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Optimal spatial-dynamic management to minimize the damages caused by aquatic invasive species

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Abstract
Invasive species have been recognized as a leading threat to biodiversity. In particular, lakes are especially affected by species invasions because they are closed systems sensitive to disruption. Accurately controlling the spread of invasive species requires solving a complex spatial-dynamic optimization problem. In this work we propose a novel framework for determining the optimal management strategy to maximize the value of a lake system net of damages from invasive species, including an endogenous diffusion mechanism for the spread of invasive species through boaters' trips between lakes. The proposed method includes a combined global iterative process which determines the optimal number of trips to each lake in each season and the spatial-dynamic optimal boat ramp fee.

Keywords Invasive species · Spatial-dynamic management · Convex optimization · Bioeconomic

JEL Classification Q20 · Q50 · Q57

1 Introduction

Globally, invasive species have long been recognized as a leading threat to biodiversity (Wilcove et al. 1998; Sala et al. 2000). Lakes are especially affected by species invasions because they are closed systems sensitive to disruption (Sala

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et al. 2000; Moorhouse and Macdonald 2015). As a result, controlling the spread of aquatic invasive species (AIS) has been a major management effort for the past two decades. Further complicating AIS management, AIS are largely spread inadvertently through the movement of recreational boaters from lake to lake (Rothlisberger et al. 2010). Therefore, management is tasked with maximizing the value of a lake system net of AIS damages by changing boating behavior and thus the spread of AIS.

In this article, we develop a novel spatial-dynamic framework to determine the optimal policy to induce the optimal number of trips to each lake in a system across many seasons to maximize the net benefits of the lake system taking into consideration the damages from the spread of invasive species. This policy must be heterogeneous across space and time. Spatially, it must depend on boating patterns (which depend on the attractiveness of lakes, substitutability across lakes, the presence of invasive species) and the ecological suitability of a lake for invasion. Temporally, it must take into account the time dependent distribution of AIS across the system. AIS spread is a vicious cycle, as an increase in invaded lakes provides more opportunity for spread to uninvaded lakes. The pathways for dispersal of AIS depend on boating behavior, which in turn depends on lake management and the presence or absence of AIS in each lake. This leads to a complicated feedback loop where the optimal policy depends on the dispersal of AIS and the number of trips to each lake, the dispersal of AIS depends on boating behavior, and boating behavior depends on policy and the dispersal of AIS [see Fig. 1]. The dispersal of AIS and boating behavior both depend on the presence or absence of AIS in every lake in the system. Therefore, the optimal policy must be globally coordinated across the system (Epanchin-Niell et al. 2010).

There are five major components to our optimal spatial-dynamic AIS management framework.

1. *Model of recreational boating decisions.* The dispersal of AIS depends on boating decisions, therefore, we need a model of how boaters choose where to boat. We rely on a standard economic model known as a random utility model (RUM) in which boaters maximize their utility by choosing where to boat.
2. *Model of AIS dispersal.* The dispersal of AIS requires (i) boaters to visit an invaded lake, (ii) inadvertently transport the invasive species out of the invaded lake, (iii) then visit an uninvaded lake while the invaded species is still alive, and finally (iv) the invasive species must become established in a suitable lake (Zipp et al. 2019). Given the dispersal of AIS, we model the probability that a lake becomes invaded given that it is not already invaded as a hazard model.
3. *Net benefits to recreational boaters.* Lakes provide benefits to boaters. Our RUM allows us to calculate the welfare benefits of boating. The net benefits of the lake system are these benefits of boating minus the damages caused by AIS. We consider both the damages from AIS to both shoreline property owners (Horsch and Lewis 2009; Provencher et al. 2012) and boaters (Lewis et al. 2015).
4. *Optimal number of boating trips to each lake.* We find the optimal number of boating trips to each lake in each season to maximize net benefits (step 3) subject to AIS dispersal (step 2) and the model of boating decisions (step 1).

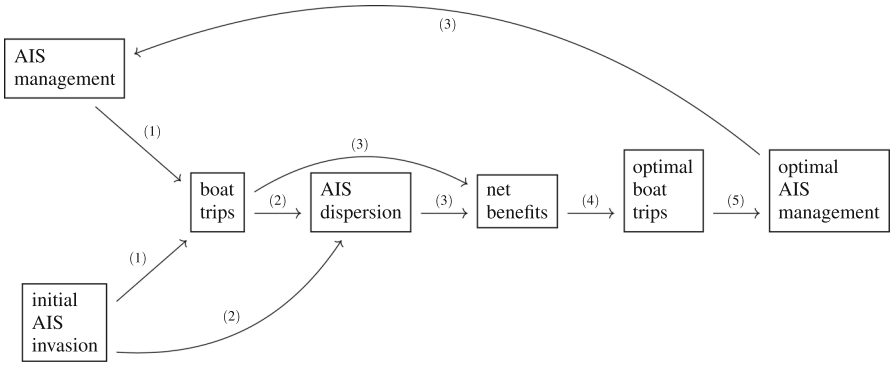

Fig. 1 Diagram of model framework

5. *Optimal policy.* Finally, we find the optimal policy that leads boaters to choose the socially optimal number of trips.

This framework makes three major contributions to the literature. First, we develop a method that, to our knowledge, has not been previously used in the emerging works on optimal spatial-dynamic policy management and we refer to (Epanchin-Niell et al. 2010; Brock and Xepapadeas 2008, 2010; Sanchirico et al. 2010; Eiswerth and van Kooten 2002; Hof 1998; Horan et al. 2005; Leung et al. 2002) and the references therein for other techniques and methods. The proposed algorithms are efficient and do not require an intractable amount of computing resources for its implementation. The main reason behind this is that model parameters are fixed and we are only concerned with the direct problem of finding optimal ramp fees. We use an economic model of recreational boating decisions to parameterize the spread of aquatic invasive species. We also parameterize the objective function (i.e. the net benefit function) with nonmarket valuation studies. By fixing our parameters from economic theory even as the state space (i.e. the number of lakes times the number of seasons) increases and our tensors become sparser we are able to calculate the spread of the invasive species and the objective function. We use an iterative method to solve for the optimal boat ramp fees that involves solving quadratic programs for which the analytic solution is known. These steps only require calculations with dimensions equal to the state space. Second, we incorporate an endogenous dispersal method. The literature on optimal spatial-dynamic bioeconomic policies typically models the dispersal of species as radially dispersing from an original location. Our model on AIS allows the dispersal mechanism to depend on the movement of boaters and thus be endogenous. Third, we have a fully-coupled natural human system where boating decisions depend on the status of invasions and the dispersal of invasions depends on boating decisions.

2 Problem Formation

The spatial-dynamic optimal policy can be determined through an iterative process with five major components (see Fig. 1), which will be explained in detail.

2.1 Model of recreational boating decisions

We model boating decisions as a repeated random utility travel cost model in which boaters $n \in \{1, \ldots, N\}$ maximize their utility U_{isnt} on day $t \in \{1, \ldots, T\}$ in season $s \in \{1, \ldots, S\}$ by either (a) visiting lake $i \in \{2, \ldots, I\}$; or (b) choosing not to go boating (denoted by $i = 1$). Let utility[1] be defined as

$$\begin{aligned} U_{isnt} &= v_{isnt} + \varepsilon_{isnt}, \quad \text{where} \\ v_{isnt} &= Z_{isnt} + \alpha(M_n - \tau_{i,s}) - \xi x_{i(s-1)}, \end{aligned} \tag{1}$$

$Z_{isnt} = \sum_m \beta_m z_{isnt,m}$ represents m boater, lake, day, or season attributes ($z_{isnt,m}$) that influence decision to go boating or stay home with parameters β_m and remains constant in our model, α is marginal utility of income, M_n is the income of boater n, τ_{is} is the boat ramp fee, ξ is the effect of AIS on boater utility, $x_{i(s-1)}$ is the status of AIS lake i at the end of the previous season $(s-1)$,[2] and ε_{isnt} are identically independently distributed Type I Extreme Value random variables with variance $\pi^2/6$. Under this assumption of error distribution, the probability that boater n chooses an alternative $i \in \{1, \ldots I\}$ on day t in season s is given by the conditional logit model (Train 2009)

$$P_{isnt} = \frac{\exp(v_{isnt})}{\sum_{j=1}^{I} \exp(v_{jsnt})}.$$

We set the total number of trips taken to lake $i \in \{2, \ldots, I\}$ (or days spent at home in the case of $i = 1$) in season s to be $b_{is} = \sum_{n=1}^{N} \sum_{t=1}^{T} P_{isnt}$ (note that $\sum_{i=1}^{I} b_{is} = NT$ by definition). Clearly, b_{is} is a function of the boat ramp fees $\{\tau_{js}\}_{j=2}^{I}$ and the invasion statuses $\{x_{j(s-1)}\}_{j=2}^{I}$.

2.2 Model of AIS dispersal

AIS are largely spread inadvertently through the movement of boaters across lakes (Chivers and Leung 2012; Rothlisberger et al. 2010). We define the status of the invasion in lake i in season s, denoted by x_{is}, as the probability that lake i is invaded at the end of season s. We assume that invasions are irreversible so that once a lake becomes invaded it remains invaded for all remaining seasons. Let $\tilde{x}_{is}$ be the probability that lake i is invaded at the end of season s conditional on being uninvaded at the end of season $s-1$. We have the following relations.

$$x_{i(s+1)} = 1 - \prod_{m=1}^{s} (1 - \tilde{x}_{im}), \quad \text{or equivalently,}$$

[1] Note that this utility function can be further generalized to allow for nonlinear impacts of income and to allow for congestion to impact boaters (i.e. independence among boaters' utilities).

[2] We assume that the invasion status is updated at the end of the season such that boaters' trip decisions depend on x_{s-1} and at the end of the season when the invasion status is updated x_s depends on the boating decisions in season s.

$$x_{i,s+1} = x_{is} + (1 - x_{is})\tilde{x}_{i(s+1)}. \tag{2}$$

We assume that $x_{i,s=1} =: x_{i1}$ is given. We use the following approximate hazard model:

$$\widetilde{x}_{is} = \mathfrak{s}(K_{is}) = \begin{cases} 0, & K_{is} < -a, \\ \frac{K_{is}+a}{2a}, & K_{is} \in [-a, a], \\ 1, & K_{is} > a, \end{cases} \tag{3}$$

which approximates the sigmoid function.[3] Here, K_{is} is defined as

$$K_{is} = \gamma L_{is} + \xi_l \lambda_{is}$$

where L_{is} is the expected number of trips carrying AIS to lake i in season s (referred to as propagule pressure), λ_{is} is a measure of the suitability of lake i to host AIS, and (γ, ξ_l) are estimated parameters related to the effects of propagule pressure and lake suitability on the probability of invasion. Propagule pressure requires (i) boaters to visit an invaded lake, (ii) inadvertently transport the invasive species out of the invaded lake, and (iii) then visit an uninvaded lake while the invaded species is still alive. Recent evidence suggests that the majority of AIS remains alive for approximately one day (Bruckerhoff et al. 2014). It is not possible to spread the invasive species from lake i to itself, so we are only concerned with trips between different lakes. Therefore, the expected probability that a boater visits any invaded lake $j \neq i$ on the previous day is $\sum_{j \neq i} P_{jsn(t-1)} x_{j(s-1)}$. Let μ be the probability of a boat leaving an invaded lake with AIS (this is a constant independent of n,i,s,t). Therefore, the propagule pressure is

$$L_{is} = \sum_{t=2}^{T} \sum_{n=1}^{N} \underbrace{\left(\mu \sum_{j \neq i} P_{jsn(t-1)} x_{j(s-1)} \right)}_{Q_{isnt}} P_{isnt}$$

where Q_{isnt} is the expected number of trips that boater n makes to lake i on day t in season s carrying an invasive species from any lake $j \neq i$.

2.3 Net benefits

Lakes provide benefits to boaters who receive utility from boating instead of staying home. Let $w_{snt} = \max\{v_{1snt}, \ldots, v_{Isnt}\}$. Without any boating, boaters just receive utility from staying home $w_{snt} = v_{1snt}$. Therefore, the value of having the option to boat at all lakes in the system $i \in 2, \ldots, I$ is worth $\mathscr{L}^s_{snt}$ per boater per trip per season:

[3] We take $a = 2.824153$ which approximates the sigmoid with ℓ_∞ error at most 0.056075.

$$\begin{aligned}\mathscr{L}^{s}_{snt} &= -\frac{1}{|\alpha|}\left(\ln\left(\exp(v_{1snt})\right) - \ln\left(\sum_{i=1}^{I}\exp(v_{isnt})\right)\right)\\ &= -\frac{1}{|\alpha|}\left(\ln\left(\frac{\exp(v_{1snt})}{\sum_{i=1}^{I}\exp(v_{isnt})}\right)\right)\\ &= -\frac{1}{|\alpha|}\left(\ln\left(P_{1snt}\right)\right) \geq 0,\end{aligned} \tag{4}$$

where α is the marginal utility of income and is used to convert measures of utility into measures of dollars and $P_{1snt} \leq 1$ is the probability of boater n choosing to stay home $i = 1$ on day t in season s. The total value of the system to all boaters across all trips in season s is the sum of $\mathscr{L}^{s}_{snt}$ over boaters n and days t:

$$\begin{aligned}\mathscr{W}^{s}_{s} &= -\frac{1}{|\alpha|}\sum_{n,t}\ln(P_{1snt}) \approx -\frac{1}{|\alpha|}\sum_{n,t}(P_{1snt} - 1)\\ &= -\frac{1}{|\alpha|}\left(b_{1s} - NT\right) = \frac{1}{|\alpha|}\sum_{i=2}^{I} b_{is}.\end{aligned} \tag{5}$$

The net benefits of the lake system are these benefits $\mathscr{W}^{s}_{s}$ minus the damages caused by AIS. Aquatic invasive species (AIS) cause damage to both shoreline property owners (Provencher et al. 2012; Horsch and Lewis 2009) and recreational users (e.g. boaters) (Lewis et al. 2015). We consider h_i shoreline properties around lake i, assuming that shoreline properties are constant across seasons. We assume that shoreline property owners incur a constant annual welfare loss, $\mathscr{L}^{h}$, per shoreline property, h_i, for each invaded lake. Therefore, we define the total welfare loss in season s from the spread of AIS to shoreline property owners as

$$\mathscr{W}^{h}_{s} = \sum_{i=2}^{I}\mathscr{L}^{h}h_i x_{is}$$

where we use that $x_{1s} = 0$ for all s.

Next, we define the welfare loss from the spread of AIS to boaters. We assume that boaters incur a constant welfare loss, $\mathscr{L}^{b}$, per trip b_{is} to an invaded lake in season s.[4] Therefore, we define the expected total welfare loss in season s from the spread of AIS to recreational boaters as

$$\mathscr{W}^{b}_{s} = \sum_{i=2}^{I}\mathscr{L}^{b}b_{is}x_{is}.$$

Therefore, the net benefits of the entire lake system is the discounted sum of the benefits of the lake system, $\widetilde{\mathscr{W}^{s}_{s}}$ minus the damages from AIS, $\mathscr{W}^{h}_{s} + \mathscr{W}^{b}_{s}$:

[4] Accounting for a non-constant welfare loss that depended on the number of lakes invaded was found to be minor in Zipp et al. (2019) (around 2%), however, future work could allow the welfare loss per boater to depend on the number of invaded lakes.

$$\mathscr{F} = \sum_s \rho^s \left(\widetilde{\mathscr{W}}_s^s - \mathscr{W}_s^h - \mathscr{W}_s^b \right) \tag{6}$$

where $\rho^s = \left(\frac{1}{1+r}\right)^s$ is the discount factor and r is the discount rate.

2.4 Optimal number of boating trips to each lake

With this net benefit function, $\mathscr{F}$, our objective is to find the optimal boat ramp fee τ_{is}^* that maximizes the net benefit of the lake system subject to the dispersal function of AIS (10). This is a difficult optimization problem to solve. We make two contributions that allow us to more easily solve this problem. First, we rewrite the objective function $\mathscr{F}(\boldsymbol{b}, \boldsymbol{x})$ as a function of the matrix of boating trips $\boldsymbol{b} = \left((b_{is})_{i=1}^I\right)_{s=1}^S$ and the matrix of invasion probabilities $\boldsymbol{x} = \left((x_{is})_{i=1}^I\right)_{s=1}^S$ in the following general form (with the specific details after Eq. (11) below)

$$\mathscr{F}(\widetilde{\boldsymbol{b}}, \boldsymbol{b}) = \langle \mathbb{D}(\widetilde{\boldsymbol{b}})\boldsymbol{b}, \boldsymbol{b}\rangle + 2\langle \boldsymbol{f}(\widetilde{\boldsymbol{b}}), \boldsymbol{b}\rangle + \langle \boldsymbol{g}(\widetilde{\boldsymbol{b}}, \boldsymbol{P}), \mathbf{1}\rangle. \tag{7}$$

The reason this is can be written as function of $\boldsymbol{b}$ is because, as discussed after Algorithm 4, the boat ramp fee $\boldsymbol{\tau}$ in the model can be determined by the number of trips $\boldsymbol{b}$, and the probabilities of invasion $\boldsymbol{x}$ are also uniquely determined from the taxes $\boldsymbol{\tau}$. It is important to note that in general we would like to maximize $\mathscr{F}(\cdot, \cdot)$ above when $\widetilde{\boldsymbol{b}} = \boldsymbol{b}$. Thus, for a fixed $\widetilde{\boldsymbol{b}}$ we have the following maximization to find the optimal number of trips to each lake $\boldsymbol{b}^*$ to maximize net benefits with three constraints:(i) AIS dispersal follows (10), (ii) the total number of trips has to equal the number of trip occasions (9), (iii) the minimum number of trips boaters can take to lake i is zero (9).

Optimization Problem

$$\text{Find} \quad \boldsymbol{\tau}_* := \arg\max_{\boldsymbol{\tau}} \mathscr{F}\left(\boldsymbol{b}(\boldsymbol{\tau}), \boldsymbol{P}(\boldsymbol{\tau})\right) = \{\langle \mathbb{D}(\boldsymbol{b})\boldsymbol{b}, \boldsymbol{b}\rangle + 2\langle \boldsymbol{f}(\boldsymbol{b}), \boldsymbol{b}\rangle + \langle g(\boldsymbol{b}, \boldsymbol{P}(\boldsymbol{\tau})), \mathbf{1}\rangle\}, \tag{8}$$

Subject to the constraints:

$$\sum\nolimits_{i=1}^I b_{is} = N \times T, \quad 0 \le b_{is} \tag{9}$$

where x_{is} is computed with given initial values $\{x_{i1}\}_{i=1}^I$ and

$$x_{is} = x_{i,s-1} + (1 - x_{i,s-1})\mathfrak{s}(K_{is}) \tag{10}$$

for $i = 2, \ldots, I$ and $s = 2, \ldots, S$.

In what follows we will use iterative approximation of $\mathscr{F}$ by quadratic forms which can be easily optimized. In such cases we evaluate all terms such as $\mathbb{D}(\boldsymbol{b})$, $\boldsymbol{f}(\boldsymbol{b})$, $\boldsymbol{g}(\boldsymbol{b}, \boldsymbol{P})$ at our previous iterates, denoted here by $\widetilde{\boldsymbol{b}}$, $\widetilde{\boldsymbol{\tau}}$. We thus have

$$\mathscr{F}(\widetilde{\boldsymbol{b}}, \boldsymbol{b}) := \langle \mathbb{D}(\widetilde{\boldsymbol{b}})\boldsymbol{b}, \boldsymbol{b}\rangle + 2\langle \boldsymbol{f}(\widetilde{\boldsymbol{b}}), \boldsymbol{b}\rangle + \langle g(\widetilde{\boldsymbol{b}}, \boldsymbol{P}(\widetilde{\boldsymbol{\tau}})), \mathbf{1}\rangle. \tag{11}$$

To be more specific, let us introduce the quantities involved in writing the explicit functional form of $\mathbb{D}$, $\boldsymbol{f}$, and $\boldsymbol{g}$. We define:

$$m_{1,is} = \frac{1 - x_{i,s-1}}{2a} w_{is} + \frac{1 + x_{i,s-1}}{2}, \quad m_{2,is} = \frac{1 - x_{i,s-1}}{2a} y_{is} A_s,$$
$$c_{1,is} = L^b m_{1,is} + L^h h_i m_{2,is}, \quad c_{2,is} = L^b m_{2,is}.$$

A direct calculation from the definition of $\mathscr{F}$ gives

$$\mathbb{D}_{is,is} = \begin{cases} -\rho^s c_{2,is}, & K_{is} \in [-a, a], \\ 0, & K_{is} \notin [-a, a], \end{cases}, \quad g_{is} = \begin{cases} -\rho^s L^h h_i m_{1,is}, & K_{is} \in [-a, a], \\ 0, & K_{is} < -a, \\ -\rho^s L^h h_i, & K_{is} > a, \end{cases}$$

$$f_{is} = \frac{1}{2} \begin{cases} \rho^s \left(c_{1,is} - \frac{1}{|\alpha|} \right), & K_{is} \in [-a, a], \\ \frac{\rho^s}{|\alpha|}, & K_{is} < -a, \\ \rho^s \left(L^b - \frac{1}{|\alpha|} \right), & K_{is} > a. \end{cases}$$

In (11) the values of $\mathbb{D}$, $\boldsymbol{f}$ and $\boldsymbol{g}$ are all evaluated at $\widetilde{b}$ which could be the previous iterate in the iterative procedure or we may have $\widetilde{\boldsymbol{b}} = \boldsymbol{b}$ and in such case $\mathscr{F}(\boldsymbol{b}, \boldsymbol{b})$ will be the objective function.

2.5 Optimal policy

To compute optimal tax values we propose the following algorithms to find the optimal boat ramp fees, τ_{is}^* for each lake i in season s, that map from the optimal number of trips to each lake, $b_{*,is}$.

Algorithm 1 Global iteration

1. Input: given an initial state of invasions $x_{i,s=1}$ (a vector with I elements) and an initial guess of the boat ramp fee $\tau_{is}^{(0)}$ (an $(I \times S)$ matrix), we propose the following iterative procedure.
   For $k = 1, 2, \ldots$ until convergence:
   (a) Compute the spread of AIS $x_{is}^{(k)}$ for all $s \in (2, \ldots, S)$ from $\tau_{is}^{(k)}$ using Algorithm 2.
   (b) Find the $b_{is}^{*(k)}$ that maximizes the net benefits via Algorithm 3 for fixed $\boldsymbol{P}$ from the previous iteration.
   (c) Find $\tau_{is}^{*(k)}$ from the computed $b_{is}^{*(k)}$ by solving the nonlinear system of equations relating these quantities via Algorithm 4.
2. **If** a convergence criteria is met, or iteration number is too large **Then** Stop

We now rewrite L_{is} as a linear function of b_{is} plus some "noise". To do this, let us introduce A_{is} − the average of Q_{isnt} with respect to n and t or the share of boater trips where the boater removes an invasive species from an invaded lake, namely.

$$A_{is} = \frac{1}{NT}\sum_n\sum_t Q_{isnt} = \frac{1}{NT}\sum_n\sum_t\left[\sum_{j\neq i}\mu x_{j(s-1)}P_{jsn(t-1)}\right].$$

Then for L_{is} we have,

$$L_{is} = A_{is}b_{is} + \underbrace{\sum_t\sum_n (Q_{isnt} - A_{is})P_{isnt}}_{g_{is}}$$

In the algorithms that follow, the second term in the right side above is lagged, i.e. taken from the previous iteration. As a result, e can write K_{is} as a linear function of $\boldsymbol{b}$:

$$K_{is} = \gamma A_{is}b_{is} + \gamma g_{is}(b, P) + \xi_l\lambda_{is}$$

where $A_{is}b_{is}$ is the average propagule pressure on lake i, i.e. the average number of trips to lake i that carry an invasive species in season s, and g_{is} is a measure of the deviance across trip occasions from the average propagule pressure.

The algorithms described below, follow the steps outlined in the Global Iteration Algorithm 1.

Algorithm 2 Compute x_{is} given τ_{is}

1. **Input**: a vector $x_{\cdot,1} = (x_{11}, x_{21}, \ldots, x_{I,1})^t \in \mathbb{R}^I$ (for $s = 1$) and $\tau_{is}^{(k)} \in \mathbb{R}^{I\times S}$.
2. **Output**: $x_{\cdot,2:S+1}^{(k)} \in \mathbb{R}^{I\times S}$ and $b_{is}^{(k)}$.
   For $s = 2, \ldots, (S+1)$

$$P_{isnt}^{(k)} = \frac{\exp\left(\sum_m z_{isnt,m}\beta_m - \xi x_{i(s-1)}^{(k)} - \alpha\tau_{is}^{(k)}\right)}{\sum_j\left\{\exp\left(\sum_m z_{jsnt,m}\beta_m - \xi x_{j(s-1)}^{(k)} - \alpha\tau_{js}^{(k)}\right)\right\}}$$

$$b_{is}^{(k)} = \sum_{n,t} P_{isnt}^{(k)}, \quad Q_{isnt}^{(k)} = \sum_{j\neq i}\mu P_{jsn(t-1)}^{(k)} x_{j(s-1)}^{(k)}$$

$$A_{is}^{(k)} = \frac{1}{NT}\sum_{n,t} Q_{isnt}^{(k)}, \quad g_{is}^{(k)} = \sum_{n,t}\left(Q_{isnt}^{(k)} - A_{is}^{(k)}\right)P_{isnt}^{(k)}$$

$$d_{is}^{(k)} = \gamma_{is}g_{is}^{(k)}, \quad K_{is}^{(k)} = \gamma A_{is}^{(k)}b_{is}^{(k)} + d_{is}^{(k)} + \xi_l\lambda_{is}$$

Now, for a given b_{is}, we update x_{is}

$$x_{is}^{(k)} = x_{i,s-1}^{(k)} + \mathfrak{s}\left(K_{is}^{(k)}\right)$$

Notice that $\mathscr{F}(\widetilde{\boldsymbol{b}}, \cdot)$ is a quadratic functional for fixed $\widetilde{\boldsymbol{b}}$ with negative definite diagonal matrix $\mathbb{D}$. The solution then to the optimization problem in Algorithm 3, is

Algorithm 3 Maximization (Find $\boldsymbol{b}$ given $\widetilde{\boldsymbol{b}}^{(k)}$ and $\boldsymbol{x}$))

1. **Input**: $\boldsymbol{x}_{\cdot,2:S+1} \in \mathbb{R}^{I\times S}$, $\widetilde{\boldsymbol{b}}^{(k)}$.
2. **Output**: $\boldsymbol{b}^{(\boldsymbol{k+1})} \in \mathbb{R}^{I\times S}$.
3. Set $\widetilde{\boldsymbol{b}} = \boldsymbol{b}^{(k)}$
4. Find the $\boldsymbol{b}^{(k+1)}$: $\boldsymbol{b}^{(k+1)} = \arg\max_{\boldsymbol{b}} \mathscr{F}(\boldsymbol{b}^{(k)}, \boldsymbol{b})$, subject to the constraints:

$$0 \le b_{is} \le \upsilon_{is} := \sum_{n,t} \frac{\exp\left(v_{isnt}(\cdot, \tau_{is}^{(k)})\right)}{\sum_{j=1}^{I} \exp\left(v_{jsnt}(\cdot, \tau_{js}^{(k)})\right)}.$$

given by (see Deutsch (2001, Theorem 4.1, p.43))

$$b_{is}^{(k+1)} = \begin{cases} 0, & [\mathbb{D}^{-1}\boldsymbol{f}]_{is} < 0 \\ [\mathbb{D}^{-1}\boldsymbol{f}]_{is}, & 0 \le [\mathbb{D}^{-1}\boldsymbol{f}]_{is} \le \upsilon_{is} \\ u_{is}, & [\mathbb{D}^{-1}\boldsymbol{f}]_{is} > \upsilon_{is}. \end{cases} \tag{12}$$

Algorithm 4 Computation of $\tau_{is}^{(k+1)}$ from $b_{is}^{(k+1)}$ and $x_{is}^{(k)}$.

1. **Input:** $\boldsymbol{b} = \boldsymbol{b}^{(k+1)}$ from Algorithm 3 and $\boldsymbol{x}^{(k)}$ from Algorithm 2.
2. **Output:** $\boldsymbol{\tau}^{(k+1)}$ which solves $\mathscr{R}(\boldsymbol{\tau}^{(k+1)}) = \boldsymbol{b}^{(k+1)}$,
where, for $i \ge 2$,
$\mathscr{R}(\tau_{is}^{(k+1)}) = \sum_n \sum_t \frac{\Theta_{isnt}^{(k)} \eta_{is}^{(k)}}{\left[\sum_j \Theta_{jsnt}^{(k)} \eta_{js}^{(k)}\right]}$ and $\Theta_{isnt}^{(k)} = \exp\left(\widetilde{Z}_{isnt}^{(k)}\right)$, $\eta_{is}^{(k)} = \exp(-\alpha\tau_{is}^{(k)})$, and,

$$\widetilde{Z}_{isnt}^{(k)} = \sum_m z_{isnt,m}\beta_m - \zeta x_{i(s-1)}^{(k)}.$$

This nonlinear system is solved by the following iterative method:

Set $\eta_{1s} = 1$ (or $\tau_{1s}^{(k)} = 0$), $S = 2, \ldots, S+1$. This reduces the number of unknowns, but number of equations is also reduced as $b_{1s} = NT - \sum_{i\ge 2} b_{is}$.
Set an initial guess $\boldsymbol{\tau}_{\boldsymbol{is}}^{(\boldsymbol{0})}$ (which gives us $\eta_{is}^{(0)}$).
For $q = 1, 2, \ldots$ until convergence:
For $s = 2, \ldots, S+1$,$i = 2, ..., I$ solve for $\eta_{is}^{(q)}$

$$\mathfrak{f}_{is}(\eta^{(q)}) := \boldsymbol{b}_{is} - \sum_n \sum_t \frac{\Theta_{isnt}\eta_{is}^{(q)}}{\Theta_{isnt}\eta_{is}^{(q)} + \sum_{j\ne i} \Theta_{jsnt}\eta_{js}^{(q-1)}} = 0 \tag{13}$$

If a convergence criteria is met, **Then** Stop

A comment on the solution of the nonlinear equations given in Algorithm 4 (see (13)) is in order. By a closer inspection of the functions $\mathfrak{f}_{is}(\cdot)$ on the right side of this

equation we observe that $\mathfrak{f}'_{is} < 0$. Furthermore, the upper bounds on $\boldsymbol{b}$ given in (9) guarantee that $\mathfrak{f}_{is}(0) > 0$ and $\mathfrak{f}_{is}(1) < 0$ whenever $\eta_{is}^{(q-1)} \in (0, 1)$. Hence, $\eta_{is}^{q} \in (0, 1)$ is unique (from the monotonicity of $\mathfrak{f}_{is}(\cdot)$) and this guarantees that τ_{is}^{q} is also unique and positive.

3 Discussion

These algorithms allow us to empirically estimate the optimal spatial-dynamic policies to maximize the value of lakes net of damages from invasive species. To interpret these results we focus on the cases where i and s are such that $K_{is} \in [-a, a]$. In these cases the optimal number of boating trips are defined as

$$b_{is}^{*} = \frac{\frac{1}{|\alpha|} - \mathscr{L}^{b}\left(\frac{(1-x_{i(s-1)})}{2a} w_{is} + \frac{1+x_{i(s-1)}}{2}\right) - \mathscr{L}^{h} h_i \left(\frac{1-x_{i(s-1)}}{2a}\right) y_{is} A_{is}}{\mathscr{L}^{b}\left(\frac{1-x_{i(s-1)}}{2a}\right) y_{is} A_{is}}. \quad (14)$$

We can simplify Eq. (14) so that the optimal number of trips to each lake occurs when

$$\mathscr{L}^{b}(x_{is} - x_{i(s-1)}) + \mathscr{L}^{h} h_i \frac{\partial x_{is}}{\partial b_{is}} = \frac{1}{|\alpha|}. \quad (15)$$

The left-hand side of Eq. (15) is the expected marginal costs from boating. $\mathscr{L}^{b}(x_{is} - x_{i(s-1)})$ is the expected marginal damages to boaters from the spread of AIS and $\mathscr{L}^{h} h_i \frac{\partial x_{is}}{\partial b_{is}}$ is the marginal damages to homeowners from boaters spreading AIS. The right-hand side of Eq. (15) is the marginal benefit of boating.

This makes intuitive sense; if AIS dispersal from boaters is small, then the optimal policy is to have no boat ramp fee. On the other hand when the benefits from boating are lower than the damages from boating then the optimal number of boating trips might reach the lower bound, which is zero.[5] The lower bound of boating trips corresponds to an optimal boat ramp fee that approaches infinity, $\tau_{is}^{*} \to \infty$. This also makes intuitive sense; if the damages from AIS are worse than the benefits from boating it makes sense to close the lake to boating.

3.1 A numerical example

Next, we provide a simple numerical illustration of how the proposed method works. We consider a system with 2 lakes, 10 boaters, 2 seasons, and with 11 days per season, i.e. $(I, S, N, T) = (3, 1, 10, 11)$. For the rest of the model parameters, we have set $\mathscr{L}^{b} = 100$, $\mathscr{L}^{h} = 1400$, $\gamma = 5$, $\xi = 0.1$, $f = 0.03$, $h_i = 1$, for $i = 1, \ldots, I$, $\lambda_{is} = -0.001$, $r = 0.5 \times 10^{-1}$, $x_{\cdot,s=1} = 0.5$, $\tau_{i,s} = 1.0$, and $\tilde{Z}_{isnt} \sim Uniform(-2, -1)$. In Table 1 we show the optimal number of boating trips, tax values, invaded probabilities and the total total benefits value. The data are for three

[5] Clearly, allowing a negative number of boating trips would not be realistic.

Table 1 Optimal number of boating trips for varying values of the parameter α

α	0.010	0.133	0.015
Upper bound (U) on $b_{i\in\{2,3\},s=2}$	36.16, 36.39	53.53, 53.81	53.27, 53.56
Lower bound (L) on $b_{i\in\{2,3\},s=2}$	0, 0	0, 0	0, 0
Optimal boating $b^*_{i\in\{2,3\},s=2}$	36.16, 36.39	34.74, 33.02	0.00, 0.00
Optimal tax $\tau^*_{i\in\{2,3\},s=2}$	0.02, 0.02	57.64, 63.72	375.04, 374.71
Invaded prob. $x^*_{i\in\{2,3\},s=2}$	0.83, 0.83	0.74 , 0.74	0.75, 0.75
$\mathscr{F}(b_{is}=L)$, $\mathscr{F}(b_{is}=U)$	-8802, -1522	-8054, -3678	-1115, -3479
$\mathscr{F}(b_{is}=b^*_{is})$	-1522	-1669	-1115

simulations with varying α in which we have the optimal value of the boating trips b_{is} to be at the upper bound, at the lower bound, or in the middle.

4 Conclusions

In conclusion, we have developed a novel method to solve for the optimal management of a complex spatial-dynamic process (the spread of AIS) with an endogenous diffusion mechanism. The benefits of our method include an analytical solution for the optimal number of trips to each lake in each season and an iterative method to solve for the spatial-dynamic optimal boat ramp fee $\boldsymbol{\tau}$ from $\boldsymbol{b}$ that is likely to converge. A careful look at the algorithms given here show that the main computational cost is in evaluating the function $\mathscr{F}$. Applying this novel framework to empirical data, to more general utility functions that allow for non-constant marginal utility of income and interdependence among boaters, to other management strategies such as boat washing stations, to other migration processes, as well as the mathematical analysis of the proposed algorithms such as the convergence and approximation properties are subject of current and future research.

Appendix: On the convergence of the algorithm

The algorithm we proposed above utilizes a sequence of quadratic programming problems which, as we have shown in Sect. 2.5, are solvable analytically. This is a novel approach and to support this design we give a brief analysis of its convergence.

We note that the goal is to maximize $\mathscr{G}(\boldsymbol{b})$ given by [(see (11)]:

$$\mathscr{G}(\boldsymbol{b}) = \left[\langle \mathbb{D}(\widetilde{\boldsymbol{b}})\boldsymbol{b}, \boldsymbol{b}\rangle + 2\langle \boldsymbol{f}(\widetilde{\boldsymbol{b}}), \boldsymbol{b}\rangle + \langle g(\widetilde{\boldsymbol{b}}, \boldsymbol{P}(\widetilde{\boldsymbol{\tau}})), \mathbf{1}\rangle\right]\Big|_{\widetilde{\boldsymbol{b}}=\boldsymbol{b}} = \mathscr{F}(\widetilde{\boldsymbol{b}}, \boldsymbol{b})\Big|_{\widetilde{\boldsymbol{b}}=\boldsymbol{b}} = \mathscr{F}(\boldsymbol{b}, \boldsymbol{b}).$$

In the equations above, one can think of $\mathscr{F}(\widetilde{\boldsymbol{b}}, \boldsymbol{b})$ as extending $\mathscr{G}$ from the "line" $\boldsymbol{b} = \widetilde{\boldsymbol{b}}$ to the "plane" $(\widetilde{\boldsymbol{b}}, \boldsymbol{b})$. It should be clear that we use the terms "line" and "plane" loosely here to identify the multidimensional analogues of such objects.

Since we are free to choose the extension $\mathscr{F}$ we may assume that we have extended the profit function so that

$$\mathscr{G}(\boldsymbol{b}) = \mathscr{F}(\boldsymbol{b}, \boldsymbol{b}) \leq \mathscr{F}(\widetilde{\boldsymbol{b}}, \boldsymbol{b}), \quad \forall(\widetilde{\boldsymbol{b}}, \boldsymbol{b}), \quad \text{satisfying constraints.} \tag{16}$$

Recall that, Algorithm 3, for a given $\boldsymbol{b}_k$ maximizes $\mathscr{F}(\boldsymbol{b}_k, \boldsymbol{c})$ with respect to $\boldsymbol{c}$, and the optimal value of $\mathscr{F}$ is at $\boldsymbol{c} = \boldsymbol{b}_{k+1}$. Note that this implies that

$$\mathscr{F}(\boldsymbol{b}_k, \boldsymbol{b}_{k+1}) \geq \mathscr{F}(\boldsymbol{b}_k, \boldsymbol{b}_*), \tag{17}$$

where $\boldsymbol{b}_*$ is the optimal solution which maximizes $\mathscr{F}(\boldsymbol{b}, \boldsymbol{b})$ (the optimal value we want to find). As we have shown, such relation holds because at $\boldsymbol{c} = \boldsymbol{b}_{k+1}$, the function $\mathscr{F}(\boldsymbol{b}_k, \boldsymbol{c})$ viewed as function of $\boldsymbol{c}$, is at a maximum. Therefore, the value of $\mathscr{F}(\boldsymbol{b}_k, \boldsymbol{b}_{k+1})$ cannot be smaller than the value of $\mathscr{F}(\boldsymbol{b}_k, \boldsymbol{b}_*)$.

If we further assume that $\lim_{k\to\infty} \boldsymbol{b}_k = \boldsymbol{b}_\infty$ (which we found numerically to be always true in all examples we tried) and take the limit on both sides. Since $\mathscr{F}$ is continuous (not necessarily differentiable, just merely continuous is enough here), we obtain that

$$\mathscr{F}(\boldsymbol{b}_*, \boldsymbol{b}_*) \geq \mathscr{F}(\boldsymbol{b}_\infty, \boldsymbol{b}_\infty) \geq \mathscr{F}(\boldsymbol{b}_\infty, \boldsymbol{b}_*) \geq \mathscr{F}(\boldsymbol{b}_*, \boldsymbol{b}_*).$$

The first inequality holds because $\boldsymbol{b}_*$ is the optimal solution, the second holds because of the limit w.r.t k in (17) and last inequality follows from (16).

Since the left and right sides of these inequalities are equal, we must have equality everywhere. In conclusion, under the simple assumptions we made above, if the sequence of iterates converges then it converges to an optimal value of the objective function.

To make the argument precise, let us point out that the sequence of *all* iterates may not converge, but may have one, two or more convergent subsequences. As is known, by Heine–Borel theorem, as long as this sequence is bounded (which it is because of the constraints), it must be a convergent subsequence. The considerations given above apply to any convergent subsequence as well. We can then conclude that the function values of the limit of any such convergent subsequence is the optimal value of the benefit function. We have the following result and its proof is an immediate consequence of the considerations above.

Lemma 1 *If the extension satisfies* (16)*, then for every convergent subsequence of iterates* $\{b_{k_j}\}_{j=1}^{J}$, $\lim_{j\to\infty} b_{k_j} = b_{\infty,j}$ *the function values converge to the optimal value, namely,*

$$\mathscr{F}(b_{\infty,j}, b_{\infty,j}) = \mathscr{F}(b_*, b_*), \quad j = 1, 2, \ldots, J.$$

We further remark that, while the optimal solutions, i.e. the limits of subsequences, $\{b_{\infty,j}\}_{j=1}^{J}$ may be different, the function values at such points are the same.

Finally, let us note that the technique of extending a function to a higher dimensional space (from one variable to two) is well known in the theory of partial differential

equations and can be viewed as a special *reguralization*. The reason is that a less regular problem can be extended to more regular and better behaved in higher dimension.

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