetic field direction is not quantized, leading to additional energy of $(\hbar k_z)^2/2m$ (where k_z is the momentum along the magnetic field direction) for LLs:

$$\varepsilon_{n,k} = \frac{\hbar eB}{m^*}\left(n - \frac{1}{2}\right) + \frac{\hbar^2 k_z^2}{2m^*} \quad (n = 1,\ 2,\ 3,\ \ldots). \qquad 6.$$

Similarly, an additional energy term due to unquantized k_z also occurs for 3D relativistic fermions:

$$\varepsilon_n = \upsilon_F \, \mathrm{sgn}(n)\sqrt{2e\hbar \,|\,B\,||\,n\,| + (\hbar k_z)^2}. \qquad 7.$$

[←**AU: (1) In Equation 7, should υ_F be v_F? Cf. Equations 5 and 7 with the line of text below Equation 5. JH: v_F (2) In Equation 7 (as in Equation 5), the "e" in $2e\hbar$ is not italicized – should it be italicized (if it is, e.g., a variable, then it should be italicized)?**] Therefore, although the zeroth LL's energy is still field independent, it is not strictly zero. Moreover, Equation 7 is valid for Dirac fermions with $n = 0, 1, 2, \ldots$. For Weyl fermions, the chirality is well defined due to the lifting of spin degeneracy, so Equation 7 needs to be modified for the zeroth LL of Weyl fermions. As discussed in Section 3.4, the chiral zeroth LL leads to one important effect for Weyl fermions, i.e., the chiral anomaly.

3.2.2. The Lifshitz–Kosevich model for de Haas–van Alphen oscillations.

For the perfect 2D case, the Landau bands are degenerate into sharp levels (**Figure 3b,d**), and the motions of all electrons at the Fermi level are in phase. For the 3D case, due to the additional energy related to unquantized k_z as shown in Equations 6 and 7, different LLs overlap in energy space, leading to a mixture of Landau bands for particular energy (**Figure 3e,f**) and a continuous energy spectrum. This is better illustrated in **Figure 3g**: Landau quantization for 3D free electrons manifests as Landau cylinders along the magnetic field direction, so an equal energy surface intersects multiple Landau cylinders. **[←**AU: Per house style, text such as the highlighted should be deleted from the main text, as such text pertains specifically to the figure. Please move the highlighted to the caption where applicable**JH: yes, we did it]**

This scenario is distinct from the 2D case (**Figure 3h**). Therefore, different models have been derived for 3D and 2D quantum oscillations.

Here we start with the dHvA oscillation because the magnetization is the derivative of the Gibbs thermodynamic potential Ω at constant temperature and chemical potential ζ,

$M = -\left(\dfrac{\partial \Omega}{\partial B}\right)_{T,\zeta}$, so that it directly reflects the LL spectrum. At the zero temperature limit, the oscillatory thermodynamic potential Ω due to Landau quantization for a 3D system can be expressed as (in CGS units) ([203]):

$$\Omega_{osc} = \left(\frac{e}{2\pi c\hbar}\right)^{3/2} \frac{e\hbar B^{5/2}}{mc\pi^2 (\partial^2 S_{extr}/\partial k_z^2)^{1/2}} \sum_{r=1}^{\infty} \frac{1}{r^{5/2}} \cos\left[2\pi r\left(\frac{F}{B}-\gamma\right)+2\pi\delta\right],$$ 8.

where S_{extr} is the extremal Fermi surface cross-section area perpendicular to the magnetic field, $\partial^2 S_{extr}/\partial k_z^2$ is the Fermi surface curvature along the k_z direction (i.e., the field direction) at the extremal cross section, and r is the harmonic index. Given several damping factors, the general formula of the magnetization oscillations for a 3D system, derived by Lifshitz & Kosevich (the LK formula) ([203], [204], [212]), is (in SI units)

$$M_{osc}^{3D} = -\left(\frac{e}{2\pi\hbar}\right)^{3/2} \frac{S_{extr}}{\pi^2 m^*} \left(\frac{B}{|\partial^2 S_{extr}/\partial k_z^2|}\right)^{1/2} \sum_{r=1}^{\infty} \frac{1}{r^{3/2}} R_T R_D R_S \sin\left[2\pi r\left(\frac{F}{B}-\gamma+\frac{\delta}{r}\right)\right].$$ 9.

R_T, R_D, and R_S are the temperature-, field-, and spin-damping factors, which are associated with the finite temperature corrections to Fermi-Dirac distribution function, the finite relaxation time due to impurity scattering, and the phase difference between the spin-up and spin-down subbands, respectively. [←**AU: OK?** JH: OK to move respectively, but may not need to use multiple "with"] These factors can be expressed as

$$R_T = \frac{raT\mu/B}{\sinh(raT\mu/B)},$$ 10.

$$R_D = \exp\left(-\frac{raT_D\mu}{B}\right),$$ 11.

$$R_S = \cos\frac{r\pi g\mu}{2},$$ 12.

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where μ is the ratio of effective cyclotron mass m^* to free electron mass m_0. T_D is the Dingle temperature that is relevant to the quantum relaxation time, and $a = (2\pi^2 k_B m_0)/(\hbar e) \approx 14.69$ T/K. [←**AU: OK as phrased? Please clarify what this is JH: the constant a is not dimensionless. T/K is the unit.**]

The sine term in Equation 9 describes the oscillation with frequency rF and phase factor $2\pi r\left(-\gamma + \dfrac{\delta}{r}\right)$, where the fundamental frequency F is linked to S_{extr} by the Onsager relation $F = \hbar S_{extr}/2\pi e$. [←**AU: italicize "e"? JH: agree**] The determination of the phase factor is of particular interest for the quantum oscillation study of topological materials since the Berry phase ϕ_B is connected to the phase factor via $\gamma = \dfrac{1}{2} - \dfrac{\phi_B}{2\pi}$. The Berry phase, which was not included in Lifshitz & Kosevich's original formalism (i.e., $\gamma = \dfrac{1}{2}$) (212), can effectively shift the phase of quantum oscillations (209, 210). The phase shift δ in Equation 9, which is determined by the dimensionality of the Fermi surface, is 0 for the 2D case and $\pm 1/8$ for the 3D case. For the 3D case, $\delta = -1/8$ (+1/8) for maximal (minimal) cross section for a 3D electron pocket (203, 204, 212) and vice versa for a 3D hole pocket. [←**AU: Please clarify what you mean by "vice versa" JH: means** +1/8(-1/8), the signs are changed**]

Although most topological semimetals are 3D, there are also some materials with layered structure and that thus display a quasi-2D electronic structure, such as ZrSiTe (156) and (Sr/Ba)Mn(Bi/Sb)$_2$ (143, 173, 177, 213). For a perfectly 2D system, the above LK formula has been modified by Shoenberg and others (203, 204, 214, 215):

$$M_{osc}^{2D} = -\left(\frac{e}{2\pi\hbar}\right)\frac{S}{\pi^2 m^*}\sum_{r=1}^{\infty}\frac{1}{r}R_T R_D R_S \sin\left[2\pi r\left(\frac{F}{B} - \gamma\right)\right], \qquad 13.$$

with the same definitions for damping factors (R_T, R_D, and R_S) and phase factor γ as the 3D model. The Fermi surface cross-section area become a constant for 2D, so S_{extr} in the 3D model (Equation 9) is replaced by S, and the phase factor δ is zero. In addition to this phase difference, the oscillation amplitude (i.e., the prefactor of the summation in Equation 13) and harmonic components ($r \neq 0$) are enhanced relative to the 3D model.

Significantly, the above 3D (Equation 9) and 2D (Equation 13) LK models are based on the

assumption of constant chemical potential, which is appropriate for a 3D system because the electron energy spectrum is continuous, as mentioned above. In this scenario, the lowest unoccupied state is always located at E_F and is independent of B (i.e., the chemical potential $= E_F$ for $T = 0$ K). In contrast, the 2D Landau quantization gives rise to discrete energy levels, so the chemical potential, which is the minimum energy needed to add an electron to the system, is pinned to the highest occupied LL and hence also oscillates with ramping magnetic field. This chemical potential oscillation will affect the quantum oscillations. Furthermore, in real materials, the interlayer coupling is not negligible in layered compounds, which is also not captured by Equation 13. [⬅**AU: Please clarify what is also not captured by Equation 13 JH: the effect of the interlayer coupling is not included in Eq. 13. For example, it may lead additional term, or some other corrections to Eq. 13.**] More comprehensive analyses can be found in References 203 and 204 and references therein.

   In practice, the oscillation frequency(ies) F can be directly resolved from the fast Fourier transform (FFT) of the oscillation pattern, and other important parameters, including effective cyclotron mass, quantum relaxation time, and Berry phase, can be obtained from the analyses with the LK formula. From FFT, one can also clarify whether the higher harmonic terms ($r > 1$) with frequency rF are significant. In principle, these terms attenuate quickly with $r^{-3/2}$ for a 3D system (Equation 9) or r^{-1} for a 2D system (Equation 13), and thus the quantum oscillations in real materials are usually dominated by fundamental frequencies ($r = 1$). If the oscillation contains only a single frequency without obvious harmonic frequency components, effective mass m^* can be obtained from the fit of the temperature dependence of the oscillation amplitude A_{osc} at a fixed magnetic field to the thermal damping factor R_T in Equation 10 [i.e., $M_{osc}(T) \propto R_T$]. In normal metals with exact parabolic bands, the band effective mass is expected to be a constant, despite the location of Fermi level. It can be easily shown that such band mass is equivalent to the cyclotron mass, which is defined as $m^* = \dfrac{\hbar^2}{2\pi}\left[\dfrac{\partial S}{\partial E}\right]_{E=E_F}$ within the semiclassical approximation, where S is the extremal area enclosed by the cyclotron orbit in momentum space. Applying the same definition to the linearly dispersed bands with an isotropic Dirac cone, one can easily find that m^* is connected to the Fermi vector k_F and velocity v_F with $m^* = \hbar k_F / v_F$. [⬅**AU: Please ensure that v_F is consistent throughout, per above query**] Thus, m^* should vanish when a Dirac point resides at E_F (where $k_F = 0$) and should increase [⬅**AU:

OK? JH: OK**] when the Dirac point is shifted away from E_F. Such a trend has been observed in various Dirac materials ([172](), [216]()). Generally, E_F is not too far away from the Dirac band crossing point in most known topological semimetals, so m^* obtained from quantum oscillation is usually small, as summarized in **Table 1**.

With a known effective mass, the Dingle temperature that is associated with the quantum relaxation time can be extracted from the fit of the field dependence of the oscillation amplitude at a fixed temperature by the field damping factor R_D in Equation 11 [i.e., $M_{osc}(B) \propto R_D$]. Because T_D is included in the exponential term of R_D, the logarithm of the oscillation amplitude normalized by $B^{1/2}R_T$ (for 3D) or R_T (for 2D) should have linear dependence on $1/B$ according to Equation 11. Thus, T_D can be obtained from the slope of the linear fit of such a Dingle plot. In practice, Dingle plots are nonlinear in some cases in which accurate T_D cannot be obtained. Such a scenario [**←**AU: Please clarify JH: meaning the situation of the nonlinearity in Dingle plot****] could be attributed to, e.g., sample inhomogeneity, magnetic field inhomogeneity, beating oscillation pattern due to the existence of two very close frequencies, or torque interaction at high fields if using torque magnetometry ([203]()).

From T_D extracted from a Dingle plot, the quantum relaxation time τ_q can be derived via $\tau_q = \hbar/(2\pi k_B T_D)$. Because τ_q affects the oscillation amplitude exponentially (Equation 11), strong dHvA oscillations present in low field ranges implies large τ_q, which is generally the case for topological semimetals (**Table 1**). It is important to distinguish the quantum relaxation time from the transport relaxation time τ_t, as discussed in Section 3.1. While both arise from the scattering by static impurities and defects, these two quantities are essentially different ([217](), [218]()): τ_q characterizes the quantum lifetime of the single-particle relaxation time of the momentum eigenstate, which determines the LL broadening of the momentum eigenstate by $\Gamma = \hbar/2\tau_q$, whereas τ_t is introduced in the classical Drude model and affects the Drude conductivity, $\sigma = ne\mu = ne^2\tau_t/m^*$. Given that τ_t measures the motion of charged particles along the electric field gradient, it is largely unaffected by the forward scattering (i.e., small-angle scattering), in contrast to τ_q, which is susceptible to momentum scattering in all directions. Therefore, τ_t is usually larger or even much larger than τ_q. Taking the form of the classical transport mobility $\mu_t = e\tau_t/m^*$, one can also define the quantum mobility by $\mu_q = e\tau_q/m^*$. Consequently, μ_q obtained from quantum oscillation is usually less than μ_t derived from magnetotransport, which has been observed in various topological semimetals, as shown in **Table 1**. [**←**AU: Does the**

highlighted apply to magnetotransport? Please clarify JH: the highlighted means the whole prior sentence, i.e., μ_q obtained from quantum oscillation is usually less than μ_t derived from magnetotransport **]

In addition to nearly zero effective mass and high quantum mobility, nontrivial Berry phase is a key signature of relativistic fermions. As indicated above, it results in the zeroth LL, which is absent in the LL spectrum of nonrelativistic electrons. In general, for a system exhibiting quantum oscillations with a single frequency, ϕ_B can be determined from the LL index fan diagram, i.e., the plot of the LL indices n versus the inverse magnetic field $1/B$ (one example is shown in **Figure 4a,b**). This method has been widely used in previous studies on topological insulators, and a proper way to construct a LL fan diagram has been established, although there had been some confusions in early studies ([202], [219]). We first consider a 2D situation. As shown in **Figure 3b**, with ramping magnetic field, the LLs successively pass through E_F. Integer LL indices are assigned when E_F lies at the middle of two adjacent LLs [i.e., minimum DOS(E_F)], while half-integer indices are assigned when E_F is right at the LL [maximum DOS(E_F)]. For a LL fan diagram established with such a definition of the [**←**AU: OK? JH: OK**] LL index, the linear extrapolation of the linear fit of $n(1/B)$ to the $\frac{1}{B} \to 0$ limit must lead to $n = 0$ for nonrelativistic electrons, but $n = 1/2$ for relativistic fermions due to the zeroth LL <mark>pinned</mark> [**←**AU: pinned? JH: Yes**] at the zero energy. This $n = 1/2$ intercept corresponds to an ideal Berry phase of π. For a 3D system, the phase of quantum oscillation is shifted by $2\pi\delta$, as mentioned above, so the linear extrapolation should intercept the n axis at $\frac{\phi_B}{2\pi} - \delta$.

<COMP: PLEASE INSERT FIGURE 4 HERE>

Figure 4 Quantum oscillations in topological semimetals. [**AU: Regarding the highlighted references in this caption: do they denote that panels a and b are reproduced from Ref 178, panel c is reproduced from Ref 192, and panels d through f are reproduced from Ref 44? Please clarify** JH: yes they are from the references] (*a*) The oscillatory component of resistance for Cd_3As_2, obtained via subtracting the smooth magnetoresistance (MR) background, as a function of $1/B$ at various temperatures ([178]). (*b*) Landau level (LL) fan diagram constructed from Shubnikov–de Haas oscillations for two Cd_3As_2 samples. (*Inset*) Intercepts of the linear extrapolations of LL indices for the two samples ([178]). (*c*) The oscillatory component of resistance for TaP, obtained via subtracting the smooth MR background, as a function of $1/B$ at various temperatures. The red solid lines show the fits of the oscillation data to the two-band

Lifshitz & Kosevich (LK) model (192). (*d*) Mixed real and momentum space representation of the Weyl orbit, which consists of the Fermi arcs at the top and bottom surface connecting the projections of Weyl nodes with opposite chirality (labeled as + and −, respectively) and the bulk states with fixed chirality (*blue* and *red*) (44). (*e,f*) MR at 2 K and its fast Fourier transform for a thin (150-nm) slab sample, for magnetic field parallel (90°) and perpendicular (0°) to the surface. In addition to the bulk frequency F_B, another oscillation frequency corresponding to the surface state (F_S) is observed for the perpendicular field (44).

Therefore, proper assignment of LL indices is critically important for guaranteeing precise determination of Berry phase. Oscillations in differential magnetic susceptibility $\chi \left(= \dfrac{\mathrm{d}M}{\mathrm{d}B} \right)$ offer a straightforward approach to determining integer LL indices; that is, the minima of χ should be assigned with integer LL indices, since they correspond to minimal DOS(E_F). This [←**AU: Please clarify**JH: mans the whole prior sentence "the minima of χ … DOS(E_F)"] can be understood as follows: As indicated above, magnetization is equal to the derivative of the Gibbs thermodynamic potential Ω at constant temperature and chemical potential ζ, $M = -\left(\dfrac{\partial \Omega}{\partial B} \right)_{T,\zeta}$. At zero temperature, Ω is indeed proportional to the total energy of electrons and is modulated by magnetic field in the form of a cosine function (Equation 8) (203). Given $\chi = \dfrac{\partial M}{\partial B} = -\dfrac{\partial^2 \Omega}{\partial B^2}$, χ and Ω would oscillate in phase when Landau quantization occurs with increasing magnetic field. Since the minima of Ω correspond to the minimal DOS(E_F), minimal χ should be assigned with integer LL indices. Given $\chi = \dfrac{\partial M}{\partial B}$, if the oscillations of magnetization are used to establish a LL fan diagram, the minima of M should be assigned with *n*-1/4 (where *n* is an integer number). [←**AU: Please clarify the highlighted – do you mean 1/4*n*?** JH: it means n minus 1/4**] With this approach, the [←**AU: OK? JH: OK**]** nontrivial Berry phase has been extracted from dHvA oscillations for several topological semimetals (156, 220–222).

Several factors can affect the value of the Berry phase in topological semimetals. First of all, the Berry phase can deviate from an ideal value of π if the band dispersion is not perfectly linear (210). Second, the Zeeman effect, which has not been considered so far, also leads to a deviation of the Berry phase obtained from a LL fan diagram (210). Therefore, the Berry phase

determination using the LL fan diagram should be performed with caution for high-field quantum oscillations or for materials with large g-factors such as Cd_3As_2 ([172], [223]) and ZrSiS ([221]). Furthermore, from the aspect of data analysis, reading the Berry phase from a LL fan diagram may bear large uncertainty in some cases. Because the Berry phase is determined by the intercept of the linear fit of $n(1/B)$, when low-LL indices cannot be reached in experiments due to high oscillation frequency, a slight change in the slope of the linear fit can lead to a large shift in the intercept, thus resulting in a large uncertainty in the extracted Berry phase. Therefore, reaching low-LL indices under high magnetic fields is necessary for obtaining a reliable Berry phase from a LL fan diagram.

In addition to magnetization measurements, dHvA oscillations can also be probed by torque magnetometry since a magnetic moment $\vec{m}$ in a magnetic field is subject to a torque $\vec{\tau} = \vec{m} \times \vec{B}$. It is convenient to perform magnetic torque measurements on topological semimetals by using a cantilever ([176], [224]–[230]) to high magnetic field, even up to 60 T. One drawback of the torque magnetometry is the torque interaction, an instrumental effect due to the feedback of the oscillating magnetic moment on the cantilever position, which leads to artificial effects in quantum oscillations under high magnetic fields ([203]).

3.2.3. Shubnikov–de Haas oscillations.

Besides dHvA oscillation, the resistivity oscillation, i.e., the SdH effect, is also widely used to study topological semimetals ([46], [141], [171], [172], [174], [178], [179], [183], [191]–[193], [231]–[233]). The extraction of the Berry phase from SdH oscillations seems straightforward. Since the SdH effect also originates from Landau quantization, the nontrivial Berry phase associated with the zeroth LL also manifests itself by a phase shift in the SdH oscillation. As stated above, integer LL indices should be assigned when E_F lies in the middle of two adjacent LLs and $DOS(E_F)$ reaches minima. The situation is less complicated in 2D integer quantum Hall systems (including the 2D surface states of the 3D topological insulators), in which the integer LL indices unambiguously correspond to the quantized Hall plateaus where the longitudinal conductance reaches minima ($S_{xx} = 0$) due to the dissipationless edge state. The proper way to build a LL fan diagram from the SdH effect for topological insulators was discussed in a previous review ([202]).

In the studies of topological semimetals, however, there have been controversies in constructing LL fan diagrams from the SdH effect. The literature contains various definitions for

integer LL indices, including resistivity minimum (141, 178, 234, 235), resistivity maximum (171, 179, 183, 191, 193, 232, 233, 236–239), and conductivity minimum (143, 172, 213). At first glance, it is natural to extend the above argument for the quantum Hall system to topological semimetals, except that the conductivity of topological semimetals cannot be directly measured through conventional transport experiments but should be obtained through inverting the resistivity tensor, $\hat{\sigma} = \hat{\rho}^{-1}$. For in-plane ($x$-$y$ plane) current I and out-of-plane (z-direction) magnetic field B (i.e., a standard Hall effect setup with $B \perp I$) applied to a 2D system, the charge carriers undergo only in-plane motion, and we have

$$\hat{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} = \hat{\rho}^{-1} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix}^{-1}. \qquad 14.$$

Here the resistivity tensor elements ρ_{ij} ($i,j = x, y$) are defined as E_i / J_j (where E_i is the electric field component along the $+i$ direction and J_j is the current density along the $+j$ direction) or, equivalently, V_i / I_j (where V_i is the voltage drop along the $+i$ direction and I_j is the current along the $+j$ direction). [**←****AU: Please clarify what cannot be directly measured **JH: just to emphasis. Can be deleted****] In fact, from this definition, ρ_{xx} and ρ_{xy} are essentially the longitudinal and transverse (Hall) resistivity. Under the assumption of isotropic scattering rate for a given 2D material, it is easy to demonstrate $\rho_{xx} = \rho_{yy}$ and $\rho_{xy} = -\rho_{yx}$. Therefore, precise conductivity can be obtained from measured ρ_{xx} and ρ_{xy} via $\sigma_{xx} = \dfrac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2}$.

However, additional considerations must be taken for 3D topological semimetals. Although the integer quantum Hall effect (QHE) also has a semiclassical interpretation based on Landau quantization, its underlying transport mechanism is distinct from the SdH effect due to its nonlocal character. As discussed in more detail in Sections 3.2.7 and 3.5, the quantized Hall conductance plateaus and the zero longitudinal conductance are associated with the dissipationless edge channels. Such scale-invariant dissipationless edge conduction in quantum Hall systems is completely different from the transport in conventional diffusive systems, where the resistance or conductance is associated with the sample dimensions and is governed by the transport relaxation rate (i.e., the scattering rate). The scattering mechanisms in real materials can be very complicated. Fortunately, a semiquantitative LK model that gives satisfactory

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descriptions for the SdH effect has been developed for 3D systems. The earlier transport theory has established that the scattering probability is proportional to the number of available states that electrons can be scattered into (47, 240) and thus [**←**AU: please clarify JH: it means the scattering probability ****] oscillates in concert with the oscillations of DOS(E_F) and gives rise to SdH oscillations (203, 204). More explicitly, $\text{DOS}(E_F)_{osc} \propto \left(\dfrac{m^* B}{S_{extr}} \right)^2 \dfrac{\partial M_{osc}}{\partial B}$. With this relation, the expression for conductivity/resistivity oscillation, i.e., the LK formula for the SdH effect, can be derived from the derivative of the magnetization oscillation (203, 204). Clearly, within the framework of this LK model based on the oscillation scattering rate, conductivity should exhibit maxima when the scattering rate reaches minima that occur at minimal DOS(E_F). Given that integer LL indices should correspond to DOS(E_F) minima as indicated above, the maxima of conductivity oscillation should be assigned with integer LL indices. However, this approach is based on the semiquantitative model for the SdH effect (203). The scattering rate in a real material depends on a number of factors and can be very complicated, particularly in multiband or anisotropic systems, which could lead the SdH oscillations to strongly deviate from the LK theory (204). As a result, a simple connection between the integer LL indices and the SdH oscillation extrema may be problematic in some cases. Therefore, to demonstrate [**←**AU: insert "a" or "the"?JH: the ****] nontrivial Berry phase, a better approach might be the oscillation of thermodynamic properties that are directly linked to [**←**AU: insert "the"? JH: OK****] LL energy spectrum, such as the dHvA effect as discussed above.

In addition, the complication of the scattering rate in the SdH oscillation also leads to inconsistency between the SdH effect and the dHvA effect. In some layered topological semimetals, dHvA oscillation is strong for arbitrary magnetic field directions, but SdH oscillation quickly attenuates when the magnetic field is tilted toward the current direction (221, 226, 232, 241, 242). In those materials, the stronger dHvA effect is also useful to distinguish the Zeeman splitting effect from the oscillation pattern [**←**AU: "… from X" – please clarify JH: from the oscillation pattern****] (221).

3.2.4. Multifrequency quantum oscillations.

The above discussions on LL fan diagram are applicable to quantum oscillations with a single frequency. However, multiple oscillation frequencies are often observed in most

topological semimetals, such as those of the TaAs family ([179](), [180](), [182](), [183](), [191]()–[193](), [227](), [243]()) and *WHM* materials with PbFCl-type structure (*W* = Zr or Hf; *H* = Si, Ge, or Sn; *M* = S, Se, or Te) ([156](), [221](), [222](), [226](), [228](), [232](), [233](), [241](), [242](), [244](), [245]()). Given $F = \hbar S_{\text{extr}}/2\pi e$, [**←**AU: should "e" be italicized? JH: yes****] the dependence of oscillation frequencies on the magnetic field orientation provides useful information on Fermi surface morphology. In the presence of multifrequency oscillations, the method used to analyze effective mass, quantum mobility, and the Berry phase differs from what is discussed for the single-frequency situation. The commonly used approach to obtain the effective mass for each frequency band is the fits of the FFT amplitudes for each frequency component by the thermal damping factor R_{T} (Equation 10). In this method, the inverse magnetic field $\dfrac{1}{B}$ in R_{T} is approximated by the average inverse field

$\left\langle \dfrac{1}{B} \right\rangle$, defined as $\left\langle \dfrac{1}{B} \right\rangle = \dfrac{1}{2}\left(\dfrac{1}{B_1} + \dfrac{1}{B_2} \right)$, where $\dfrac{1}{B_1}$ and $\dfrac{1}{B_2}$ are the upper and lower inverse fields used for FFT analyses. However, this method may lead to large errors for the fitted effective mass in some cases, since the obtained effective mass may depend on the range of the inverse magnetic field ($\dfrac{1}{B_1} \rightarrow \dfrac{1}{B_2}$) used for FFT. For example, for the NLSM ZrSiS, the effective mass obtained from the fit of the FFT amplitude is greatly increased when a narrower field range is used for the FFT analysis. When the inverse magnetic field range is taken as 0.143 – 1.5 T^{-1}, the fitted effective mass is small for the $F_\beta = 240$ T band, ~0.052 m_0 (221) [**←** Add the current ref. 221 here, Hu, J. PRB 96, 045127 (2017) ****]. However, when the inverse field range is reduced to 0.3 – 0.5 T^{-1}, the fitted effective mass is increased to 0.17 m_0 (348) [**←** ref. 348 is a new reference, it should be: "Antony Carrington, private communication"****]. Since the two quantum oscillation frequencies observed in ZrSiS are far apart (i.e. 8.4T and 240T), the effective masses corresponding to these oscillation components can also be obtained by fitting the temperature dependence of the oscillation amplitude probed at a certain field. The effective mass obtained using such a method is 0.18 m_0 for the 240T oscillation component. This example shows a narrower inverse field range for FFT may improve the accuracy of the fitted effective mass (348) [**←** ref. 348 is a new reference, it should be: "Antony Carrington, private communication"****]. However, this is not always true.

Therefore, one must be extremely careful when using FFT amplitude to extract the effective mass. For multi-frequency oscillations, if the frequencies are far apart, it may be possible to obtain accurate effective mass by directly reading the oscillation amplitudes as discussed above. On the other hand, if the frequencies are close to each other, several approaches may be used to double check effective mass (348) [**⬅** ref. 348 is a new reference, it should be: "Antony Carrington, private communication"**]. Firstly, as demonstrated above, accurate effective masses may be obtained from the FFT analyses within a narrow field range. Secondly, it may be possible to use Fourier filter to separate multi-frequency oscillations to several single frequency oscillations, which may allow for obtaining accurate effective mass for each frequency. In this method, the data near the two ends of the magnetic field range should be excluded after applying the Fourier filter, since the end effect could induce artificial signal. To minimize the errors in effective mass, the combination of the above methods, together with a simulation of the oscillation pattern using LK formula after obtaining the effective mass, may be helpful.

The Dingle temperature and Berry phase can be extracted through fitting the oscillation pattern to the generalized multiband LK formula, with the assumption that the quantum oscillations of different bands are additive. This method was previously used for the $LaAlO_3/SrTiO_3$ heterostructure (246) and was first employed for analyzing the SdH oscillations of TaP (**Figure 4c**) in the study of topological semimetals (192) and was then proven to be effective in characterizing topological fermion properties for many other multiband topological semimetals (143, 156, 221, 226, 230, 245, 247–249). For the multiband LK fit, it is important to include all major frequency components, as well as the higher harmonic ($r > 1$ in Equations 9 and 13) terms if they are significant in the FFT spectrum, although there is a trade-off for accuracy due to the [**⬅**AU: insert "an" or "the"?JH: "the"**] increased number of parameters. R_S is field independent (see Equation 12) and can thus be treated as a constant for the fit; it takes effects in modulating [**⬅**AU: OK as phrased? JH: OK**] the amplitude for the harmonic component, as it contains r. Furthermore, R_S can be used to extract the Landé g-factor of a 2D/quasi-2D system via the spin-zero method; that is, the oscillation amplitude vanishes at some field orientation due to the interference of spin split Fermi surfaces. This provides an alternative method to evaluate the g-factor in addition to the direct measurement of the separation of the

split oscillation peaks. Such analysis has been reported for ZrSiS ([221]) and WTe$_2$ ([250]).

3.2.5. Magnetic breakdown.

Multiple oscillation frequencies usually result from multiple Fermi surface extremal cross-section areas perpendicular to the field. Additionally, charge carriers may tunnel from one cyclotron orbit to another and jump back to the original one to form a bigger cyclotron orbit, hence leading to an additional frequency or frequencies equal to the sum or difference of two or more fundamental frequencies ([203], [251]). This phenomenon, termed magnetic breakdown, becomes more pronounced at high fields because the tunneling probability scales exponentially with the inverse field $1/B$ as $e^{-\alpha/B}$, where α is a material-dependent parameter relevant to the k-space separation of the orbits ([203]). The additional frequencies ascribed to magnetic breakdown have been observed in high-field quantum oscillation studies on several topological semimetals ([171], [226], [252]).

In type II WSMs, the magnetic breakdown has been predicted to be associated with the Klein paradox, which was in 1929 and which states that the tunneling barrier is nearly "transparent" for relativistic fermions when its height exceeds the electron's rest energy mc^2 ([253]). This relativistic effect is attributed to the positron or electron emission by a potential barrier when the barrier is sufficiently high ([254]–[256]). The matching between electron and positron wave functions across the barrier leads to high-probability tunneling ([257]). However, the requirement of the high potential barrier ($\sim mc^2$) imposes a great challenge for the experimental observation of this phenomenon in particle physics. Fortunately, the (rest) [←**AU: "rest of the"? JH: here the mass means the "rest mass, it is massless (zero mass)"**] massless relativistic fermions discovered in condensed matter provide a realistic platform, given that, in principle, there is no theoretical requirement of the potential barrier for massless relativistic fermions. Klein tunneling has been demonstrated in graphene, with a potential barrier created by a local gate ([257], [258]). A similar effect is expected in topological semimetals with massless relativistic fermions. Recent theoretical work has predicted a momentum space counterpart of Klein tunneling in quantum oscillations for type II WSMs ([259]). In the scenario of magnetic breakdown, quantum tunneling through different momentum space orbits naturally mimics real space tunneling of carriers [e.g., in graphene ([257], [258])], which is expected to lead to an unusual dependence of the FFT amplitude on magnetic field orientation ([259]).

3.2.6. Quantum oscillation due to Weyl orbits.

The unusual surface Fermi arc is one distinct property of topological WSMs. For a DSM whose Dirac node can be viewed as the superposition of two Weyl nodes with opposite chirality, its surface state exhibits two sets of Fermi arcs curving in opposite directions on two opposite surfaces, as shown in **Figure 4d**. It has been predicted that under magnetic fields, electrons can transport on a cyclotron orbit that connects one surface Fermi arc to the opposite Fermi arc by coupling to bulk states (**Figure 4d**) ([43], [260]). Such an unconventional Weyl orbit manifests itself by an additional frequency in quantum oscillations (**Figure 4e,f**), with 2D character that can be verified by the measurement of the field orientation dependence of oscillation frequency (i.e., $F \propto 1/\cos\theta$). Quantum oscillations due to Weyl orbits exhibit anomalous properties such as a sample thickness–dependent oscillation phase shift. To observe such a Weyl orbit, it is necessary to reduce the sample size to suppress the contribution of the bulk states. This has been demonstrated in nanostructures of Cd_3As_2 (**Figure 3e,f**) ([44], [261]) and WTe_2 ([262]).

3.2.7. Other anomalous transport signatures originating from the zeroth Landau level.

As indicated above, the field-independent zeroth LL of relativistic fermions leads to a phase shift in quantum oscillations from which the Berry phase can be inferred. In some layered topological semimetals, the zeroth LL has been probed more directly by several transport techniques such as QHE and interlayer tunneling.

The concept for QHE for 2D Dirac fermions has already been established for graphene and topological insulators ([216], [263]–[265]). Under a magnetic field, Landau quantization gives rise to quantized electron cyclotron orbits. Semiclassically, under sufficiently strong field, the electrons are pinned to these quantized small radii orbits, which causes a bulk insulating state. However, electrons that are close enough to the edges cannot complete cyclotron motions but rather get bounced back by the edges. Given the direction of the Lorentz force, the reflected electrons have to move forward until they are reflected by the edge again. This creates the so-called skipping orbit at the edge that carries current, i.e., the edge channel (**Figure 5a**). Given that the skipping orbit originates from [⬅**AU: insert "the"? JH: OK**] cyclotron orbit, the number of the edge conduction channels is determined by the number of the quantized cyclotron motion states that electrons can occupy, which is the number of the filled LLs below E_F. This gives rise to quantized Hall conductance of $G_{xy} = nG_0$, where $G_0 = e^2/h$ is the conductance quantum. In the

language of band theory, the internal (bulk) of the 2D system is gapped when E_F locates in between LLs. At the sample edge, the confining electrostatic potential that keeps electrons inside the sample bends the LLs upward, as illustrated in **Figure 5a**. The bent LLs that cross E_F form the edge channels, giving rise to quantized Hall conductance. From the above edge channel interpretation for the QHE, the QHE is a direct manifestation of Landau quantization of electron energy states. This is in contrast with SdH oscillation, which arises from the oscillating scattering rate and is thus an indirect probe of LLs. In other words, the QHE is a nonlocal transport phenomenon due to LLs, while SdH effect **[←**AU: edit to "the SdH effect" or "SdH oscillation"? JH: effect**]** is a manifestation of LLs in local transport. Furthermore, the QHE also has a topological interpretation, which is discussed in Section 3.5.

<COMP: PLEASE INSERT FIGURE 5 HERE>

Figure 5 Direct manifestations of the zeroth Landau level (LL). **[**AU: To clarify the highlighted refs in this caption: do you mean that panels b and c are reproduced from Ref 177 and panels d/e/f are reproduced from Ref 247? If not, please clarify JH: yes they are from the references **]** (*a*) Schematic of the real space Landau levels for relativistic electrons in a finite-size 2D sample. (*b*) Crystal structure of EuMnBi$_2$ (177). (*c*) Normalized inverse Hall resistivity ρ_{xy}^{0}/ρ_{xy} versus B_F/B measured at 1.4 K for two EuMnBi$_2$ samples, where B_F is the SdH oscillation frequency and $B = \mu_0(H + M)$ is the magnetic induction (177). (*d*) Schematic of the interlayer tunneling of the zeroth LLs' relativistic fermions in YbMnBi$_2$ (247). (*e*) Experimental setup for the measurement of the angular dependence of interlayer magnetotransport (247). (*f*) Angular-dependent interlayer resistance (AMR) measured under different fields up to 31 T and at $T = 2$ K, using the setup in panel *e*. The black curves superimposed onto the data represent the fits to the tunneling model. The inset shows the $\sin^2\theta$ dependence at low field (247).

Given the existence of the field-independent zeroth LL pinned at the band crossing point (**Figure 3d,f**), there is always an edge channel formed by the zeroth LL, as shown in **Figure 5a**. Since the zeroth LL is evenly shared by both electrons and holes (**Figures 3f** and **5a**), the contribution of the zeroth LL to edge conduction is half the contribution of nonzero LLs, leading to the so-called half-integer quantization, i.e.,

$$G_{xy} = G_0 \left(n + \frac{1}{2} \right). \quad 15.$$

This half-integer quantization can also be understood in terms of **[←**AU: insert "a"? JH: "a"**]** Berry phase of π for relativistic fermions and has been observed in graphene (216, 263),

zero-gap HgTe quantum wells ([266]), and 3D topological insulators ([264], [265]). In real materials, an integer factor may be applied for G_0 due to degeneracy, such as graphene with a factor of 4 <mark>originating from spin and valley degeneracies</mark> ([216], [263]). [**←**AU: Please clarify what originates from spin and valley degeneracies JH: the factor of 4 for graphene originates from spin and valley degeneracies. **]

Given the difference in Landau quantization in 2D and 3D systems as mentioned in Section 3.2.1, it is challenging to probe the half-integer QHE in 3D topological semimetals. One approach is to pursue their 2D nanostructures, but only the integer QHE has been observed so far in nanostructures of Cd_3As_2 and WTe_2 ([261], [267], [268]), probably due to the quantum confinement effect, which gaps the Dirac cone ([267]). Masuda et al. ([177]) reported a half-integer QHE in a bulk DSM $EuMnBi_2$ with a layered structure (**Figure 5b**). This material exhibits coexistence of two AFM orders, one formed by the Mn sublattice and the other by the Eu sublattice. Application of a magnetic field induces a spin flop transition for the Eu AFM order, resulting in a canted AFM state, which significantly reduces interlayer coupling so that Dirac fermions generated by Bi square-net layers are more confined within the plane (i.e., are [**←**AU: OK? JH: OK**] quasi-2D) and exhibit signatures of the half-integer QHE. As seen in **Figure 5c**, $1/\rho_{xy}$ normalized by $1/\rho_{xy}^0$ (where ρ_{xy}^0 is the step size of successive plateaus) displays quantized plateaus with half-integers. However, the quantum limit corresponding to $(1/\rho_{xy})/(1/\rho_{xy}^0) = 1/2$ could not be reached in this system due to the fact that the canted AFM state of Eu sublattice exists only in a limited field range.

In another structurally similar compound, $YbMnBi_2$, the zeroth LL was probed via interlayer transport ([247]). In this material, the Bi layers that host relativistic fermions are separated by the relatively insulating Yb-MnBi-Yb blocks, leading to a quasi-2D electronic state. As shown in **Figure 5d**, given that two linear bands cross right at E_F in this material ([269]), 2D Landau quantization leads to the zeroth LL to be pinned to E_F, regardless of magnetic field strength. Therefore, increasing magnetic field leads to a monotonic increase in DOS(E_F), which further enhances tunneling of electrons of neighboring Bi layers through the Yb-MnBi-Yb barrier when an interlayer electric field is applied. Because 2D Landau quantization in $YbMnBi_2$ is governed by the magnetic field component perpendicular to the Bi plane, such exotic quantum tunneling of the zeroth LL carriers is sensitive to the magnetic field direction and can be detected in angular-dependent magnetotransport such as interlayer MR and the interlayer Hall effect. For example,

for the experimental setup shown in **Figure 5*e***, at low field when LLs are not well separated, LL broadening and thermal excitations smear out discrete LLs, which leads to conventional $(\sin\theta)^2$ dependence for the angular-dependent interlayer resistance (AMR) (**Figure 5*f***, inset). In contrast, when the magnetic field is strong enough to establish the above quantum tunneling scenario, AMR reaches a broad minimum, with θ being approximately 0° due to strong quantum tunneling, but sharply increases for the in-plane field orientation when 2D Landau quantization is suppressed. This causes a surprising strong peak centered at $\theta = 90°$ in AMR, which can be well fitted by the model that includes tunneling of the zeroth LL's carriers (**Figure 5*f***) ([270]).

3.2.8. Beyond the quantum limit.

When magnetic field is strong enough to push all LLs above E_F except for the lowest LL, all electrons are condensed to the lowest LL; such a state is generally referred to as a quantum limit. From this definition, one can find that the critical field needed to reach a quantum limit is at least comparable to the quantum oscillation frequency. The quantum limit is not accessible under a moderate magnetic field for most materials with high carrier density (i.e., large Fermi surface and large quantum oscillation frequency). A system under a quantum limit or an ultraquantum limit may show unusual properties, which has been a long-standing topic of interest even for conventional materials. For instance, a fractional QHE can occur near or in the ultraquantum limit of a 2D electron gas ([271]). In topological semimetals, the dramatically enhanced degeneracy for the lowest LL, combined with the unique nature of relativistic fermions, may lead to some new exotic phenomena. Indeed, a mass enhancement in the quantum limit has been observed for $ZrTe_5$ ([272]). This was interpreted as the dynamic mass generation accompanied by density wave formation, which is due to the nesting of the zeroth LL driven by enhanced electron correlation ([272]). Another example of unusual transport in the quantum limit due to degeneracy enhancement is the aforementioned quantum tunneling of relativistic fermions in $YbMnBi_2$ ([247]). Because the zeroth LL is pinned at E_F ([269]), the quantum limit can be reached in relatively low fields in this material ([247]).

Another phenomenon directly associated with electron condensation to the zeroth LL in topological semimetals is anomalous magnetization ([224]). The Landau quantization for a 3D WSM yields energy spectra of

$$\varepsilon_{n,k} = \begin{cases} \upsilon_{\mathrm{F}} \, \mathrm{sgn}(n)\sqrt{2e\hbar \, |B| \, |n| + \hbar^2 k_z^2}, & n \neq 0, \\ \chi \hbar \upsilon_{\mathrm{F}} k_z, & n = 0, \end{cases} \qquad 16.$$

where $\chi = \pm 1$ represents the chirality of the Weyl points. [←**AU: In Equation 16, should the "e" be italicized? JH: Yes Also, again, please check consistency of υ_{F} (which appears twice in Equation 16) versus v_F throughout text and equations JH: use v_F**] At the quantum limit, magnetization is entirely due to the zeroth LL states, with $M_{n=0} = -\partial \varepsilon_{n=0,k} / \partial B$. Taking the derivatives of Equations 6 and 16, one can find that the magnetization per electron should saturate to a constant in a trivial metal but should vanish in the Weyl case. Therefore, one can expect a collapse of magnetization for topological semimetals crossing the quantum limit. Indeed, the magnetic torque anomaly, which has been observed in NbAs, can be quantitatively described by the topological character of the electronic dispersion (224). [←**AU: OK? JH: OK**]

High magnetic field may also lead to annihilation of a Weyl state. The recent studies on TaP have shown that the two counterpropagating chiral modes of the lowest LL (represented by $\chi = \pm 1$ in Equation 16) may hybridize and open up an energy gap, leading to a magnetic tunneling–induced Weyl node annihilation in TaP that manifests as a sharp reversal of the Hall signal (**Figure 6***a*) (273).

<COMP: PLEASE INSERT FIGURE 6 HERE>

Figure 6 Anomalous transport behavior beyond the quantum limit. (*a*) Magnetic field dependence of the longitudinal (ρ_{xx}) and transverse (ρ_{xy}) resistivity at 1.5 K and 4.2 K for TaP. A steep drop and sign reversal for ρ_{xy} are seen at high field (273). [←**AU: In panel a, what does the "T6" at top right denote? Can it simply be deleted? (If so, our illustration editor will make the change.) JH: it is the sample number and can de deleted**] (*b*) The oscillatory component of resistance ΔR at 4.2 K of three ZrTe$_5$ samples (s6, s7, and s9) with log(B) period (274). [←**AU: Do the highlighted references denote that panel a is reproduced from Ref 273 and panel b is reproduced from Ref 274? Please clarify if not JH: yes they are from the references**]

In addition to the above phenomena associated with the properties of the relativistic Dirac or Weyl fermions on the zeroth LL, new quantum states in the quantum limit regime have been proposed (274, 275). For ZrTe$_5$, whose carrier density varies with different crystal growth techniques, its quantum limit can be reached under a very small magnetic field (~0.2 T) for low-

carrier-density samples. In the quantum limit, surprising resistivity oscillations periodic in $\log(B)$ have been observed (**Figure 6b**) ([274](#)), and these oscillations are believed to be associated with the discrete scale invariance and formation of [**←****AU: insert "a" or "the"?JH: the**] two-body quasi-bound state ([274](#), [275](#)).

Another long-known but intensively investigated transport behavior in the quantum limit is linear MR. As discussed in Section 3.1, orbital MR stemming from the Lorentz effect should exhibit quadratic or nearly quadratic field dependence. In the quantum limit, however, MR grows linearly with B ([195](#)). Such linear MR was discovered in a number of materials ([276](#)–[280](#)) before the establishment of the theory for topological quantum states. Linear MR has been widely observed in many of the recently reported topological semimetals ([45](#), [172](#), [173](#), [175](#), [234](#), [235](#), [281](#)–[283](#)). However, linear MR for those materials begins to develop at a field much lower than the critical field needed to reach their quantum limits ([45](#), [172](#), [173](#), [175](#), [234](#), [235](#), [281](#)–[283](#)). An alternative proposition is that the linear MR in Cd_3As_2 may arise from spatial fluctuations of the magnitude of and direction for local current density in disordered systems ([172](#)), which appears to be applicable for other topological semimetals with linear MR. [**←****AU: Please clarify what appears to be applicable for other topological semimetals with linear MR JH: the above interpretation using the spatial fluctuations appears to be applicable **]

3.3. The Intrinsic Anomalous Hall Effect

In the last section, [**←****AU: please specify section number: 3.2? JH: yes the entire section 3.2**] we intensively discuss the phenomena related to the Landau quantization and the zeroth LL in topological semimetals. As indicated above, the unique zeroth LL originates from the Berry phase of the band character of relativistic fermions. In this section, we review another important phenomenon in magnetic topological semimetals, i.e., the intrinsic AHE, which also stems from Berry phase physics.

AHE, the enhanced Hall signal that couples with the magnetization of magnetic materials, has been intensively studied, as discussed in previous reviews (e.g., [284](#)). Generally, the total Hall resistivity ρ_{xy} in a FM material has an anomalous contribution proportional to sample magnetization M ($\rho_{xy}^{AH} = R_s M$) ([284](#)). Anomalous Hall resistivity can originate from extrinsic mechanisms such as skew scattering ([285](#)) and side jumps ([286](#)) and from intrinsic mechanisms due to the topological properties of bands ([56](#), [287](#)–[289](#)).

One important feature of magnetic WSMs is their intrinsic AHE. Such an intrinsic Hall

component can be understood in terms of the Berry curvature $\vec{\Omega}$ of the electronic Bloch states, which leads to an anomalous electron group velocity perpendicular to the longitudinal electric field $\left[(e/\hbar)\vec{E}\times\vec{\Omega} \right]$ (288). In a magnetic WSM, a pair of Weyl nodes with opposite chirality can be seen as monopole sources of Berry curvature. In this case, the AHE is purely intrinsic and tunable by the separation of paired Weyl nodes (54). The intrinsic AHE current is dissipationless (55, 56, 284, 289) and fully spin polarized (289–291) and therefore has great potential for spintronic applications.

A time reversal symmetry (TRS)-breaking Weyl state has also been predicted or established in many magnetic compounds. [←**AU: Please define TRS (at first use here); please check whether the spelled-out term appears above. If it does, then TRS should be defined above at first use JH: time reversal symmetry.**] An incomplete list includes Co-based Heusler alloys Co$_2$XZ (X = IVB or VB; Z = IVA or IIIA) (95–99), half-metallic Co$_3$Sn$_2$S$_2$ (93, 94, 292), half-Heusler compounds RPtBi (R = Gd and Nd) with AFM orders (108–110), and chiral antiferromagnets MnSn$_3$ and MnGe$_3$ (102, 103). The FM Co$_2$XZ compounds are known to be half-metallic ferromagnets, and some of them have Curie temperatures above room temperature, high spin polarization, and large Seebeck coefficient (293, 294). It has been theoretically predicted that the locations of the Weyl points of these compounds in momentum space can be tuned by the magnetization direction (96, 97). These properties, together with the predicted giant anomalous Hall conductivity (98, 293), make these materials potentially useful for spintronic and thermoelectric applications. These predictions are awaiting experimental verification. A large intrinsic AHE and a giant anomalous Hall angle were recently reported in FM Co$_3$Sn$_2$S$_2$ (94, 292), for which the existence of Weyl fermions has been demonstrated by the observation of surface Fermi arcs (93).

The topological nontrivial states in half-Heusler compounds attracted significant attention even before the discoveries of topological semimetals (108, 295–297). The recent observations of chiral anomaly—a unique feature of Weyl fermions—together with band structure calculations suggest a magnetic field–driven Weyl state in AFM RPtBi (109, 110). Although different mechanisms such as Zeeman splitting (109) and exchange field (110) have been proposed for the formation of a TRS-breaking Weyl state in these AFM zero-gap semiconductors with quadratic band touching, the intrinsic AHE associated with the magnetic field–driven Weyl

state has been probed (**Figure 7*a***), with a very large anomalous Hall angle of ~0.15 comparable to the largest observed in bulk ferromagnets (**Figure 7*b***) ([110](#), [298](#)).

<COMP: PLEASE INSERT FIGURE 7 HERE>

Figure 7 Anomalous Hall effect. (*a*) Magnetic field dependence of the transverse (Hall) resistivity ρ_{xy} for GdPtBi, with field along the [001] direction (298). (*b*) Anomalous Hall angle $\Delta\sigma_{xy}/\sigma_{xx}$ at different temperatures for GdPtBi (298). (*c*) Magnetic field dependence of the Hall resistivity ρ_H for Mn_3Sn (100). [←**AU: Does the highlighted denote that panels a and b are reproduced from Ref 298 and panel c is reproduced from Ref 100? Please clarify JH: yes they are from the references **]

The chiral antiferromagnets Mn_3Sn and Mn_3Ge exhibit large anomalous Hall resistivity in the AFM-ordered state, with a sharp and narrow hysteresis loop in magnetic field sweeps (**Figure 7*c***) ([100](#), [101](#)). In particular, Mn_3Sn is the first antiferromagnet to be discovered to exhibit such a surprising large room temperature AHE ([100](#)). Furthermore, remarkable anomalous behavior has also been observed in its Nernst effect ([57](#)). [←**AU: Please clarify what "its" refers to JH: means this material, Mn3Sn**]** These anomalous transport features have been ascribed to a magnetic Weyl state, which was subsequently demonstrated both theoretically ([102](#)) and experimentally ([103](#)).

Although the intrinsic AHE results from magnetic Weyl states, the strong intrinsic AHE does not exclusively occur in magnetic Weyl systems. [←**AU: OK? Please clarify as necessary JH: it is OK**]** Other magnetic systems such as FM kagomé metal Fe_2Sn_3 ([299](#)), FM spinel $CuCr_2Se_{4-x}Br_x$ ([289](#)), and magnetic semiconductors ([288](#), [291](#)) have also been reported to display the intrinsic AHE.

3.4. [←**AU: "The"? JH: OK to use The**]** Chiral Anomaly

As a hallmark of WSMs, the chiral anomaly is particularly important, as it bridges Weyl fermions in condensed matter physics and in high-energy physics. [←**AU: OK? JH: OK**]** Generally, the numbers of left- and right-handed Weyl fermions are conserved. This individual conservation of particles with opposite chirality is violated in the presence of parallel electric and magnetic fields. This effect, which was originally proposed in particle physics and termed the Adler–Bell–Jackiw effect or chiral anomaly ([17](#)), leads to exotic transport behaviors in condensed matter, i.e., negative longitudinal MR, AMR narrowing, and the planar Hall effect (PHE), which are discussed in detail below.

3.4.1. The chiral magnetic effect and negative longitudinal magnetoresistance.

Negative longitudinal MR (i.e., the increase in magnetic field parallel to the electrical current leading to a decrease of resistivity) related to chiral anomaly has been discovered in several topological semimetal systems, as shown below. Chiral anomaly is the manifestation of the chiral magnetic effect: the generation of electric current under magnetic field induced by the chirality imbalance. The mechanism of this phenomenon is well established ([10], [11], [52], [53]). Here we give a brief overview on its relevant physics. We consider the quantum limit, **[←**AU: OK to add comma here? JH: OK**]** where only the zeroth LL is occupied. As described in Equation 16 and illustrated in **Figure 8a**, the 3D Landau quantization of a WSM leads to counterpropagating zeroth LLs for a pair of Weyl cones, which disperse only along the magnetic field direction. This direction is also the direction for electrons to have coherent motion when an external electric field $\mathbf{E}$ is applied. Such electric field–driven motion leads to electron pumping between Weyl nodes with a rate $\propto -\mathbf{E}\cdot\mathbf{B}$ ([10], [11], [53]), which results in imbalanced population of carriers between the two zeroth LLs of the paired Weyl cones. As a result, the chirality becomes imbalanced. In condensed matter, this charge pumping process is finally relaxed by inter-Weyl node scattering, and a steady state is reached, with a chiral current $j_{\mathrm{c}} \propto B\mathbf{E}\cdot\mathbf{B}\tau_{\mathrm{int}}$, where τ_{int} is the internode relaxation time ([10], [11], [53]). Clearly, this chiral current contributes to negative MR when $\mathbf{E}//\mathbf{B}$. Aside from this quantum mechanical interpretation based on only the zeroth LL, a semiclassical approach based on the Boltzmann equation also yields the same result; with this approach, it **[←**AU: please clarify "it" JH: "it" means the formulism of the chiral anomaly**]** can also be generalized to the semiclassical regime that involves multiple LLs ([10], [11], [53]).

<COMP: PLEASE INSERT FIGURE 8 HERE>

Figure 8 Chiral anomaly and negative longitudinal magnetoresistance (MR). (*a*) Schematic of chiral charge pumping between two Weyl cones with opposite chiralities under parallel magnetic and electric fields ([105]). (*b*) Magnetic field–induced Weyl state by lifting the spin degeneracy of a Dirac cone due to the Zeeman effect ([105]). (*c*) Longitudinal ρ_{xx} at various temperatures for Na_3Bi. Negative longitudinal MR is observed at lower temperatures ([105]). (*d*) Longitudinal ρ_{xx} at various temperatures for $ZrTe_5$. Negative longitudinal MR is observed at lower temperatures ([107]). **[←**AU: Do the highlighted references denote that panels a through c are reproduced from Reference 105 and panel d is reproduced from Reference 107? Please clarify JH: yes they are from those references**]**

Although the negative longitudinal MR originating from chiral magnetic effect occurs in both the quantum limit and semiclassical regime, the actual field dependence of MR can be material dependent. Generally, the negative MR is expected to be linearly dependent on B in the quantum limit while being $\propto B^2$ in the low-field range. But the real situation can be more complex if the internode scattering that relaxes the chiral charge pumping becomes field dependent. This is possible in the quantum limit at high field, as shown below. In real materials, the situation can be further complicated by positive orbital MR due to the Lorentz effect, which is determined by the magnetic field component perpendicular to current, as discussed in Section 3.1. Ideally, such positive orbital MR should vanish when $\mathbf{E}//\mathbf{B}$, but finite orbital MR may arise from a [←**AU: insert "an" or "the"? JH: a**] anisotropic Fermi surface for $\mathbf{E}//\mathbf{B}$ (300). Given such orbital effects, the longitudinal MR may show quadratic field dependence in the low-field range but becomes negative when the chiral magnetic effect dominates.

It is also worth noting that the chiral magnetic effect is not limited to the case of exact $\mathbf{E}//\mathbf{B}$, since the chiral charge pumping rate is finite for nonorthogonal electric and magnetic fields. Therefore, negative MR may be observed in a range of field orientation angles and vanishes when it is compensated by the positive orbital MR component, which is determined by the transverse magnetic field component. If the negative MR is too sensitive to field orientation (e.g., it disappears when the magnetic field is deviated by 1° or 2° from the parallel direction), it may suggest a classical origin of current jetting, which is discussed below.

The chiral magnetic effect was first observed in Dirac systems such as $Bi_{0.97}Sb_{0.03}$ (301), Na_3Bi (105), Cd_3As_2 (**Figure 8c**) (45, 106), and $ZrTe_5$ (**Figure 8d**) (107) before the experimental discovery of WSMs. This effect [←**AU: OK? JH: yes**] can be attributed to the fact that the Dirac point in a 3D DSM can be viewed as a superposition of two paired Weyl nodes with opposite chirality. Such two overlapping Weyl nodes can be separated in momentum space by magnetic field, which breaks time-reversal symmetry (**Figure 8b**). Half-Heusler RPtBi is another group of materials that exhibits the [←**AU: OK? JH: OK**] magnetic field–induced chiral magnetic effect (109, 110). As mentioned in Section 3.3, these materials are zero-gap semiconductors, and their Weyl points are believed to be caused by the external field–induced Zeeman splitting (109) or by the [←**AU: OK? JH: OK**] exchange field from $4f$ electrons (110). It has been proposed that their Weyl points can be induced for any magnetic field orientation, and the induced Weyl points do not necessarily reside on the axis parallel to the field

([104](#)). For these field-induced Weyl states, the separation of Weyl points in momentum space may be dependent on magnetic field, so the negative longitudinal MR could display nonuniversal field dependence. For example, a quadratic field dependence of negative MR anticipated for a non–quantum limit regime has been observed for most of the above materials ([107](#), [110](#), [301](#)). However, a saturation behavior is seen in Na_3Bi (**Figure 8*c***), which is attributed to the field-dependent internode relaxation time in the quantum limit ([105](#)).

Since the experimental discoveries of the WSM state in materials such as TaAs class (type I) materials ([22](#), [23](#), [25](#), [27](#), [39](#), [86](#)–[92](#)) and (W/Mo)Te$_2$ (type II) materials ([28](#), [111](#)–[122](#)), many groups have reported observation of negative longitudinal MR in those materials and have attributed it to the chiral magnetic effect ([109](#), [110](#), [179](#)–[181](#), [183](#), [192](#), [225](#), [302](#), [303](#)). Although chiral anomaly is usually viewed as smoking gun evidence for a Weyl state, one must be cautious before attributing the observed negative longitudinal MR to chiral anomaly, since a classical effect, current jetting, can also lead to negative longitudinal MR ([47](#)). Current jetting is simply due to the rule that the current flows predominately along the **[←**AU: insert "the" or "a"? JH: the**]** high-conductance direction. Once large-conductance anisotropy exists, equipotential lines are strongly distorted, and the current thus forms jets. For materials with large transverse MR, which is the case for most DSMs and WSMs, magnetic field causes very strong conductance anisotropy between the along-current and perpendicular-to-current directions. Therefore, with increasing magnetic field, the voltage drop between voltage contacts may even decrease for asymmetric point–like electrical contacts and irregular sample shape, leading to negative longitudinal MR ([10](#), [304](#), [305](#)). To minimize such a classical effect, it is important to use a perfect bar-shape sample with a large aspect ratio and well-separated, symmetric voltage contacts. Current jetting is also expected to be weak in materials with small transverse MR [e.g., GdPtBi ([304](#))] due to reduced-conductance anisotropy under magnetic fields. More comprehensive discussions of the current jetting effect in topological semimetals can be found in References [304](#) and [305](#).

For type II WSMs such as (W/Mo)Te$_2$ ([28](#), [111](#)–[122](#)), chiral anomaly shows a different situation. Given the strongly titled Weyl cones in such WSMs, **[←**AU: OK? JH: OK. Change "of" to "in"**]** Landau quantization sensitively depends on the orientation of magnetic field, and the Landau spectrum is gapped for some field directions. Therefore, their negative longitudinal MR is strongly anisotropic ([28](#), [306](#), [307](#)); this has been observed in WTe$_2$ ([302](#),

303). Further studies also found that, in the classical limit characterized by $\omega_c \tau \ll 1$ (as opposed to the quantum limit or semiclassical limit, where $\omega_c \tau \gg 1$, where ω_c is the cyclotron frequency and τ is the transport relaxation time), negative longitudinal MR in type II WSM becomes isotropic, similar to that in type I semimetals (303, 308).

3.4.2. The Planar Hall effect.

In addition to generating negative MR in longitudinal transport, the chiral anomaly also leads to a nontrivial transverse (Hall) signal under in-plane magnetic field (**Figure 9a**). Intuitively, an in-plane Hall signal is not expected under in-plane magnetic field due to the absence of electron accumulation on the sample edges. However, in-plane Hall voltage can be generated in the presence of coplanar electric and magnetic fields (**Figure 9a**) due to chiral anomaly, leading to the so-called PHE (309–315).

<COMP: PLEASE INSERT FIGURE 9 HERE>

Figure 9 The planar Hall effect (PHE) and angular-dependent interlayer resistance (AMR) narrowing. (*a*) Experimental setup for the PHE. The magnetic field is rotated within the sample plane (the *x-y* plane). (*b*) Experimental setup for the conventional Hall effect. The magnetic field is rotated from the out-of-plane direction toward the sample plane (the *y-z* plane). (*c,d*) Angular dependence of the (*c*) planar (ρ_{xy}^{PHE}) and (*d*) conventional (ρ_{xy}) Hall resistivity in GdPtBi at 9 T and 2 K, using the setup shown in panels *a* and *b*, respectively. A twofold symmetry is observed for the PHE, in contrast with a onefold symmetry for the conventional Hall effect (313). [**←****AU: Does the highlighted denote that panels a through d are reproduced from Ref 313? Please clarify **JH: yes they are****] (*e,f*) Magnetic field orientation dependence of the magnetoconductivity [$\Delta\sigma_{xx} = \sigma_{xx}(B,\phi) - \sigma_{xx}(B,90°)$] of Na$_3$Bi at 4.5 K, measured at (*e*) low and (*f*) high magnetic fields. [**←****AU: OK? **JH: OK****] The insets show the same data in polar representation. The peak profiles in the angular dependence are clearly narrower at high fields (105). [**←****AU: Does the highlighted denote that panels e and f are reproduced from Ref 105? Please clarify **JH: yes they are** **]

The PHE, a well-known phenomenon observed in ferromagnets, is due to the resistivity anisotropy caused by anisotropic magnetization (316). Although topological semimetals have the same in-plane angular dependence in Hall resistivity ρ_{xy} as do ferromagnets, the PHE in topological semimetals occurs in the absence of magnetic order, with a significantly enhanced amplitude (309, 310). With coplanar electric and magnetic fields, the transverse resistance ρ_{xy} of the PHE is (309)

$$\rho_{xy} = \frac{(\rho_\| - \rho_\perp)}{2}\sin 2\varphi, \qquad 17.$$

where $\rho_\|$ and $\rho_\perp$ denote resistivity with current flowing along and perpendicular to the direction of the magnetic field, respectively, and φ is the angle between the current flow and magnetic field orientation (**Figure 9a**). As discussed in Section 3.2, in the Drude model, the orbital MR for *B//I* is strictly zero unless a multiband effect is involved. Therefore, $\rho_\| - \rho_\perp$ represents the resistivity anisotropy caused by chiral anomaly.

In experimental studies on DSMs and WSMs, an abnormal Hall signal under in-plane magnetic field was first reported in ZrTe$_5$ ([317](#)). A strict $\sin 2\varphi$ dependence was later observed in a number of materials, including ZrTe$_5$, Cd3As$_2$, GdPtBi, WTe$_2$, and VAl$_3$ ([311](#)–[315](#)). With rotating in-plane field (**Figure 9a**) and out-of-plane field (**Figure 9b**), the twofold anisotropy of the PHE (**Figure 9c**) clearly differs from the onefold symmetry seen for the conventional Hall effect (**Figure 9d**) ([313](#)). Unlike the conventional Hall effect, the PHE does not satisfy antisymmetry; i.e., $\rho_{xy} \neq -\rho_{yx}$. This is because the PHE does not originate from the Lorentz force ([309](#), [310](#)).

3.4.3. Narrowing of angular-dependent interlayer resistance.

With the above definition of $\rho_\|$ and $\rho_\perp$, the longitudinal resistivity can be expressed as ([309](#))

$$\rho_{xx} = \rho_\perp + (\rho_\| - \rho_\perp)\cos^2\varphi. \qquad 18.$$

Another unusual property that can be derived from Equation 18 is the narrowing of the AMR peak at high magnetic field ([309](#)). For simplicity, the magnetoconductivity with sweeping in-plane angle φ may be expressed as $\dfrac{1}{\rho_{xx}(B,\varphi)} - \dfrac{1}{\rho_{xx}(0,\varphi)}$ (a stricter process requires tensor conversion). At a small angle, the angular dependence of magnetoconductivity has a Lorentzian profile with angular width ([309](#)):

$$\Delta\varphi \approx \left(\frac{\varepsilon_F}{\hbar v_F / l_B}\right)^2 \sqrt{\frac{\tau}{\tau_c}}, \qquad 19.$$

where $l_B = \sqrt{\hbar / eB}$ is the magnetic length, τ_c is the relaxation time for chiral charge diffusion,

and τ is the conventional momentum relaxation time. At low fields, LLs are wiped out by energy level broadening and thermal excitation. In this case, the parameters involved in Equation 19 are field independent except for l_B, indicating a narrowing of angular width with B that has been observed in Na$_3$Bi (**Figure 8e,f**) (176). When a strong magnetic field drives the system to the quantum limit, the field dependence of each parameter in Equation 19 leads to the saturation of $\Delta\varphi$, as shown in **Figure 9e,f** (176).

3.5. Quantum Hall States in the 2D Limit

3.5.1 Classifications of the various quantum Hall states [JH: this subheading is added to ensure there will be at least two subheadings in this sections]

In the 2D limit, one intriguing aspect of topological semimetals is the potential to generate various quantum Hall states. In Section 3.2.8, we mention that the QHE in the 3D layered topological semimetal EuMnBi$_2$ is caused by the formation of 2D electronic states due to restriction of electron motion in the 2D Bi plane (177). Here we discuss two other quantum Hall states in the 2D limit that have potential applications in electronics and spintronics: the QSHI (i.e., 2D topological insulator) state and the QAHI state. [←**AU: OK? (Note that QSHI and QAHI are now defined above in text, at the first use of each of these terms) JH: OK since they have been defined in the introduction**]

The 2D quantum Hall states for both nonrelativistic and relativistic electrons reflect the fundamental topological properties of materials. For example, the integer QHE, an established [←**AU: OK? JH: OK**] phenomenon that was well understood in terms of the Landau quantization, now has a topological interpretation based on the topological invariant of the Chern number, which opens up the field of topological electronic states in condensed matter. As shown in **Figure 10a** and mentioned in Section 3.2.7, an integer quantum Hall system under sufficiently strong fields is characterized by an insulating bulk state with electrons pinned to quantized small radii orbits and a conducting, dissipationless chiral edge state formed by skipping orbits. [←**AU: OK? JH: remove the second "by" before the** conducting, dissipationless chiral edge ****]** The superposition of two copies of time-reversal integer quantum Hall systems in the quantum limit leads to the QSHI, i.e., the 2D topological insulator, which displays a pair of counterpropagating, spin-polarized edge states due to spin-orbit locking (**Figure 10c**). Apparently, the magnetic field necessary to produce an integer quantum Hall system is no longer

needed for a QSHI system ([76], [79]), as the magnetic field is cancelled out when the time-reversal copies of integer quantum Hall systems are brought together. [**←**AU: Edits to the last sentence OK? JH: OK****] Another modification of the integer quantum Hall system that does not require an external magnetic field is the QAHI state, [**←**AU: OK? JH: OK****] in which spontaneous magnetization leads to the dissipationless chiral edge state (**Figure 10***b*) and the formation of LLs is not required ([76], [79]).

<COMP: PLEASE INSERT FIGURE 10 HERE>

Figure 10 Quantum Hall effects in various topological phases. (*a*–*c*) Schematic for (*a*) the integer quantum Hall insulator (IQHI) state, (*b*) the quantum anomalous Hall insulator (QAHI) state, and (*c*) the quantum spin Hall insulator (QSHI) state. (*d*–*f*) The (*d*) 1H, (*e*) 1T, and (*f*) 1T′ structures of monolayer transition metal dichalcogenides. Panels *d*–*f* reproduced from Reference [72]. [**←**AU: OK? JH: OK****] (*g*) Gate voltage dependence of the differential conductance of the monolayer WTe$_2$ at difference temperatures. Panel g reproduced from Reference [74]. [**←**AU: OK? JH: OK ****]

The QSHI and QAHI states [**←**AU: OK? JH: OK****] also provide significant insights into topological physics beyond simple modification of the integer quantum Hall system ([76]). The QAHI and [**←**AU: insert, e.g., "insulators in the"? JH: do not inset that****] the integer quantum Hall system are essentially 2D Chern insulators characterized by nonzero Chern numbers, in contrast with a trivial insulator with $C = 0$. With TRS, the Chern number must vanish, but another topological invariant, the Z_2 number, can be introduced to clarify the 2D insulators, becoming 0 for trivial insulators and 1 for a symmetry-protected topological insulator (QSHI) ([318]). Simple stacking of these 2D building blocks leads to a 3D weak Chern insulator or a weak topological insulator that is not robust against disorder ([319]). It is also possible to extend the topological classification of a QSHI to 3D and create a strong 3D topological insulator ([319]). However, the extension of the 2D Chern insulator to 3D cannot produce a strong 3D Chern insulator. Instead, this development [**←**AU: OK? JH: OK****] results in a metallic phase: the topological semimetal ([76]). The above discussions show how quantum Hall systems, the QSHIs, the QAHIs, [**←**AU: OK? JH: see my edits****] 3D topological insulators, and topological semimetals are closely connected in terms of the topological properties, which implies the possibility of conversion between these states.

From the experimental aspect, QSHIs and QAHIs are expected to display unusual nonlocal

transport (320, 321). [←**AU: Referring to the insulators (text would remain as is) or the systems/states (in which case please add "systems" or "states" after "QSHI and QAHI")? JH: the insulators**] The resistance or conductance of conventional diffusive systems is dependent on the dimensions of the sample and is determined by the local resistivity or conductivity (Ohm's law). However, in quantum Hall systems, due to scale-invariant dissipationless edge conduction, transport is nonlocal, and the concepts of resistivity or conductivity are thus meaningless. The Hall conductance can be obtained from the Chern number C by $G_{xy} = Ce^2 / h$; a half-quantized Hall conductance is also expected for massless relativistic fermions, as discussed in Section 3.2.8 (Equation 15). For a QSHI, $G_{xy} = 0$ due to $C = 0$ in a TRS system, which can be attributed to the fact that the pair of time-reversed chiral edge states cancels each other (**Figure 10c**). For the longitudinal conductance G_{xx}, the measurement results strongly depend on the configuration of the contact electrodes. This is because an ideal contact attached to the edge of the sample acts as a reservoir that draws electrons and emits them from and to the edge channels. [←**AU: OK? JH: OK**] The spin information of an electron is smeared out during this process. For an integer quantum Hall system or a QAHI system, the edge state is chiral (**Figure 10a,b**), and the electrons emitted from the contact have to flow along the same direction, which should lead to zero longitudinal conductance and hence zero longitudinal resistance according to resistivity and conductivity tensor conversion. However, for a QSHI [←**AU: insert "state" or "system? QSHI can refer to one type of insulator, so "state" or "system" is not needed here **] with time-reversed spin-polarized edge states, the spin of the emitted electrons has half probability to be reversed, [←**AU: OK as phrased? JH: OK**] corresponding to the back-moving edge channel with opposite spin. Therefore, a finite resistance depending on the number and configuration of contacts can be expected (320, 321).

3.5.2. Material realizations for the QSHI and QAHI states. [**AU: (1) Edits OK? JH: OK (2) If using subheadings in a section, there need to be at least two, per house style. Please either add another subheading to the section (e.g., add a Section 3.5.2) or remove this one here. JH: another subheading is added at the beginning of this section, so there are two subheadings now**]

The QSHI state [←**AU: OK? JH: OK. QSHI can refer to the actual insulator or the quantum state**] has been proposed in the monolayer form of the layered 1T′- transition metal dichalcogenides MX_2 (M = W, Mo; X = S, Se, Te) (72) and WHM (322). The structure of

monolayer MX_2 is formed from the stacking of X-M-X layers, with its physical properties being determined by the type of stacking. A hexagonal H structure with *ABA* stacking (**Figure 10***d*) results in the well-known direct-band-gap semiconductors ([323](#)). For a rhombohedral 1T phase with *ABC* stacking (**Figure 10***e*), the structure is unstable and undergoes a spontaneous lattice distortion to the 1T′ phase (**Figure 10***f*), which consequently leads to a QSHI state in the presence of SOC ([72](#)). The QSHI state in monolayer 1T′-MX_2 was first demonstrated in WTe_2, as this material naturally has the 1T′ structure in the bulk form. There is transport ([74](#), [75](#)) and spectroscopic ([73](#)) evidence of the QSHI state in WTe_2 monolayers prepared using mechanical exfoliation or molecular beam epitaxy (MBE) growth. For example, upon [**←****AU: OK? **JH: OK****] sweeping the gate voltage, a conductance plateau associated with the 1D edge state of a QSHI is observed in a WTe_2 monolayer (**Figure 10***g*) but is absent in bilayer or few-layer samples ([74](#), [75](#)). More importantly, the temperature at which the conductance plateau starts to develop is as high as 100 K (**Figure 10***g*), which is greatly higher than the operating temperature of other well-established QSHIs in semiconductor quantum wells ([324](#)) and could be ascribed to the large bulk band gap of the 1T′-WTe_2 monolayer [which was predicted to be 100 meV ([72](#)) and found to be 55 ± 20 meV for MBE-grown samples ([73](#))]. This finding has great potential for practical device applications. Furthermore, under one proposal, the horizontal electric field may break the inversion symmetry and may induce strong Rashba splitting of the bands near E_F, which closes the bulk gap at some critical electric fields. Such gap closing leads to a topological phase transition to a trivial phase; this transition occurs very rapidly and can thus be used for topological field effect transistors ([72](#)).

The tetragonal layered *WHM* compounds have also been predicted to become QSHIs in the monolayer form ([322](#)). Different from WTe_2, which is a type II topological WSM in the bulk form ([28](#), [111](#)–[113](#), [117](#)), bulk *WHM* is predicted to be a weak topological insulator formed from the stacking of QSHIs ([322](#), [325](#)); this is a long-sought topological quantum state ([326](#)). In *WHM*, C_{2v} symmetry ensures nodal-line crossings near E_F in the absence of SOC, but this symmetry cannot prevent SOC gap opening ([154](#)). Because the Fermi level crosses the gapped cones and the band dispersion is extremely linear over a wide energy range, *WHM*s have been established as topological NLSMs ([78](#), [85](#), [154](#)). To realize the predicted QSHI state, one possible route is to exfoliate the bulk *WHM*s to their monolayers. Although the interlayer coupling in *WHM*s is not van der Waals type ([322](#), [327](#)), the weak coupling strength in some

*WHM*s allows for mechanical exfoliation, as has been demonstrated ([156](#)). One possible advantage of using *WHM*s as a platform for searching **[←**AU: "for"? JH: can change "searching" to "realizing"**]** QSHIs is the variable SOC gap with various combinations of *W*, *H*, and *M* ([226](#)); such a gap offers the opportunity to design different QSHIs.

As mentioned above, a QAHI system is in principle similar to the integer quantum Hall system, but the former occurs without an external magnetic field and LLs ([76](#), [79](#)) and thus carries great promise for possible applications in spintronics. **[←**AU: OK? JH: OK**]** Furthermore, a QAHI system **[←**AU: insert "state" or "system"? JH: system**]** also provides a promising platform for the creation, manipulation, and utilization of Majorana fermions, the hypothetical particles that are their own antiparticles ([328](#), [329](#)). The QAHI state was first experimentally demonstrated in magnetically doped topological insulators ([330](#)–[332](#)). However, it has so far been realized only at very low temperatures (<1 K) ([330](#)–[332](#)). Room temperature QAHIs, **[←**AU: "QAHI state" or "QAHIs"?, JH: QAHIs**]** if realized, will have the potential to revolutionize information technology through dissipationless spin-polarized chiral edge transport in spintronic devices. Recent studies have revealed a new possible route to the realization of high-temperature QAHIs: 3D FM WSMs can evolve into large-gap QAHIs when the dimensionality is reduced from 3D to 2D, due to the confinement-induced quantization of low-energy states ([21](#)). **[←**AU: In the last sentence, change QAHI to "QAHIs"? JH: OK**]** One possible candidate material is $HgCr_2Se_4$ ([21](#)), which is awaiting experimental verification. In addition to these two approaches, there are other proposals for the realization of QAHIs ([76](#)).

4. SUMMARY AND PERSPECTIVE

Above we review distinct electronic transport phenomena associated with nontrivial band topology in different types of topological semimetals and discuss how to extract the fundamental properties of Dirac/Weyl fermions such as effective mass, quantum mobility, and the Berry phase from dHvA or SdH quantum oscillation measurements. The above discussion shows that topological semimetals exhibit a rich variety of exotic properties that are not seen in nonrelativistic electron systems. These properties include chiral anomaly and the PHE in WSMs, the intrinsic AHE in time-reversal symmetry–breaking WSMs, quantum oscillations due to Weyl orbits and AMR peak narrowing under high magnetic fields in DSMs, the half-integer QHE and

quantum tunneling of the zeroth LLs in layered magnetic DSMs, and [**←**AU: OK? JH: OK**] vanishing magnetization and dynamic mass generation in the quantum limit of DSMs/WSMs. We discuss how these properties are connected with nontrivial band topology, although the mechanisms for some of these properties are not fully understood. Furthermore, we discuss how DSMs/WSMs are linked with the QSHI and QAHI states [**←**AU: OK? JH: OK**] and how these two quantum Hall states can be approached by reducing NLSMs/FM WSMs to 2D thin layers.

As previous reviews have noted (10, 11), one challenge in this field is the experimental realization of ideal model systems like graphene (10) or the hydrogen atom (11) for various types of topological semimetal phases. An ideal model system should contain only the topological band(s), with the same types of Dirac or Weyl points being symmetrically related, located at the Fermi energy level, and well separated in momentum space. For the material aspect, such a system should be stable in the ambient environment and have minimal defects (10, 11). As noted above, the topological semimetals discovered so far are probably the tip of the iceberg. Given that topological semimetals can be predicted by band structure calculations, we believe that many new topological semimetal phases and candidate materials will be discovered and that some of them may serve as model systems. There have been recent breakthroughs in topological phase screening and database development for topological quantum materials (37, 325, 333–336). With new simple model systems, the trivial bands will not mask or interfere with the contributions from exotic phenomena arising from the nontrivial bands, and novel knowledge of various topological semimetal phases can be further revealed.

Topological quantum materials have stimulated great interest because of not only their connection with high-energy particle physics but also their great potential in future technology applications. As discussed above, both the QSHI and QAHI states [**←**AU: OK? JH: OK**] can be obtained by reducing the dimension of NLSMs/FM WSMs to 2D, and these two states can support dissipationless transport through their topological spin-polarized edge states. [**←**AU: Please spell out DNLS (it appears only once in text - in the last sentence) JH: should be NLSMs**] Therefore, they carry great promise for applications for spintronic devices and quantum computation. Although both the QSHI and QAHI [**←**AU: OK? JH: remove "states"**] have been demonstrated experimentally, these states currently occur only in the low-temperature range. Pushing their operation temperature to room temperature is another great

challenge in the field. Achieving this goal requires discoveries of new topological materials with better properties, along with integrated efforts in theoretical modeling, computation, synthesis, characterization, and device demonstrations.

[**AU: Please insert your Disclosure of Potential Bias statement, covering all authors, here. If you have nothing to disclose, please confirm that the statement below may be published in your review. Fill out and return the forms sent with your galleys, as manuscripts CANNOT be sent for page proof layout until these forms are received. JH: the statement below is OK with me (Jin Hu). Please verify with other authors as well**]

DISCLOSURE STATEMENT

The authors are not aware of any affiliations, memberships, funding, or financial holdings that might be perceived as affecting the objectivity of this review.

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Table 1 Parameters obtained from transport and quantum oscillation experiments at base [←**AU: OK as phrased? JH: change "based" to "base"**] temperatures (1.5–5 K), including magnetoresistance (MR) at 9 T, residual resistivity ρ_{res}, transport mobility μ_T, quantum relaxation time τ_q, quantum mobility μ_q, and effective mass ratio m^*/m_0 [←**AU: OK to delete the hyphen beneath the asterisk here and in the table (header row, second-to-last column)? JH: yes it is OK. In fact in my version there is no such hyphen under the asterisk. Also, for the table below, each cell should have content in it. For each blank cell, please specify text that should go in it – e.g., "NA" (if so, please define NA as, e.g., not applicable or not available) JH: OK**]

	MR at 9 T	ρ_{res} ($\mu\Omega$ cm) [←**AU: Should there be a multiplication dot (·) between these units? Please clarify JH: with or without dot, both are fine**]	μ_T [cm^2/(V·s)]	τ_q (ps)	μ_q [cm^2/(V·s)]	m^*/m_0	Reference(s)
Cd$_3$As$_2$	34.5–1,336	0.032–46.5	4×10^3–8.7×10^6	0.03–0.21	4,700–6,000	0.023–0.26	45, 172, 178, 236, 237, 272
Na$_3$Bi	5.69–97.1	1.72–87	5,500–78,900	0.0816	NA[**AU: Blank cell; please specify content JH: NA, not available***]	0.11	105, 176
TaAs	3–30,000	0.63–1.9	18,000–	0.038–1.1	32,000	0.021–0.68	46, 179–

family			10,000,000				183, 191–194, 225, 227, 243
WTe₂	4,000–25,000	0.39–1.9	24,000–176,000	NA [**AU: Blank cell; please specify content**]	NA	0.41–0.46	48, 184, 186, 231
MoTe₂	2,653	28	16,000–58,000	NA [**AU: Blank cell; please specify content**]	NA [**AU: Blank cell; please specify content**]	0.8–2.9	337–340
PtSn₄	1,000–2,100	NA [**AU: Blank cell; please specify content**]	NA [**AU: Blank cell; please specify content**]	NA [**AU: Blank cell; please specify content**]	14,257–15,809	0.05–0.36	341–343
PtBi₂	12,000	NA [**AU: Blank cell; please specify content**]	NA [**AU: Blank cell; please specify content**]	NA [**AU: Blank cell; please specify content**]	NA [**AU: Blank cell; please specify content**]	NA [**AU: Blank cell; please specify content**]	171
Pt(Te/Se)₂	A few tens [←**AU: Possible to be more specific, e.g., "~30–40"? JH: the original	NA [**AU: Blank cell; please specify content**]	3,600–5,500	NA [**AU: Blank cell; please specify content**]	NA [**AU: Blank cell; please specify content**]	0.11–3.6	229, 344

	papers states like that. They do not provide a number or number range **]						
PdTe$_2$	A few tens [←**AU: Possible to be more specific, e.g., "~30–40"? JH: the original papers states like that. They do not provide a number or number range **]**	NA [**AU: Blank cell; please specify content**]	NA [**AU: Blank cell; please specify content**]	0.18–0.65	1,293–6,209	0.04–1.16	229, 248
AMn(Sb/Bi)$_2$ (A = Ca, Sr, Ba, Yb)	1	NA [**AU: Blank cell; please specify content**]	1,500–3,400	NA [**AU: Blank cell; please specify content**]	NA [**AU: Blank cell; please specify content**]	NA [**AU: Blank cell; please specify content**]	141, 143, 173, 175, 213, 234, 235, 238, 239, 247, 283, 345, 346
WHM[a]	1.3–140,000	0.052	2,000–28,000	0.025–0.35	209–10,000	0.025–0.27 1.32[b] [←**AU: Do these represent two different number ranges?	156, 222, 226, 228, 232, 233, 245, 347

						Please clarify JH: the second one (1.32) is very unusual and have other specific origins, as denoted below (the mass enhancement)**]	

^aMR, effective mass, and quantum relaxation time widely vary in different *WHM* materials, possibly due to the spin-orbit coupling gap, which varies with the atomic number.

^bCaused by the mass enhancement at low temperatures ([245]). [←**AU: OK? Or "Mass is enhanced at low temperatures (245)"? JH: see my edits**]