Wu *et al.* Reply: We offer a reply to the preceding Comment on our recent search for axionlike dark matter with a liquid-state nuclear spin comagnetometer [1]. We are grateful to the authors for their interest in our work and the insightful analysis of Fig. 3 of our paper.

Our experiment [2] searches for an oscillatory signal due to coupling of an axionlike dark matter field to nuclear spins via the axion-wind interaction. The axion field oscillates at the Compton frequency. For Compton periods τ_a longer than T, the span of the data, since we do not sample long enough to observe an oscillation of the axion field, the detected signal would be modulated primarily by the rotation of Earth. In the Comment, the authors point out that, for an oscillating signal with a frequency smaller than $1/T$, the sensitivity of a null measurement to the axionlike dark matter field significantly decreases as compared to measurements at higher frequencies. In order to prove this argument, the authors fit the data with two basis functions, $\sin(2\pi ft)$ and $\cos(2\pi ft)$, where f is the frequency of the oscillating field and t is the time of the measurement. The amplitude of the oscillating field is the quadrature amplitude of the sine and cosine functions. We fully agree with the authors on this point.

The amplitude of the signal with frequencies smaller than $1/T$ is equal to the amplitude at Ω_{sid} of the amplitude spectrum, which is a positive value and is thus proportional to $|g_{aNN}\sin(\phi)|$, where Ω_{sid} is the sidereal frequency, g_{aNN} parametrizes the coupling strength between axion and nucleon, and ϕ is the phase of the axion field. In our work, we took the weighted average value of $|\sin(\phi)|$ for $\phi \in [0, 2\pi]$. Instead, as the authors of the preceding Comment correctly point out, the proper way to interpret our null experimental result is to marginalize over the uniform phase distribution: For such low frequencies, our experiment samples one particular random value of the phase and does not average over the phases. The constraints must be weakened due to the phase issue and should be corrected for signals with frequencies smaller than $1/T$, shown at the left-hand side of Fig. 3 [2].

Let us also point out an additional issue beyond the phase problem described in the Comment. The assumption of a constant axion-field amplitude and velocity may not be adequate even for the simplest case of random virialized dark matter [3]. In general, there are three distinct regimes depending on how the span of the data (T) compares to two different timescales: the axion field oscillation period $\tau_a = 1/m_a$ and the coherence time, which for the case of a random field is $\tau_c \approx 10^6\tau_a$.

Regime 1: $T \gg \tau_c \gg \tau_a$.—Since the span of the data (T) is much longer than the coherence time of the axion field, the stochastic axion field amplitude and velocity are well sampled. It is thus appropriate to use the average values of the axion field amplitude and velocity. Besides, since the span of the data is much longer than the oscillation period, which means that we would observe many cycles of oscillation, the initial phase does not change the derived signal amplitude.

Regime 2: $\tau_c \gg T \gg \tau_a$.—Here, we sample one particular amplitude of the axion field from the stochastic distribution of amplitudes according to the Rayleigh distribution and one particular velocity [3]. To interpret null results in terms of constraints, one must marginalize over the distribution of possible amplitudes and velocities. For example, the axion field amplitude could be extremely small (around 0) for the whole period of measurement time. It is thus not suitable to simply use the average value of the axion field amplitude for setting the constraint. It appears that, to date, all other axion-search experiments working in this regime have ignored the stochastic amplitude and velocity distributions. However, similar to regime 1, the initial phase is irrelevant to the derived signal amplitude, since many oscillation cycles are sampled.

Regime 3: $\tau_c \gg \tau_a \gg T$.—This is the condition that is relevant to the preceding Comment. The amplitude and velocity issue is the same as that for regime 2. For the phase, since we would not sample a complete oscillation of the axion field, it is not suitable to use the average value of $|\sin(\phi)|$, which should be as well marginalized over the distributions and should weaken the exclusion limit at the left-hand edge of Fig. 3 [2]. Revised constraint plots for this experiment, including correct marginalization over the phase, velocity, and amplitude in the appropriate regimes, are presented in Ref. [3].

Teng Wu,[1] John W. Blanchard,[1] Gary P. Centers,[1]
Nataniel L. Figueroa,[1] Antoine Garcon,[1] Peter W. Graham,[2]
Derek F. Jackson Kimball,[3] Surjeet Rajendran,[4]
Yevgeny V. Stadnik,[1] Alexander O. Sushkov,[5]
Arne Wickenbrock[1] and Dmitry Budker[1,4,6]

[1]Helmholtz-Institut Mainz, Johannes Gutenberg University 55128 Mainz, Germany
[2]Department of Physics, Stanford Institute for Theoretical Physics, Stanford University California 94305, USA
[3]Department of Physics, California State University–East Bay Hayward, California 94542-3084, USA
[4]Department of Physics, University of California at Berkeley California 94720-7300, USA
[5]Department of Physics, Boston University Boston, Massachusetts 02215, USA
[6]Nuclear Science Division, Lawrence Berkeley National Laboratory Berkeley, California 94720, USA

Received 22 August 2019; published 14 October 2019
DOI: 10.1103/PhysRevLett.123.169002

[1] E. G. Adelberger and W. A. Terrano, preceding Comment, Comment on "Searches for Axionlike Dark Matter with a Liquid-State Nuclear Spin Comagnetometer", Phys. Rev. Lett. **123**, 169001 (2019).

[2] T. Wu, J. W. Blanchard, G. P. Centers, N. L. Figueroa, A. Garcon, P. W. Graham, D. F. Jackson Kimball, S. Rajendran, Y. V. Stadnik, A. O. Sushkov, A. Wickenbrock, and D. Budker, Searches for Axionlike Dark Matter with a Liquid-State Nuclear Spin Comagnetometer, Phys. Rev. Lett. **122**, 191302 (2019).

[3] G. P. Centers, J. W. Blanchard, J. Conrad, N. L. Figueroa, A. Garcon, A. V. Gramolin, D. F. Jackson Kimball, M. Lawson, B. Pelssers, J. A. Smiga, Y. Stadnik, A. O. Sushkov, A. Wickenbrock, D. Budker, and A. Derevianko, Stochastic fluctuations of bosonic dark matter, arXiv:1905.13650v3.