IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received August 7, 2019, accepted September 21, 2019, date of publication October 29, 2019, date of current version January 6, 2020.

Digital Object Identifier 10.1109/ACCESS.2019.2949995

Distributed Dynamic State Estimation Considering Packet Losses in Interconnected Smart Grid Subsystems: Linear Matrix Inequality Approach

MD MASUD RANA[1] AND AMIR SHAHIRINIA[2,3]

[1]Department of Electrical and Computer Engineering, Missouri University of Science and Technology, Rolla, MO 65409, USA
[2]Department of Electrical Engineering
[3]Center of Excellence for Renewable Energy, University of the District of Columbia, Washington, DC 20008, USA

Corresponding authors: Md Masud Rana (mrana928@yahoo.com) and Amir Shahirinia

This work was supported by the National Science Foundation (NSF) under Grant 1900462.

ABSTRACT This paper proposes a distributed dynamic state estimation algorithm considering packet losses in interconnected smart grid subsystems. Particularly, the distributed filter structure is developed in an interconnected way where the packet dropouts occur in communication links between them. The system error dynamic between true and estimate state is written in a compact form after combining all estimation errors. It can be transformed into the linear matrix inequality framework after introducing semidefinite programming variables. Finally, the local and neighboring gains for the distributed estimator are computed after solving the convex optimization problem. The explore method is applied to the IEEE 14-bus system. In doing this, the state-space model of IEEE 14-bus is obtained using the Holt-Winters method. Simulation results are demonstrated considering packet losses and cyber attacks.

INDEX TERMS Communication networks, distributed dynamic state estimation, Holt-Winters method, interconnected subsystems, linear matrix inequality, packet dropouts, smart grids.

I. INTRODUCTION

Due to climate change, global warming and energy crisis, the renewable energy resources (DERs) such as solar cells and wind turbines are integrated into the grid. Even though it reduces energy losses, carbon dioxide emissions, and tariff rate, but the significant challenges arise for network monitoring and maintaining its stability [1]. Consequently, the electricity network requires to monitor in a distributed way as the microgrid (for an example) is located in consumer premises or remote areas such as mountain and river sites. Interestingly, the unprecedented growth of the signal processing and communication technology can assist the vision of the distributed monitoring and stabilizing challenges effectively.

Actually, the system state estimation is necessary for monitoring the distribution networks to achieve a stable and reliable grid operations [2]. However, most of the traditional dynamic state estimation method for smart grids is centralised [3]. It not only requires a massive computational and communication resources but also vulnerable for single paint failure. From the flexible and convenient point of view, the distributed state estimation has attracted considerable attention in recent years because of the ubiquitous applications in industry, utility operator, sensor networks and reliable energy management system design [4]. Unfortunately, without coordination between adjacency estimators/packet losses, the estimation provides by the different estimators may not converge [5]. It motivates the introduction of information sharing among estimators, so enhancing the smart grid state estimation accuracy. In other words, this paper addresses the distributed state estimation problem for a discrete-time system under the condition of packet dropouts among the neighbouring estimators without any significant approximation.

A. RELATED WORK

In wireless sensor networks [6], the stability of continuous-time system is analysed, where it is taken into account the communication delays and packet dropouts. From the engineering perspective, the stability analysis for a continuous-time system is easy, while the discrete-time system is easy to implement in the digital platforms. Early work on

The associate editor coordinating the review of this manuscript and approving it for publication was Qinghua Guo.

Kalman filter (KF)-based centralised estimation of discrete-time systems include [7], in which the sensing measurements are lost where the Bernoulli distribution is considered as a packet dropout sequence. It derives the convergence condition which is the maximum and minimum packet arrival probabilities corresponding to the updated and predicted error covariance matrices (upper and lower bound). The stability analysis of networked control systems using the standard linear quadratic Gaussian (LQG) approach is proposed in [3], [8]. Moreover, the weak convergence analysis of the KF algorithm subject to the intermittent observations is presented in [9]. Even though it considers the Markov process as a packet loss model, but these algorithms are suitable for centralised power system state estimation only. Furthermore, the extended KF-based power system dynamic state estimation (centralised) is suggested in [10], [11] which considers the unreliable link between sensors and estimators.

In the diffusion distributed state estimation, the global estimation is combined with local estimation results with a set of weighting parameters. First of all, the diffusion least mean square (LMS)-based distribution estimation for sensor networks is presented in [12]. In order to reduce the computational complexity, the sparsity-aware LMS algorithm is proposed over distributed sensor networks [13]. It exploits the l_1-norm regularisation approach through adaptive combination of neighbouring agent weights. Even though it algorithmic complexity is reasonable from the practical implementation point of view, but the performance is not so good compared with the diffusion KF algorithm.

Basically, the KF-based diffusion state estimator for sensor networks is put forward in [12], [14]. The concept is then extended in [15]–[19] where it considers the packet losses and delays in measurements. Additionally, the covariance intersection-based fusion algorithm is commonly used where the weighting coefficients can be determined by taking the reciprocal of locally estimated error covariances [20]–[22]. The accuracy and consistent of this method is analysed in [20]. Essentially, the Bayesian approach to compute the cross-covariance between two estimators and information fusion is suggested in [23]. When there is more than two estimators, the method is inapplicable as it can not compute the cross-correlation between estimators. Overall, it is very difficult to obtain the optimum weighting factors for diffusion approach; the estimation performance depends on it. Besides, it is quiet difficult to analysis the stability of this method as the local and global estimation is computed separately. From the prevalent point of view, the interconnected distributed state estimation method has appreciable attention in the research community.

From the signal processing perspective, the belief prorogation-based distributed message passing algorithm is proposed in [24]. However, the performance of this method is quite similar with the KF method, but it needs enormous computational complexity and difficult to analysis the convergence [25]. The game theory-based optimal distributed approach for target tracking is proposed in [26]. This approach considers only the neighboring estimators (including itself) and apply it for target tracking in sensor networks. Thirdly, the event-triggered based distributed H_∞ state estimation is explored in [27]. It considers the neighboring estimator is an event which occurs synchronous and asynchronous fashion depending on the event-triggered condition. In [28], proposes a distributed H_∞ consensus-based estimation technique using the dissipativity theory is the sufficient condition to guarantee the stability is derived for a continuous-time system subject to perfect communication between estimators. Last but not least, the distributed receding horizon estimation subject to random packet dropouts is put forward in [29]. The sufficient conditions that guarantee the consensus on estimation are developed, but it only applicable if the packet dropout process satisfies certain conditions.

Recently, the consensus analysis in sensor networks with periodic sensing and different switching topologies is presented in [30], but it is only suitable for a continuous-time linear system. From the digital implementation point of view, the consensus analysis for multi-agent systems with delay and occasional packet dropout is proposed in [31], [32]. However, the searching feedback gain computation algorithm is designed specifically for second order systems and applied for stabilising the system state which assumes to be perfectly known. The fully state feedback concept is then extended in [33] for higher-order systems without noise, but it can be applied for stabilising the system where it has different states in each agents. Moreover, the distributed consensus-based state feedback approach is proposed in [34], and it derives the upper bound of convergence rate by selecting appropriate quantizer parameters. Finally, a survey of recent progress in networked control systems considering coding rate for stabilising the system in the presence of packet losses, topology coordination problem and communication constraints is presented in [35]–[37]. All of the aforementioned papers, it assumes that the system state is available, then it designs a state feedback gain computation method for regulating the system states. Unfortunately, the power system state such as rotor angle and flux is usually unknown in practice, so firstly it requires to apply a state estimation method, then the control algorithm can be applied for stabilizing the system states if needed. Motivated by the aforementioned research gaps, this paper proposes a distributed state estimation algorithm considering packet dropouts among adjacency estimators without any important approximation [38], [39].

B. KEY CONTRIBUTIONS

The specific contributions of the paper are summarized as follows:

- Considering the packet losses in communication links between the distributed estimators, the system error function between true and estimate state is written in a compact form. Inspired by the Laplacian operator from the graph theory, the dynamic error function is combined.

- The local and neighbouring gains are determined based on the convex semidefinite programming (SDP) approach. Using the Schur complement, the designed error dynamic is transformed into the liner matrix inequality (LMI). After congruence transformation, the desired optimum gains for distributed estimation process are determined.
- The performance of the developed method is demonstrated using the IEEE 14-bus system. In doing this, the state-space model of smart grid is obtained using the Holt-Winters method. It shows that the proposed method can be well estimated the system states within 0.1 second after sharing information between the estimators despite the packet losses among them.

Paper Outline: The remainder of this paper is organized as follows. A system and observation model are illustrated in Section II. In Section III, the problem is formulated for the smart grid distributed state estimation. The proposed algorithm is derived in Section IV. The power system including the IEEE 14-bus as well as the numerical simulations are demonstrated in Section V. This paper ends with conclusion in Section VI.

Notations: Bold face lower and upper case letters are used to represent vectors and matrices, respectively. Superscripts $\boldsymbol{x}'$ denotes the transpose of $\boldsymbol{x}$, $E(\bullet)$ denotes the expectation operator and $\boldsymbol{I}$ denotes the identity matrix with appropriate dimension. The quantity $\rho(\bullet)$ denotes the spectral radius of a matrix and $\otimes$ denotes the Kronecker product. Symbol $\star$ denotes the symmetric terms in a symmetric matrix.

II. SYSTEM AND OBSERVATION MODEL

For developing estimation scheme, consider the following system at time instant k:

$$\boldsymbol{x}(k+1) = A_d\boldsymbol{x}(k) + B_d u(k) + n_d(k), \tag{1}$$

where $\boldsymbol{x}(k) \in \mathbb{R}^q$ is the system state, $u(k) \in \mathbb{R}^r$ is the control input, $\boldsymbol{n}_d(k) \in \mathbb{Q}^q$ is the Gaussian process noise with zero mean $Q(k)$ covariance matrix, $\boldsymbol{A}_d \in \mathbb{R}^{q\times q}$ and $\boldsymbol{B}_d \in \mathbb{R}^{q\times r}$ are constant state and input matrices.

To get measurements, the service providers deploy a set of sensors around the physical object whose measurements are described by *i-th* estimator:

$$\boldsymbol{y}^i(k) = \boldsymbol{C}\boldsymbol{x}(k) + w^i(k), \quad i = 1, 2, \cdots, n \tag{2}$$

where $\boldsymbol{y}^i(k)$ is the measurement, $\boldsymbol{C}$ is the observation matrix and $\boldsymbol{w}^i(k)$ is the Gaussian measurement noise with zero mean $\boldsymbol{R}^i(k)$ covariance matrix. Generally, the estimator exchanges information with neighbour estimators through a lossy communication network which causes packet dropouts [27], [30]. This is due to the fact that the packet loss is induced by the transmission errors, delays, fading and link failures [37].

III. PROBLEM FORMULATION BASED ON INTERCONNECTED FILTER STRUCTURE

Consider there are packet losses during exchanging information among the neighbouring estimators in Fig. 1 [27],

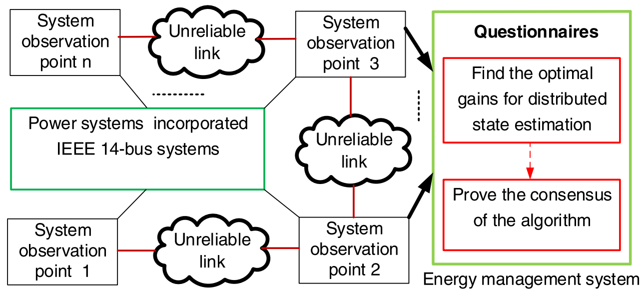

FIGURE 1. Smart grid interconnected subsystems and its research questionnaires.

[30], [31], [33]. Let N_i denotes neighbouring estimators and $\alpha^{ij}(k) \in \{0, 1\}$ is the Bernoulli distribution and it is considered as a packet loss model as follows [7]:

$$\alpha^{ij}(k) = \begin{cases} 1, & \text{probability of } \lambda^{ij}(k), \\ 0, & \text{probability of } 1-\lambda^{ij}(k). \end{cases} \tag{3}$$

Here, $\lambda^{ij}(k)$ is the packet dropout probability between estimator i and j at time instant k. Inspired by [30], [4], [38], [39] the proposed estimator is written as follows:

$$\hat{\boldsymbol{x}}^i(k+1) = \boldsymbol{A}_d\hat{\boldsymbol{x}}^i(k) + \boldsymbol{B}_d\boldsymbol{u}(k) + \boldsymbol{K}[\boldsymbol{y}^i(k) - \boldsymbol{C}\hat{\boldsymbol{x}}^i(k)] + \boldsymbol{L}\sum_{j\in N_i}\alpha^{ij}(k)[\hat{\boldsymbol{x}}^j(k) - \hat{\boldsymbol{x}}^i(k)]. \tag{4}$$

Here, $\hat{\boldsymbol{x}}^i(k+1)$ is the estimated state, $\hat{\boldsymbol{x}}^i(k)$ is estimated state of the previous step, $\boldsymbol{K}$ and $\boldsymbol{L}$ are the local and neighbouring gain to be designed. Overall, the pictorial view of the system model and its research questionnaires are depicted in Fig. 1. In future, the consensus of the proposed algorithm will develop.

IV. PROPOSED STATE ESTIMATION ALGORITHM USING LINEAR MATRIX INEQUALITY APPROACH

The estimation error $\boldsymbol{e}^i$ define as follows [38], [39]:

$$\boldsymbol{e}^i(k) = \boldsymbol{x}(k) - \hat{\boldsymbol{x}}^i(k). \tag{5}$$

$$\boldsymbol{e}^i(k+1) = \boldsymbol{x}(k+1) - \hat{\boldsymbol{x}}^i(k+1). \tag{6}$$

Putting (4) into (6), and using (1) as well as (2) we can obtain:

$$\begin{aligned} \boldsymbol{e}^i(k+1) &= \boldsymbol{x}(k+1) - \boldsymbol{A}_d\hat{\boldsymbol{x}}^i(k) - \boldsymbol{B}_d\boldsymbol{u}(k) - \boldsymbol{K}[\boldsymbol{y}^i(k) \\ &\quad - \boldsymbol{C}\hat{\boldsymbol{x}}^i(k)] - \boldsymbol{L}\sum_{j\in N_i}\alpha^{ij}(k)[\hat{\boldsymbol{x}}^j(k) - \hat{\boldsymbol{x}}^i(k)] \\ &= [\boldsymbol{A}_d - \boldsymbol{K}\boldsymbol{C}]\boldsymbol{e}^i(k) + \boldsymbol{L}\sum_{j\in N_i}\alpha^{ij}(k)[\boldsymbol{e}^j(k) - \boldsymbol{e}^i(k)] \\ &\quad + \boldsymbol{n}_d(k) - \boldsymbol{K}\boldsymbol{w}^i(k). \end{aligned} \tag{7}$$

Define $\boldsymbol{G}_m = [g_m^{ij}]_{n\times n}$ is the Laplacian matrix with

$$g_m^{ij} = \begin{cases} \sum_{j\in N_i} \alpha_m^{ij} g_m^{ij}, & \text{if i = j } (\textit{diagonal element}), \\ -\alpha_m^{ij} g_m^{ij}, & \text{if } (i,j) \in E, i \neq \text{j } (\textit{adjacence}), \\ 0, & \text{otherwise } ((i,j) \notin E, \textit{disjoint}). \end{cases} \tag{8}$$

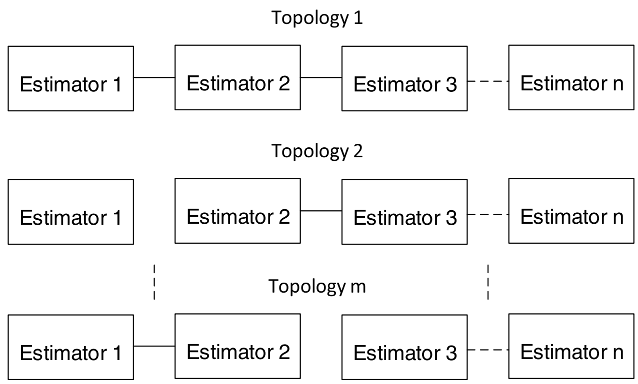

FIGURE 2. Topologies due to packet losses in communication networks.

The Laplacian $\boldsymbol{G}_{\sigma(k)} \in \boldsymbol{G}_m$, $\sigma(\bullet) : Z^+ \rightarrow \{1, 2, \cdots, S\}$ is a stochastic process driven by an independent and identically distributed sequence, $m = 1, 2, \cdots, S$ where $S = 2^{\sum_{i=1}^{n} |N_i|/2}$ is the total number of possible topologies and $|N_i|$ is the number of estimators in the neighbouring set N_i [38], [39]. To illustrate, Fig. 2 shows the different switching topologies. It is noticed that the interconnected topology is changed because of the unreliable communication links between the physical connected estimators as shown in Figs. 1 and 2 [30], [31], [33]. In this case, the packet loss probability is given by:

$$\phi_m = \prod_{i=1,\ i<j}[\alpha_m^{ij}(1-\lambda^{ij}) + (1-\alpha_m^{ij})\lambda^{ij}]. \tag{9}$$

Now we can define the following augmented vectors for estimator $i = 1, 2, \ldots, n$:

$$\boldsymbol{e}(k) = [\boldsymbol{e}'^1(k) \ldots \boldsymbol{e}'^n(k)]', \boldsymbol{w}(k) = [\boldsymbol{w}'^1(k) \ldots \boldsymbol{w}'^n(k)]'. \tag{10}$$

The combine error dynamic is written in a compact form:

$$\begin{aligned}\boldsymbol{e}(k+1) = [\boldsymbol{I}_n \otimes (\boldsymbol{A}_d - \boldsymbol{KC}) - \boldsymbol{G}_{\sigma(k)} \otimes \boldsymbol{L}]\boldsymbol{e}(k) \\ + \boldsymbol{I}_n \boldsymbol{n}_d(k) - (\boldsymbol{I}_n \otimes \boldsymbol{K})\boldsymbol{w}(k).\end{aligned} \tag{11}$$

Here the symbol $\otimes$ denotes the Kronecker product. The following approach is used to compute the gains for distribute estimation.

The system error dynamic is stable after minimizing the estimation error, if the following linear matrix inequality (LMI) holds:

$$\boldsymbol{A}'_{cl}\boldsymbol{X}\boldsymbol{A}_{cl} - \boldsymbol{X} < \boldsymbol{0}, \quad \boldsymbol{X} > \boldsymbol{0}, \ \rho(\boldsymbol{A}_{cl}) < 1. \tag{12}$$

Here, $\boldsymbol{A}_{cl} = \boldsymbol{I}_n \otimes (\boldsymbol{A}_d - \boldsymbol{KC}) - \boldsymbol{G}_{\sigma(k)} \otimes \boldsymbol{L}$. Introducing semidefinite programming variables, $\boldsymbol{M} = \boldsymbol{KX}$ and $\boldsymbol{N} = \boldsymbol{LX}$, it can be rewritten into the LMI form as follows:

$$\begin{bmatrix} -\boldsymbol{X} & (\boldsymbol{I}_n \otimes \boldsymbol{A}'_d)\boldsymbol{X} - \boldsymbol{I}_n \otimes (\boldsymbol{C}'\boldsymbol{M}') - \boldsymbol{G}_k \otimes \boldsymbol{N}' \\ \star & -\boldsymbol{X} \end{bmatrix} < \boldsymbol{0}.$$

Using the above inequality, the optimization variables $\boldsymbol{X}$, $\boldsymbol{M}$ and $\boldsymbol{N}$ are computed. Finally, the optimum gains are determined by:

$$\boldsymbol{K} = \boldsymbol{X}^{-1}\boldsymbol{M}, \quad \boldsymbol{L} = \boldsymbol{X}^{-1}\boldsymbol{N}. \tag{13}$$

After computing the gains by (13), the system state estimation is obtained using (4). The main characteristic of this proposed method is that it does not need to approximate the error function for computing the optimum gains. In reality, it does not also require to calculate the computational intensive predicted/updated error covariance matrix as it is virtually replaced by optimization variable $\boldsymbol{x}$. Precisely, the gains computation are accurately reflected the original filter structure in (4). This is because the error function in (11) can assist to trace back to the original filter structure as it does not need any approximation. In contrast, the *suboptimal filter* structure in [4], it assumes that there are no neighbouring terms ($\boldsymbol{L} = \boldsymbol{0}$) in the error function (7) for computing the error covariance and gain in the stability analysis. This means the gain/error covariance expression for estimation is totally different from the stability analysis.

V. IEEE 14-BUS STATE SPACE MODEL USING HOLT-WINTERS METHOD AND SIMULATION RESULTS

The proposed method is applied to the IEEE 14-bus system. In doing this, the state-space model of IEEE 14-bus is obtained using the Holt-Winters method. Simulation results are demonstrated considering packet losses and cyber attacks The simulation has been carried out using the Matlab, Matpower [40] and YALMIP softwares [41]. Similar with [31], [33], we consider there are n=4 interconnected estimators as shown in Fig. 2. Obviously, there are $m = 1, 2, \cdots, 8$ possible topologies and their Laplacian matrix are described by:

$$\boldsymbol{G}_1 = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}, \quad \boldsymbol{G}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$$\boldsymbol{G}_3 = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}, \quad \boldsymbol{G}_4 = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{G}_5 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}, \quad \boldsymbol{G}_6 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{G}_7 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{G}_8 = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

For instance, the Laplacian matrix $\boldsymbol{G}_1$ and $\boldsymbol{G}_2$ are corresponding to the topology 1 and 2 in Fig. 2, respectively. For simulation, the Laplacian matrix is considered static.

The IEEE 14-bus system is employed to demonstrate the performance of the proposed approach. A single-line diagram of the IEEE 14-bus is depicted in Fig. 3. The system has total 10 generators and 11 loads. It has the total generation capacity 772.4 MW and load capacity 259 MW. Basically, the system state vector is comprised of bus voltage magnitudes

TABLE 1. Simulation parameters.

Symbols	Values	Symbols	Values
Δt	0.015 sec	λ^{12}	0.95
λ^{23}	0.6	λ^{34}	0.1
$\mathbf{R}^1$	0.0000002*$\mathbf{I}$	$\mathbf{R}^2$	0.0000003*$\mathbf{I}$
$\mathbf{R}^3$	0.0000004*$\mathbf{I}$	$\mathbf{R}^4$	0.0000005*$\mathbf{I}$

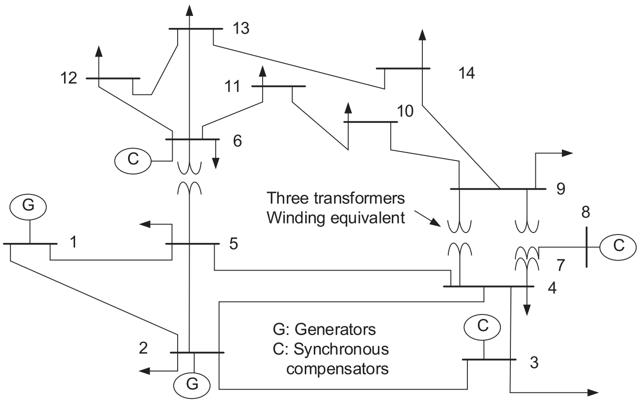

FIGURE 3. Single-line diagram of the IEEE 14-bus system [39], [42].

TABLE 2. The nominal values of the IEEE 14-bus system.

Bus No.	Phase angle Θ_M (degree)	Voltage magnitude V_M (per unit)
1	0.000	1.060
2	-4.983	1.045
3	-12.725	1.010
4	-10.313	1.018
5	-8.774	1.020
6	-14.221	1.070
7	-13.360	1.062
8	-13.360	1.090
9	-14.939	1.056
10	-15.097	1.051
11	-14.791	1.057
12	-15.076	1.055
13	-15.166	1.050
14	-16.034	1.036

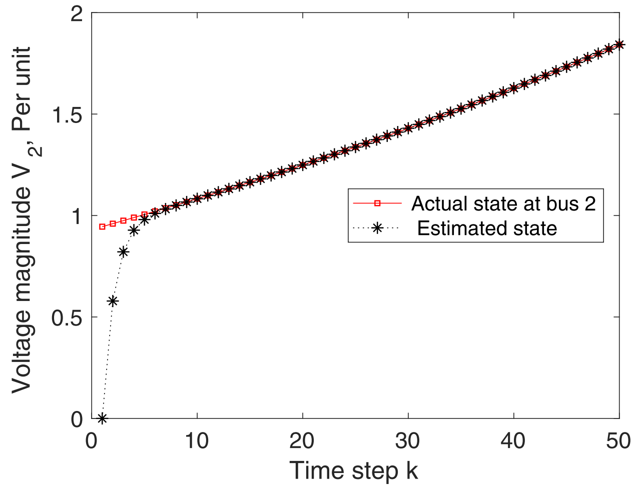

FIGURE 4. Voltage magnitude V_2 and its estimation.

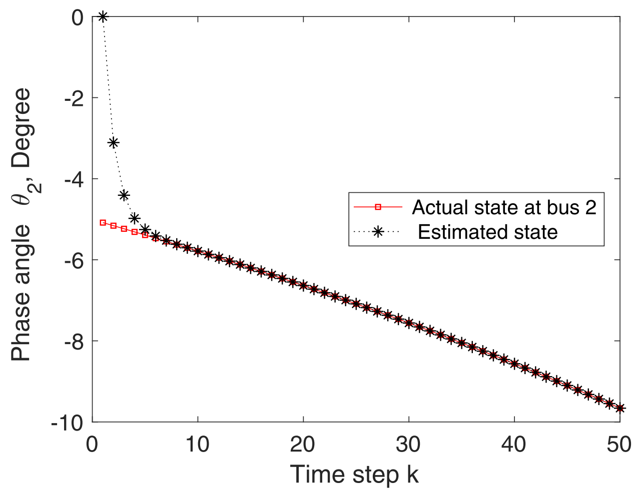

FIGURE 5. Voltage angle θ_2 and its estimation.

and phase angles. Generally, for a power system with M buses, the system state vector $\boldsymbol{x}$ can be defined as $\boldsymbol{x} = [\Theta_1\ \Theta_2 \cdots \Theta_M,\ V_1\ V_2\ \cdots\ V_M]'$, where Θ_i is the i-th phase angle and V_i is the i-th bus voltage magnitude. The bus 1 is a slack bus. The simulation parameters of the IEEE 14-bus system can be found in [40]. The nominal phase angles and bus voltage magnitudes are shown in Table 2, where the optimal power flow is evaluated by Matpower tool using the Newton's method.

In order to apply the proposed method in the linear state-space model for estimating the system states, it requires the system state matrix $\boldsymbol{A}$ and input matrix $\boldsymbol{B}$ in (1). For computing them, this paper adopts the Holt-Winters method [24], [43], [44] which is summarised by:

$$\boldsymbol{A} = \gamma(1+\beta)\boldsymbol{I}. \tag{14}$$

$$\begin{aligned}\boldsymbol{B} = diag[&(1+\beta)(1-\gamma)\hat{\boldsymbol{x}}(t-1) - \beta\boldsymbol{a}(t-1) \\ &+ (1-\beta)\boldsymbol{b}(t-1)].\end{aligned} \tag{15}$$

Here, the coefficient $\gamma, \beta \in (1, 0)$, the system tuning parameters $\boldsymbol{a}(t)$ and $\boldsymbol{b}(t)$ are recursively computed by:

$$\begin{aligned}\boldsymbol{a}(t) &= \gamma\boldsymbol{x}(t) + (1-\gamma)\hat{\boldsymbol{x}}(t). \\ \boldsymbol{b}(t) &= \beta[\boldsymbol{a}(t) - \boldsymbol{a}(t-1)] + (1-\beta)\boldsymbol{b}(t-1).\end{aligned} \tag{16}$$

Here, $\boldsymbol{a}(t-1)$ and $\boldsymbol{b}(t-1)$ are the initial coefficients, $\boldsymbol{x}$ and $\hat{\boldsymbol{x}}$ are the nominal and predicted states of the system.

For computing $\boldsymbol{A}$ and $\boldsymbol{B}$ in (14) and (15), this paper uses $\gamma = 0.5$, $\beta = 0.8$, $\boldsymbol{a}(t-1) = \mathbf{0}$, $\boldsymbol{b}(t-1) = \mathbf{0}$, $\hat{\boldsymbol{x}}$ is flat started (unit value) and $\boldsymbol{x}$ is nominal values which are mentioned in Table 2. Similar with [24], this paper uses the nominal voltage magnitudes and phase angles. For computing the system parameters $\boldsymbol{A}$ and $\boldsymbol{B}$, the simulation is conducted at time t=1 to 10. It is evident from the dynamic responses in Figs. 4-5 that the estimation results precisely match the actual system states within few time steps. This clearly implies that the explored method can well reject the system impairments, and accurately monitor the large-scale power systems. It is worth to mention that the sharing information

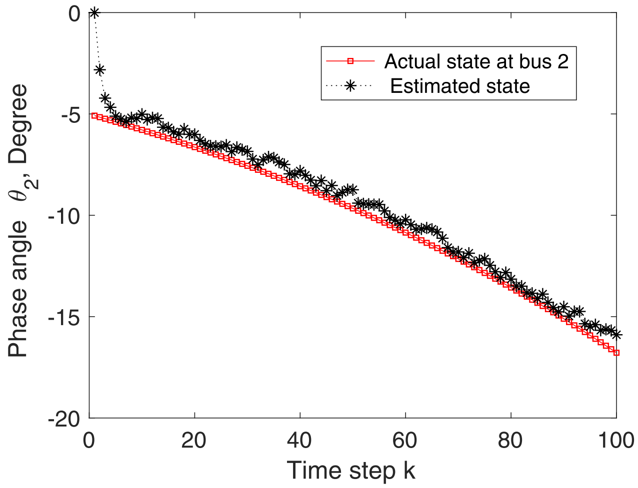

FIGURE 6. Voltage angle θ_2 and its estimation considering cyber attacks.

between the connected estimators also influences to reach it consensus on estimation despite unreliable communication links among them. Sometime the optimal KF approach is used to compute the local gain in case of infeasibility [45], [46].

Sometimes there is a cyber attack in the sensing measurements. Considering random false data injection attacks, the simulation result is presented in Fig. 6. It can be seen that the proposed algorithm requires more time to recover the system state.

VI. CONCLUSION AND FUTURE WORK

This paper proposes a distributed dynamic state estimation algorithm based on the linear matrix inequality approach for smart grids. Inspired by the control theory, matrix properties of the Kronecker product and the Laplacian operator, the proposed framework is developed. The desired gains are determined by the convex optimization process, and the algorithm is applied to IEEE 14-bus system. It shows that the estimated state converges to the true state within 0.1 second. Therefore, these findings are valuable for green communication, households, and provides knowledge towards the distributed energy management system design. Future and ongoing research includes the investigation of delays on the state estimation performance and stabilizing the system states in a distributed way. In future, the consensus of the proposed algorithm will develop.

REFERENCES

[1] H. Li, F. Li, Y. Xu, D. T. Rizy, and J. D. Kueck, "Adaptive voltage control with distributed energy resources: Algorithm, theoretical analysis, simulation, and field test verification," *IEEE Trans. Power Syst.*, vol. 25, no. 3, pp. 1638–1647, Aug. 2010.

[2] V. Kekatos and G. B. Giannakis, "Distributed robust power system state estimation," *IEEE Trans. Power Syst.*, vol. 28, no. 2, pp. 1617–1626, May 2013.

[3] A. K. Singh, R. Singh, and B. C. Pal, "Stability analysis of networked control in smart grids," *IEEE Trans. Smart Grid*, vol. 6, no. 1, pp. 381–390, Jan. 2015.

[4] R. Olfati-Saber, "Kalman-consensus filter: Optimality, stability, and performance," in *Proc. Int. Conf. Decis. Control*, Dec. 2009, pp. 7036–7042.

[5] Y. Sun and D. B. Work, "A distributed local Kalman consensus filter for traffic estimation," in *Proc. Conf. Decis. Control*, Dec. 2014, pp. 6484–6491.

[6] C. Han, W. Wang, and H. Zhang, "Optimal filtering on continuous-time systems with Markovian communication delays and packet dropouts," in *Proc. World Congr. Intell. Control Automat.*, Jul. 2012, pp. 2027–2032.

[7] B. Sinopoli, L. Schenato, M. Franceschetti, K. Poolla, M. I. Jordan, and S. S. Sastry, "Kalman filtering with intermittent observations," *IEEE Trans. Autom. Control*, vol. 49, no. 9, pp. 1453–1464, Sep. 2004.

[8] N. Xiao, L. Xie, and M. Fu, "Kalman filtering over unreliable communication networks with bounded Markovian packet dropouts," *Int. J. Robust Nonlinear Control*, vol. 19, no. 16, pp. 1770–1786, 2009.

[9] S. Kar, B. Sinopoli, and J. M. F. Moura, "Kalman filtering with intermittent observations: Weak convergence to a stationary distribution," *IEEE Trans. Autom. Control*, vol. 57, no. 2, pp. 405–420, Feb. 2012.

[10] L. Hu, Z. Wang, I. Rahman, and X. Liu, "A constrained optimization approach to dynamic state estimation for power systems including PMU and missing measurements," *IEEE Trans. Control Syst. Technol.*, vol. 24, no. 2, pp. 703–710, Mar. 2016.

[11] X. Tai, D. Marelli, and M. Fu, "Power system dynamic state estimation with random communication packets loss," in *Proc. Int. Symp. Adv. Control Ind. Process.*, May 2011, pp. 359–362.

[12] F. S. Cattivelli and A. H. Sayed, "Diffusion strategies for distributed Kalman filtering and smoothing," *IEEE Trans. Autom. Control*, vol. 55, no. 9, pp. 2069–2084, Sep. 2010.

[13] S. Lu, V. H. Nascimento, J. Sun, and Z. Wang, "Sparsity-aware adaptive link combination approach over distributed networks," *Electron. Lett.*, vol. 50, no. 18, pp. 1285–1287, 2014.

[14] J. Hu, L. Xie, and C. Zhang, "Diffusion Kalman filtering based on covariance intersection," *IEEE Trans. Signal Process.*, vol. 60, no. 2, pp. 891–902, Feb. 2012.

[15] B. Chen, W.-A. Zhang, and L. Yu, "Distributed fusion estimation with missing measurements, random transmission delays and packet dropouts," *IEEE Trans. Autom. Control*, vol. 59, no. 7, pp. 1961–1967, Jul. 2014.

[16] C.-L. Su and C.-N. Lu, "Interconnected network state estimation using randomly delayed measurements," *IEEE Trans. Power Syst.*, vol. 16, no. 4, pp. 870–878, Nov. 2001.

[17] M. Moayedi, Y. K. Foo, and Y. C. Soh, "Adaptive Kalman filtering in networked systems with random sensor delays, multiple packet dropouts and missing measurements," *IEEE Trans. Signal Process.*, vol. 58, no. 3, pp. 1577–1588, Mar. 2010.

[18] L. Schenato, "Optimal estimation in networked control systems subject to random delay and packet drop," *IEEE Trans. Autom. Control*, vol. 53, no. 5, pp. 1311–1317, Jun. 2008.

[19] M. Sahebsara, T. Chen, and S. L. Shah, "Optimal filtering with random sensor delay, multiple packet dropout and uncertain observations," *Int. J. Control*, vol. 80, no. 2, pp. 292–301, 2007.

[20] J. Cong, Y. Li, G. Qi, and A. Sheng, "An order insensitive sequential fast covariance intersection fusion algorithm," *Inf. Sci.*, vol. 367, pp. 28–40, Nov. 2016.

[21] M. M. Rana, L. Li, and S. Su, "Distributed state estimation using RSC coded smart grid communications," *IEEE Access*, vol. 3, pp. 1340–1349, 2015.

[22] G. Rigatos, P. Siano, and N. Zervos, "A distributed state estimation approach to condition monitoring of nonlinear electric power systems," *Asian J. Control*, vol. 15, no. 3, pp. 849–860, 2013.

[23] Z. Weng and P. M. Djurić, "A Bayesian approach to covariance estimation and data fusion," in *Proc. Eur. Signal Process. Conf.*, 2012, pp. 2352–2356.

[24] P. Chavali and A. Nehorai, "Distributed power system state estimation using factor graphs," *IEEE Trans. Signal Process.*, vol. 63, no. 11, pp. 2864–2876, Mar. 2015.

[25] T. Sui, D. Marelli, and M. Fu, "Convergence analysis for Guassian belief propagation: Dynamic behaviour of marginal covariances," in *Proc. Int. Conf. Acoust., Speech Signal Process.*, Mar. 2016, pp. 2599–2602.

[26] D. Gu, "A game theory approach to target tracking in sensor networks," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 41, no. 1, pp. 2–13, Feb. 2011.

[27] D. Ding, Z. Wang, B. Shen, and H. Dong, "Event-triggered distributed H_∞ state estimation with packet dropouts through sensor networks," *IET Control Theory Appl.*, vol. 9, no. 13, pp. 1948–1955, Aug. 2015.

[28] V. Ugrinovskii and C. Langbort, "Distributed H_∞ consensus-based estimation of uncertain systems via dissipativity theory," *IET Control Theory Appl.*, vol. 5, no. 12, pp. 1458–1469, 2011.

[29] H. Li, W. Yan, and H. Yang, "Receding horizon consensus estimation of sensor networks with packet dropouts," in *Proc. Chin. Control Conf.*, Jul. 2015, pp. 7659–7664.
[30] Y. Zhang and Y.-P. Tian, "Consensus tracking in sensor networks with periodic sensing and switching connected topologies," *Syst. Control Lett.*, vol. 84, pp. 44–51, Oct. 2015.
[31] Y. Zhang and Y.-P. Tian, "Consensus of data-sampled multi-agent systems with random communication delay and packet loss," *IEEE Trans. Autom. Control*, vol. 55, no. 4, pp. 939–943, Apr. 2010.
[32] Y. Zhang and Y.-P. Tian, "Maximum allowable loss probability for consensus of multi-agent systems over random weighted lossy networks," *IEEE Trans. Autom. Control*, vol. 57, no. 8, pp. 2127–2132, Jan. 2012.
[33] W.-M. Zhou and J.-W. Xiao, "Dynamic average consensus and consensusability of general linear multiagent systems with random packet dropout," *Abstr. Appl. Anal.*, vol. 2013, Aug. 2013, Art. no. 412189.
[34] L. Zhenxing and J. Haibo, "Quantized consensus for linear discrete-time multi-agent systems," in *Proc. Int. Conf. Control Autom. Robot. Vis.*, Dec. 2012, pp. 1472–1476.
[35] K.-Y. You and L.-H. Xie, "Survey of recent progress in networked control systems," *Acta Autom. Sinica*, vol. 39, pp. 101–117, Feb. 2013.
[36] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *Proc. IEEE*, vol. 95, no. 1, pp. 215–233, Jan. 2007.
[37] L. Schenato, B. Sinopoli, M. Franceschetti, K. Poolla, and S. S. Sastry, "Foundations of control and estimation over lossy networks," *Proc. IEEE*, vol. 95, no. 1, pp. 163–187, Jan. 2007.
[38] M. M. Rana, W. Xiang, and E. Wang, "IoT-based state estimation for microgrids," *IEEE Internet Things J.*, vol. 5, no. 2, pp. 1345–1346, Apr. 2018.
[39] M. M. Rana, W. Xiang, and B. J. Choi, "Distributed state estimation for smart grids considering packet dropouts," in *Proc. 15th Int. Conf. Control, Autom., Robot. Vis.*, Nov. 2018, pp. 2009–2013.
[40] R. D. Zimmerman, C. E. Murillo-Sánchez, and R. J. Thomas, "MATPOWER: Steady-state operations, planning, and analysis tools for power systems research and education," *IEEE Trans. Power Syst.*, vol. 26, no. 1, pp. 12–19, Feb. 2011.
[41] J. Löfberg, "YALMIP: A toolbox for modeling and optimization in MATLAB," in *Proc. Int. Symp. Comput. Aided Control Syst. Design*, 2004, pp. 284–289.
[42] A. J. Wood and B. F. Wollenberg, *Power Generation, Operation, and Control*. Hoboken, NJ, USA: Wiley, 2012.
[43] A. M. L. da Silva, M. B. D. C. Filho, and J. F. de Queiroz, "State forecasting in electric power systems," *IEE Proc. C Gener., Transmiss. Distrib.*, vol. 130, no. 5, pp. 237–244, Sep. 1983.
[44] N. Kashyap, S. Werner, T. Riihonen, and Y.-F. Huang, "Reduced-order synchrophasor-assisted state estimation for smart grids," in *Proc. Int. Conf. Smart Grid Commun.*, Nov. 2012, pp. 605–610.
[45] H. Y. and M. M. Rana, "Least mean square fourth based microgrid state estimation algorithm using the Internet of Things technology," *PLoS ONE*, vol. 12, no. 5, 2017, Art. no. e0176099.
[46] F. Zhang, *Matrix Theory: Basic Results and Techniques*. New York, NY, USA: Springer, 2011.