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A Real-Time Optimal Eco-driving Approach for Autonomous Vehicles Crossing Multiple Signalized Intersections*

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Abstract— **This paper develops a methodology for obtaining an optimal acceleration/speed profile for a single autonomous vehicle crossing multiple signalized intersections without stopping in free flow mode. We aim to minimize an objective function that involves a trade-off between travel time and energy consumption of autonomous vehicles. Our design approach differs from most existing approaches based on numerical calculations: it begins with identifying the structure of the optimal acceleration profile and then showing that it is characterized by several parameters, which are used for design optimization. Therefore, the infinite dimensional optimal control problem is transformed into a finite dimensional parametric optimization problem, which enables a real-time online analytical solution. We include simulation results to show quantitatively the advantages of considering multiple intersections jointly rather than dealing with them individually. Based on mild assumptions, the optimal eco-driving algorithm is readily extended to include interfering traffic.**

Index Terms— **Autonomous vehicles, interior-point constraints, optimal control, parametric optimization, vehicle-to-infrastructure communication**

I. INTRODUCTION

The alarming state of existing transportation systems has been well documented. From a control and optimization standpoint, the challenges stem from requirements for increased safety, increased efficiency in energy consumption, and lower congestion both in highway and urban traffic. Connected and automated vehicles (CAVs), commonly known as self-driving or autonomous vehicles, provide an intriguing opportunity for enabling users to better monitor transportation network conditions and to improve traffic flow. Their proliferation has rapidly grown, largely as a result of Vehicle-to-X (or V2X) technology [1] which refers to an intelligent transportation system where all vehicles and infrastructure components are interconnected with each other. Such connectivity provides precise knowledge of the traffic situation across the entire road network, which in turn helps optimize traffic flows, enhance safety, reduce congestion, and minimize emissions. Controlling a vehicle to improve energy consumption has been studied extensively, e.g., see [2]–[5]. By utilizing road topography information, an energy-optimal control algorithm for heavy diesel trucks is developed in [4]. Based on Vehicle-to-Vehicle (V2V) communication, a minimum energy control strategy is investigated in car-following scenarios in [5]. Another important line of research focuses on coordinating vehicles at intersections to increase traffic flow while also reducing energy consumption. Depending on the control objectives, work in this area can be classified as dynamically controlling traffic signals [6] and as coordinating vehicles [7], [8], [9], [10]. More recently, an optimal control framework is proposed in [11] for CAVs to cross one or two adjacent intersections in an urban area. The state of art and current trends in the coordination of CAVs is provided in [12].

Our focus in this paper is on an optimal control approach for a single autonomous vehicle approaching multiple intersections in free flow mode in terms of energy consumption and taking advantage of traffic light information. The term “ECO-AND” (short for “Economical Arrival and Departure”) is often used in the literature to refer to this problem [13]. Its solution is made possible by vehicle-to-infrastructure (V2I) communication, which enables a vehicle to automatically receive signals from upcoming traffic lights before they appear in its visual range. For example, such a V2I communication system has been launched in Audi cars in Las Vegas by offering a traffic light timer on their dashboards: as the car approaches an intersection, a red traffic light symbol and a “time-to-go” countdown appear in the digital display and reads how long it will be before the traffic light ahead turns green [14]. Clearly, an autonomous vehicle can take advantage of such information in order to go beyond current “stop-and-go” to achieve “stop-free” driving. Along these lines, the problem of avoiding red traffic lights is investigated in [15]–[18]. The purpose in [15] is to track a target speed profile, which is generated based on the feasibility of avoiding a sequence of red lights. The approach uses model predictive control based on a receding horizon. Avoiding red lights with probabilistic information at multiple intersections is considered in [16], where the time horizon is discretized and deterministic dynamic programming is utilized to numerically compute the optimal control input. The work in [17] devises the optimal speed profile given the feasible target time, which is within some green light interval. A velocity pruning algorithm is proposed in [18] to identify feasible green windows, and a velocity profile is calculated numerically in terms of energy consumption. Most existing work solves the eco-driving problem with traffic light constraints numerically, invoking methods such as dynamic programming [16], [19], and predictive control [15]. To enable the real-time use of such eco-driving methods, it

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is desirable to have an on-line analytical solution [20]. Even though the analytical solution may not be able to handle uncertainties, it provides a performance lower bound, which quantifies the performance gap between numerical solutions and analytical solutions. On the other hand, the analytical solution provides a reference trajectory.

It is clear that a need exists for developing new methods for eco-driving of autonomous vehicles with traffic light constraints. This paper aims to address this need by proposing an extension to our previous approach from a single signalized intersection [21], [22] to multiple intersections [23], [24]. We show explicitly that the optimal acceleration profile has a piecewise linear form, similar to the results in [21], [22], that includes all state equality and temporal inequality constraints involved. It follows from the theoretical analysis that the optimal acceleration profile can be parameterized by a piecewise linear function of time, which offers a real-time analytical solution to eco-driving of autonomous vehicles crossing multiple signalized intersections without stopping. We illustrate the effectiveness of the proposed optimal parametric approach through simulations and show that it yields better results compared with our previous eco-driving approach [21], [22] applied to each intersection individually. We also show that the optimal eco-driving algorithm can be adjusted to handle the case with interfering traffic under the assumption of the availability of some traffic information.

II. Problem Formulation

The vehicle dynamics are modeled by a double integrator

$$\dot{x}(t) = v(t), \tag{1}$$
$$\dot{v}(t) = u(t), \tag{2}$$

where $x(t)$ is the travel distance of the vehicle relative to some origin on the road, which may include turns, $v(t)$ the velocity, and $u(t)$ the acceleration/deceleration. At t_0, the initial position and velocity are given by $x(t_0) = x_0$ and $v(t_0) = v_0$, respectively. On-road vehicles have to obey traffic rules, such as the minimum and maximum speed permitted $0 \le v_{\min} \le v(t) \le v_{\max}$. The physical constraints on acceleration and deceleration are determined by vehicle parameters $u_{\min} \le u(t) \le u_{\max}$, where $u_{\min} < 0$ and $u_{\max} > 0$ denote the maximum deceleration and acceleration, respectively.

Assume that there are N intersections. Each intersection i is equipped with a traffic light, which is dictated by the square wave $f_i(t)$ defined below

$$f_i(t) = \begin{cases} 1, & \text{when } kT_i \le t \le kT_i + D_iT_i, \\ 0, & \text{when } kT_i + D_iT_i < t < kT_i + T_i, \end{cases}$$

where $f_i(t) = 1$ indicates that the traffic light is green, and $f_i(t) = 0$ indicates that the traffic light is red. The parameter $0 < D_i < 1$ is the fraction of the time period T_i during which the traffic light is green, and $k = 0, 1, \ldots,$ are non-negative integers. Assume that there is no offset among the signals. Our algorithm also supports dynamically actuated traffic signals if the time until green/red can be determined and communicated to the autonomous vehicle. Then we can re-solve the problem with the new timing information.

Let $\{t_i\}_{i=1}^N$ be a sequence of intersection crossing times with $t_{i+1} > t_i$, that is, $x(t_i) = \sum_{j=1}^{i} l_j$, where l_j is the length of road segment j. To ensure stop-free intersection crossing, t_i must be within the green light interval, that is, $kT_i \le t_i \le kT_i + D_iT_i$ for some non-negative integer k.

Our objective is the eco-driving of autonomous vehicles crossing multiple intersections in terms of both time and energy efficiency. Therefore, our problem formulation is given below:

Problem 1: ECO-AND Problem

$$J = \min_{u(t)} \rho_t (t_p - t_0) + \rho_u \int_{t_0}^{t_p} u^2(t)\, dt$$

subject to

$$(1) \text{ and } (2) \tag{3}$$
$$x(t_i) = \sum\nolimits_{j=1}^{i} l_j, \; i = 1, \ldots, N \tag{4}$$
$$v_{\min} \le v(t) \le v_{\max} \tag{5}$$
$$u_{\min} \le u(t) \le u_{\max} \tag{6}$$
$$k_iT_i \le t_i \le k_iT_i + D_iT_i, \; i = 1, \ldots, N \tag{7}$$

for some non-negative integers k_i, where ρ_t and ρ_u are the weight parameters, and $t_p = t_N$ is the time when the vehicle arrives at the last intersection.

Procedures for normalizing these two terms ρ_t and ρ_u for the purpose of a well-defined optimization problem can be found in [21], [22] by properly determining weights ρ_t and ρ_u. In Problem 1, the term $J^t = t_p - t_0$ is the travel time while $J^u = \int_{t_0}^{t_p} u^2(t)\, dt$ captures the energy consumption; see [25].

III. Main Results

Before proceeding further, let us first introduce a lemma, which will be used frequently throughout the following analysis.

Lemma 1: Consider the vehicle's dynamics (1) and (2) with initial conditions x_0 and v_0. If the acceleration profile of the vehicle has the form $u(t) = at + b$ during the time interval $[t_0, t_1]$, where a and b are two constants, then

$$\begin{aligned} v(t_1) =& v_0 + b(t_1 - t_0) + \frac{a}{2}\left(t_1^2 - t_0^2\right), \\ x(t_1) =& x_0 + v_0(t_1 - t_0) + \frac{1}{2}b(t_1 - t_0)^2 \\ &+ \frac{a}{6}\left(t_1^3 + 2t_0^3 - 3t_0^2 t_1\right), \\ J^u =& \frac{a^2}{3}\left(t_1^3 - t_0^3\right) + ab\left(t_1^2 - t_0^2\right) + b^2(t_1 - t_0). \end{aligned}$$

The proof is obtained by using the kinematic equations of the vehicle (1) and (2) and the definition of J^u. Due to space constraints, the details are omitted.

Remark 1: We will show in Theorem 1 below that in fact the *optimal* acceleration profile for Problem 1 is of the form $u(t) = at + b$, which captures most acceleration profiles used in the literature and vehicle simulation software [26]. When $a = b = 0$, the vehicle travels at a constant speed.

When $a = 0$, the acceleration profile becomes either constant acceleration ($b > 0$) or constant deceleration ($b < 0$). When $a \neq 0$, the resulting linear acceleration profile is also called "smooth jerk" [26].

In order not to overshadow the main idea, we consider the case of only two consecutive intersections here, where $t_p = t_2$. We will show how the proposed framework can include our previous result on a single intersection [21], [22] as a special case in Subsection IV-A.

The main challenge of extending the result from one intersection [21], [22] to multiple intersections lies in the interior-point constraints $x(t_1) = l_1$, which is a spatial equality constraint and $kT_1 \leq t_1 \leq kT_1 + D_1T_1$, which is a temporal inequality constraint. Other constraints, such as state, acceleration/deceleration, and terminal constraints, have been thoroughly studied in our previous work [21], [22]. The following theorem shows the form of the optimal acceleration profile when the interior-point constraints are present.

Theorem 1: The optimal acceleration profile $u^*(t)$ of Problem 1 has the form $u^*(t) = a(t)t + b(t)$, where $a(t)$ and $b(t)$ are piece-wise constant and $u^*(t_p^*) = 0$.

The proof of Theorem 1 can be found in [27]. Theoretically speaking, $u^*(t)$ may have a jump at time t_1 only when $v(t_1) = v_{\max}$ or $v(t_1) = v_{\min}$, where t_1 is the intersection crossing time. If such cases occur, we impose a continuity constraint on $u^*(t)$ at t_1 so as to avoid the jerk effect that results in discomfort for both driver and passengers.

Based on Theorem 1, we know that the optimal acceleration profile has the form $u^*(t) = a(t)t + b(t)$, where $a(t)$ and $b(t)$ are piece-wise constant. For example, we have $a(t) = 0$, $b(t) = u_{\max}$ for $u^*(t) = u_{\max}$, and $a(t) = 0$, $b(t) = u_{\min}$ for $u^*(t) = u_{\min}$. For the case that $u(t) = 0$, we could set $a(t) = b(t) = 0$. Most of the time, $a(t) = a$ and $b(t) = b$ are just constants. In addition, there are only a few time instants when $a(t)$ and $b(t)$ switch from a constant to another. Such instants include the time when the maximum acceleration starts to decrease, the maximum deceleration starts to increase, the vehicle reaches the maximum or the minimum allowed speed limits, or at the first intersection crossing time t_1. Therefore, we could parameterize the *optimal* acceleration profile by a sequence of linear functions of time.

IV. Parametric Optimization

Based on the analysis of the last section, the optimal acceleration profile can be parameterized by a sequence of linear functions of time, such as the one shown in Fig. 1. Now let us split the analysis into two parts: $[t_0, t_1]$ and $[t_1, t_2]$, where t_1 is the first intersection crossing time. The optimal acceleration profile at most has four switches at $\tau_1, \tau_3, \tau_5, \tau_6$ as shown in Fig. 1. The acceleration profile shown in Fig. 1 is the most complicated acceleration profile possible, starting with the maximum acceleration, which can be obtained based on the following facts:

- $u^*(t_2^*) = 0$, which can be seen from Theorem 1.
- Whenever $v(t) = v_{\min}$ or $v_{\max}$, $u^*(t) = 0$.
- $u^*(t)$ is continuous without jumps.
- $\dot{\lambda}_2(t)$ may change sign only at t_1.

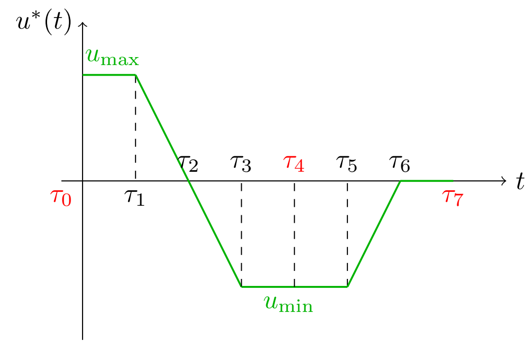

Fig. 1. Optimal acceleration profile for two intersections

A similar optimal acceleration profile can be drawn when it starts with the maximum deceleration. The second fact above corresponds to the interval $[\tau_6, \tau_7]$ in Fig. 1, in which $v(t) = v_{\min}$ for $t \in [\tau_6, \tau_7]$. The fourth fact above can be visualized in Fig. 1 as well. Before τ_4, the acceleration decreases monotonically; and after τ_4 it increases monotonically. Even though there are five linear functions in Fig. 1, seven linear functions are needed to parameterize the acceleration profile, as explained next. Over the interval $[\tau_1, \tau_3]$ in Fig. 1, there is only one linear function. In order to guarantee that the acceleration profile during each interval contains either acceleration or deceleration but not both, which is ensured by the constraint (9) below, we consider that there is a switch at τ_2 between acceleration and deceleration. Therefore, two linear functions are used to parameterize the optimal acceleration profile. By doing so, the speed either increases or decreases during each interval. Therefore, the constraint (8) below ensures that the speed is within the minimum and maximum bounds all the time.

We can thus parameterize the optimal acceleration profile as follows:

$$u^*(t) = \begin{cases} a_1t + b_1 & \text{for } t \in [\tau_0, \tau_1] \\ a_2t + b_2 & \text{for } t \in [\tau_1, \tau_2] \\ a_3t + b_3 & \text{for } t \in [\tau_2, \tau_3] \\ a_4t + b_4 & \text{for } t \in [\tau_3, \tau_4] \\ a_5t + b_5 & \text{for } t \in [\tau_4, \tau_5] \\ a_6t + b_6 & \text{for } t \in [\tau_5, \tau_6] \\ a_7t + b_7 & \text{for } t \in [\tau_6, \tau_7] \end{cases}$$

where $\tau_0 = t_0$, $\tau_4 = t_1$, and $\tau_7 = t_2$.

Remark 2: The optimal acceleration profile is parameterized by the triplets (a_i, b_i, τ_i), $i = 1, 2, \ldots, 7$, 21 variables in total. The number of variables can be reduced when the properties of $u^*(t)$ are considered. The advantage of the parametric form of the optimal controller is that it simplifies the complicated analysis through a computationally efficient scheme suitable for real-time implementation. Also note that vehicles may experience both acceleration and deceleration during a single road segment, which is different from the optimal acceleration profile for a single intersection [21], [22].

We have now shown that Problem 1 is equivalent to this parametric optimization problem:

Problem 2: ECO-AND problem

$$\min \rho_t \tau_7 + \rho_u \sum_{i=1}^{7} J_i^u$$

subject to

$$\begin{aligned} & v_{\min} \le v(\tau_i) \le v_{\max}, && (8) \\ & (a_i \tau_i + b_i)(a_i \tau_{i-1} + b_i) \ge 0, && (9) \\ & u_{\min} \le a_i \tau_i + b_i \le u_{\max}, && (10) \\ & u_{\min} \le a_i \tau_{i-1} + b_i \le u_{\max}, && (11) \\ & \tau_{i-1} \le \tau_i, && (12) \\ & i = 1, \ldots, 7, && \\ & k_1 T_1 \le \tau_4 \le k_1 T_1 + D_1 T_1, && (13) \\ & x(\tau_4) = l_1 && (14) \\ & k_2 T_2 \le \tau_7 \le k_2 T_2 + D_2 T_2, && (15) \\ & x(\tau_7) = l_1 + l_2 && (16) \end{aligned}$$

where J_i^u is the energy cost during the interval $[\tau_{i-1}, \tau_i]$, which can be expressed as

$$J_i^u = \frac{a_i^2}{3}\left(\tau_i^3 - \tau_{i-1}^3\right) + a_i b_i \left(\tau_i^2 - \tau_{i-1}^2\right) + b_i^2 (\tau_i - \tau_{i-1})$$

from Lemma 1,

$$v(\tau_i) = v(\tau_{i-1}) + b_i(\tau_i - \tau_{i-1}) + \frac{a_i}{2}\left(\tau_i^2 - \tau_{i-1}^2\right)$$

and

$$\begin{aligned} x(\tau_i) \;=\; & x(\tau_{i-1}) + v(\tau_{i-1})(\tau_i - \tau_{i-1}) \\ & + \frac{b_i}{2}(\tau_i - \tau_{i-1})^2 + \frac{a_i}{6}\left(\tau_i^3 + 2\tau_{i-1}^3 - 3\tau_{i-1}^2 \tau_i\right). \end{aligned}$$

Remark 3: Problem 2 is equivalent to Problem 1, where the continuous velocity constraint (5) is ensured by (8) and (9). The continuous acceleration constraint (6) is ensured by (10) and (11). The constraint (12) is needed to ensure the correct order of the critical times defining the linear segments of the optimal acceleration profile.

Remark 4: The parametric optimization framework is very general so that it can be used to solve many different eco-driving problems. By taking into consideration the driving comfort, we can just add the constraints $|a_i| \le a_J$, where a_i corresponds to the jerk profile, and a_J is the limit of jerk tolerance [13]. The parametric optimization framework can also easily incorporate an initial acceleration condition, interior and terminal velocity/acceleration constraints by adding additional equality or inequality constraints.

In the following, we will show how this optimal parametric framework includes our previous result [21], [22] as a special case.

A. Single Intersection

Based on our analysis for a single intersection [21], [22], the optimal acceleration profile can be parameterized as

$$u^*(t) = \begin{cases} a_1 t + b_1 & \text{for } t \in [\tau_0, \tau_1] \\ a_2 t + b_2 & \text{for } t \in [\tau_1, \tau_2] \\ a_3 t + b_3 & \text{for } t \in [\tau_2, \tau_3] \end{cases}$$

where $\tau_0 = t_0$, and $\tau_3 = t_1$. The optimal parameters (a_i, b_i, τ_i) for $i = 1, 2, 3$ can be obtained by solving the following optimization problem:

Problem 3: ECO-AND problem

$$\min \rho_t \tau_3 + \rho_u \sum_{i=1}^{3} J_i^u$$

subject to

$$\begin{aligned} & v_{\min} \le v(\tau_3) \le v_{\max} && (17) \\ & u_{\min} \le a_1 \tau_0 + b_1 \le u_{\max} && (18) \\ & \tau_{i-1} \le \tau_i, \quad i = 1, \ldots, 3 && (19) \\ & kT \le \tau_3 \le kT + DT && (20) \\ & x(\tau_3) = l, && (21) \end{aligned}$$

where J_i^u is the energy cost during the interval $[\tau_{i-1}, \tau_i]$, which can be expressed as

$$J_i^u = \frac{a_i^2}{3}\left(\tau_i^3 - \tau_{i-1}^3\right) + a_i b_i \left(\tau_i^2 - \tau_{i-1}^2\right) + b_i^2 (\tau_i - \tau_{i-1})$$

according to Lemma 1, and

$$\begin{aligned} x(\tau_i) \;=\; & x(\tau_{i-1}) + v(\tau_{i-1})(\tau_i - \tau_{i-1}) \\ & + \frac{b_i}{2}(\tau_i - \tau_{i-1})^2 + \frac{a_i}{6}\left(\tau_i^3 + 2\tau_{i-1}^3 - 3\tau_{i-1}^2 \tau_i\right) \\ v(\tau_i) \;=\; & v(\tau_{i-1}) + b_i(\tau_i - \tau_{i-1}) + \frac{a_i}{2}\left(\tau_i^2 - \tau_{i-1}^2\right). \end{aligned}$$

Note that we do not include the constraint (9) here since we have established the result in [21], [22] that the optimal acceleration profile contains either acceleration or deceleration, but not both. Therefore, the terminal velocity constraint (17) can replace the velocity constraint (8). Also based on the analysis in [21], [22], the initial acceleration constraint (18) is sufficient instead of using (10) and (11).

V. EXTENSION TO CASES WITH INTERFERING TRAFFIC

In the above results, we assume that a single vehicle operates in free flow mode. We also assume that autonomous vehicles cannot change lanes. There is a recent line of work which deals with multi-lane eco-driving of autonomous vehicles crossing signalized [28] and non-signalized [29] intersections. We will show that the proposed method can be easily extended to traffic conditions where other road users may affect the movement of the autonomous vehicle. Therefore, a safety constraint has to be enforced at all times, that is,

$$x_h(t) - x(t) \ge \alpha v(t) + \beta \tag{22}$$

where x_h is the position of the preceding vehicle, α and β are two scalars representing dynamic and static gaps, respectively [30]. The following assumptions are made:

- On the road, the future speed and acceleration profiles of the preceding vehicle can be estimated accurately enough by the autonomous vehicle.
- At the intersection, the queue information and stopped vehicle lengths are also available to the autonomous vehicle.

When the proposed approach without considering interfering traffic is applied, the safety constraint in (22) may be violated. The first case is that the preceding vehicle will cross the intersection at some $t \in [kT_i, kT_i+D_iT_i]$ while the autonomous vehicle will not. In this case, the autonomous vehicle becomes the leading vehicle on the road, and the intersection crossing time is set as the beginning of the next green light interval $t_i = kT_{i+1}$ for some positive integer k. The safety constraint may not be violated since the preceding vehicle will accelerate or cruise through the intersection while the autonomous vehicle will decelerate to approach the intersection in the next green light interval. The second case is that both the preceding vehicle and the autonomous vehicle will cross the intersection at the same green light interval. In this case, we can simply decrease the maximum speed of the autonomous vehicle to $\theta v_{\max}$ with $0 < \theta < 1$ so that the safety constraint is satisfied at all times. The third case is that the preceding vehicle stopped before the intersection. In this case, the autonomous vehicle will cross the intersection after the preceding vehicle with a certain time gap σ when the traffic signal turns to green so that the safety constraint is not violated, that is, $t_i = kT_i + \sigma$.

VI. SIMULATION EXAMPLES

We evaluate the proposed solution by testing the following scenario with two intersections, and Problem 2 and Problem 3 are solved by `fmincon` in Optimization Toolbox in MATLAB. The length for each road segment is 200 meters. Each intersection is equipped with a traffic light. Two phases were set up for each signal. The total cycle is 40 seconds, where the green time is set to 20 seconds. The speed limits are set as $v_m = 2.78\ m/s$, and $v_M = 20\ m/s$. The maximum acceleration and deceleration are $u_M = 2.5\ m/s^2$ and $u_m = -2.9\ m/s^2$. We assume that the vehicle starts with $v_0 = 0$. Once the vehicle's speed reaches v_m, it will never drop below v_m. We use our previous approach for a single intersection [21], [22] as the baseline scenario, which solves the eco-driving problem for each road segment individually, and compare the proposed solution and the baseline scenario. Table I shows the performance by using our previous approach [21], [22] and the optimal parametric approach, where J is defined in Problem 1, J_1 and J_2 corresponds the performance on $[t_0\ t_1]$ and $[t_1\ t_2]$, respectively. Even though our previous approach [21], [22] calculates the optimal performance for each road segment, it is not the optimal solution for the *combined* two segments as a whole. Overall, the optimal multi-intersection parametric approach outperforms [21], [22] by 10.29%.

TABLE I

PERFORMANCE COMPARISON

Method	**Performance**		
	J_1	J_2	J
[21], [22]	0.1366	0.1793	0.3159
Optimal Parametric Approach	0.1494	0.1340	0.2834
Improvement	-9.67%	25.26%	10.29%

A. *Example with Interfering Traffic*

In the following, we consider a scenario where the autonomous vehicle will not be obstructed on the road but there is a vehicle which will stop before the second intersection. We assume that such information is available to the autonomous vehicle at time t_0. It is infeasible for the autonomous vehicle to cross the second intersection at 40 seconds as before. We assume that the feasible intersection crossing time is $t \in [44, 60]$, where the four seconds include driver's reaction time, headway time, and time for the stopped vehicle to clear the intersection. Figures 2-4 depict the acceleration profiles, speed profiles, and distance profiles, respectively, for both the cases with and without interfering traffic. It can be seen from the figures that the first intersection crossing times are the same for both cases. However, when there is interfering traffic, the autonomous vehicle has to start with a higher acceleration and decelerate more before the first intersection compared with the case without interfering traffic.

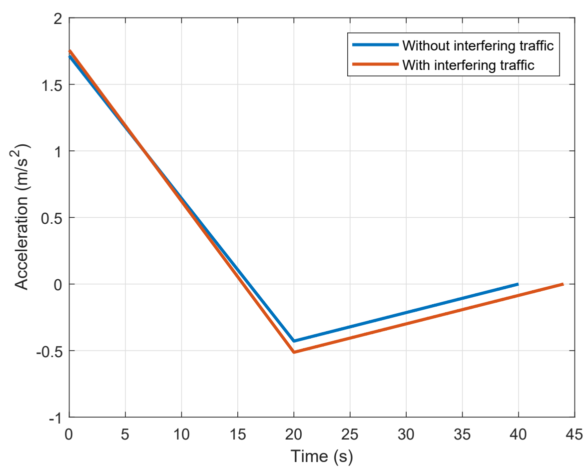

Fig. 2. Acceleration profiles of the optimal solution with and without interfering traffic.

Here we do not compare the results with those of human-driven vehicles due to space constraints. Such a comparison was done in our previous work for a single intersection [21], [22], where 2%-10% performance improvement was shown in terms of travel time and fuel consumption.

VII. CONCLUSIONS

In this paper, we solve an eco-driving problem of autonomous vehicles crossing multiple intersections without stopping. Spatial equality constraints and temporal inequality constraints are used to capture the traffic light constraints. The optimal acceleration profile is proved to have a piecewise linear parametric structure. We illustrate the effectiveness of the proposed parametric approach through simulation examples. The results show that the performance is significantly improved by using the proposed optimal parametric approach compared with our previous approach which is optimal for each individual intersection decoupled from the

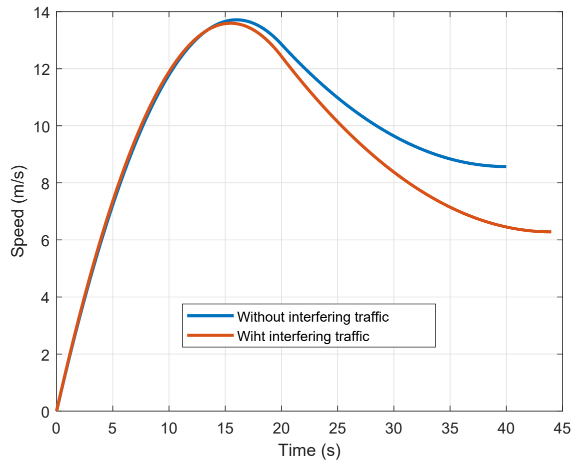

Fig. 3. Speed profiles of the optimal solution with and without interfering traffic.

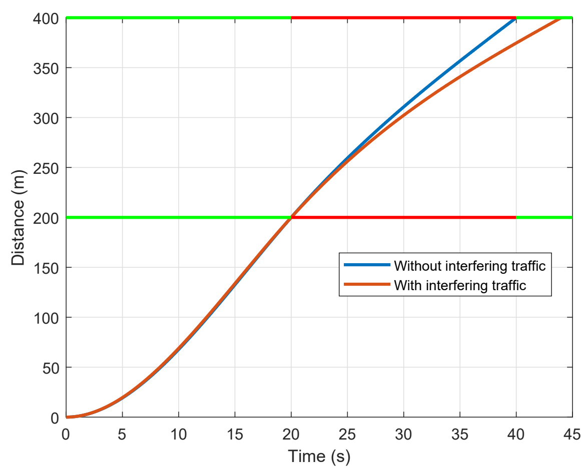

Fig. 4. Distance profiles of the optimal solution with and without interfering traffic.

other. We also show that the optimal eco-driving algorithm is capable of dealing with interfering traffic.

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