On Observations of Correlation Functions and Power Spectra in Rain: Obfuscation by Advection and Sampling

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Often the spatial and temporal structures are statistically independent.

Because drops evolve in the vertical, that dimension is taken to represent time so that the changes in optical brightness can be considered to be representative of temporal variations of rain intensity while the spatial distribution of the rain shafts represents the spatial variations in rain structure.
With advection the total variance is then the sum of the temporal and the spatial variances so that the total power spectra are the sums of each power spectrum.

Observations also involve filtering of the data. Temporal averaging amounts to a low pass filter while the finite dimension of a network acts as a high pass filter. In this work the effects of both the advection of the rain and the observational filtering are considered for wide-sense statistically stationary and homogeneous rain along one-dimension for rain exponentially correlated in both space and time.

Temporal with Spatial Advection

In time, the power spectrum is

$$S_{total}(\omega)=\frac{1}{2}\left[\frac{2\mathfrak{T}^2}{\pi(1+\mathfrak{T}^2\omega^2)^2}+\frac{2\mathfrak{T}_s^2}{\pi(1+\mathfrak{T}_s^2\omega^2)^2}\right]\frac{\mathrm{Sinc}^2(\frac{T\omega}{2})}{2\pi}$$

where ω is the frequency, $\mathfrak{T}$ is the time to 1/e decorrelation, $\mathfrak{T}_s$ is the effective time to decorrelation of the spatial distance to decorrelation $\mathfrak{L}$ such that $\mathfrak{T}_s = \mathfrak{L} / v$ where *v* is the advection velocity along the one dimension, *T* is the temporal sample interval and time *t* = *s* / *v* where s is the separation distance between two locations along the line of instruments.

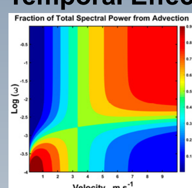
Spatial with Temporal Advection

In space the power spectra is (Jameson, 2020)

$$S_{total}(\kappa)=\left[\frac{2\mathfrak{L}^2}{\pi(1+\mathfrak{L}^2\kappa^2)^2}+\frac{\mathfrak{L}_t^2\mathrm{Sinc}^2\left(\frac{Tv\kappa}{2}\right)}{\pi^2(1+\mathfrak{L}_t^2\kappa^2)^2}\right]\times\left[1-\mathrm{Sinc}^2(4\pi\mathfrak{D}\kappa^2)\right]$$

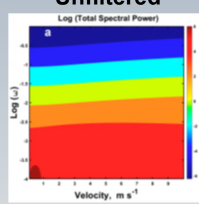
where κ is the wave number, $\mathfrak{D}$ is the dimension of the network and the effective spatial distance to decorrelation of the advected temporal structure is $\mathfrak{L}_t = \mathfrak{T} \times v$.

Temporal Effects of Spatial Advection

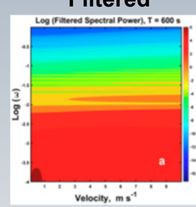


As the advection velocity increases, the advection contribution increases the power at higher frequencies (shorter wavelengths).

Unfiltered

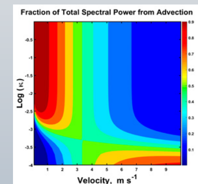


Filtered



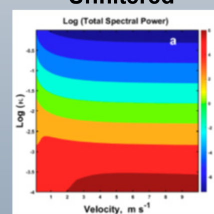
The temporal filtering introduces oscillations while suppressing the power at higher frequencies thus in part counteracting the effects of advection.

Spatial Effects of Temporal Advection



As the advection velocity, increases, the advection contribution increases the power at smaller wavenumbers (longer wavelengths).

Unfiltered



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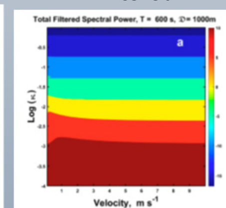
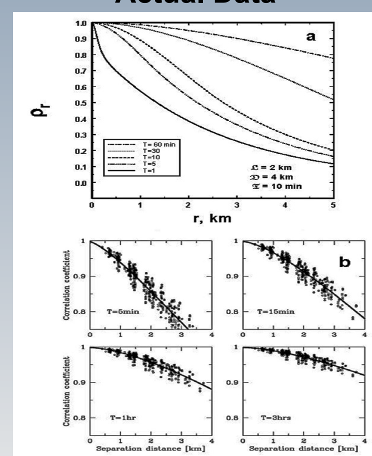


The spatial filtering adds to the the effects of advection by suppressing further the power at higher wavenumbers (shorter wavelengths).

Actual Data



From (Jameson, 2017) (a) The advection (velocity) effects for various observation intervals for an intrinsic exponential temporal correlation function having $\mathfrak{T}$ = 10 min and a mean advection velocity of 2 m s^{-1} combined with the intrinsic exponential spatial correlation function having $\mathfrak{L}$ = 2 km as observed using a 4-km network. The advection distorts the intrinsic spatial correlation function with increasing *T* by enhancing the importance of longer wavelengths that, in turn, leads to enhanced correlation. This mimics observations (b) over a network of 25 rain gages covering an area of about 3 km by 3 km.
[from (Ciach and Krajewski 2006) with permission from Elsevier and the authors.]

Implications

It is found that advection and filtering can significantly shift the spectral and correlation portrayal of the rain from those of the true structures. Consequently, rainfall characterizations from observations should be carefully considered and not be over-generalized to inapplicable situations.

References

Ciach, G. J., and W. F. Krajewski, 2006: Analysis and modeling of spatial correlation structure in small-scale rainfall in Central Oklahoma. *Adv. Water Resour.*, 29, 1450–1463, https://doi.org/10.1016/j.advwatres.2005.11.003.

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