

Individual Differences in Math Problem Solving and Executive Processing

Among Emerging Bilingual Children

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Abstract

This study identified cognitive processes that underlie individual differences in working memory (WM) and mathematical problem-solution accuracy in emerging bilingual children (English Learners). A battery of tests was administered in both English and Spanish that assessed problem solving, achievement, and cognitive processing in children in first ($N = 155$), second ($N = 129$) and third grades ($N = 110$). The results were that (a) the executive component of WM predicted solution accuracy of word problems independent of measures of vocabulary, reading, calculation, knowledge of algorithms, processing speed, short-term memory, and inhibition, (b) L1 (Spanish) measures that contributed unique variance in predicting L2 math problem solving accuracy were latent measures of Spanish STM, Spanish inhibition, and Spanish WM, and (c) L1 (Spanish) measures of the executive component of WM and magnitude comparisons were major predictors of math problem solving accuracy in both languages. The results support the notion that the executive system of WM is a unique predictor of emerging bilingual children's math problem solving accuracy in both languages.

Key words: Executive processing, working memory, math word problems, English learners, cross-sectional

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

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According to the National Mathematics Advisory Panel (2008), and to PISA (Programme for International Student Assessment; OCED 2012a,b), U.S. children show substantial weaknesses in math relative to other achievement domains and in comparison to other industrialized countries, which suggests a need to understand and develop interventions to improve math performance. This problem is especially compounded in children learning English as a second language. Spanish-speaking English Learners (EL, also referred to as emerging bilinguals) make up a large percentage (73.1%) of the English language learners (August & Hakuta, 1997; National Assessment of Educational Progress, 2011; 2017; National Center for Education Statistics, 2011) and represent a substantial number of students who do not demonstrate proficiency in mathematics. In math, one of the key areas found particularly difficult for emerging bilingual children whose first language is Spanish is solving word problems (e.g., Macizo, Herra, Roman, & Marin, 2011; Martiniello, 2008; 2009; Ockey, 2007). This is because math word problems require complex processes beyond basic computation, such as reading comprehension, the use of linguistic information, identifying relevant information, and constructing the appropriate problem statement. (Note. The term emerging bilingual rather than EL will be used throughout the manuscript).¹

Clearly, not all emerging bilingual children experience math problem solving difficulties and therefore it is important to determine some of the cognitive strengths of these children compared to their lower performing peers. Although some of the difficulties in math experienced by emerging bilingual children with Spanish as a first language have been partially attributed to cross language transfer, oral language, linguistic complexity, and reading skill (e.g., Farnia &

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

Geva, 2011; Han, 2012, Macizo et al., 2011; Martiniello, 2008; 2009; Ockey, 2007; Vukovic & Lesaux, 2013), other processes besides language and reading may play a critical role in such children's math difficulties (MD). Therefore, it is important to determine some of the cognitive measures that predict success on math problem solving measures for Spanish speakers so intervention programs can be developed and tested.

There is recent evidence to suggest that one domain-general cognitive process, working memory (WM), plays a significant role in math for monolingual children who suffer from math difficulties (e.g., Authors et al., 2008; David, 2012; Geary, Nicholas, Li, & Sun, 2017; Menon, 2016; Mammarella, Caviola, Giofr  & Sz cs, 2018; Passolunghi & Mammarella, 2010; Peng, Namkung, Barnes, & Sun, 2016; Sz cs, Devine, Soltesz, Nobes, & Gabriel, 2013). Working memory is defined as consisting of a limited capacity system of temporary stores, functions related to the preservation of information while simultaneously processing other information and attention control related to these functions (e.g., Baddeley, 2012).² One component of WM that has been associated with math difficulties is executive processing (e.g., David, 2012; Menon, 2016; Passolunghi & Mammarella, 2010; Peng, Namkung, Barnes & Sun, 2016).

No doubt, based on previous research with monolingual children, significant correlations between the executive component of WM and math problem solving among emerging bilinguals are likely to be found. However, it is critical to expand our understanding of problem solving performance in the child's second language by examining the potential variables that may also contribute to math problem solving accuracy and/or may supersede the importance of WM. This is because some studies demonstrate a bilingual benefit in WM (e.g., Kudo & Swanson, 2014; Morales, Calvo & Bialystok, 2013), whereas as other do not (e.g., de Abreu, 2011, de Bruin, Barbara & Della Sala, 2015; Namazi & Thordardottir, 2010). Thus, an important question

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

emerges as to whether executive component of WM plays an important role when performance from other processing (e.g., STM, processing speed), academic (e.g., reading) and/or language (e.g., vocabulary) domains are considered. We consider at least three processing domains, besides executive processing, that may play an important role in predictions of math problem solving accuracy in emerging bilingual children.

Language and Reading Proficiency

Vocabulary and reading have been clearly related to word problem solving accuracy (e.g. author et al., 2014; Bjork & Bowyer-Crane, 2013; Fuchs et al. 2006, 2016; Gorman, 2012; Harvey & Miller, 2017; Lee, Ng, Ng, & Lim, 2004). Several studies show that L1 performance in vocabulary (e.g., Farnia & Geva, 2011) and reading (e.g. author et al., 20016; Lonigan, Allan, Goodrich & Farrington, 2017) is related to comprehension, which in turn is highly correlated with math problem solving accuracy (e.g., Fuchs, Fuchs, Compton, Hamlett & Wang, 2015). For example, previous studies have identified the instrumental role of language in mathematical understanding, reasoning, problem-solving and expression of solutions (Bjork & Bowyer-Crane, 2013; Vukovic & Lesaux, 2013). In terms of reading, the research with monolingual children shows that children with math difficulties manifest deficits that are hard to differentiate from children with reading disabilities (e.g. Landerl & Moll, 2010). This is because MD co-occurs with reading disabilities more frequently than expected by chance (e.g., Branum-Martin, Fletcher, & Stuebing, 2013).

A logical prediction from this research suggests that emerging bilingual children with better L1 vocabulary and L1 reading will outperform children with the lesser vocabulary and reading skills on math problem solving measures. This prediction is consistent with the assumption that

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

mastery of the first language plays a major role toward identifying the cognitive processes that influence achievement (math) (Cummins, 1979; Kempert et al, 2011; Lonigan, Allan, Goodrick, Farrington & Phillips, 2017; Rosselli, Aridila, Lalwani & Velez-Urbe, 2016).

Domain Specific Processes

Domain specific knowledge also plays a significant role in math problem solving performance in emerging bilingual children. For example, specific numerical knowledge activated from long-term memory (LTM), such as magnitude judgment and number line estimation, can mediate the relationship between executive component of WM and word problem-solving (e.g., Fuchs et al. 2012; Geary, 2011; Simmons et al. 2012). The model would further assume that strengths within the child's first language in accessing domain specific knowledge would predict second language problem solving performance.

In the present study, we assess whether the retrievability of domain specific contents in LTM underlie the relationship between WM and math problem solving. One of the core competencies of children in math performance is their ability to map Arabic numerals onto representations of small quantities or large quantities (referred to as magnitude comparisons; e.g., Booth & Siegler, 2008; Kolkman, Kroesbergen, & Leseman, 2014). Various forms of symbolic magnitude comparisons discussed in the literature include judgments of number magnitude (e.g., which is larger: 8 or 5; distinguishing between an approximate number of dots or objects, e.g., Rousselle & Noël, 2007). Other forms focus on *estimation*. This process involves judging measurements and assigning numbers without counting. For example, a number-line estimation task requires children to estimate the position of target numbers on a line within numerals at end points (e.g., 0 and 100). Geary (2011) and several studies (e.g., Fuchs et al., 2012; Rousselle & Noël, 2007) have found that accuracy in making placement on a mathematical

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

number line was uniquely predictive of math skills. Other specific forms of knowledge accessed from LTM may play a role in math problem solving include recognizing propositions within word problems. The specific propositions of interest within math word problems are related to accessing numerical, relational, question, and extraneous information, as well as accessing the appropriate operations and algorithms for the solution (e.g., Mayer & Hegarty, 1996).

Phonological Storage

Another variable of interest that may contribute to individual differences in WM and math word problem solving is phonological short-term memory (STM). Phonological STM is an important component of second-language vocabulary acquisition and achievement (e.g., Engle de Abreau & Gathercole, 2012; Thorn & Gathercole, 1999). Theoretically, STM involves passive storage, in which the learner mentally rehearses verbal and/or visual-spatial information (Baddeley, 2007). Although WM or complex span tasks share the same processes (e.g., rehearsal, updating, controlled search) as STM or simple span tasks, STM tasks have a greater reliance on phonological processes than complex span (WM) tasks (see Unsworth & Engle, 2007, pp. 1045-1046, for a review).

The majority of studies that have compared children who vary in language and math skills assume that STM measures tap a phonological system or what Baddeley (2007) denotes as the phonological loop. The phonological loop has been referred to as STM because it involves two major components discussed in the STM literature: a speech-based phonological input store and a rehearsal process (see Baddeley, 2012, for review). Research to date indicates that children with difficulties in second-language acquisition have difficulties on tasks requiring the short-term retention of ordered information (e.g., Thorn & Gathercole, 1999; Thorn, Gathercole, & Frankish, 2002), which in turn may play a role in math problem solving performance.

Purpose of Study

In summary, we investigate whether the executive component of WM contributes unique variance in math problem solving accuracy when the aforementioned variables (e.g., vocabulary, reading, numeracy, STM) are entered into a regression analysis. What is at issue is not whether second language math problem solving relates to some of the aforementioned variables, but whether differences in math problem solving within or between the two language systems can also be attributed to executive component of WM above the well-attested contribution of these aforementioned processes. We operationalize the executive component of WM as the unique variance that predicts math problem solving accuracy after the aforementioned variables are partialled in a regression analysis. The executive component involves both verbal and visuospatial WM measures. Our definition is consistent studies that assume that when phonological STM is partialled out from the effects of WM on math problem solving, the remaining residual variance reflects the executive component (controlled attention) of WM (e.g., Bayliss, Jarrold, Gunn, & Baddeley, 2003; Engle, Tuholski, Laughlin, & Conway, 1999).

This study also explores the influence of L1 skills on L2 math problem solving performance. No study we are aware of has compared the influence of L1 and L2 on the array of measures included in this study. The majority of these studies test emerging bilingual children primarily in their second language (English), and therefore the influence of code-switching (moving between L1 and L2) and/or cross-language transfer on math problem solving difficulties cannot be evaluated.

Therefore, a second objective of this study was to investigate the relationship between WM and math problem solving in emerging bilingual children within both language systems (i.e., Spanish and English). We predict because children are emerging bilinguals in this study an

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

asymmetrical effect will emerge such that L1 measures are relied upon in predicting L2 math problem solving performance. Relevant to this hypothesis is a study by Costa and Santesteban (2004) who compared participants with varying language backgrounds and levels of L2 proficiency who performed a language switching task (shown a picture naming task that alternated between L1 and L2 in an unpredictable manner). They observed for L2 learners, switching from L2 to L1 carried a higher cost than switching from L1 to L2. This finding suggested that participants inhibited L1 more than the L2, so the switching from the L2 to the L1 placed greater cognitive demands on the participant. Thus, an asymmetrical effect emerged that showing that predictions of L2 performance relied on the strength of L1 and/or a mixed of L1 and L2 predictors. However, it was also important to note in this study that a cross language (or asymmetrical) effect was “not” found with bilinguals. Participants who were relatively proficient in L1 and L2 (i.e., bilinguals) were able to achieve a language specific selection “without” inhibiting their L1 language. That is, when both language systems could be accessed, L1 predictor measures were related to L1 criterion measures and L2 predictor measures were related to L2 criterion measures. Thus, this study explored whether emerging bilingual children’s L2 problem solving performance reflects a mixed of both language systems or reflects the language of the criterion measure (e.g., L1 measures predict L1 problem solving or L2 measures predict L2 problem solving). An answer to this question has implications for designing interventions since the literature is unclear as to whether emerging bilingual children draw upon a L1 executive system and/or L2 executive system when confronted with L2 math problem solving measures.

In summary, two primary questions that directed this study were:

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

1: Does the executive component of WM contribute unique variance to math problem solving accuracy when vocabulary, reading, domain specific processing and phonological storage are entered into the analysis?

2: Do L1 vocabulary, achievement and cognitive measures, rather than L2 measures, predict L2 math problem solving?

Method

Participants

Three hundred and ninety-four ($N=394$) students in grades 1 ($n=155$), 2 ($n=129$) and 3 ($n=110$) from two large urban school districts in the southwest United States participated in this study. The sample was the first testing wave of a four year federally funded longitudinal study assessing executive performance among emerging bilingual children (authors et al.). Ninety-eight (98) percent of the children in sample participated in a full or reduced Federal lunch program and were drawn from neighborhoods with high Hispanic representation ($> 95\%$). The children in this study were designated as EL or emerging bilingual by their school and were selected from 30 elementary classrooms. The final sample included 192 boys and 202 girls who returned signed consent forms. School records indicated children's primary home spoken language was Spanish (80%). All children were selected from dual language classrooms in which math instruction was in both English and Spanish. The math programs in each school district placed a heavy emphasis on math facts and word identification. No significant differences in gender representation emerged across the grades, $\chi^2(df=2, N=394)=2.88, p=.23$.

Measures

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

The study included group and individual administrations of a battery of tests. The series of tests were counterbalanced into one of four presentation orders. No Spanish and English versions of the same test (except for the Expressive One-word Picture Vocabulary Test, Spanish-Bilingual Edition; Brownell, 2001) were presented simultaneously. Several cognitive measures that required Spanish-translated versions were developed from an earlier study (see Authors et al, 2014, 2017, for further discussion). All participants were administered both English and Spanish versions of each measure by bilingual graduate students and staff researchers. Instructions were given in Spanish for all tasks requiring Spanish responses unless noted otherwise. There were some tasks (e.g., pseudoword reading) that required calibration for task difficulty. Three native Spanish speakers made judgments on the difficulty of the items in relation to the task presented in English. Interrater agreements exceeded 90%. A brief description is provided for commercial measures, whereas experimental measures are discussed in more detail. The psychometric properties are described in detail in previous studies (author et al., 2004; 2008; 2015). Raw scores and reliabilities for each measures for the current sample described below are provided in Appendix A.

Criterion Measures

Applied Problem Solving. The Applied Math Problem Solving subtest from the *Woodcock Johnson III* (Woodcock, Grew, & Math, 2001) was administered for the English presentation and the *Problemas Aplicados* from the *Batería III Woodcock-Muñoz* (Muñoz-Sandoval Woodcock, McGrew, & Mather, 2005) was administered to establish normed referenced math levels in Spanish. Both of these subtests are individually administered and assess children's early mathematical operations (e.g., counting, addition, and subtraction) through practical problems. In order to solve each problem, the subject must listen to the

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

formulation, recognize the procedures that must be followed, and then perform relatively simple calculations.

Mental computation of word problems. This individually administered task was taken from the English and Spanish arithmetic subtest of the *Wechsler Intelligence Scale for Children, Third Edition* (WISC-III; Psychological Corporation, 1991). The task was selected since it has been found predictive of math problem solving (e.g., Author et al., 2008). For this task, each word problem was orally presented and was solved without paper or pencil.

Measures Assumed Related to Criterion Measures

Measures of Achievement

Woodcock-Muñoz Language Survey-Revised (WMLS-R). The WMLS-R Spanish and English Word Identification and Passage Comprehension subtests were administered. This test assessed children's reading level in English and Spanish (Woodcock-Muñoz –Sandoval & Alverado, 2005). The subtests were administered individually to students in both English and Spanish. For the Word Identification Subtest, children were tested individually by presenting them with a list of words, which gradually increased in difficulty. The words followed regular spelling patterns in this non-timed test. For the passage comprehension subtest, children identified specific words that go in the blank spaces of various passages. Early passages were accompanied by a corresponding picture, and sentences gradually increased in complexity.

Pseudoword reading task. This measure was developed from the Word Attack subtest of the Woodcock Reading Mastery Test (Woodcock, 1998). The test was administered in English according to the standardized instructions. The measure required the child to orally read the list of pseudowords arranged in increasingly difficult order. A Spanish version of the task was also

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

administered using the same rules.

Peabody Picture Vocabulary Test (PPVT). The Peabody Picture Vocabulary test (Dunn & Dunn, 2007) was administered in English. In this task, children were presented with four pictures and asked to select the picture that matched the word read aloud in English. Word presentation gradually increased in difficulty.

Test de Vocabulario en Imagenes (TVIP). This measure is similar to the PPVT in the presentation and administration, except that words are read aloud in Spanish (Dunn, Lugo, Padilla & Dunn, 1986).

Expressive One-Word Picture Vocabulary Test -Spanish-Bilingual Edition. The Expressive One-Word Picture Vocabulary Test -Spanish-Bilingual Edition (EOWPVT-SBE: Brownell, 2001) was used as a measure of syntax and assesses English and Spanish speaking vocabulary.

Fluid (nonverbal) intelligence. The Colored Progressive Matrices (Raven, 1976) was used as an indicator of nonverbal reasoning or Fluid intelligence. Children were given a booklet with patterns on each page, each pattern revealed a missing piece. For each pattern, six possible replacement pattern pieces were displayed.

Measures of Domain Specific Math Processes

Arithmetic calculation. The calculation subtests from the *Woodcock Johnson III* (Woodcock, Grew, & Math, 2001) were administered for the English presentation and Cálculo from the *Batería III Woodcock-Muñoz* (Muñoz-Sandoval Woodcock, McGrew, & Mather, 2005) were administered to establish normed referenced math levels in Spanish. The Arithmetic subtest from the Wide Range Achievement Test (WRAT-3; Wilkinson, 2003) was also administered to assess arithmetic computational skills. The subtest required written computation of problems

that increased in difficulty.

Word problem-solving components. This experimental task assessed the child's ability to identify propositions within word problems (author et al., 2008). Each booklet contained three problems that included pages assessing the recall of text from the word problems. To control for reading problems, the examiner orally read each problem and all multiple-choice response options as the students followed along. After the problem was read, students were instructed to turn to the next page on which they were asked a series of multiple-choice questions requiring them to identify the correct propositions related to (1) question (2) number, (3) goal, (4) operation and (5) algorithm of each story problem. Children were also to identify the extraneous propositions for each story problem.

Estimation. Two number line estimation tasks adapted from Siegler and Opfer's (2003) and Siegler and Booth's (2004) study, were administered. For set 1 of the Estimation task, children were asked to examine five straight lines that were 25-cm long. Each line was identical in length and was marked with a zero at one end and one hundred on the other end, creating a blank number line. A single number (e.g., 50, 75, 45, 32, 6, 22) was placed above the center of each line. Children were asked to estimate where they thought the number presented should be placed on the line and indicated this by marking an X on the line. For set 2, children were asked to examine another set of five straight lines. For this set, however, each line was of a different length (25cm, 20cm, 12cm, 30cm, and 20cm) with end points of 0 and 100. The reason to manipulate the length of the line was related to issues raised as to whether spatial information or magnitude judgment underlines problem in estimation (Chew, Forte et al., 2016).

For each of the 10 lines (set 1 and 2), the point of accuracy was calibrated for each line. Accuracy was calculated by using a transparency template and counting how many units of

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

measure the X was from the correct answer. For the five lines in Set 1, the distance from the accuracy point was computed for each $\frac{1}{4}$ inch. For set 2, arithmetically equivalent distances were used to count off the distance between the participant's X and the where actual placement the correct answer should be on the line. The difference score (number of units from the exact point) was computed for each set. Thus, our estimate of the number line estimation varied from that of Siegler and Opfer (2003), in that they used group level median placements fitted to linear analog models to make inferences about the children's placements.

Magnitude comparisons. Two sets of digits were presented in 25 rows with three columns. Each row had the same number of digits (1 digit, 2 digits, and 3 digits) in each column. In the first set, children were asked to circle the largest number in each row as fast as they could in 30 seconds. The second set also had an additional 25 rows of numbers with three numbers in each row. Children were asked to circle the smallest number in each group as fast as they could in 30 seconds.

Measures of Phonological Storage and Naming Speed

Short-term memory (STM) measures (phonological loop). Four measures of STM were administered in Spanish and English: Forward Digit Span, Backward Digit Span, Word Span, and Pseudoword Span. The Forward and Backward Digit Span task (taken from the WISC-III; Wechsler, 1991) and a Spanish translated version were administered. The Forward Digit Span task required children to recall sequentially ordered sets of digits that increased in number, which were spoken by the examiner. The Backward Digit Span task required children to recall sets of digits, but in reverse order. Backward span was included as part of STM since it loads on the same factor as forward digits and is associated with phonological memory (e.g.,

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

Colom, Shih, Flores-Mendoza & Quiroga, 2006 ; Rosen & Engle, 1997). Dependent measures for both tasks were the largest set of items recalled in order (range = 0 to 8 for Digits Forward; range = 0 to 7 for Digits Backward). For the translated Spanish versions of the Digit Span subtest (both Digits Forward and Backward), identical numbers were presented in the same order as the English version. There were no deviations in procedure, except for language use.

The Word Span and Pseudoword Span tasks were presented in the same manner as the Forward Digit Span task. The Word Span task, the examiners read lists of one or two-syllable, high frequency words that included unrelated nouns and then asked the children to recall the words. The Pseudoword Span task (Phonetic Memory Span task) uses strings of one-syllable nonsense words, which are presented one at a time in sets of 2 to 6 nonwords (e.g., DES, SEEG, SEG, GEEZ, DEEZ, DEZ). A parallel version was developed in Spanish for the Word Span and Pseudoword Span tests. The dependent measure for all STM measures was the highest set of items retrieved in the correct serial order (range 0 to 7).

Rapid Naming of digits and letters. Several models assume that operations related to academic achievement (e.g. math) are time-related (e.g., Bonifacci, Giombini, Bellocchi & Contento, 2011; Georgiou, Tziraki, Manolitsis, & Fella, 2013), and therefore naming speed was assessed. The administration followed those specified in the manual of the Comprehensive Test of Phonological Processing (CTOPP; Wagner, Torgesen, & Rashotte, 2000). For this task, the examiner presented the child with an array of items (e.g., six letters or six digits). Children were asked to name the items, speaking in either English or Spanish, as quickly as possible for each stimulus set. The dependent measure was the amount of combined time it took for students to complete each set. Number of errors was also taken into account in creating the final score.

Measures of Inhibition and Executive Component of WM

Random generation. The use of Random Generation tasks has been well articulated in the literature as a measure of inhibition (e.g., Towse & Cheshire, 2007). The task required children to actively monitor candidate responses and suppress responses that would lead to well learned sequences, such as 1-2-3-4 or a-b-c-d (Baddeley, 2007). For this study, each child was asked to write numbers (or letters) as quickly as possible, first in sequential order and then in a random non-systematic order. For example, children were first asked to write numbers from 0 to 10 in order (i.e., 1, 2, 3, 4) as quickly as possible in a 30-second period. They were then asked to write numbers as quickly as possible “out of order” within a 30-second period. After correcting for information redundancy and percentage of paired responses, the dependent measure was the number of random items written.

Executive WM. The Conceptual Span, Listening Sentence Span, Digit Sentence and Updating task were administered in English and Spanish to capture the executive component of WM. The WM tasks required children to hold increasingly complex information in memory while simultaneously responding to a question about the task. For example, after children listened to a list of words in the Conceptual Span task they were asked, “Which word from the list did I say, X or Y?” They were then asked to recall words from the list. This balance of simultaneous storage and processing is consistent with a number of studies of WM processing, including Daneman and Carpenter's (1980) seminal WM measure.

Specifically, the Conceptual Span task was used as an indicator of WM processing that involves the ability to organize sequences of words into abstract categories. Children listened to a set of words that, when re-organized, could be grouped into meaningful categories. For example, they were told a word set, such as, “shirt, saw, pants, hammer, shoes, nails.” After answering a distracter question, they were asked to recall the words that “go together” (i.e., shirt,

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

pants, and shoes; saw, hammer, and nails). The range of set difficulty was two categories containing two words each to four categories with four words each. A Spanish- translated version was also administered. Care was taken in the development of the measure to keep the abstract categories the same in both languages (e.g., clothes and tools). The dependent measure for both versions was the number of items recalled correctly.

The children's adaptation (author et al, 2013) of Daneman and Carpenter's (1980) Listening Sentence Span task was also administered. This task required the presentation of groups of sentences, read aloud, for which children tried to simultaneously understand the sentence contents and to remember the last word of each sentence. The digit sentence task measured the participant's ability to recall numerical information that is embedded within a short sentence. The numerical information referenced either a location or address. On this test, the examiner reads a sentence and then asks the examinee a process question and then asked the examinee to recall the numbers in the sentence.

Because WM tasks were assumed to tap a measure of controlled attention referred to as updating (e.g., Miyake, Friedman, Emerson, Witzki, & Howerter, 2000), an experimental Updating task, adapted from author et al., (2004), was also administered. A series of one-digit numbers was presented that varied in set length from 3, 5, 7, and 9. No digit appeared twice in the same set. The examiner told the child that the length of each list of numbers might be 3, 5, 7, or 9 digits long. Children were then told that they should only recall the last three numbers presented. Each digit was presented at approximately one-second intervals. After the last digit was presented the child was asked to name the last three digits, in order. The dependent measure was the total number of sets correctly repeated (range 0 to 16).

Visuospatial working memory. Some studies have noted that difficulties in solving

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

word problems are related to measures that tap the visual-spatial sketchpad (e.g., Ashkenazi et al. 2013). Thus, two measures were administered to assess visual-spatial WM: Visual Matrix and Mapping & Directions (Author, 1992; 2013). The Visual Matrix task assessed the ability of participants to remember visual sequences within a matrix. Participants were presented a series of dots in a matrix and were allowed 5 seconds to study the matrix. The matrix was then removed and participants were asked, in both English and Spanish, "Are there any dots in the first column?" After answering the process question, students were asked to draw the dots they remembered seeing in the corresponding boxes of their blank matrix response booklet. The task difficulty ranged from a matrix of 4 squares and 2 dots to a matrix of 45 squares and 12 dots. The dependent measure was the number of matrices recalled correctly (range of 0 to 11).

The Mapping and Directions task required the child to remember a sequence of directions on a map (author, 1992; 2013). The experimenter presented a street map with dots connected by lines; the arrows illustrated the direction a bicycle would go to follow this route through the city. The dots represented stoplights, while lines and arrows mapped the route through the city. The child was allowed 10 seconds to study the map. After the map was removed, the child was asked a process question [i.e., "Were there any stop lights on the first street (column)?"]. The child was then presented a blank matrix on which to draw the street directions (lines and arrows) and stop lights (dots). Difficulty ranged on this subtest from 4 dots to 19 dots. The dependent measure was the highest set of correctly drawn maps (range = 0 - 9).

Procedures

Children were tested individually and in groups after informed consent forms were obtained for participation. Group testing occurred over the course of two consecutive days for approximately one hour each day and included small groups of children. Individual testing also

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

included two sessions, lasting thirty to sixty minutes for each session.

The small group administered tests (measures of calculation, random number generation, estimation, identifying word problem solving components, estimation) were evenly divided into two batteries administered across two sessions such that a balanced of Spanish and English administrations occurred for each battery. Within each battery, the presentation order of English and Spanish measures for each type of task was counterbalanced across groups. No Spanish or English administration of the same measure occurred within the battery.

The remaining measures were administered individually. One of four presentation orders related to the individually administered tasks was randomly assigned to each child. In addition, the presentation orders of Spanish and English tests were counterbalanced across all participants. No Spanish or English administration of the same measure followed each other.

Research Design and Approach to Analyses

Children in grades 1, 2 and 3, designated as English learners from their schools, were tested on the aforementioned battery of achievement, language and cognitive measures in English and Spanish. The results are organized into three parts. First, we determined if our categorization of the tasks provided a good fit to the data. Thus, a confirmatory factor model was computed for measurement purposes (latent variables control for measurement error as different variables have different weightings on a construct) and also for practical reasons: some constructs (e.g., WM, STM) included several tasks. The model for the confirmatory analysis was based on an earlier study with monolingual children (e.g., author et al., 2016, 2018). Overall, the factors include math problem solving (WJ-applied problems, WISC-III word problems), reading (word identification, passage comprehension, WRMT-word attack), calculation (WJ-calculation, WRAT-Calculation), Vocabulary (PPVT receptive, EOWVT- expressive), Magnitude

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

comparisons (large numbers, small numbers), Estimation (line estimation-same line length, line length varied), STM (span=nonword or pseudoword, real words, digit forward, digit backward), naming speed (letters, numbers), executive component of WM (conceptual span, sentence span, listening span, updating), and inhibition (random generation= letters and numbers). For the present study, separate factors were established in the same analysis for the English and Spanish presentation. Also included in the model were factors for nonverbal reasoning (Raven Colored Progressive Matrices Test-sections A, AB, and B) and visual-spatial WM (Mapping/directions, visual matrix). For these last two factors, the administration was in both English and Spanish. We used the SAS CALIS (2012) program to create factor scores (latent variables) for each set of measures with two or more variables. This procedure allowed us to calculate standardized beta weights. Task weightings for the latent measures used in the analysis (as well as means and *SDs* and task sample reliabilities) are presented in Appendix A. Based on the standardized loadings in Appendix A, latent scores were computed by multiplying the z-score of the target variable by the standardized factor loading weight based on the total sample (see Nunnally & Bernstein, 1994, p. 508 for calculation procedures). All measures were scaled to have a mean of 0 and standard deviation of 1. Normality of the data was considered. Except for the naming speed tasks, measures met standard criteria for univariate normality with skewness for all measures less than 3 and kurtosis less than 4. Transformations were conducted on these variables and the results of each model with and without these transformations were compared. Since there were no substantial differences in the model results, the untransformed scores were used for analyses.

Second, mixed regression analyses determined those aforementioned latent scores that predicted increases in problem solving accuracy in both English and Spanish. (We used a mixed regression analysis for a simpler presentation since we had an extensive number of measures and

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

the complexity of a visual diagram presentation in SEM made interpretation difficult.) Age in months was a covariate in all models. The mixed regression analyses included computing an unconditional means model and three conditional models. The unconditional means model took into consideration the age-related effects on problem solving accuracy. An unconditional means model on the criterion measures can be shown as:

$$\text{Word problem solving}_{ti} = \beta_{0i} + \beta_{1i}(\text{age}_{ti}) + e_{ti} \quad (\text{Eq. 1})$$

$$e_{ti} \sim N(0, \sigma^2)$$

Where a child's word problem solving accuracy score (i) at age t . Word problem solving $_{ti}$, was a function of an individual-specific intercept parameter (β_{0i}), and (β_{1i}) is the individual-specific age-related parameter and the residual error (e_{ti}). For all models, the intercept (β_{0i}) reflected the average sample performance on the criterion measure (latent measure of math word problem solving accuracy) at age 9. In contrast, β_{1i} , reflected age-related effects on math word problem solving accuracy. To examine the extent to which age-related-child variance in the criterion measures (math word problem solving) was influenced by performance on the latent cognitive measures (explanatory variables), several conditional models were considered. The explanatory variables in this analysis were grand mean centered. The full HLM model represented the intercept and covariates in the form:

$$\begin{aligned} \beta_{0i} = & \gamma_{00} + \gamma_{01}(\text{Age}) + \gamma_{02}(\text{English vocabulary}) + \gamma_{03}(\text{Spanish Vocabulary}) + \gamma_{04} \\ & (\text{English reading}) + \gamma_{05}(\text{Spanish-reading}) + \gamma_{06}(\text{Fluid intelligence}) + \gamma_{07}(\text{English calculation}) + \\ & \gamma_{08}(\text{Spanish calculation}) + \gamma_{09}(\text{English magnitude comparisons}) + \gamma_{010}(\text{Spanish magnitude} \\ & \text{comparisons}) + \gamma_{011}(\text{English naming speed}) + \gamma_{012}(\text{Spanish naming speed}) + \gamma_{013}(\text{English STM}) \end{aligned}$$

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

+ γ_{014} (Spanish STM) + γ_{015} (English inhibition) + γ_{016} (Spanish inhibition) + γ_{017} (English WM-executive) + γ_{018} (Spanish WM-executive) + γ_{019} (visual-spatial WM) + u_{0i} ,

$$\beta_{1i} = \gamma_{10} + \gamma_{11} \text{ (English vocabulary)} + \gamma_{12} \text{ (Spanish Vocabulary)} + \gamma_{13} \text{ (English reading)} + \gamma_{14} \text{ (Spanish-reading)} + \gamma_{15} \text{ (Fluid intelligence)} + \gamma_{16} \text{ (English calculation)} + \gamma_{17} \text{ (Spanish calculation)} + \gamma_{18} \text{ (English magnitude comparisons)} + \gamma_{19} \text{ (Spanish magnitude comparisons)} + \gamma_{20} \text{ (English naming speed)} + \gamma_{21} \text{ (Spanish naming speed)} + \gamma_{21} \text{ (English STM)} + \gamma_{22} \text{ (Spanish STM)} + \gamma_{23} \text{ (English inhibition)} + \gamma_{24} \text{ (Spanish inhibition)} + \gamma_{25} \text{ (English WM-executive)} + \gamma_{26} \text{ (Spanish WM-executive)} + \gamma_{27} \text{ (visual-spatial WM)} + u_{1i}$$

The final conditional model provided a parsimonious fit to the data. This was done by removing parameters that were not significant in the full model. All models were fit to the data using the SAS PROC Mixed software (SAS, 2010). Maximum likelihood (ML) procedures were used to determine the parameter estimates; because the ML estimation procedure has several advantages over other missing data-techniques (see Peugh & Enders, 2004, for discussion). The model accommodated nesting effects (children nested within math classroom) and missing data points. To account for the influence of children nested within classrooms, the multilevel regression model included children's assignment to the various math classroom/teachers. Robust standard errors (Huber-white) were used to allow for the nonindependence of observations from children nested within classrooms. The conditional models were compared to each other, as well as to an unconditional means model. This comparison was done by determining the differences between the deviance values (i.e., the likelihood value for the correspondence between model and data) from the unconditional (baseline) and conditional models. An additional index of model fit was the Akaike's Information Criterion (AIC) which allowed for a comparison of models that were not nested (Hox, 2010, pp. 47-50). In general, models with lower deviance and

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

AIC values fit better than models with higher values. Because of the clear chronological age effects, we set for interpretation purposes the intercept (latent measure of problem solving accuracy) at age 8 (the end point in our cross-sectional study).

Results

Normative Sample Representation

Table 1 shows the means and standard deviations on normed referenced measures as a function of the total sample and grade level. The normative scores yielded three important patterns. First, variations emerged on receptive and expressive vocabulary scores. Receptive language scores in Spanish were higher than receptive language scores in English, while the reverse pattern emerged for expressive language scores. Overall, the mean vocabulary scores were in the below average range and/or low average range suggesting the sample is best characterized as emerging bilinguals rather than Spanish dominant. Second, the means for the norm-referenced scores were substantially higher on basic skill measures (calculation, word identification) than on higher level skill measures such as problem solving accuracy and reading comprehension. Finally, a general pattern across the grade levels was that normative scores in problem solving accuracy were lower in grade 3 than grades 1 and 2.

Data Preparation

To determine if our a priori categorization of measures fit the data, the Comparative Fit Index (CFI), Bentler-Bonett non-normed fit index (NNFI), and root mean square error of approximation (RMSEA) were computed. Values at .90 and over on the CFI, and NNFI and RMSEA values of .05 or less indicate an acceptable fit. The model provided an acceptable fit to the data (CFI = .92, NNFI=.90 RMSEA = .044; 90% CI: .039 to .048). Clearly, a second order

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

model could have been tested that reflected constructs that overlapped between the two languages (i.e., language independent constructs), however, as stated in our research question we were primarily interested in individual differences in accessing information within and across the language systems and if such differences played a significant role in the predictions of math problem solving accuracy. Thus, we assumed that individual differences that emerge on each measure were related to the ease of access within the preferred language, and not a language specific cognitive system.

Given these preliminary analyses, the following analysis addressed our first research question.

1. Does the executive component of WM contribute unique variance to math problem solving accuracy when vocabulary, reading, domain specific processing and phonological storage are entered into the analysis?

To answer this question, several mixed regression analyses were computed to determine those variables that significantly predicted problem solving accuracy. The mixed regression models are shown in Table 2 for predictions of English word problem solving accuracy and Table 3 for predictions of Spanish word problem solving accuracy. Thus, as shown in Table 2, the average z-score for English math problem solving accuracy for an 8 year-old in the total sample for the unconditional means model was .92. Because the intercept was measured in terms of z-scores, the average level of performance for 8-year-olds yielded an estimated advantage in math problem solving accuracy of approximately 1 standard deviation relative to children in the younger grades. The average increment (slope) in problem solving accuracy as function of age in month was .02 units. That is, because the predictor (explanatory) variables

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

were grand mean centered, the age variable suggested that children who differed by 1 point in chronological age (in months), differed by .02 points in English math problem solving accuracy.

For the second part of mixed regression models in Table 2, the random effects (error) for the unconditional means model included the intercept variance between classrooms (.44) and the residual error (1.33). The intraclass correlation for the unconditional means model in predicting English math problem solving accuracy was $.25[.44/.44 + 1.33]$.

Conditional models. Table 2 and 3 also shows the results for the three conditional models. The first conditional model in Table 2 included executive processing measures (Model 2) in predicting English math problem solving accuracy. Visual-spatial WM was included in the executive processing model because of its association with the executive component of WM (e.g., Miyake, Friedman, Rettinger, Shah & Hegarty, 2001).

As shown in Table 2 and 3, the second Conditional Model (Full Model) entered all the explanatory variables into the analysis for predicting English and Spanish math problem solving accuracy, respectively. The results show that in the full model that the executive component of WM uniquely predicts math problem solving in both English and Spanish math problem solving accuracy. The mixed regression analysis shown in Tables 2 and 3 also addressed our second question.

2. Do L1 vocabulary, achievement and cognitive measures, rather than L2 measures, predict L2 math problem solving?

As shown in Table 2 and 3 for the Full Regression Model (Model 2), the key variable that predicted both L1 and L2 math problem solving were measures of Spanish magnitude comparisons and Spanish WM. Unique predictors were also found within each language system.

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

For predicting the latent measure of English math problem solving, the significant explanatory measures were English vocabulary, English calculation, Spanish magnitude comparisons, Spanish STM, Spanish inhibition and English and Spanish WM. Thus, several L1 measures uniquely predicted L2 problem solving accuracy. For predicting the latent measure of Spanish math problem solving, the significant explanatory variables were Spanish vocabulary, Spanish magnitude comparisons, Spanish Estimation, English naming speed, and Spanish WM. In general, the results support the notion that for emerging bilingual children there is a reliance on LI processes (in this case Magnitude comparisons and WM) in predicting L2 math problem solving. These findings are consistent with the notion that emerging bilingual children yield an asymmetrical effect in that L2 performance relies on the strength of L1 and/or a mixed of L1 and L2 predictors.

Parsimonious Model

As shown in Table 2 and 3, Model 3 (reduced model) entered only the significant parameters from the full model. As shown, the reduced model for predicting English math problem solving provided a better fit to the data ($AIC=779.2$) than the unconditional model ($AIC=1190.8$). Likewise, the reduced model when predicting Spanish math problem solving provided a better fit to the data ($AIC=909.9$) than the Unconditional Model ($AIC =1248.6$). In summary, the important findings were that support was found for the notions that the executive component of WM was significantly and uniquely predictive of math problem solving accuracy in both languages.

A follow-up analysis, we determined whether the reduced Model (Model 3) captured a substantial amount of the variance in predictions of math problem solving accuracy. An

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

analogous measure to R^2 was computed comparing the Model 3 (reduced model) to the unconditional means model using a formula by Snijders and Bosker (1999). The R_1^2 was computed by considering the residual variance and intercept variance. Model 3 (reduced model) accounted for 58% of the variance in English problem solving accuracy, $R_1^2 = (1 - [(.10 + .56) / (.44 + 1.13)]) \times 100$ and 68% of the variance in Spanish problem solving, $R_1^2 = (1 - [(.08 + .71) / (1.21 + 1.25)]) \times 100$. To determine if we have enough power in our analysis, we computed the effective sample size using a formula discussed in Hox (2010, p. 241-242) as:

$$\eta_{eff} = \eta / [1 + (\eta_{clus} - 1)\rho] \quad (\text{Eq. 2})$$

where η_{eff} is the effective sample size, η_{clus} is the cluster size (30 in this case), and η is the total sample size ($N=394$), and ρ is the intraclass correlation. The intraclass correlations for the full conditional models (Model 2) were .09 and .085 in predicting English and Spanish word problem solving, respectively. The conditional intraclass correlation from Model 2 rather than unconditional intraclass correlation was used since it includes all the independent variables. The effective sample size was 109 for predicting English word problem solving and was 114 for predicting Spanish word problem solving. Therefore we assume our analysis was adequately powered to determine cognitive variables that predict problem solving accuracy.

Summary

The important findings related to the mixed regression analyses were that the executive component of WM was significantly related to math problem solving accuracy within both language systems. Spanish WM and magnitude comparisons measures predicted both English and Spanish problem solving measures. These findings emerged when measures related to reading, vocabulary, domain specific math processes, and phonological storage processes were

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

entered into the mixed regression analyses. Other important findings were related to cross language transfer. For predicting the latent measure of L2 (English) math problem solving in the Full Model, L1 explanatory measures that contributed unique variance in predicting L2 problem solving accuracy were measures of Spanish STM, Spanish inhibition, and Spanish WM.

Discussion

This study determined the cognitive processes that were significantly related to math problem solving accuracy in emerging bilingual children. To this end, children in grades 1, 2 and 3 were assessed in both Spanish and English on a large array of cognitive and achievement measures. Three important findings emerged. First, the results directly supported the notion that the executive component of WM uniquely predicted problem solving accuracy when latent measures of vocabulary, reading, domain specific and phonological storage were entered into the regression model. Although the executive component of WM was not the only significant predictor, the results clearly indicated that the executive component of WM in Spanish uniquely predicted word problem solving accuracy in both English and Spanish.

Second, code-switching occurred in predictions of English and Spanish problem solving accuracy. Spanish latent measures of magnitude comparisons, STM, inhibition and WM significantly predicted English math problem solving. Interestingly, the reduced model found that none of L2 measures predicted L1 math problem solving. The only measures that uniquely predicted both English and Spanish math problem solving accuracy were Spanish measures of magnitude comparisons and the executive component of WM. The results are consistent with the notion that emerging bilingual children, in contrast to fully bilingual children, have difficulty alternating between L1 and L2 demands (e.g., Costa & Santesteban, 2004). That is, a

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

symmetrical effect (language specific selection “without” inhibiting their L1 language) did not occur with the present sample. Thus, predictions of L2 math problem solving performance relies on the strength of L1 and/or a mixed of L1 and L2 predictors.

A related interpretation of the findings suggest that our sample of emerging bilinguals experienced delays in accessing context-appropriate language while inhibiting the other language as it relates to math problem solving. That is, when emerging bilinguals use one of their languages, both languages are cognitively active (e.g., Kroll et al., 2014), and therefore the high incidence of code-switching in our sample reflected a weakness in controlled attention within the less dominant language system (English is this case). Simply stated, the monitoring of the two language systems for this sample was not at a level to directly influence problem solving within the required language (English in this case).

Although delays in acquiring some aspects of language, such as vocabulary, have been found among bilingual children (e.g., Bialystok, Craik, Green, & Gollan, 2009), as in our sample, extensive recent research has suggested cognitive advantages of learning two languages, particularly in the area of executive function (Bialystok, 2015; Engel de Abreu, Cruz-Santos, Tourinho, Martin, & Bialystok, 2012). Thus, our finding are consistent with others suggesting that executive processing plays a unique role in emerging bilingual children’s math problem solving. Although the results support the unique role the executive component of WM plays in math problem solving among emerging EL students, some comments are necessary related to other variables included in the regression analysis.

Two observations related to the mixed regression modeling are of interest. First, L1 vocabulary and L1 reading did not uniquely predicted L2 problem solving accuracy in the full regression model. Rather, the results showed that measures of English vocabulary predicted

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

English problem solving accuracy whereas measures of Spanish vocabulary uniquely predicted Spanish problem solving accuracy. However, the entry of the latent variables related to vocabulary into the analysis did not partial out the relationship between WM and problem solving performance. The general finding from this model was that only the latent measures of vocabulary within the language systems were found to uniquely predict math problem solving accuracy.

Second, only a few domain specific process uniquely predicted problem solving accuracy. Domain specific abilities assessed in this study were measures of calculation skill, measures identifying propositions with word problems, magnitude judgments, and line estimation. However, one domain specific variable significantly related to problem solving accuracy in both languages was magnitude comparisons (i.e., Spanish measure of magnitude judgments). The outcomes from the mixed regression modeling showed that magnitude comparisons (judging the magnitude of number) administered in Spanish played a significant role predicting solution accuracy on both English and Spanish measures of problem solving. Thus, it appears individual differences in magnitude comparisons within the Spanish language system were an important contributor to problem solving accuracy across both language systems. Interestingly, the number-line test administered in Spanish uniquely predicted Spanish word problem-solving accuracy, however, the number line task administered in English did not contribute unique variance to solution accuracy. Although children's ability to accurately map numbers onto a mathematical number line is clearly an important foundation for later math learning (e.g., Geary, 2011), we are unsure why it does not appear to play a primary role in predicting English math word problem solving difficulties among emerging bilingual children.

Theoretical Implications

What is the theoretical importance of the findings? We provide three applications to theory. First, our findings qualify bottom-up models suggesting that low-order processes (e.g., phonological STM) mediate the influence of executive processing (WM) on children mathematical problem solving performance. Although our findings indicate that math problem solving is associated with the phonological loop (i.e. STM), this latent measure did not completely account for the influence of the executive component of WM on math problem solving performance. Thus, the influence of low-order processes on executive processing was minimal for children in this study.

A second implication is that only two of the components in Baddeley's three components model independently predicted math problem solving performance. This is an important finding because WM and STM tasks have been suggested as tapping the same construct (e.g., load onto the same factor, Hutton & Towse, 2001) and other studies have found such components as nonsignificant predictors when measures of reading have been entered into the analysis (e.g., Fuchs et al., 2005; Peng et al., 2016).

Finally, the findings do not align well with the hypothesis that the relationship between WM and problem solving are completely moderated by vocabulary and/or proficiency in reading (e.g., Fuchs et al., 2005; Peng et al., 2016). The influence of WM in predicting math problem solving was not partialled out when vocabulary, reading, STM, and inhibition measures were entered into the regression analysis. What the results do suggest is that high performance on the executive component of WM is related to increased performance on problem solving measures.

Practical Implications

There are two practical implications related to our findings. The first is that there is a

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

language specific effect that influences performance on math word problem-solving measures.

We find that Spanish measures of magnitude comparisons and the executive component of working memory are particularly well-suited to identify children at risk for problem solving difficulties who are emerging bilinguals and/or learning English as a second language. This finding is important because confounds exist in the assessment of children who are second language learners. These confounds are due in part to attributing difficulties in second language acquisition and reading acquisition to the same cognitive processes that are involved in mathematical problem-solving. In practice, these confounds may lead to English language learners being inappropriately diagnosed with MD and placed in special education.

The second is that two Spanish variables (i.e., magnitude comparisons and the executive component of WM) need to be considered in developing interventions for problem solving difficulties. Intervention studies related to improving numeracy (e.g., Toll & Van Luit, 2014) and the executive component of WM (e.g., Peng & Fuchs, 2017) and academic performance are reported in the literature. However, the influence of direct instruction in these areas is unclear as to whether they can effectively bolster math problem solving performance. Thus, important intervention work towards determining the instructional links between magnitude comparisons, executive processing, and problem solving accuracy is needed.

Limitations

There are at least three limitations to this study. First, the design of the study was cross-sectional instead of longitudinal. In order to investigate language dominance and language shift in emerging bilingual children, longitudinal studies in which the development of linguistic skills is monitored in the course of time are necessary. Second, the sample yielded minimal variance in

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

SES (98% of the sample was on Federal assistance programs) and therefore the influence of high versus low SES could not be evaluated.

Finally, the sample reflected sequential bilingualism (L2 follows L1 development) and therefore may not reflect bilingualism when two languages are learned simultaneously. It is important to note that the majority of these studies on executive processing and bilingualism have focused on children who learned L1 and L2 simultaneously. However, emerging bilingual children in U.S. public schools frequently represent children who learn L1 first and L2 later (as they enter school). Thus, few studies have focused on sequential bilinguals (who learn their L1 first, then L2 later) with different levels of language proficiency on executive processing and math.

Summary

Taken together, we interpret our findings as suggesting that math problem solving performance in emerging bilingual children is directly tied to the WM system. The results suggest that when the effects of vocabulary, reading, Fluid intelligence, naming speed and inhibition were partialled out, the executive system component of WM was related to problem solving ability. Further, the results suggested that performance within the Spanish language system (magnitude comparisons and the executive component of WM) played a significant role in predicting English math problem solving.

References

- Ashkenazi, S., Rosenberg-Lee, M., Metcalfe, A. W. S., Swigart, A. G., & Menon, V. (2013). Visuo-spatial working memory is an important source of domain-general vulnerability in the development of arithmetic cognition. *Neuropsychologia*, 51(11), 2305-2317. doi:[10.1016/j.neuropsychologia.2013.06.031](https://doi.org/10.1016/j.neuropsychologia.2013.06.031)
- August, D., & Hakuta, K. (1997). *Improving schooling for minority-language children: A research agenda*. Washington, DC: National Academy Press.
- Author, 1992; 2013; Authors et al., 2004; Authors et al., 2008; Authors et al. 2014; Authors et al. 2015; Authors et al. 2016; Authors 2018
- Baddeley, A. (2007). *Working memory, thought, and action*. New York, NY, US: Oxford University Press. doi:10.1093/acprof:oso/9780198528012.001.0001
- Baddeley, A. (2012). Working memory: Theories, models, and controversies. *Annual Review of Psychology*, 63, 1-29. doi: [10.1146/annurev](https://doi.org/10.1146/annurev)
- Baddeley, A., Gathercole, S., & Papagno, C. (1998). The phonological loop as a language-learning device. *Psychological Review*, 105, 158-173. doi:10.1037//0033-295X.105.1.158
- Baddeley, A. D., & Logie, R. H. (1999). The multiple-component model. In A. Miyake & P. Shah (Eds.), *Models of working memory: Mechanisms of active maintenance and executive control* (pp. 28-61). Cambridge, U.K.: Cambridge University Press.
- Bayliss, D. M., Jarrold, C., Gunn, D. M., & Baddeley, A. D. (2003). The complexities of complex span: Explaining individual differences in working memory in children and adults. *Journal of Experimental Psychology: General*, 132(1), 71-92. doi: 10.1037/0096-3445.132.1.71

- Bialystok, E. (2009). Bilingualism: The good, the bad, and the indifferent. *Bilingualism: Language and Cognition*, 12(1), 3-11. doi:10.1017/S1366728908003477
- Bialystok, E. (2011). Coordination of executive functions in monolingual and bilingual children. *Journal of Experimental Child Psychology*, 110, 461-468. doi:10.1016/j.jecp.2011.05.005
- Bialystok, E., Craik, F. I. M., & Luk, G. (2008). Lexical access in bilinguals: Effects of vocabulary size and executive control. *Journal of Neurolinguistics*, 21(6), 522-538. doi:/10.1016/j.jneuroling.2007.07.001
- Bialystok, E., Craik, F. I. M., Green, D. W., & Gollan, T. H. (2009). Bilingual minds. *Psychological Science in the Public Interest*, 10(3), 89-129. doi:/10.1177/1529100610387084
- Bialystok, E., & Feng, X. (2009). Language proficiency and executive control in proactive interference: Evidence from monolingual and bilingual children and adults. *Brain and language*, 109(2), 93-100. doi: 10.1016/j.bandl.2008.09.001
- Bjork, I. M., & Bowyer-Crane, C. (2013). Cognitive skills used to solve mathematical word problems and numerical operations: A study of 6- to 7-year-old children. *European Journal of Psychology of Education*, 28(4), 1345-1360. doi:/10.1007/s10212-012-0169-7
- Bonifacci, P., Giombini, L., Bellocchi, S., & Contento, S. (2011). Speed of processing, anticipation, inhibition and working memory in bilinguals. *Developmental Science*, 14(2), 256-269.
- Booth, J. L., & Siegler, R. S. (2008). Numerical magnitude representations influence arithmetic learning. *Child Development*, 79(4), 1016-1031. doi:/10.1111/j.1467-8624.2008.01173.x

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

- Branum-Martin, L., Fletcher, J. M., & Stuebing, K. K. (2013). Classification and identification of reading and math disabilities: The special case of comorbidity. *Journal of Learning Disabilities, 46*(6), 490-499.
- Brownell, K. (2001). *Expressive One-Word Picture Vocabulary Test* (3rd Edition). New York: Academic Therapy Publications.
- Censabella, S., & Noel, M. P. (2008). The inhibition capacities of children with mathematical disabilities. *Child Neuropsychology, 14*(1), 1-20. doi:10.1080/09297040601052318
- Chew, C. S., Forte, J. D., & Reeve, R. A. (2016). Cognitive factors affecting children's nonsymbolic and symbolic magnitude judgment abilities: A latent profile analysis. *Journal of Experimental Child Psychology, 152*, 173-191. doi:[10.1016/j.jecp.2016.07.001](https://doi.org/10.1016/j.jecp.2016.07.001)
- Colom, R., Shih, P. C., Flores-Mendoza, C., & Quiroga, M. Á. (2006). The real relationship between short-term memory and working memory. *Memory, 14*(7), 804-813. doi:10.1080/09658210600680020
- Costa, A., & Santesteban, M. (2004). Lexical accessing in bilingual speech production: Evidence from language switching in highly proficient bilinguals and L2 learners. *Journal of Memory and Language, 50*, 491-511. doi: 10.1016/j.jml.2004.02.002,
- Cummins, J. (1979). Linguistic interdependence and the educational development of bilingual children. *Review of Educational Research, 49*, 222-251. doi:10.3102/00346543049002222
- Daneman, M., & Carpenter, P. A. (1980). Individual differences in working memory and reading. *Journal of Verbal Learning and Verbal Behavior, 19*, 450-466. doi:10.1016/S0022-5371(80)90312-6

David, C. V. (2012). Working memory deficits in math learning difficulties: A meta-analysis.

International Journal of Developmental Disabilities, 58(2), 67-84.

doi:[10.1179/2047387711Y.00000000007](https://doi.org/10.1179/2047387711Y.00000000007)

de Abreu, Pascale M. J. Engel. (2011). Working memory in multilingual children: Is there a bilingual effect? *Memory*, 19(5), 529-537. doi:10.1080/09658211.2011.590504

de Bruin, A., Barbara, T., & Della Sala, S. (2015). Cognitive advantage in bilingualism: An example of publication bias? *Psychological Science*, 26(1), 99-107. doi:

10.1177/0956797614557866

Dunn, L. M., & Dunn, L. M. (2007). *The Peabody Picture Vocabulary Test-4*. NY: Pearson.

Dunn, L. M., Lugo, D. E., Padilla, E. R., & Dunn, L. M. (1986). *Test de Vocabulario Imágenes Peabody*. Circle Pines, MN: American Guidance Service.

Engle de Abreau, P. M. & Gathercole, S. E. (2012). Executive and phonological processes in second language acquisition. *Journal of Educational Psychology*, 104(4), 974-986.

doi 10.1037/a0028390

Engel, D. A., Cruz-Santos, A., Tourinho, C. J., Martin, R., & Bialystok, E. (2012). Bilingualism enriches the poor: Enhanced cognitive control in low-income minority children. *Psychological Science*, 23(11), 1364-1371. doi:/10.1177/0956797612443836

Engle, R. W. (2002). Working memory capacity as executive attention. *Current Directions in Psychological Science*, 11(1), 19-23. doi: 10.1111/1467-8721.00160

Engle, R. W., Tuholski, S. W., Laughlin, J. E., & Conway, A. R. (1999). Working memory, short-term memory, and general fluid intelligence: A latent variable approach. *Journal of Experimental Psychology: General*, 128, 309-331. doi:10.1037//0096-3445.128.3.309

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

Farnia, F., & Geva, E. (2011). Cognitive correlates of vocabulary growth in English language learners. *Applied Psycholinguistics*, 32, 711-738. doi:10.1017/S0142716411000038

Friedman, N. P., Haber, B. C., Willcutt, E. G., Miyake, A. Young, S. Corely, R. P. & Hewitt, J. K. (2007). Greater attention problems during childhood predict poorer executive functioning in late adolescence. *Psychological Science*, 18, 893-900.
doi:10.1111/j.1467-9280.2007.01997

Fuchs, L. S., Compton, D. L., Fuchs, D., Paulsen, K., Bryant, J. D., & Hamlett, C. L. (2005). The prevention, identification, and cognitive determinants of math difficulty. *Journal of Educational Psychology*, 97(3), 493-513. doi:/10.1037/0022-0663.97.3.493

Fuchs, L. S., Compton, D. L., Fuchs, D., Powell, S. R., Schumacher, R. F., Hamlett, C. L., . . . Vukovic, R. K. (2012). Contributions of domain-general cognitive resources and different forms of arithmetic development to pre-algebraic knowledge. *Developmental Psychology*, 48(5), 1315-1326. doi:/10.1037/a0027475

Fuchs, L. S., Fuchs, D., Compton, D. L., Powell, S. R., Seethaler, P. M., Capizzi, A. M., Schatschneider, C., & Fletcher, J. M. (2006). The cognitive correlates of third-grade skill in arithmetic, algorithmic computation, and arithmetic word problems. *Journal of Educational Psychology*, 98(1), 29-43. doi:10.1037/0022-0663.98.1.29

Fuchs, L. S., Fuchs, D., Compton, D. L., Hamlett, C. L., & Wang, A. Y. (2015). Is word-problem solving a form of text comprehension? *Scientific Studies of Reading*, 19(3), 204-223. doi:10.1080/10888438.2015.1005745

Fuchs, L. S., Gilbert, J. K., Powell, S. R., Cirino, P. T., Fuchs, D., Hamlett, C. L., . . . Tolar, T. D. (2016). The role of cognitive processes, foundational math skill, and calculation

accuracy and fluency in word-problem solving versus prealgebraic knowledge.

Developmental Psychology, 52(12), 2085-2098. doi:[/10.1037/dev0000227](https://doi.org/10.1037/dev0000227)

Gathercole, S. E., Pickering, S. J., Ambridge, B., Wearing, H. (2004). The structure of working memory from 4 to 15 years of age. *Developmental Psychology*, 40, 177-190.

doi:[10.1037/0012-1649.40.2.177](https://doi.org/10.1037/0012-1649.40.2.177)

Geary, D. C. (2011). Cognitive predictors of achievement growth in mathematics: A 5-year longitudinal study. *Developmental Psychology*, 47(6), 1539-1552. doi:[/10.1037/a0025510](https://doi.org/10.1037/a0025510)

Geary, D. C., Nicholas, A., Li, Y., & Sun, J. (2017). Developmental change in the influence of domain-general abilities and domain-specific knowledge on mathematics achievement: An eight-year longitudinal study. *Journal of Educational Psychology*, 109(5), 680-693.

doi:[/10.1037/edu0000159](https://doi.org/10.1037/edu0000159)

Georgiou, G. K., Tziraki, N., Manolitsis, G., & Fella, A. (2013). Is rapid automatized naming related to reading and mathematics for the same reason(s)? A follow-up study from kindergarten to grade 1. *Journal of Experimental Child Psychology*, 115(3), 481-496. doi:[10.1016/j.jecp.2013.01.004](https://doi.org/10.1016/j.jecp.2013.01.004)

Gorman, B. K. (2012). Relationships between vocabulary size, working memory, and phonological awareness in Spanish-speaking English language learners. *American Journal of Speech-Language Pathology*, 21(2), 109-123. doi:[/10.1044/1058-0360\(2011/10-0063](https://doi.org/10.1044/1058-0360(2011/10-0063)

Gray, S., Green, S., Alt, M., Hogan, T., Kuo, T., Brinkley, S., & Cowan, N. (2017). The structure of working memory in young children and its relation to intelligence. *Journal of Memory and Language*, 92, 183-201. doi:[10.1016/j.jml.2016.06.0](https://doi.org/10.1016/j.jml.2016.06.0)

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

Han, W. (2012). Bilingualism and academic achievement. *Child Development*, 83(1), 300-321.

doi: 10.1111/j.1467-8624.2011.01686.x

Harvey, H. A., & Miller, G. E. (2017). Executive function skills, early mathematics, and vocabulary in head start preschool children. *Early Education and Development*, 28(3), 290-307. doi:/10.1080/10409289.2016.1218728

Hecht, S. A., Torgesen, J. K., Wagner, R. K., & Rashotte, C. A. (2001). The relations between phonological processing abilities and emerging individual differences in mathematical computational skills: A longitudinal study from second to fifth grades. *Journal of Experimental Child Psychology*, 79(2), 192-227. doi:[10.1006/jecp.2000.2586](https://doi.org/10.1006/jecp.2000.2586)

Hox, J. (2010). *Multilevel Analysis: Techniques and Applications* (2nd Ed.) New York, NY: Routledge/Taylor & Francis.

Hutton, U. M. Z., & Towse, J. N. (2001). Short-term memory and working memory as indices of children's cognitive skills. *Memory*, 9(4-6), 383-394. doi:[10.1080/09658210042000058](https://doi.org/10.1080/09658210042000058)

Kempert, S., Saalbach, H., & Hardy, I. (2011). Cognitive benefits and costs of bilingualism in elementary school students: The case of mathematical word problems. *Journal of Educational Psychology*, 103(3), 547-561. doi:/10.1037/a0023619

Kolkman, M. E., Kroesbergen, E. H., & Leseman, P. P. M. (2014). Involvement of working memory in longitudinal development of number–magnitude skills. *Infant and Child Development*, 23(1), 36-50. doi:/10.1002/icd.1834

Kroll, J. F., Bobb, S. C., & Hoshino, N. (2014). Two languages in mind: Bilingualism as a tool to investigate language, cognition, and the brain. *Current Directions in Psychological Science*, 23(3), 159-163. doi:/10.1177/0963721414528511

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

- Kudo, M., & Swanson, H. L. (2014). Are there advantages for additive bilinguals in working memory tasks? *Learning and Individual Differences*, 35, 96-102. doi: 10.1016/j.lindif.2014.07.019
- Landerl, K., & Moll, K. (2010). Comorbidity of learning disorders: Prevalence and familial transmission. *Journal of Child Psychology and Psychiatry*, 51(3), 287-294. doi:[10.1111/j.1469-7610.2009.02164.x](https://doi.org/10.1111/j.1469-7610.2009.02164.x)
- Lee, K., Ng, S-F., Ng, E-L, & Lim, Z-Y. (2004). Working memory and literacy as predictors of performance on algebraic word problems. *Journal of Experimental Child Psychology*, 89(2), 140-158. doi:10.1016/j.jecp.2004.07.001
- Li, Y., & Geary, D. C. (2017). Children's visuospatial memory predicts mathematics achievement through early adolescence. *PLoS One*, 12(2) doi:[10.1371/journal.pone.0172046](https://doi.org/10.1371/journal.pone.0172046)
- Lonigan, C. J., Allan, D. M., Goodrich, J. M., Farrington, A. L., & Phillips, B. M. (2017). Inhibitory control of Spanish-speaking language-minority preschool children: Measurement and association with language, literacy, and math skills. *Journal of Learning Disabilities*, 50(4), 373-385. doi:[10.1177/0022219415618498](https://doi.org/10.1177/0022219415618498)
- Macizo, P., Herra, A., Roman, P., & Marin, M.C. (2011). Proficiency in a second language influences the processing of number words. *Journal of Cognitive Psychology*, 23, 915-921. Doi:10.1080/20445911.2011.586626

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

- Mammarella, I. C., Caviola, S., Giofrè, D., & Szűcs, D. (2017). The underlying structure of visuospatial working memory in children with mathematical learning disability. *British Journal of Developmental Psychology*, doi:[10.1111/bjdp.12202](https://doi.org/10.1111/bjdp.12202)
- Mammarella, I. C., Lucangeli, D., & Cornoldi, C. (2010). Spatial working memory and arithmetic deficits in children with nonverbal learning difficulties. *Journal of Learning Disabilities*, 43(5), 455-468. doi:10.1177/0022219409355482
- Martin, R. B., Cirino, P. T., Barnes, M. A., Ewing-Cobbs, L., Fuchs, L. S., Stuebing, K. K., & Fletcher, J. M. (2013). Prediction and stability of mathematics skill and difficulty. *Journal of Learning Disabilities*, 46(5), 428-443. Retrieved from <https://search.proquest.com/docview/1651841294?accountid=14521?accountid=14521>
- Martiniello, M. (2008). Language and the performance of ELL in math word problems. *Harvard Educational Review*, 78(2), 333-368. doi:[10.17763/haer.78.2.70783570r1111t32](https://doi.org/10.17763/haer.78.2.70783570r1111t32)
- Martiniello, M. (2009). Linguistic complexity, schematic representations, and differential item functioning for English language learners in math tests. *Educational Assessment*, 14(3-4), 160-179. doi:[10.1080/10627190903422906](https://doi.org/10.1080/10627190903422906)
- Mayer, R. E., & Hegarty, M. (1996). The process of understanding mathematical problem solving. In R.J. Sternberg & T. Ben-Zeev (Eds.). *The Nature of Mathematical Thinking* (pp.29-54). Mahwah, NJ: Erlbaum.
- Menon, V. (2016). Working memory in children's math learning and its disruption in dyscalculia. *Current Opinion in Behavioral Sciences*, 10, 125-132. doi:[10.1016/j.cobeha.2016.05.014](https://doi.org/10.1016/j.cobeha.2016.05.014)
- Miyake, A., Friedman, N. P., Emerson, M. J., Witzki, A. H., & Howerter, A. (2000). The unity and diversity of executive functions and their contributions to complex “frontal lobe”

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

tasks: A latent variable analysis. *Cognitive Psychology*, 41(1), 49-100.

doi:10.1006/cogp.1999.0734

Miyake, A., Friedman, N. P., Rettinger, D. A., Shah, P., & Hegarty, M. (2001). How are visuospatial working memory, executive functioning, and spatial abilities related? A latent-variable analysis. *Journal of Experimental Psychology: General*, 130(4), 621-640.
doi:10.1037/0096-3445.130.4.621

Morales, J., Calvo, A., & Bialystok, E. (2013). Working memory development in monolingual and bilingual children. *Journal of Experimental Child Psychology*, 114(2), 187-202.
doi:/10.1016/j.jecp.2012.09.002

Namazi, M., & Thordardottir, E. (2010). A working memory, not bilingual advantage, in controlled attention. *International Journal of Bilingual Education and Bilingualism*, 13(5), 597-616. doi:10.1080/13670050.2010.488288

National Mathematics Advisory Panel (2008). The final report of the National Mathematics Advisory Panel. U.S. Department of Education. Retrieved September, 2014, from <http://www2.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf>

National Assessment of Educational Progress (2011). Achievement gap: How Hispanics and white students in public schools perform in mathematics and reading on the national assessment of educational progress. Washington DC: US Department of Education.

National Assessment of Educational Progress (2013). Math report Card. Washington DC: US Department of Education.

National Assessment of Educational Progress (2017). The condition of education (update 2017) Washington DC: US Department of Education.

Nunnally, J. C., & Bernstein, I. H. (1994). *Psychometric Theory* (3rd Ed.). NY: McGraw-Hill.

OECD (2012a). Programme for International Student assessment (PISA): Results from PISA

2012. Retrieved September, 2014, from <http://www.oecd.org/pisa/keyfindings/PISA-2012-results-US.pdf>

OECD (2012b). PISA 2012 results in focus. Retrieved September, 2014, from

<http://www.oecd.org/pisa/keyfindings/pisa-2012-results-overview.pdf>

Ockey, G. J. (2007). Investigating the validity of math word problems for English Language Learners with DIF. *Language Assessment Quarterly*, 4(2), 149-164.

doi:[10.1080/15434300701375717](https://doi.org/10.1080/15434300701375717)

Passolunghi, M. C., & Mammarella, I. C. (2010). Spatial and visual working memory ability in children with difficulties in arithmetic word problem solving. *European Journal of Cognitive Psychology*, 22(6), 944-963. doi:/10.1080/09541440903091127

Passolunghi, M. C., & Siegel, L. S. (2001). Short-term memory, working memory, and inhibitory control in children with difficulties in arithmetic problem solving. *Journal of Experimental Child Psychology*, 80, 44-57. doi:10.1006/jecp.2000.2626

Peña, E. D. (2007). Lost in translation: Methodological considerations in cross-cultural research. *Child Development*, 78(4), 1255-1264. doi:[10.1111/j.1467-8624.2007.01064.x](https://doi.org/10.1111/j.1467-8624.2007.01064.x)

Peña, E. D., Bedore, L. M., & Kester, E. S. (2016). Assessment of language impairment in bilingual children using semantic tasks: Two languages classify better than one. *International Journal of Language & Communication Disorders*, 51(2), 192-202. doi:[10.1111/1460-6984.12199](https://doi.org/10.1111/1460-6984.12199)

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

- Peng, P., Barnes, M., Wang, C., Wang, W., Li, S., Swanson, H. L., . . . Tao, S. (2018). A meta-analysis on the relation between reading and working memory. *Psychological Bulletin*, 144(1), 48-76. doi:/10.1037/bul0000124
- Peng, P., Namkung, J., Barnes, M., & Sun, C. (2016). A meta-analysis of mathematics and working memory: Moderating effects of working memory domain, type of mathematics skill, and sample characteristics. *Journal of Educational Psychology*, 108(4), 455-473. doi:/10.1037/edu0000079
- Peugh, J. L., & Enders, C. K. (2004). Missing data in educational research: A review of reporting practices and suggestions for improvement. *Review of Educational Research*, 74, 525-556. doi:10.3102/00346543074004525
- Raven, J. C. (1976). *Colored Progressive Matrices*. London, England: H. K. Lewis & Co. Ltd.
- Roberts, G., & Bryant, D. (2011). Early mathematics achievement trajectories: English-language learner and native English-speaker estimates, using the early childhood longitudinal survey. *Developmental Psychology*, 47(4), 916-930. doi:/10.1037/a0023865
- Rosen, V. M., & Engle, R. W. (1997). Forward and backward serial recall. *Intelligence*, 25(1), 37-47.
- Rosselli, M., Ardila, A., Lalwani, L. N., & Vélez-Urbe, I. (2016). The effect of language proficiency on executive functions in balanced and unbalanced Spanish–English bilinguals. *Bilingualism: Language and Cognition*, 19(3), 489-503. doi:10.1017/S1366728915000309
- Rousselle, L., & Noël, M. (2007). Basic numerical skills in children with mathematics learning disabilities: A comparison of symbolic vs non-symbolic number magnitude. *Cognition*, 102(3), 361-395. doi:/10.1016/j.cognition.2006.01.005

SAS Institute. (2010). *SAS/STAT software: Changes and Enhancements through release 9.3*.

Cary, NC: SAS Institute Inc.

Schmiedek, F., Hildebrandt, A., Lövdén, M., Wilhelm, O., & Lindenberger, U. (2009). Complex span versus updating tasks of working memory: The gap is not that deep. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 35(4), 1089-1096.
doi:[10.1037/a0015730](https://doi.org/10.1037/a0015730)

Siegler, R. S. & Booth, J. (2004). Development of numerical estimation in young children. *Child Development*, 75(2), 428-444. doi:10.1111/j.1467-8624.2004.00684.x

Siegler, R. S., & Opfer, J. (2003). The development of numerical estimation: Evidence For multiple representation of numerical quantity. *Psychological Science*, 14(3), 237-243. doi:10.1111/1467-9280.02438

Snijders, T., & Bosker, R. (1999). *Multilevel modeling: An introduction to basic and advanced multilevel modeling*. Thousand Oaks, CA: Sage Press. Swanson, H. L. (1992). Generality and modifiability of working memory among skilled and less skilled readers. *Journal of Educational Psychology*, 84, 473-488. doi:10.1037//0022-0663.84.4.473

Szücs, D., Devine, A., Soltesz, F., Nobes, A., & Gabriel, F. (2013). Developmental dyslexia in related to visuo-spatial memory and inhibition impairment. *Cortex*, 49, 2674-88. doi: 10.1016/j.cortex.2013.06.007

Thorn, A. S., & Gathercole, S. (1999). Language-specific knowledge and short-term memory in bilingual and non-bilingual children. *Quarterly Journal of Experimental Psychology, Section A*, 52(2), 303-324. doi: 10.1080/713755823

Thorn, A. S., Gathercole, S., Frankish, C.R. (2002). Language familiarity effects in short-term memory: The role of output delay and long-term knowledge. *Quarterly Journal of Experimental Psychology, Section A*, 55, 1363-1383.

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

- Toll, S. W. M., Van, d. V., Kroesbergen, E. H., & Van Luit, Johannes E. H. (2011). Executive functions as predictors of math learning disabilities. *Journal of Learning Disabilities, 44*(6), 521-532. doi:[10.1177/0022219410387302](https://doi.org/10.1177/0022219410387302)
- Toll, S. W. M., & Van Luit, Johannes E. H. (2014). Effects of remedial numeracy instruction throughout kindergarten starting at different ages: Evidence from a large-scale longitudinal study. *Learning and Instruction, 33*, 39-49. doi:/10.1016/j.learninstruc.2014.03.003
- Towse, J., & Cheshire, A. (2007). Random generation and working memory. *European Journal of Cognitive Psychology, 19* (3), 374-394. doi:10.1080/09541440600764570
- Unsworth, N., & Engle, R. W. (2007). On the division of short-term and working memory: An examination of simple and complex span and their relation to higher order abilities. *Psychological Bulletin, 133*(6), 1038-1066. doi:10.1037/0033-2909.133.6.1038
- Vukovic, R. K., & Lesaux, N. K. (2013). The language of mathematics: Investigating the ways language counts for children's mathematical development. *Journal of Experimental Child Psychology, 115*(2), 227-244. doi: 10.1016/j.jecp.2013.02.002
- Wagner, R., Torgesen, J., & Rashotte, C. (2000). *Comprehensive Test of Phonological Processes*. Austin TX: Pro-ED.
- Wechsler, D. (1991). *Wechsler Intelligence Scale for Children-Third Edition*. San Antonio, TX: Psychological Corporation.
- Wilkinson, G. S. (2003). *The Wide Range Achievement Test*. Wilmington DE: Wide Range, Inc.
- Woodcock, R. W. (1998). *Woodcock Reading Mastery Test- Revised* (Form G). Circle Pines, MN: American Guidance Service, Inc.
- Woodcock, R. W., Muñoz-Sandoval, A. F. & Alverado, C. G. (2005). *Woodcock-Muñoz Language Survey*. Itasca, IL: Riverside Publishing.

Footnote

¹The literature is unclear as to what terms appropriately capture our sample (e.g., English language learners, English learners, limited English-proficient, balance vs. unbalanced bilingual, emerging bilinguals). We used the term English learner to align with the literature, but realize the sample is best described as emerging bilinguals to emphasize children's strengths as well as their language proficiency.

²One framework to capture executive processing in emerging bilingual children is Baddeley's multicomponent WM model (Baddeley & Logie, 1999). This multicomponent model characterizes WM as comprising a central executive controlling system that interacts with a set of two subsidiary storage systems: the speech-based phonological loop and the visual-spatial sketchpad. According to Baddeley (Baddeley, 2012; Baddeley & Logie, 1999), the central executive coordinates the two systems, focusing and switching attention, and activating representations within long-term memory (LTM). This model has been revised to include an episodic buffer (Baddeley, 2012), but support for the tripartite model has been found across various age groups of children (Gathercole Pickering, Ambridge & Wearing, 2004; Gray et al., 2017). Thus, this study will focus on the three components of WM consistent with Baddeley's earlier model.

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

Table 1

Descriptive Information on Norm Reference Measures for Total Sample and Grade Level.

Variable	N	Total Sample		N	Grade=1		N	Grade=2		N	Grade=3	
		Mean	SD		Mean	SD		Mean	SD		Mean	SD
Age (Mos)	394	92.46	13.43	155	81.10	8.19	129	94.33	8.76	110	106.26	9.28
Fluid Intell	360	101.09	16.63	141	98.99	19.21	122	103.05	16.19	97	101.69	12.4
English												
Problem Solving & Calculation												
WJ-AP	380	96.77	13.67	149	102.24	10.94	125	94.57	12.3	106	91.67	15.89
WISC-III	382	92.33	14.91	150	93.6	15.14	126	89.37	14.99	106	94.06	14.08
WJ-Cal	382	101.55	18.00	150	105.21	19.25	126	100.51	15.51	106	97.61	18.1
WRAT	381	106.85	10.99	150	111.3	8.1	125	106.2	9.37	106	101.3	13.43
Reading												
WordID	379	103.6	16.73	150	104.38	16.63	125	100.24	17.85	104	106.52	14.85
E-Comp	379	92.98	17.23	150	95.49	19.54	125	91.35	16.22	104	91.30	14.31
Word attack	383	95.5	17.33	151	98.07	13.88	123	94.84	18.93	109	92.67	19.29
Vocabulary												
E-ppvt	375	90.94	17.39	148	89.44	17.21	122	88.7	17.28	105	95.65	17.03
E-expressive	379	98.85	24.74	150	98.00	27.93	124	96.42	24.41	105	102.92	19.42
Spanish												
Problem Solving & Calculation												
BWJ-AP	383	101.42	13.95	151	105.79	14.05	123	99.25	12.01	109	97.82	14.36
WISC-III	381	88.71	19.42	150	86.93	20.98	123	86.26	18.87	108	93.98	16.78
BWJ-Cal	383	96.21	12.65	151	105.49	7.03	125	91.73	12.81	107	88.33	10.29
WRAT	367	107.18	10.96	150	113.38	7.53	113	103.97	11.13	104	101.72	10.61
Reading												
Word-ID	383	117.98	17.23	151	125.55	18.43	123	113.46	15.39	109	112.61	13.36
S-Comp	378	91.81	16.39	149	98.83	17.47	120	89.56	14.48	109	84.69	12.81
Vocabulary												
S-ppvt	382	92.73	15.46	152	92.26	15.89	121	94.88	14.11	109	91	16.15

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

S-expressive	379	74.93	17.06	150	75.38	19.44	123	73.55	15.9	106	75.91	14.65
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Note. Note. E-English; S-Spanish; Fluid intelligence=Raven Colored Progressive Test; WJ=Woodcock Johnson; AP=applied problem subtest; WISC-III=Wechsler Intelligence Scale for Children, Arithmetic subtest; BWJ=Bateria; WJ Cal=calculation subtest; Word ID = Letter Word identification subtest from Woodcock-Muñoz Language Survey-Revised; WRAT= computation subtest from Wide Range Achievement Test; E-Comp = English Passage Comprehension subtest from Woodcock-Muñoz Language Survey-Revised; Word attack = Word attack subtest from Woodcock-Muñoz Language Survey-Revised; PPVT = Peabody Picture Vocabulary Test in English; E-Expressive= Expressive One-Word Picture Vocabulary Test-English-Bilingual Edition; S-ppvvt = Test de Vocabulario en Imagenes; S-Comp = Spanish Passage Comprehension subtest from Woodcock-Muñoz Language Survey-Revised; S-Expressive= Expressive One-Word Picture Vocabulary Test -Spanish-Bilingual Edition (EOWPVT-SBE: Brownell, 2001).

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

Table 2

Mixed Regression Models Predicting English Math Word Problem Solving

	Unconditional Model		Model 1		Model 2		Model 3	
Fixed	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE
Intercept	0.92***	0.17	0.88***	0.12	0.71***	0.09	0.62***	0.07
Age(mos)	0.02***	0.005	0.02***	0.004	0.004	0.004		
E-Vocab					0.15***	0.04	0.27***	0.04
S-Vocab					0.02	0.05		
S-Read					-0.03	0.05		
E-Read					0.04	0.03		
FIQ					0.02	0.03		
E-cal					0.13*	0.05	0.22***	0.04
S-cal					0.03	0.05		
E-Num					0.009	0.05		
S-Num					0.08*	0.04	0.12**	0.03
E-Estim					0.004	0.04		
S-Estim					-0.05	0.05		
E-STM					-0.03	0.04		
S-STM					0.11**	0.04	0.08**	0.03
E-Speed					-0.08	0.06		
S-Speed					0.02	0.05		
E-Inhib			0.08	0.06	0.02	0.04		
S-Inhib			0.05	0.07	-0.14*	0.06	-0.08	0.06
E-WM			0.33***	0.05	0.08*	0.03	0.11**	0.03
S-WM			0.09**	0.03	0.09**	0.03	0.08**	0.03
Vis-WM			0.15**	0.05	0.04	0.05		
Random	Variance	SE	Variance	SE	Variance	SE	Variance	SE
Intercept	0.44**	0.17	0.14*	0.07	0.05*	0.03	.10*	0.05
Residual	1.13***	0.08	0.78***	0.07	0.50***	0.05	.53***	0.04
Fit Indices								
Deviance	1186.8		917		665.2		779.2	
AIC	1190.8		921		669.2		783.2	

SE= standard error, * $p < .05$, ** $p < .01$, *** $p < .001$, Model 1=executive processing measures-only, Model 2=full model with all measures, Model 3=parsimonious model-only measure found significant in full model. Deviance= Chi-square value for the correspondence between model and data, AIC= Akaike's Information Criterion. Note. E-English, S-Spanish Vocab=vocabulary, Read=Reading, FIQ=fluid intelligence, Cal=calculation, num=magnitude number comparisons, Estim=estimation, , STM=short-term memory or phonological loop, speed=naming speed, inhib=inhibition, WM=executive component of Working memory, Vis-WM=visuospatial working memory.

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

Table 3

Mixed Regression Models Predicting Spanish Math Word Problem Solving

	Unconditional		Model 1		Model 2		Model 3	
Fixed	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE
Intercept	0.91***	0.22	0.92***	0.16	0.79***	0.08	0.83***	0.07
Age(mos)	0.01**	0.006	0.009	0.005	-0.001	0.004		
E-Vocab					0.05	0.05		
S-Vocab					0.31***	0.05	0.39***	0.05
S-Read					0.06	0.06		
E-Read					0.09	0.06		
FIQ					0.02	0.03		
E-cal					0.02	0.07		
S-cal					0.04	0.08		
E-Num					-0.0005	0.07		
S-Num					0.11*	0.05	0.20***	0.03
E-Estim					-0.008	0.04		
S-Estim					-0.11*	0.05	-0.13**	0.03
E-STM					0.03	0.04		
S-STM					0.06	0.05		
E-Speed					-0.11*	0.05	-0.009	0.03
S-Speed					0.01	0.06		
E-Inhib			-0.04	0.07	-0.11*	0.05	-0.01	0.06
S-Inhib			0.23**	0.08	0.10	0.08		
E-WM			0.15**	0.04	0.01	0.05		
S-WM			0.36***	0.04	0.22***	0.04	0.28***	0.03
Vis-WM			0.12*	0.05	0.05	0.05		
Random			Variance	SE	Variance	SE	Variance	SE
Intercept	1.21**	0.41	0.41**	0.18	0.06*	0.04	.08*	0.04
Residual	1.25***	0.01	0.88***	0.07	0.65***	0.06	.71***	0.06
Fit Indices								
Deviance	1244.6		971.1		722		909.9	
AIC	1248.6		975.1		726		913.9	

SE= standard error, * $p < .05$, ** $p < .01$, *** $p < .001$, Model 1=executive processing measures-only, Model 2=full model with all measures, Model 3=parsimonious model-only measure found significant in full model. Deviance= Chi-square value for the correspondence between model and data, AIC= Akaike's Information Criterion. Note. E-English, S-Spanish Vocab=vocabulary, Read=Reading, FIQ=fluid intelligence, Cal=calculation, num=magnitude number comparisons, Estim=estimation, , STM=short-term memory or phonological loop, speed=naming speed, inhib=inhibition, WM=executive component of Working memory, Vis-WM=visuospatial working memory.

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

Appendix A

Descriptive Information on Raw Scores, Reliability, and Standardized Estimates.

Appendix A

	M	SD	Kurtosis	Skewness	KR ₂₀	Estimates	SE	t-ratio
Math Problem Solving								
English								
WJ-AP	21.31	6.13	1.79	0.43	0.78	0.77	0.04	20.51**
WISC-III	10.87	3.42	1.14	-0.97	0.67	0.70	0.04	16.74**
Spanish								
WJ-AP	23.76	6.65	0.61	-0.52	0.83	0.86	0.03	34.36**
WISC-III	10.05	4.55	-0.47	-0.62	0.83	0.78	0.03	24.67**
Math Calculation								
English								
WJ-Cal	15.32	4.91	1.11	-0.55	0.78	0.83	0.03	32.25**
WRAT	20.61	3.01	0.52	0.53	0.42	0.87	0.02	39.11**
Spanish								
WJ-Cal	9.29	3.38	0.29	-0.45	0.71	0.85	0.02	37.50**
WRAT	20.81	2.91	-0.2	0.36	0.44	0.89	0.02	45.16**
Vocabulary								
English								
Receptive	103.89	29.09	-0.29	-0.14	0.96	0.84	0.03	31.89**
Expressive	49.46	19.36	0.1	-0.08	0.95	0.87	0.02	35.08**
Spanish								
Receptive	50.57	15.64	1.03	-0.07	0.92	0.85	0.04	21.45**
Expressive	30.8	15.31	-0.77	-0.18	0.96	0.56	0.05	10.58**
Reading								
English								
Word-ID	32.45	14.61	-1.07	0.18	0.95	0.96	0.01	86.83**
Compreh.	11.51	5.98	-0.97	-0.04	0.90	0.89	0.02	50.62**
Word-Att	12.48	10.82	-0.52	0.67	0.96	0.7	0.04	18.88**
Spanish								
Word-ID	35.34	10.58	0.41	0.2	0.89	0.81	0.03	29.62**
Compreh.	10.72	5.03	3.29	0.68	0.80	0.79	0.03	26.22**
Word-Att	21.62	11.31	-1.03	-0.57	0.95	0.69	0.04	17.83**
Fluid Intelligence								
Eng/Span								
RAV_A	8.29	2.05	3.26	-1.27	0.39	0.65	0.05	13.05**
RAV_AB	7.45	2.94	-0.25	-0.62	0.73	0.77	0.04	18.79**
RAV_B	5.88	3.01	-0.5	0.04	0.79	0.79	0.04	19.57**
Comparisons								
English								
PS-Components	11.67	4.23	0.03	-0.58	0.80	0.51	0.05	9.39**
Num-L	7.52	4.81	-0.13	0.26	0.87	0.88	0.02	39.76**

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

Num-S	5.78	4.36	-0.09	0.58	0.87	0.83	0.03	32.47**
Spanish								
PS-Components	10.73	4.19	0.08	-0.58	0.81	0.51	0.05	9.26**
Num-L	7.72	4.66	-0.14	0.15	0.86	0.87	0.02	37.07**
Num-S	5.74	4.07	0.48	0.65	0.85	0.83	0.03	31.49**
Estimation								
English								
Set 1-	39.36	29.06	-1.32	0.39	0.99	0.84	0.02	34.70**
Set 2-	21.43	18.78	-0.55	0.96	0.98	0.85	0.02	34.95**
Spanish								
Set 1-	40.39	26.29	-1.37	0.19	0.98	0.77	0.04	21.24**
Set 2-	20.69	16.24	-0.85	0.74	0.98	0.89	0.03	30.12**
STM-Phonological Loop								
English								
Non words	5.64	2.98	-0.03	0.26	0.76	0.51	0.06	8.68**
Real Words	9.82	3.79	-0.11	0.22	0.77	0.67	0.05	13.97**
Digit-Forward	6.46	1.97	0.73	0.55	0.42	0.67	0.05	13.93**
Digit-Backward	2.92	1.42	0.14	0.14	0.47	0.42	0.06	6.56**
Spanish								
Non words	4.87	2.91	0.21	0.38	0.79	0.54	0.06	9.49**
Real Words	7.42	3.57	-0.68	-0.22	0.80	0.75	0.04	17.57**
Digit-Forward	6.10	1.76	2.23	-0.86	0.30	0.56	0.06	10.02**
Digit-Backward	2.86	1.5	2.01	0.47	0.41	0.51	0.06	8.75**
Naming Speed								
English								
Letters	59.41	24.61	5.29	1.91	0.96	0.94	0.02	40.65**
Digits	55.28	22.85	8.97	2.41	0.95	0.77	0.03	22.92**
Spanish								
Letters	69.98	25.94	4.41	1.59	0.96	0.79	0.03	23.58**
Digits	55.13	18.62	8.52	2.25	0.94	0.87	0.03	30.29**
WM-Executive Processing								
English								
Concept-Span	3.59	3.42	2.71	1.63	0.84	0.43	0.06	6.92**
Sentence-Span	2.42	2.96	3	1.63	0.85	0.62	0.05	11.88**
Listen-Span	1.21	1.38	5.23	1.67	0.52	0.50	0.06	8.41**
Update	3.08	2.27	1.35	0.96	0.80	0.50	0.06	8.52**
Spanish								
Concept-Span	3.29	2.86	1.11	1.17	0.83	0.64	0.05	13.00**
Sentence-Span	2.84	3.31	2.59	1.6	0.86	0.63	0.05	12.7**
Listen-Span	0.92	1.06	1.16	1.04	0.52	0.52	0.06	9.21**
Update	3.22	2.21	2.27	1.13	0.70	0.52	0.06	9.23**
Inhibition								
English								
Letters	6.09	3.25	-0.48	0.05	0.80	0.51	0.07	7.40**
Numbers	3.43	2.63	0.05	0.58	0.77	0.66	0.07	9.33**

MATH PROBLEM SOLVING AND EXECUTIVE PROCESSING

Spanish								
Letters	3.52	2.72	-0.48	0.49	0.81	0.47	0.08	5.80**
Numbers	5.8	3.4	-0.4	0.2	0.82	0.43	0.08	5.45**
Visual-Spatial Sketchpad								
Eng/Span								
Matrix	9.11	7.77	-0.33	0.66	0.95	0.74	0.07	11.17**
Mapping	3.29	3.11	7.21	2.30	0.80	0.63	0.06	9.68**

*Note. Eng/Span=administer in both languages. ** $ps < .01$

E-English, S-Spanish, Fluid intelligence=three parts of the Raven Colored Progressive Test, WJ-=Woodcock Johnson; AP=applied problem subtest; WISC-III=Wechsler Intelligence Scale for Children, Arithmetic subtest; WJ Cal=calculation subtest; WRAT= computation subtest from Wide Range Achievement Test; E-Receptive = Peabody Picture Vocabulary Test in English; S-Receptive = Test de Vocabulario en Imagenes; Expressive= Expressive One-Word Picture Vocabulary Test -Spanish-Bilingual Edition (EOWPVT-SBE: Brownell, 2001), Word ID = Letter Word identification subtest from Woodcock-Muñoz Language Survey-Revised; Word-att= Word attack subtest from Woodcock Reading Mastery Test; PS-Components=Problem solving components, Num=magnitude number comparisons, L-=letters, N-=numbers. The Table for the intercorrelations among all measures can be accessed on the web at****

