RFI Mitigation for One-Bit UWB Radar Systems

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***Abstract*—Radio frequency interference (RFI) mitigation is critical to the proper operation of ultra-wideband (UWB) radar systems. This paper considers RFI mitigation for a one-bit UWB radar system, with its measurements obtained via a low-cost and high-rate sampling strategy using a known threshold varying with slow-time. We first establish a data model for the RFI sources. Then we present a relaxation based algorithm to estimate the parameters of the RFI sources from the signed measurements and thresholds. Next, a sparse method is introduced to recover the desired UWB radar echoes using the estimated RFI parameters. Finally, numerical examples are presented to demonstrate the effectiveness of the proposed method.**

***Index Terms*—Signed measurements, one-bit sampling, slow-time-varying thresholds, UWB radar, RFI mitigation**

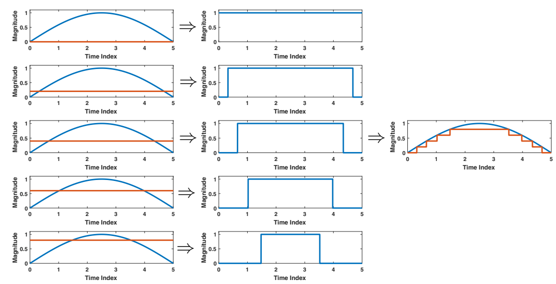

Fig. 1: The procedure of CTBV sampling.

I. Introduction

Ultra-wideband (UWB) radar has been used in a wide range of applications including sleep monitoring, contact-less vital sign measurements, through-wall imaging and landmine detection. Due to the large bandwidth, for example, of over 10 GHz, of an impulse UWB radar system, the conventional analog-to-digital converter (ADC) at its receiver can significantly increase its cost and power consumption. For the UWB application, low resolution quantization is attractive due to its low cost, low power consumption and its ability to achieve ultra-high sampling rates. For instance, the NVA6100 impulse radar system [1], a low-cost single-chip UWB radar from Novelda, adopts the so-called Continuous Time Binary Value (CTBV) technology [1], [2] to achieve a very high sampling rate of 39 GHz and a 13-bit quantization resolution with a simple circuit design and a low power consumption. CTBV is a one-bit sampling strategy that obtains signed measurements with one-bit ADC and a known threshold varying with slow-time, i.e., from one pulse repetition interval (PRI) to another. High-precision samples can be acquired from these signed measurements via a simple digital integration (DI) procedure [2]. The procedure of CTBV sampling is shown in Figure 1. We refer to the CTBV based system as the one-bit UWB radar system.

One significant challenge for the proper operations of a UWB radar system is the radio frequency interference (RFI) mitigation since there are many competing users within the ultra-wideband frequency range they operate in. RFI can cause severe reduction of the signal-to-noise ratio (SNR), resulting in the degradation of the target detection and parameter estimation performance. Therefore, effective RFI mitigation is critically important for the proper functioning of a UWB radar system. Many RFI mitigation methods have been developed for radar systems using high-resolution ADCs [3]–[6]. However, it appears that RFI mitigation for one-bit UWB radar systems has not been addressed before in the literature and the existing high resolution sampling based methods are not directly applicable.

We introduce in this paper an RFI mitigation method for one-bit UWB systems, in particular the NVA6100 impulse radar system. We first establish a proper data model for the RFI sources and assume that the frequency components of the RFI sources remain the same within a coherent processing interval (CPI). Then we present a relaxation based algorithm to find the maximum likelihood (ML) estimates of the parameters of the RFI sources from the signed measurements. Next, a sparse method is introduced to recover the desired UWB radar echoes based on the estimated RFI parameters. Finally, numerical examples are presented to demonstrate the effectiveness of the proposed method.

Notation: We denote vectors and matrices by boldface lower-case and upper-case letters respectively. $(\cdot)^T$ denotes the transpose operation. $\hat{(\cdot)}$ denotes the estimated result of the related value. $\mathbf{X} \in \mathbb{R}^{N\times M}$ denotes a real-valued $N \times M$ matrix and $\mathbf{x} \in \mathbb{R}^N$ denotes a real-valued vector with N elements. $\mathrm{X}[n,m]$ denotes the (n,m)th element of matrix $\mathbf{X}$. $\mathbf{X}[n,:]$ and $\mathbf{X}[:,m]$ denote the nth row and mth column of the matrix $\mathbf{X}$, respectively. $\mathrm{x}[n]$ denotes the nth element of vector $\mathbf{x}$. For a matrix or a vector, $||\cdot||_p$ means the ℓ_p element-wise norm of this matrix or vector, i.e., $||\mathbf{X}||_p = (\sum_{m=1}^{M}\sum_{n=1}^{N}|\mathrm{X}[n,m]|^p)^{1/p}$ and $||\mathbf{x}||_p = (\sum_{n=1}^{N}|\mathrm{x}[n]|^p)^{1/p}$. $||\mathbf{X}||_{1,2} = \sum_{n=1}^{N}||\mathbf{X}[n,:]||_2$ denotes the $\ell_{1,2}$ norm of the matrix $\mathbf{X}$.

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II. Problem Formulation

Consider a one-bit impulse UWB radar system with its signed measurement matrix $\mathbf{Y} \in \mathbb{R}^{N\times M}$ obtained via comparing the signal from different PRIs with different thresholds as follows:

$$\mathbf{Y} = \mathrm{sign}(\mathbf{R}_{\boldsymbol{\theta}} + \mathbf{S} + \mathbf{E} - \mathbf{H}) \in \mathbb{R}^{N\times M}, \tag{1}$$

where N denotes the number of fast-time samples per PRI, M denotes the number of PRIs or slow-time samples within the CPI, $\mathbf{E}$ denotes the noise and other disturbances, $\mathbf{S}$ denotes the desired radar echo signal, which is assumed invariant for all the PRIs within the CPI, i.e., $\mathbf{S}[:,1] = \mathbf{S}[:,2] = \cdots = \mathbf{S}[:,M] = \mathbf{s}$, and $\mathbf{H}$ denotes the threshold matrix, which varies with slow-time, i.e., $\mathrm{H}[n,m] = -h + 2(m-1)h/(M-1), h > 0, n = 1,2,\cdots,N, m = 1,2,\cdots,M$, with h denoting the maximum threshold among all PRIs, $\mathbf{R}_{\boldsymbol{\theta}}$ denotes the RFI matrix, and $\mathrm{sign}(\cdot)$ is the element-wise sign operator defined as:

$$\mathrm{sign}(x) = \begin{cases} 1, & x \geq 0 \\ -1, & x < 0 \end{cases}. \tag{2}$$

It has been shown in [6] that a sum of sinusoids can be used to model the RFI sources in the fast-time with their frequencies fixed over the slow-time within the CPI. Thus, each element of $\mathbf{R}_{\boldsymbol{\theta}}$ can be expressed as follows:

$$\begin{aligned} \mathrm{R}_{\boldsymbol{\theta}}[n,m] &= \sum_{k=1}^{K} A_{k,m}\sin(\omega_k(n-1) + \phi_{k,m}) \\ &= \sum_{k=1}^{K} a_{k,m}\cos(\omega_k(n-1)) + b_{k,m}\sin(\omega_k(n-1)) \\ & n = 1,\cdots,N, m = 1,\cdots,M, \end{aligned} \tag{3}$$

where K denotes the number of sinusoids or RFI sources, $\omega_k \in [0,\pi)$ denotes the frequency of the kth RFI source, $A_{k,m} \in \mathbb{R}^+$ and $\phi_{k,m} \in [0, 2\pi)$ denote the amplitude and phase of the kth RFI source during the mth PRI, respectively. The unknown parameter vector of the RFI is denoted by $\boldsymbol{\theta} = [a_{1,1}, b_{1,1}, \ldots, a_{1,M}, b_{1,M}, \omega_1, \ldots, a_{K,1}, b_{K,1}, \ldots, a_{K,M}, b_{K,M}, \omega_K]^T \in \mathbb{R}^{(2M+1)K}$ with $a_{k,m} = A_{k,m}\sin\phi_{k,m} \in \mathbb{R}$ and $b_{k,m} = A_{k,m}\cos\phi_{k,m} \in \mathbb{R}$. Our goal is to recover the desired radar echo vector $\mathbf{s}$ from the signed measurement matrix $\mathbf{Y}$ while mitigating the impact of the RFI.

III. Maximum Likelihood Approach for RFI Parameter Estimation

A. Maximum likelihood estimation

We first assume that $\mathbf{S}+\mathbf{E}$ obeys *i.i.d* Gaussian distribution with zero-mean and unknown variance σ^2. The numerical examples in Section V show that the proposed algorithm is insensitive to this assumption. Because of the desirable properties like consistency and asymptotic efficiency, the estimator is a theoretically appealing approach to solving the RFI parameter estimation problem. The ML estimate of the parameter vector $[\boldsymbol{\theta}^T, \sigma]^T$ can be obtained by minimizing the negative log-likelihood function as follows:

$$\begin{aligned} \hat{\tilde{\boldsymbol{\beta}}} &= \arg\min_{\tilde{\boldsymbol{\beta}}} l(\tilde{\boldsymbol{\beta}}) \\ &= \arg\min_{\tilde{\boldsymbol{\beta}}} \sum_{m=1}^{M}\sum_{n=1}^{N} -\log\Bigg[\Phi\Bigg(\mathrm{Y}[n,m]\Bigg(\sum_{k=1}^{K}\tilde{a}_{k,m}\cos(\omega_k(n-1)) \\ &\quad + \tilde{b}_{k,m}\sin(\omega_k(n-1)) - \lambda \mathrm{H}[n,m]\Bigg)\Bigg)\Bigg], \end{aligned} \tag{4}$$

where $\Phi(x)$ denotes the cumulative distribution function of the standard normal distribution, $\lambda = \frac{1}{\sigma}$, $\tilde{a}_{k,m} = \frac{a_{k,m}}{\sigma}$, $\tilde{b}_{k,m} = \frac{b_{k,m}}{\sigma}$, and $\tilde{\boldsymbol{\beta}} = [\tilde{\boldsymbol{\theta}}^T, \lambda]^T$ is the modified unknown parameter vector with $\tilde{\boldsymbol{\theta}} = [\tilde{a}_{1,1}, \tilde{b}_{1,1}, \ldots, \tilde{a}_{1,M}, \tilde{b}_{1,M}, \omega_1, \ldots, \tilde{a}_{K,1}, \tilde{b}_{K,1}, \ldots, \tilde{a}_{K,M}, \tilde{b}_{K,M}, \omega_K]^T$.

To obtain the ML estimate, we could perform a K-dimensional search of $\boldsymbol{\omega} = [\omega_1, \cdots, \omega_K]^T$ over the space of angular frequencies $[0,\pi)^K$. At each search point, we could compute $\{\hat{\tilde{a}}_{k,m}\}_{k=1,m=1}^{K,M}$, $\{\hat{\tilde{b}}_{k,m}\}_{k=1,m=1}^{K,M}$ and $\hat{\lambda}$ by the Newton's method [7], [8]. Finally, the parameter vector corresponding to the minimum negative log-likelihood cost value could be selected as the ML estimate. This ML estimation requires a K-dimensional search on $[0,\pi)^K$, which is extremely time-consuming, especially as the number of RFI sources K and the number of PRIs M increase. More efficient algorithms similar to those in [7]–[9] can be considered.

B. 1bMMRELAX

1bMMRELAX is a majorization-minimization (MM) [10] based algorithm proposed for sinusoidal parameter estimation from a time sequence of signed measurements. Similar to the relaxation-based algorithm in [11], 1bMMRELAX maximizes the likelihood function iteratively by estimating the parameters of one sinusoid at a time. Moreover, by using the computationally efficient MM technique, 1bMMRELAX transforms the original problem into a sequence of simple infinite precision sinusoidal parameter estimation problems, which can be efficiently solved via FFTs. To estimate the RFI parameters for our problem, we modify the 1bMMRELAX algorithm and refer to the modified algorithm still as 1bMMRELAX for simplicity.

We start by applying the MM-based method to minimizing the negative log-likelihood function $l(\tilde{\boldsymbol{\beta}})$ in (4). The majorizing function can be easily obtained by using Lemma 1 in [7]. In the following, two iteration procedures will be used, an MM algorithm and a cyclic minimization (CM) algorithm [12]. Let the iteration counter of the MM algorithm be i and let the iteration counter of the CM algorithm be j. After some calculations [7], [9], the updating formula at the $(i+1)$th MM iteration, i.e., the minimization of the majorizing function at

$\tilde{\boldsymbol{\beta}}^i$, the estimated parameter vector obtained at the ith MM iteration, can be simplified as:

$$\min_{\tilde{\boldsymbol{\theta}},\lambda} \tilde{G}\Big(\tilde{\boldsymbol{\theta}},\lambda|\tilde{\boldsymbol{\beta}}^i\Big)=\sum_{m=1}^{M}\sum_{n=1}^{N}\Big[\mathrm{R}_{\tilde{\boldsymbol{\theta}}}[n,m]-\lambda\mathrm{H}[n,m]-\tilde{\mathrm{Z}}_{\tilde{\boldsymbol{\beta}}^i}[n,m]\Big]^2 \tag{5}$$

where $\tilde{\mathrm{Z}}_{\tilde{\boldsymbol{\beta}}}[n,m] = \mathrm{Y}[n,m]\Big(\mathrm{X}_{\tilde{\boldsymbol{\beta}}}[n,m]-f'(\mathrm{X}_{\tilde{\boldsymbol{\beta}}}[n,m])\Big)$ are elements of an auxiliary matrix, with $f(x)=-\log\Phi(x)$, and $\mathrm{X}_{\tilde{\boldsymbol{\beta}}}[n,m] = \mathrm{Y}[n,m]\left(\mathrm{R}_{\tilde{\boldsymbol{\theta}}}[n,m]-\lambda\mathrm{H}[n,m]\right)$, $n = 1,\cdots,N$, $m = 1,\cdots,M$. At each MM iteration, the optimization of the problem in (5) can be solved by the cyclic algorithm [12], which minimizes $\tilde{G}\left(\tilde{\boldsymbol{\theta}},\lambda|\tilde{\boldsymbol{\beta}}^i\right)$ with respect to λ for given $\tilde{\boldsymbol{\theta}}$ and minimizes $\tilde{G}\left(\tilde{\boldsymbol{\theta}},\lambda|\tilde{\boldsymbol{\beta}}^i\right)$ with respect to $\tilde{\boldsymbol{\theta}}$ for given λ cyclically. The first step of the $(j+1)$th CM iteration within the $(i+1)$th MM iteration can be easily solved in the following closed form:

$$\lambda_{j+1}^{i+1}=\max\left(0,\frac{\sum_{m=1}^{M}\sum_{n=1}^{N}\mathrm{H}[n,m]\Big[\mathrm{R}_{\tilde{\boldsymbol{\beta}}_j^{i+1}}[n,m]-\tilde{\mathrm{Z}}_{\tilde{\boldsymbol{\beta}}^i}[n,m]\Big]}{\sum_{m=1}^{M}\sum_{n=1}^{N}\mathrm{H}^2[n,m]}\right). \tag{6}$$

Assuming that the frequency components of the RFI sources are fixed within a CPI, the second step of the cyclic minimization algorithm can be interpreted as the infinite-precision direction-of-arrival estimation (DOA) problem. With $\{\mathrm{V}_{\tilde{\boldsymbol{\beta}}^i,\lambda}[n,m] = \lambda\mathrm{H}[n,m] + \tilde{\mathrm{Z}}_{\tilde{\boldsymbol{\beta}}^i}[n,m]\}_{n=1,m=1}^{N,M}$ as the input data, $\{\{\tilde{a}_{k,m,j+1}^{i+1},\tilde{b}_{k,m,j+1}^{i+1}\}_{m=1}^{M},\omega_{k,j+1}^{i+1}\}$ can be solved efficiently by using a well-known DOA estimation method, such as the RootMusic algorithm [13]. It is worth mentioning that a function of RootMusic is provided by Matlab.

Then, the 1bMMRELAX algorithm can be summarized as follows:

Step 1: Assume $K=1$. Obtain $\{\{\hat{\tilde{a}}_{1,m},\hat{\tilde{b}}_{1,m}\}_{m=1}^{M},\hat{\omega}_1\}$ and $\hat{\lambda}$ by solving (4) via the exhaustive search over the frequency domain followed by the MM procedure.

Step 2: Assume $K=2$. Obtain $\{\{\hat{\tilde{a}}_{2,m},\hat{\tilde{b}}_{2,m}\}_{m=1}^{M},\hat{\omega}_2\}$ by solving (4) via the exhaustive search over the frequency domain followed by the MM procedure with $\{\{\tilde{a}_{1,m},\tilde{b}_{1,m}\}_{m=1}^{M},\omega_1\}$ and λ replaced by their most recent estimates $\{\{\hat{\tilde{a}}_{1,m},\hat{\tilde{b}}_{1,m}\}_{m=1}^{M},\hat{\omega}_1\}$ and $\hat{\lambda}$.

Next, redetermine $\{\{\hat{\tilde{a}}_{1,m},\hat{\tilde{b}}_{1,m}\}_{m=1}^{M},\hat{\omega}_1\}$ and $\hat{\lambda}$ by solving (4) via the MM procedure with $\{\{\tilde{a}_{2,m},\tilde{b}_{2,m}\}_{m=1}^{M},\omega_2\}$ replaced by their most recent estimates $\{\{\hat{\tilde{a}}_{2,m},\hat{\tilde{b}}_{2,m}\}_{m=1}^{M},\hat{\omega}_2\}$.

Iterate the previous two MM procedures until practical convergence, i.e. the relative change of cost function is small enough.

Remaining steps: Continue until the desired or estimated model order is reached.

As mentioned above, when initializing the MM approach via exhaustive search, a $2M$-dimensional convex problem needs to be solved L times if L grid points are used over $[0,\pi)$. For large M, this step is rather computationally expensive and a faster initialization algorithm is desired.

C. Fast frequency initialization

To improve the efficiency of 1bMMRELAX, we propose a fast initialization algorithm to estimate the frequencies of the RFI sources by exploiting the sparsity property of the RFI spectrum. This method allows us to avoid the exhaustive search across the frequency domain. The initialization method for the Kth step of 1bMMRELAX is presented below.

Let $\{w_q = \frac{q\pi}{N}\}_{q=1}^{N-1}$ denote a grid that covers $[0,\pi)$. Assuming that the grid is fine enough such that the frequencies (normalized by the sampling frequency) corresponding to the RFI sources are on this grid (or practically, close to the grid), the data model (1) can be rewritten as:

$$\mathbf{Y} \approx \mathrm{sign}(\mathbf{F}\mathbf{A}^K + \hat{\mathbf{R}}^{K-1} - \mathbf{H}), \tag{7}$$

where $\hat{\mathbf{R}}^{K-1}$ corresponds to the $(K-1)$ estimated RFI sources from Step $(K-1)$, and $\mathbf{F}\mathbf{A}^K$ denotes the Kth RFI source with $\mathbf{F} \in \mathbb{R}^{N\times 2(N-1)}$ expressed as follows:

$$\mathbf{F} = \begin{bmatrix} 1 & \dots & 1 & 0 & \dots & 0 \\ \cos(w_1) & \dots & \cos(w_{N-1}) & \sin(\omega_1) & \dots & \sin(w_{N-1}) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \cos(w_1(N-1)) & \dots & \cos(w_{N-1}(N-1)) & \sin(w_1(N-1)) & \dots & \sin(w_{N-1}(N-1)) \end{bmatrix}. \tag{8}$$

The location of the dominant peak in each column of $\mathbf{A}^K \in \mathbb{R}^{2(N-1)\times M}$ provides the frequency estimate of the Kth RFI source within each PRI.

By making use of the group sparsity of the RFI in the frequency domain and the sign agreements between the signed measurements $\mathbf{Y}$ and infinite-precision signal $(\mathbf{F}\mathbf{A}^K + \hat{\mathbf{R}}^{K-1} - \mathbf{H})$ [14], we can establish an optimization problem as follows:

$$\min_{\mathbf{A}^K} \rho_1||\mathbf{A}^K||_{1,2} + ||g_1(\mathbf{Y}\odot(\mathbf{F}\mathbf{A}^K + \hat{\mathbf{R}}^{K-1} - \mathbf{H})||_1 \tag{9}$$

where $\odot$ denotes the element-wise matrix product, ρ_1 is a user parameter controlling the balance between the two penalty terms. To penalize the sign disagreements between the signed measurements $\mathbf{Y}$ and the estimate of $(\mathbf{F}\mathbf{A}^K + \hat{\mathbf{R}}^{K-1} - \mathbf{H})$, we can choose the element-wise function $g_1(x)$ as:

$$g_1(x) = \begin{cases} \frac{x^2}{2}, & x<0, \\ 0, & x\geq 0. \end{cases} \tag{10}$$

Since (9) is a convex minimization problem, it can be easily solved by using the subgradient descent method [15]. Denoting the result of (9) as $\hat{\mathbf{A}}^K$, we define $\tilde{\mathbf{A}}^K = \sqrt{\hat{\mathbf{A}}^K[1{:}N-1,:]^2+\hat{\mathbf{A}}^K[N{:}2(N-1),:]^2} \in \mathbb{R}^{(N-1)\times M}$. The square and square root here are both element-wise operations. Since in the practical applications, the frequencies of RFI sources are usually not close to zero, we assume that the RFI frequency is not on or close to the grid point of π/N. Thus, the frequency of the Kth RFI source corresponds to the row index of $\tilde{\mathbf{A}}^K[2:(N-1),:]$ with the largest ℓ_1 norm.

The objective function (9) tends to find RFI frequencies close to zero especially when the absolute value of the threshold is large relative to the RFI. To avoid this, we can use PRIs with thresholds close to zero, instead of all PRIs within the CPI, to estimate the frequency of the Kth RFI source.

IV. Radar Echo Signal Recovery

The desired UWB radar echo vector $\mathbf{s}$ can be recovered by exploiting its sparsity, based on the estimated RFI sources and enforcing the sign agreement between $\mathbf{Y}$ and the corresponding signal model. Since the signal is invariant over all PRIs within the CPI, (1) can be rewritten as follows:

$$\begin{aligned}\mathbf{Y}[:,m] &\approx \text{sign}(\mathbf{D}\boldsymbol{\gamma} - \mathbf{U}[:,m]), m = 1,\dots,M \\ \mathbf{U} &= \mathbf{H} - \hat{\mathbf{R}}_{\hat{\boldsymbol{\theta}}} \in \mathbb{R}^{N\times M},\end{aligned} \tag{11}$$

where $\hat{\mathbf{R}}_{\hat{\boldsymbol{\theta}}}$ is the RFI estimate, $\mathbf{D} \in \mathbb{R}^{N\times N}$ denotes the dictionary whose columns are time-shifted digitized versions of the transmitted impulse and $\boldsymbol{\gamma} \in \mathbb{R}^N$ is a sparse vector containing information of the magnitudes and positions of the radar echoes. To recover $\boldsymbol{\gamma}$, we solve the following convex optimization problem [14]:

$$\min_{\boldsymbol{\gamma}} \ \rho_2||\boldsymbol{\gamma}||_1 + \sum_{m=1}^{M} ||g_2(\mathbf{Y}[:,m] \odot (\mathbf{D}\boldsymbol{\gamma} - \mathbf{U}[:,m]))||_1 \tag{12}$$

where ρ_2 is a user-parameter and similar to (9), $g_2(x) = \max\{-x, 0\}$, which is used to penalize the sign disagreement.

The convex objective function (12) can be minimized efficiently by using the subgradient descent method [15]. With $\boldsymbol{\gamma}$ estimated as $\hat{\boldsymbol{\gamma}}$, the estimated UWB radar echo signal vector is $\hat{\mathbf{s}} = \mathbf{D}\hat{\boldsymbol{\gamma}}$.

Note that for radar echo signal recovery, linear penalties on both the signal amplitude (to promote sparsity) and the sign agreement constraint (to promote data-model fitting) are used. This is different from (9), where a linear sparse penalty is used with a quadratic sign disagreement penalty $g_1(x)$. In (9), we focus on finding an initial estimate of RFI frequency, and the smooth quadratic penalty $g_1(x)$ offers computational advantages due to its continuous derivative and faster convergence. In (12), the linear penalty $g_2(x)$ makes the selection of ρ_2 invariant with the data scaling.

V. Numerical Examples

In this section, we present numerical examples to illustrate the performance of our proposed algorithm for RFI mitigation and radar signal recovery for the Novelda one-bit impulse UWB radar system, as compared to the DI method [2]. The DI method is used as a benchmark, which obtains $\hat{\mathbf{s}}$ as follows:

$$\begin{aligned}\hat{\mathrm{s}}^{\mathrm{DI}}[n] = \left[\Delta h \sum_{m=1}^{M} \frac{1}{2}(\mathrm{Y}[n,m] + 1)\right] - h - \Delta h, \\ \Delta h = 2h/(M-1), n = 1,\dots,N.\end{aligned} \tag{13}$$

The simulated dataset contains 8192 PRIs with 512 fast-time samples per PRI, i.e., $M = 8192, N = 512$. The transmitted impulse is the first order derivative of a Gaussian pulse with length 21, and the assumed UWB radar echoes are shown in Figure 2. The signal-to-noise ratio (SNR) of the UWB radar echo matrix is expressed as follows:

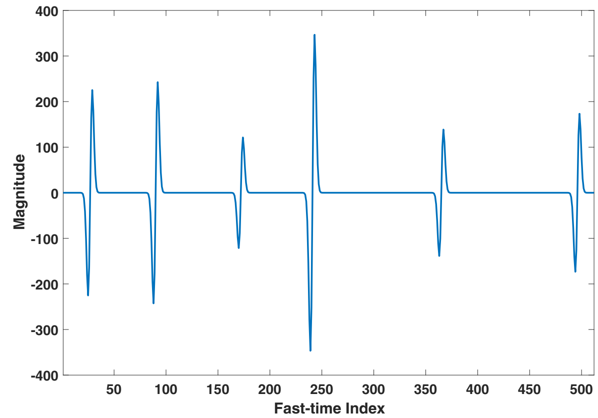


Fig. 2: Simulated radar echoes for one PRI.

$$\text{SNR} = 20\log_{10}\frac{||\mathbf{S}||_2}{||\mathbf{E}||_2}(\text{dB}). \tag{14}$$

We add Gaussian white noise with $\text{SNR} = 10$ dB to the UWB radar echo matrix. The signal-to-interference ratio (SIR) is expressed as follows:

$$\text{SIR} = 20\log_{10}\frac{||\mathbf{S}||_2}{||\mathbf{R}_{\boldsymbol{\theta}}||_2}(\text{dB}). \tag{15}$$

We fix the magnitude of the desired radar signal and add simulated RFI with different magnitudes to obtain contaminated data with different SIRs. The thresholds used for one-bit sampling change under different SIR conditions. Within the CPI, the magnitudes of the RFI sources usually do not change greatly with the slow-time. Hence, in the numerical examples, we simulate the RFI sources as a sum of sinusoids with amplitudes and frequencies fixed from one PRI to another within the CPI and the phases varying randomly with slow-time.

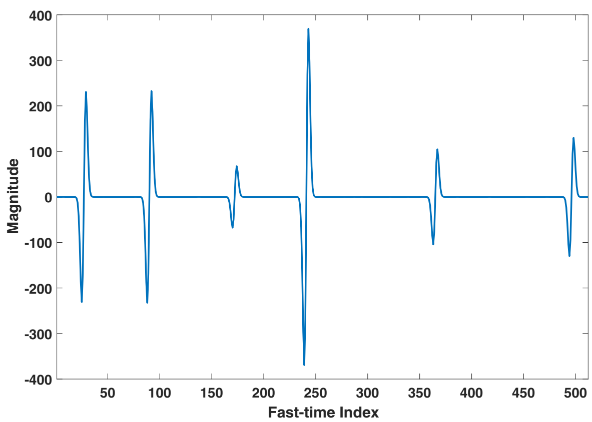
When solving (12), we set $\rho_2 = 192$. When solving (9), we set $\rho_1 = 1$. To reduce the effect of the threshold and the computational cost, only 4096 PRIs with thresholds close to zero are used to obtain the initial frequency estimate in each step of 1bMMRELAX. The radar signal recovery performance can be measured by the normalized recovery error (NRE) of the recovered radar signal vector:

$$\text{NRE} = 20\log_{10}\frac{||\mathbf{s} - \hat{\mathbf{s}}||_2}{||\mathbf{s}||_2}(\text{dB}), \tag{16}$$

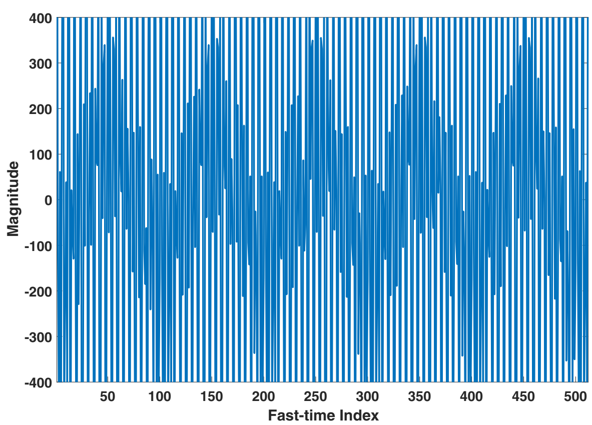
where $\hat{\mathbf{s}}$ is the recovered UWB radar signal. The detailed parameter settings of the experiment and the radar echo recovery results of the proposed algorithm and the DI method are shown in Table I. The original and the estimated radar echo signals obtained by using the proposed method and the DI method are shown in Figure 3. From Table I and Figure 3, we can see that the proposed algorithm significantly outperforms the DI method. Also, the user-parameters ρ_1 and ρ_2 work well for all the SIR values ranging from -20 dB to -40 dB.

TABLE I: Dataset Parameters and Radar Echo Recovery Results.

SIR (dB)	-20	-25	-30	-35	-40
RFI Frequencies	$[0.18, 0.35] \times 2\pi$				
RFI Amplitudes Ratio	$9/10$				
Maximum Absolute Value of RFI $\max \lvert \mathbf{R}_{\boldsymbol{\theta}} \rvert$	1050	1867	3320	5905	10500
Maximum Absolute Value of Thresholds (h)	400	700	1300	2400	4000
Output NRE (dB) of Proposed Algorithm	-25.68	-22.94	-18.99	-18.46	-16.49
Output NRE (dB) of DI	0.81	5.07	10.15	15.36	19.85



(a)



(b)

Fig. 3: Radar echo recovery results obtained for SIR = -40 dB by using a) the proposed method and b) the DI method.

VI. Conclusions

We have considered the problem of RFI mitigation for a one-bit impulse UWB radar system. We have presented a relaxation based algorithm to efficiently estimate the RFI parameters by modeling the RFI sources within each PRI as a sum of sinusoids and assuming their frequencies fixed from one PRI to another within the CPI. Then, by exploiting the s-parsity property of the UWB radar echoes, we have introduced a sparse signal recovery method to estimate the desired UWB radar echoes based on the estimated RFI sources. Finally, numerical examples have been provided to demonstrate that our proposed method significantly outperforms the DI method for RFI mitigation and radar echo recovery.

References

[1] Novelda AS. (2017) Xethru: Single-chip radar sensors with sub-mm resolution. [Online]. Available: https://www.xethru.com/

[2] H. A. Hjortland, D. T. Wisland, T. S. Lande, C. Limbodal, and K. Meisal, "Thresholded samplers for UWB impulse radar," *IEEE International Symposium on Circuits and Systems, New Orleans, LA, USA*, pp. 1210–1213, May 2007.

[3] H. Subbaram and K. Abend, "Interference suppression via orthogonal projections: a performance analysis," *IEEE Transactions on Antennas and Propagation*, vol. 41, no. 9, pp. 1187–1194, Sep. 1993.

[4] T. Miller, L. Potter, and J. McCorkle, "RFI suppression for ultra wide-band radar," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 33, no. 4, pp. 1142–1156, Oct 1997.

[5] F. Zhou and M. Tao, "Research on methods for narrow-band interference suppression in synthetic aperture radar data," *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing*, vol. 8, no. 7, pp. 3476–3485, July 2015.

[6] J. Ren, T. Zhang, J. Li, L. H. Nguyen, and P. Stoica, "RFI mitigation for UWB radar via hyperparameter-free sparse SPICE methods," *IEEE Transactions on Geoscience and Remote Sensing*, to appear in 2019.

[7] J. Ren, T. Zhang, J. Li, and P. Stoica, "Sinusoidal parameter estimation from signed measurements via majorization-minimization based RELAX," *IEEE Transactions on Signal Processing*, vol. 67, no. 8, pp. 2173–2186, April 2019.

[8] C. Gianelli, L. Xu, J. Li, and P. Stoica, "One-bit compressive sampling with time-varying thresholds: Maximum likelihood and the Cramér-Rao bound," *50th Asilomar Conference on Signals, Systems and Computers, Pacific Grove, CA, US*, pp. 399–403, Nov 2016.

[9] J. Ren, T. Zhang, J. Li, and P. Stoica, "Sinusoidal parameter estimation from signed measurements obtained via time-varying thresholds," *52nd Asilomar Conference on Signals, Systems, and Computers, Pacific Grove, CA, USA*, pp. 1111–1115, Oct 2018.

[10] D. R. Hunter and K. Lange, "A tutorial on MM algorithms," *The American Statistician*, vol. 58, no. 1, pp. 30–37, 2004.

[11] J. Li and P. Stoica, "Efficient mixed-spectrum estimation with applications to target feature extraction," *IEEE Transactions on Signal Processing*, vol. 44, no. 2, pp. 281–295, Feb 1996.

[12] P. Stoica and Y. Selen, "Cyclic minimizers, majorization techniques, and the expectation-maximization algorithm: a refresher," *IEEE Signal Processing Magazine*, vol. 21, no. 1, pp. 112–114, Jan 2004.

[13] A. Barabell, "Improving the resolution performance of eigenstructure-based direction-finding algorithms," *IEEE International Conference on Acoustics, Speech, and Signal Processing, Boston, Massachusetts, USA, USA*, pp. 336–339, Apr 1983.

[14] P. T. Boufounos and R. G. Baraniuk, "1-bit compressive sensing," *42nd Annual Conference on Information Sciences and Systems*, pp. 16–21, Mar 2008.

[15] N. Z. Shor, *Minimization methods for non-differentiable functions*. Springer Science & Business Media, 2012, vol. 3.