A Hierarchical Heuristic Approach for Solving Air Traffic Scheduling and Routing Problem with a Novel Air Traffic Model

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***Abstract*—Efficient flight routing and scheduling plays an important role in air traffic flow management (ATFM), which aims to maximize the utilization of airport and enroute capacities to ensure safety and efficiency of air transportation. In this paper we first propose a novel discrete-time flow dynamic model for an air traffic network, consisting of airports, waypoints and air links, upon which we formulate an air flow routing and scheduling problem as an integer linear programming problem. Considering the NP-hard nature of the problem, we present a novel hierarchical flow routing and scheduling approach, where the hierarchical architecture is derived naturally from the network containment relationship, and computation is carried out in a bottom-up manner, which relies on an incremental strategy. Upon resulting flow routes and schedules, a heuristic algorithm is carried out to determine flight plans for individual aircraft. The effectiveness of the proposed hierarchical approach is illustrated by air traffic data in four ASEAN Flight Information Regions (FIRs).**

***Index Terms*—air traffic flow management, flow dynamic model, hierarchical architecture, heuristic algorithm, integer linear programming**

Nomenclature

Abbreviation/Unit

ATC	Air Traffic Control
ATFM	Air Traffic Flow Management
ATFRSP	Air Traffic Flow Routing and Scheduling Problem
ACC	Area Control Center
FIR	Flight Information Region
UAV	Unmanned Aerial Vehicles
RNP	Required Navigation Performance
O-D	Origin-Destination
FIFO	First-In-First-Out
NM	Nautical Mile
FL	Flight Level

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I. Introduction

With the increasing world-wide trading and traveling, air traffic becomes an important component in the transportation system. The efficiency of the air traffic system is a major concern since air traffic delays have been costing billions of dollars to airlines each year [1]. Two critical issues are involved in a modern air traffic system, i.e., air traffic control (ATC) and air traffic flow management (ATFM). ATC works at a tactical level as a control strategy to provide information to pilots and ground controller and keep safe separations for aircraft, while ATFM works at a strategic level to distribute air traffic flows and improve utilization of en-route air traffic network capacities. In this paper, we focus on formulating and solving an ATFM problem to improve the efficiency of an air traffic system by reducing schedule deviations involved in departure and arrival processes.

In terms of modeling, there have been several major types proposed in the literature, as shown in [2]. Lagrangian models [3][4] are used to describe flight trajectories of individual aircraft. However, this model is usually computationally infeasible for a large network. Aggregate traffic models and Eulerian models [5] [6] are used to describe average behaviours of a group of aircraft, as described by the concept of *flows*. An aggregation approach typically provides a lower order, fixed-resolution model of the airspace, while the Eulerian approach provides a flexible resolution model [7]. In addition, the aggregate approach is only able to control the number of aircraft for each air traffic control center but not the aircraft in each link. Thus, it is not suitable for the en-route air traffic flow management problem. A multicommodity Eulerian-Lagrangian large-capacity cell transmission model for en-route air traffic is proposed in [8], which adopts the basic idea of cell transmission models described in [9] [10] within a standard multi-commodity flow model equipped with origin-destination (O-D) pairs to avoid difficulties in describing flow merging and diverging. Some stochastic models can be found in [11]. With regard to the decomposition methodologies, Dantzig-Wolfe Decomposition [12] is applied in the traffic flow scheduling on the Bertsimas and Stock-Patterson (BSP) model[13] [14] as well as on the cell transmission model [15].

ATFM is a planning problem aiming to arrange air traffic flows in a proper manner and is typically formulated as a mathematical programming problem and solved with some conventional optimization solvers [13][16][17][18]. Recent research works on the ATFM problem include, firstly, to optimize the utilization of airport capacity for the ground-holding problem [19] [20]; secondly, to alleviate the airborne delays [21]; thirdly, to minimize the ground-holding waiting time and airborne delays in a single optimization problem [18] [22]. Some work based on en-route air traffic network is presented in [23], [24], [25], [26]. The Eulerian or Lagrangian model for ATFM based on an en-route air traffic network are proposed in [24] and [25], respectively, and a distributed solution for the Eulerian model is proposed in [23] [26].

In this paper, we adopt a novel flow dynamics model for an en-route air traffic network. One of the major motivations of developing this model is to reduce the computational complexity. For example, this model is insensitive to the length of each air route (or link), in contrast to a cell transmission model used in [26] (where each air route needs to be partitioned into relatively small segments, whose number depends on the length of the air route), but rather sensitive to the difference between the maximum and minimum feasible speeds, which happens not to be big. Thus, the new model will lead to a much smaller number of decision variables than the number of variables in the cell transmission model used in [26]. To achieve this, the validity of the model relies on a FIFO assumption, which, from a practical application point of view, is approximately true. We consider a variety of aircraft types such as small, medium and large passenger/cargo aircraft and unmanned aerial vehicles (UAVs) that are expected to be more popular in the near future, and take the aircraft speed lower and upper limits into account when applying air route capacity constraints on air route volume dynamics. Based on the fact that most flights will not travel among different flight information regions, a hierarchical heuristic approach is provided to solve the ATFM problem for a multiple-Flight Information Region (FIR) air traffic system. The hierarchical architecture consists of two levels, i.e., the primary level for flow management in a single FIR while the secondary level for flow management in multiple FIRs. For the secondary level, to further reduce the computational complexity, we propose a distributed scheduling strategy based on Lagrangian relaxation and the subgradient method, aiming for a trade-off between the quality of scheduling and the computational complexity. Compared with existing Eulerian model for ATFM, we have made the following contributions: (1) an ATFM formulation is proposed with a novel flow dynamics model and more types of constraints, (2) a hierarchical heuristic algorithm is presented for solving a multiple-FIR ATFM problem to reduce the computational burden, (3) we also propose a structure to handle heterogeneous aircraft types.

The rest of the paper is organized as follows. An air traffic flow routing and scheduling (ATFRSP) problem is formulated in Section II. A hierarchical heuristic approach for solving the ATFRSP problem is proposed in Section III. Simulation results with a case study based on the air traffic data in the ASEAN region are shown in Section IV. Conclusions are drawn in Section V.

II. The Problem Statement

A. Introduction to an En-route Air Traffic System

In this paper we focus on a network-based en-route air traffic system, which is considered compatible with the Required Navigation Performance (RNP) flight navigation strategy. Unlike a sector-based model, aircraft in the system must follow predefined *links* to form columns. The connection of two consecutive links is a *vertex*. Different kinds of vertices are present in an air traffic system, e.g., waypoint, report point, aerodrome. To make notations simple, we use *waypoint* to denote all the *vertices* in the air traffic system. The components of such a system are shown in Fig. 1. In the current practice of the aviation industry, multiple Area Control Centers (ACCs) operate in parallel to control individual aircraft in their own responsible airspace (usually defined as Flight Information Regions, FIRs), whereas communicating with other neighboring ACCs to relay important messages. A sketch of this architecture is shown in Fig. 2, which consists of four major constituents: the FIR,

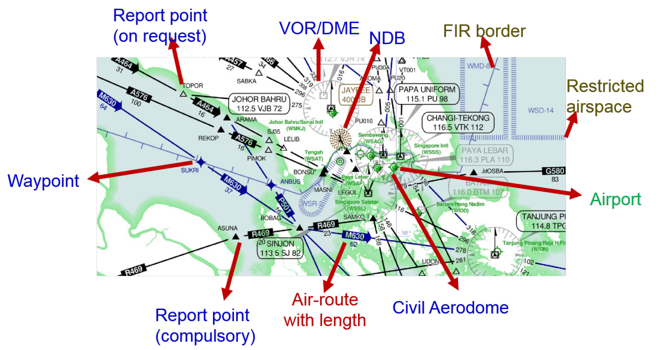

Fig. 1. An example of the en-route air traffic system (map downloaded from SkyVector, https://skyvector.com/)

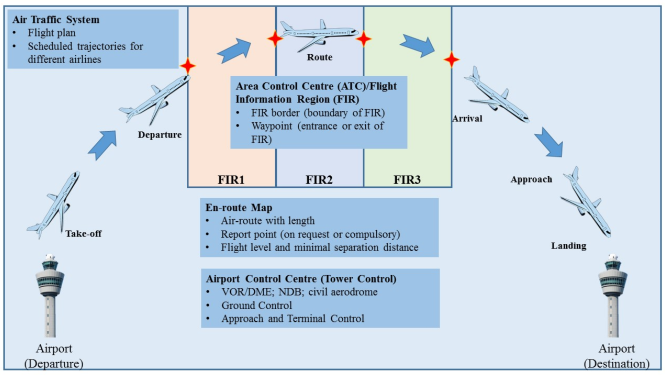

Fig. 2. An overview of Air Traffic Control System

waypoints, air links and the airport control center. To facilitate technical development, some notations are introduced below, which will be applied throughput the paper.

- $\mathbb{N}, \mathbb{R}^+$ – The sets of natural numbers and nonnegative real numbers, respectively.
- $G = (V, E)$ – The directed graph is to describe the air traffic network, where vertices and edges denote waypoints (together with all segmentation points) and air

links respectively. From now on, we will use link and segment interchangeably.

- $\mathcal{A}$ – The set of all concerned airports.
- $\phi \in \mathcal{F}$ – The set of aircraft types.
- $\underline{S}(\phi)$ – The lower speed limit of type ϕ aircraft.
- $\overline{S}(\phi)$ – The upper speed limit of type ϕ aircraft.
- Δ – The sampling interval.
- $H_p \subseteq \mathbb{N}$ – The index set of time discretization. By default, $H_p := \{1, 2, \cdots, |H_p|\}$. t is an integer time index such that $t \in H_p$. The notation H_p refers to the number of sampling points involved in the prediction horizon, which, by multiplying the sampling period, is equal to the length of prediction horizon.
- $\mathcal{C} := \mathcal{A} \times \mathcal{A}$ – The set of origin-destination (O-D) pairs. For each $P = (a, a') \in \mathcal{C}$, let $P[1] = a$ and $P[2] = a'$.
- $N_i^P(\phi, t) \in \mathbb{N}$ – The volume of link i, i.e., the number of aircraft, with type ϕ and O-D pair P at time interval t.
- $f_{ij}^P(\phi, t) \in \mathbb{N}$ – The flow of aircraft with type ϕ from link i towards link j with O-D pair P at time interval t. The flow describes the aircraft's movements within a certain time interval, captured by the number of aircraft passing through air links within that interval.

According to the ICAO Doc 4444 [27], a *flight plan* generally includes basic information such as departure and arrival points, estimated time en-route, alternate airports in case of bad weather, type of flight, the pilot's information, number of people on board and information about the aircraft itself. In this paper we only focus on routes and schedules, and define a flight plan simply as a finite sequence of tuples: $\{(d, t_d), (v_1, t_1), (v_2, t_2), \ldots, (v_{n-1}, v_n), (a, t_a)\}$, where $(d, a) \in \mathcal{C}$ is an O-D pair with the expected departure time $t_d \in \mathbb{R}^+$ and arrival time $t_a \in \mathbb{R}^+$, $v_i \in V$ for $i \in \mathbb{N} : 1 \leq i \leq n$, which represents the sequence of passing waypoints with the corresponding passing time $t_i \in \mathbb{R}^+$. An FIR is considered as a subgraph of G in this paper, where each waypoint can only belong to one FIR, and a link may be shared by two FIRs. From now on, given an FIR S, we use $V(S)$ to denote all waypoints and airports in S, and $\mathcal{L}(S)$ for all links, each of which has at least one end node in S. For a given air traffic network, assume that the set of all FIRs $\mathcal{S}$ is given. For each link $i \in E$, let L_i be the *length* of link i, $C_i(t)$ the *capacity* of link i at time interval t, which could be time-variantly affected by various factors such as the weather conditions. $U_i, D_i \subseteq E$ are denoted as the direct *upstream* link sets and *downstream* link sets for link i, which refer to the set of links whose ending (starting) waypoints are the same as the starting (ending) waypoint of link i, respectively.

In practice the airport control involves group operations such as tower control, approach control and terminal control. The flow dynamics in the civil aerodrome can be very complex owing to the safety requirements. Since the airport ground control is not our concern in this paper, we simply treat each airport as both a source and a sink of the concerned air traffic network, and denote it by two points in the network, i.e., the *departure* point, which supplies aircraft into the network, and the *arrival* point, which absorbs aircraft from the network. In this paper we assume that the aircraft handling capacities for departure and arrival flights in each airport are known in advance.

B. Air Traffic System and heterogeneous aircraft types

An air traffic system holds tremendous promise in applications ranging from search and rescue to agriculture, pipeline inspection, firefighting and freight delivery. Heterogeneous aircraft types with various cruising speeds such as small, medium and large passenger/cargo aircraft and unmanned aerial vehicles (UAVs) are involved in the air traffic system. We use the First-In-First-Out (FIFO) principle to describe the flow dynamics on each air link and the overtaking is strictly not allowed. Each O-D pair is associated with different aircraft characterized by different cruising speeds in the system. Then with these categories we put these O-D pairs into different flight levels so that they can obtain different cruising speeds as well as sampling times. In other words, aircraft with significantly different speeds will be permanently separated into disjoint traffic subnetworks and treated separately.

TABLE I
A TABLE WITH THE MAXIMUM CRUISING SPEED FOR DIFFERENT CRUISING ALTITUDES

Levels	ICAO Standard
Up to 6000ft inclusive	230 KT
Above 6000ft to 14000ft inclusive	230 KT
Above 14000ft to 20000ft inclusive	240 KT
Above 2000ft to 24000ft inclusive	265 KT
Above 24000ft to 34000ft inclusive	265 KT
Above 34000ft	M 0.83

A list of the maximum cruising speeds is shown in Table I. From this chart we can see that different flight levels (FLs) will absorb aircraft with different cruising speeds. We make some assumptions on this flight level determination procedure. We assume that (1) the aircraft of an identical type travel with similar cruising speeds; (2) different types of aircraft assigned to the same flight level share similar cruising speeds, which are determined by the flight level; (3) when the flight level increases, the expected cruising speed goes up. Note that for various flight levels, the minimal separation distance, or equivalently, the capacity of a specific air link, could be variant because of the cruising speed difference.

Some benefits we could gain from this heterogeneous setting include, firstly, flight levels introduce different cruising speeds, which leads to the possibility of taking heterogeneous aircraft types into consideration; secondly, calculations from different flight levels can run in parallel, which can reduce the computational complexity; thirdly, a trade-off between the accuracy and complexity can be achieved by tuning the time scale for different flight levels.

C. Air traffic flow dynamics model

Aircraft in the same sector must be well separated from each other to ensure safety. Each aircraft must keep a safe distance from other aircraft ahead, above, under or aside. In this paper we assume that the layout of the air traffic network has well taken care of the vertical and side-by-side separations. So we only focus on the head-and-tail separation within each link. The current practice usually enforces a separation of 5-50 nautical miles depending on the actual flying space, e.g., over land with good radar coverage or over sea with little radar coverage. This separation will result in specific link capacities. In this paper we mainly focus on how to handle a large number of aircraft within the network, thus, we adopt 5 nautical miles uniformly in our setup, although in principle the proposed framework can handle any specific separation distances, as long as they are known in advance.

We adopt a discrete-time cell transmission link dynamic model in this paper. The maximum and minimum traversing times, denoted as $\overline{T}(i,\phi)$ and $\underline{T}(i,\phi)$, respectively, required by a type-ϕ aircraft to fly through an air link i is determined by the minimum and maximum cruising speeds, denoted as $\underline{S}(\phi)$ and $\overline{S}(\phi)$), respectively, i.e.,

$$\underline{T}(i,\phi) = \frac{L_i}{\overline{S}(\phi)} \leq t \leq \frac{L_i}{\underline{S}(\phi)} = \overline{T}(i,\phi), \tag{1}$$

where t is the actual traversing time for the aircraft. To ensure that constraint 1 holds, we make the following assumption.

Assumption 1: $\lceil \underline{T}(i,\phi) \rceil \leq \lfloor \overline{T}(i,\phi) \rfloor$, where $\lceil\cdot\rceil$ and $\lfloor\cdot\rfloor$ denote the ceiling and floor functions, respectively. □

This assumption is rather mild and can be satisfied by reducing the discrete sampling time interval, or equivalently, increasing the sampling rate. For example, suppose the length of link i is $50NM$ and the speed limits for aircraft type ϕ is $[450NM/h, 500NM/h]$. If the sampling time is chosen as 1 hour, then

$$\lceil \underline{T}(i,\phi) \rceil = \left\lceil \frac{50}{500} \right\rceil = 1,$$
$$\lfloor \overline{T}(i,\phi) \rfloor = \left\lfloor \frac{50}{450} \right\rfloor = 0,$$

Assumption 1 is violated. However, if the sampling time is chosen as 6 minutes, then

$$\lceil \underline{T}(i,\phi) \rceil = \left\lceil \frac{50}{500} * \frac{60}{6} \right\rceil = 1,$$
$$\lfloor \overline{T}(i,\phi) \rfloor = \left\lfloor \frac{50}{450} * \frac{60}{6} \right\rfloor = 1.$$

Thus, *Assumption 1* can always hold at a cost of introducing more decision variables, owing to a high sampling rate.

The outgoing flow at time k from link i satisfies:

$$\begin{aligned} 0 &\leq \sum_{j\in D_i}\sum_{P\in\mathcal{C}} f_{ij}^P(\phi,t) \\ &\leq \sum_{j\in D_i}\sum_{P\in\mathcal{C}} \sum_{\lceil \underline{T}(i,\phi)\rceil \leq t \leq \lfloor \overline{T}(i,\phi)\rfloor} f_{ij}^P(\phi,k-t). \end{aligned} \tag{2}$$

The upper bound of the outgoing flow is formed by considering all possible flows arriving at the end of link i at time k, i.e., all aircraft entering during $\lceil \underline{T}(i,\phi)\rceil \leq t \leq \lfloor \overline{T}(i,\phi)\rfloor$ are able to leave link i at time k by adjusting their speeds within the lower and upper speed limits. The lower bound of the outgoing flow is the number of aircraft that must leave this air route at the end of time k. The following constraints provide more details on flow dynamics.

Let the mapping $\delta(k,\beta,i,\phi) \in [0,1]$ denote the portion of the air traffic flow consisting of aircraft type ϕ entering link i at time β and leaving the link at time k. Considering the minimum and maximum traveling times, the exiting time k should be within the interval $[\beta + \lceil \underline{T}(i,\phi)\rceil, \beta + \lfloor \overline{T}(i,\phi)\rfloor]$, i.e.,

$$\beta + \lceil \underline{T}(i,\phi)\rceil \leq k \leq \beta + \lfloor \overline{T}(i,\phi)\rfloor, \tag{3}$$

or

$$\lceil \underline{T}(i,\phi)\rceil \leq k - \beta \leq \lfloor \overline{T}(i,\phi)\rfloor, \tag{4}$$

Thus the value domain of the mapping $\delta(k,\beta,i,\phi) \in [0,1]$ is as follows,

$$\begin{aligned} &\delta(k,\beta,i,\phi) \in [0,1], \text{ if } \beta + \lceil \underline{T}(i,\phi)\rceil \leq k \leq \beta + \lfloor \overline{T}(i,\phi)\rfloor \\ &\delta(k,\beta,i,\phi) = 0, \text{ otherwise} \end{aligned} \tag{5}$$

Then the exiting flow of type ϕ aircraft from link i at time k, denoted as $f_{i,out}(\phi,k)$, can be described as follows:

$$\begin{aligned} &f_{i,out}(\phi,k) \\ &= \sum_{\beta:\lceil \underline{T}(i,\phi)\rceil \leq k-\beta \leq \lfloor \overline{T}(i,\phi)\rfloor} \delta(k,\beta,i,\phi) f_{i,in}(\phi,\beta), \end{aligned} \tag{6}$$

where $f_{i,in}(\phi,\beta)$ denotes all type-ϕ aircraft entering link i at time β. The mapping δ satisfies the following constraints.

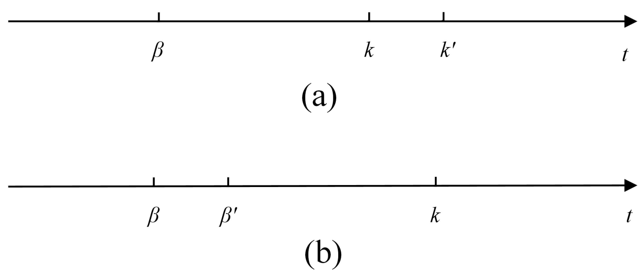

Fig. 3. Descriptions for Condition 1

Condition 1: (Weak FIFO condition)

(1-a) If a group of aircraft enters link i at time β, and one portion of the flow exits at time k and another portion exits at k', where $\beta + \underline{T}(i,\phi) \leq k, k' \leq \beta + \overline{T}(i,\phi)$, then

$$\beta < k < k' \Rightarrow \sum_{\beta\leq t\leq k} \delta(t,\beta,i,\phi) \leq \sum_{\beta\leq t'\leq k'} \delta(t',\beta,i,\phi), \tag{7}$$

namely the number of aircraft leaving link i from the same group is monotonically nondecreasing with respect to time. Since the portion of the air traffic flow $\delta(k,\beta,i,\phi) \in [0,1]$ has a non-negative value, the summation function of $\delta(k,\beta,i,\phi)$ is a monotonically nondecreasing function with respect to time t, i.e., if $\beta < k < k'$, then

$$\sum_{\beta\leq t\leq k} \delta(t,\beta,i,\phi) \leq \sum_{\beta\leq t\leq k'} \delta(t,\beta,i,\phi). \tag{8}$$

Thus this condition is redundant and could be removed.

(1-b) For two groups of aircraft entering link i at time β and β', respectively, if portions of these two flows exit at the same time k, then

$$\beta < \beta' < k \Rightarrow \sum_{\beta\leq t\leq k} \delta(t,\beta,i,\phi) \geq \sum_{\beta'\leq t'\leq k} \delta(t',\beta',i,\phi), \tag{9}$$

namely at any time t a higher percentage of aircraft leave link i from an earlier-entering group than a later-entering one. Conditions (1-a) and (1-b) are direct consequences of the FIFO condition. Nevertheless, they are not sufficient to ensure the FIFO condition in each air link, as the FIFO condition essentially says that no overtaking can take place among aircraft from different groups within a link. In reality, aircraft overtaking within one air link is quite rare, namely the FIFO condition holds most of the time. Thus, Condition 1 is normally valid. In case a user would like to explicitly model the condition of non-overtaking among different groups, the following strong FIFO condition can be used.

Condition 2: (Strong FIFO condition)

$$[\beta < \beta'] \wedge [\delta(k,\beta',i,\phi) > 0] \Rightarrow \sum_{t\le k} \delta(t,\beta,i,\phi) = 1, \quad (10)$$

namely, if aircraft from a group entering at β' starts to leave link i, then all aircraft in all earlier-entering groups must have already left link i.

Condition 3: (All exiting condition)

$$(\forall \beta \in H_p) \sum_{t} \delta(t,\beta,i,\phi) = 1, \quad (11)$$

namely, any aircraft entering a link will exit it eventually.

Proposition 1: Condition 2 and *3* imply *Condition 1*. □

Proof: For $\beta < \beta'$, we consider two cases.

For case 1, $\sum_{\beta'\le t\le k} \delta(t,\beta',i,\phi) = 0$, then clearly *Condition (1-b)* holds.

For case 2, $\sum_{\beta'\le t\le k} \delta(t,\beta',i,\phi) > 0$, then there exists at least one $\delta(t,\beta',i,\phi)$ greater than 0, we denote the non-zero $\delta(t,\beta',i,\phi)$ with the largest time interval t as $\delta(\hat{t},\beta',i,\phi)$. With *Condition 2* we know,

$$[\beta < \beta'] \wedge [\delta(\hat{t},\beta',i,\phi) > 0] \Rightarrow \sum_{t\le \hat{t}} \delta(t,\beta,i,\phi) = 1. \quad (12)$$

With *Condition 3* we know the maximum summation for $\delta(t,\beta,i,\phi)$ is 1. Thus the following inequality holds,

$$\sum_{\beta'\le t'\le \hat{t}} \delta(t',\beta',i,\phi) \le 1, \quad (13)$$

which shows *Condition (1-b)* holds. Thus *Proposition 1* holds. □

By *Prop. 1* we know that, up to different applications, either *Condition 1* or *Condition 2* can be applied, but not both.

D. Statement of air traffic flow routing and scheduling problem (ATFRSP)

1) Constraints: We consider the following constraints: the network dynamics constraints, the link capacity constraints, the air traffic flow limits constraints, as described below.

C1-Network dynamics constraints:

The network dynamics describe the relationship between the air traffic flows and the air link volumes. For all $t \in H_p$, $i \in E$, $\phi \in \mathcal{F}$, and $P \in \mathcal{C}$,

$$N_i^P(\phi,t+1) = N_i^P(\phi,t) + \left[f_{i,in}^P(\phi,t) - f_{i,out}^P(\phi,t)\right], \quad (14)$$

where $f_{in}^P(\phi,t)$ and $f_{out}^P(\phi,t)$ denote respectively the total incoming and outgoing air traffic flows of type ϕ aircraft in air link i with O-D pair P at time t. More explicitly,

$$f_{i,in}^P(\phi,t) = \sum_{j\in U_i} f_{ji}^P(\phi,t), \quad (15\text{-}1)$$

$$f_{i,out}^P(\phi,t) = \sum_{k\in D_i} f_{ik}^P(\phi,t). \quad (15\text{-}2)$$

C2-Link capacity constraints:

Due to the head-and-tail separation requirement imposed on all aircraft in the network, each link (or segment) has its own capacity. In general, different links may have different separation requirements, captured by a variable $m_{sep}(t)$, which also suggests that the separation distance is time-varying, due to possibly the time-varying weather conditions. For example, $m_{sep}(t)$=5 NM is common in en-route airspace, while $m_{sep}(t)$=3 NM is common in terminal airspace at lower altitudes. This time variant separation distance function is assumed known in advance in this paper for a routing and scheduling purpose, from which the link capacity can be defined as follows:

$$C_i(t) = \frac{L_i}{m_{sep}(t)}, \quad (16)$$

where $C_i(t)$ is the capacity of link i at t and L_i is the length of the link i. The total number of all types of aircraft in link i should not be greater than the link capacity at any time interval t, i.e.,

$$(\forall t \in H_p) \sum_{P\in\mathcal{C}} \sum_{\phi\in\mathcal{F}} N_i^P(\phi,t) \le C_i(t), \quad (17)$$

C3-Flow dynamics constraints:

As mentioned in the previous section, by *Condition 1* (or *Condition 2*) and *Condition 3*, the flow dynamics constraints are provided by Eqn. (6), (7)-(9) (or (10)) and (11). Note that in [26], a segment-based model is adopted, which aims to capture the wave-propagation-style air traffic flow dynamics on each air link, similar to the standard cell transmission model, but inevitably increases the computational complexity, which could be fatal for a real-time application. In this work, we replace this segment-based model with the FIFO model to describe the flow dynamics, which could significantly reduce the decision variables.

C4-Flow rate limit constraints:

From constraint (2) the exit flow rate in each link is constrained by the minimum and maximum possible flows arriving at the end of the link during each time interval, which is related to the cruising speed limits and the length of the link.

C5-Flight density limit constraints:

The density of the merging air flows from the upstream links should be no larger than the entrance density of the downstream link. Given a link $i \in E$, we have the following:

$$\sum_{j\in\mathcal{U}_i} \sum_{\phi\in\mathcal{F},P\in\mathcal{C}} \frac{f_{ji}^P(\phi,t)}{L_j} \le \frac{C_i(t)}{L_i}, \quad (18)$$

where $C_i(t)/L_i$ denotes the entering air traffic flow density of link i and $f_{ji}^P(\phi,t)/L_j$ denotes the exiting air traffic flow density from link j to link i, where $j \in \mathcal{U}_i$ shows that link j belongs to the direct upstream link set of link i

2) Objective function: Our objective is to minimize the total deviation from the original arrival and departure schedules as well as the chances of landing in airports different from the originally planned ones. Based on the aforementioned functions, the objective function can be formulated below:

$$\sum_{t \in H_p, \phi \in \mathcal{F}, P \in \mathcal{C}} \Bigg\{ \sum_{a \in A} \left[f_{a,in}^{P}(\phi, t) - r_a^P(\phi, t) \right]^2 + \sum_{a \in A} \left[f_{a,out}^{P}(\phi, t) - s_a^P(\phi, t) \right]^2 + \sum_{a \in A: P[2] \neq a} M f_{a,in}^{P}(\phi, t) \Bigg\} \quad (19)$$

The first term in the cost function denotes the total deviation from the original arrival schedules, and the second term for the total deviation from the original departure schedules. $r_a^P(\phi, t)$ and $s_a^P(\phi, t)$ denote the scheduled arrival and departure air traffic flow rate, respectively. The third term M is a very large positive constant, denoting the extremely high penalty on landing an aircraft to an airport different from their originally planned destination.

We now summarize what we have developed so far and state the Air Traffic Flow Routing and Scheduling Problem (ATFRSP) below, the decision variables in this problem are the traffic flow assignments, i.e., $f_{i,in}^P(\phi, t)$ and $f_{i,out}^P(\phi, t)$.

Problem 1: **Air Traffic Flow Routing and Scheduling Problem (ATFRSP)**

$$\begin{aligned}
&\text{minimize } (19) \\
&\text{subject to} \\
&\qquad \text{C1-Network dynamics } (14), (15-1), (15-2) \\
&\qquad \text{C2-Link capacity } (17) \\
&\qquad \text{C3-Flow dynamics } (6), (7)-(9) \text{ and } (11) \\
&\qquad\qquad \text{or } (6), (10) \text{ and } (11) \\
&\qquad \text{C4-Flow limits } (2) \\
&\qquad \text{C5-Density limits } (18)
\end{aligned} \quad (20)$$

Problem 1 is a non-linear programming problem as the constraints on the flow dynamics are described by linear constraints ((2), (14), (11), (15-1), (15-2), (17) and (18)), quadratic constraints / objectives ((6) and (19)) and logic constraints ((7)-(9) and (10)). To convert the problem formulation into a mixed integer programming format, the logic constraints are required to transferred into mixed integer constraints by introducing some binary variables via transformation techniques specified in [28].

As an example, we show the conversion of constraint (10). By introducing some binary variables, it could be transferred to the mixed integer linear form as follows,

$$(\beta - \beta') \geq \varepsilon + (m_1 - \varepsilon)\delta_1 \quad (21\text{-}1)$$
$$-\delta(k, \beta', i, \phi) \geq \varepsilon + (m_2 - \varepsilon)\delta_2 \quad (21\text{-}2)$$
$$-\delta_1 + \delta_3 \leq 0 \quad (21\text{-}3)$$
$$-\delta_2 + \delta_3 \leq 0 \quad (21\text{-}4)$$
$$\delta_1 + \delta_2 - \delta_3 \leq 1 \quad (21\text{-}5)$$
$$\delta_3 + \delta_4 \leq 0 \quad (21\text{-}6)$$
$$\sum_{t' \leq k} \delta(t, \beta, i, \phi) + \delta_3 \leq 0 \quad (21\text{-}7)$$
$$\delta_3 + \sum_{t \leq k} \delta(t, \beta, i, \phi) - \delta_4 \leq 1 \quad (21\text{-}8)$$
$$\delta_1, \delta_2, \delta_3, \delta_4 \in \{0, 1\} \quad (21\text{-}9)$$

Thus, Problem 1 can be equivalently converted into a mixed integer quadratic programming problem, which is NP-hard to solve. Note that formulation (20) is a general formulation for ATFRSP. Uncertainties could be added to this formulation, e.g., by changing the link capacities $C_i(t)$ or the lower and upper bounds for traveling times, i.e., t_{lb} and t_{ub} as in [29], owing to the presence of weather uncertainty, departure time deviations, speed deviations, etc.

III. A Hierarchical Heuristic Approach for Air Flow Routing and Scheduling Problem

A. A Hierarchical Structure for ATFRSP

1) The motivation behind a hierarchical structure: Owing to the NP-hardness of Problem 1, in this section we present a hierarchical heuristic algorithm in order to seek an approximate solution for this problem efficiently. Fig. 4 [30] shows the FIRs all around the world. Most domestic flights around the world are only transferred in one or a few connected FIR(s) and this feature can be adopted to reduce the computational complexity for local air traffic systems.

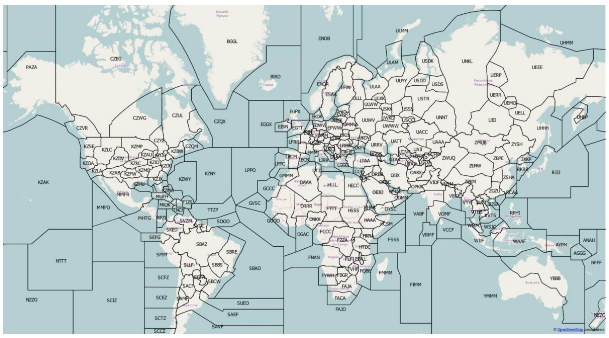

Fig. 4. FIRs around the world

As an example, assume that we have an air traffic system with four FIRs, i.e., $\mathcal{S}_1$, $\mathcal{S}_2$, $\mathcal{S}_3$ and $\mathcal{S}_4$ as shown in Fig. 5. We can first group them into different subsets, i.e., W_1 to W_5 and then put them into a hierarchical structure, where subsets with few FIRs are put in lower levels and subsets with more FIRs are put higher. Flight plans which cover specific FIRs will be solved within corresponding subsets, which will

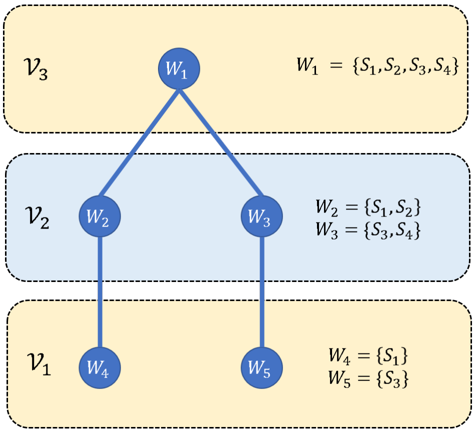

Fig. 5. An example of the hierarchical structure

reduce the computation complexity for solving the ATFRSP. A rigorous definition of the hierarchical structure is proposed in the following section.

2) The definition of a hierarchical structure: Recall that an air traffic network can be modeled by a directed graph $G = (V, E)$. Given two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, we say G_1 is a *subgraph* of G_2, denoted as $G_1 \subseteq G_2$, if $V_1 \subseteq V_2$ and $E_1 \subseteq E_2$. The union of G_1 and G_2 is defined as $G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2)$. Each subgraph of G can be considered as one *region*. Considering that each FIR is a specified region of airspace in which a flight information service and an alerting service (ALRS) are provided, and it is the largest regular division of airspace in use in the world today, we consider each FIR as our "atomic" region and develop a hierarchical architecture based on the collection of all given FIRs, which is denoted as $\mathcal{S}$. Nevertheless, the theory itself can certainly handle regions which are either bigger or smaller than FIRs. We assume that the network G is partitioned into FIRs. A directed graph is *connected* with respect to an O-D pair $P = (v_1, v_2) \in \mathcal{C}$ if there exists at least one directed path in the graph from v_1 to v_2. Given a set S we use $|S|$ to denote its cardinality. For each O-D pair $P \in \mathcal{C}$, let $\mathcal{S}_P \subseteq \mathcal{S}$ be a minimal set of FIRs such that $\cup_{S \in \mathcal{S}_P} S$ is connected with respect to P and for any other $\mathcal{S}'_P \subseteq \mathcal{S}$ with $\cup_{S' \in \mathcal{S}'_P} S'$ is connected with respect to P, we have $|\mathcal{S}_P| \leq |\mathcal{S}'_P|$.

Given the set $\Omega(\mathcal{C}) := \{\mathcal{S}_P | P \in \mathcal{C}\}$, a *hierarchical structure* with respect to $\Omega(\mathcal{C})$ is a directed graph $\mathcal{H}(\mathcal{C}) := (V_\mathcal{H}, E_\mathcal{H})$, where $V_\mathcal{H} = \Omega(\mathcal{C})$, i.e., the set of minimal sets of FIRs related to O-D pairs, and $E_\mathcal{H} \subseteq V_\mathcal{H} \times V_\mathcal{H}$ such that for all $(W, W') \in V_\mathcal{H} \times V_\mathcal{H}$, $(W, W') \in E_\mathcal{H}$ if and only if the following two conditions hold:

- $W' \subseteq W$, [Set Containment]
- $(\forall W'' \in V_\mathcal{H})\, (W'', W') \in E_\mathcal{H} \Rightarrow W'' \nsubseteq W$, namely if $(W, W') \in E_\mathcal{H}$ then there cannot exist another set $W'' \in V_\mathcal{H}$ such that $W' \subsetneq W'' \subsetneq W$.

For each $(W, W') \in E_\mathcal{H}$, W is called a *parent node* of W', and W' is the *child node* of W. Note that the definition of the minimal set $\mathcal{S}_P$ for each $P \in \mathcal{C}$ involves a heuristic choice, which restricts a possible route for P to the corresponding FIRs in $\mathcal{S}_P$. In reality, there usually exists more than one such choice. Nevertheless, such a heuristic choice for each O-D pair P can significantly reduce the search space for online air flow routing and scheduling, which will be described shortly. Owing to the set containment relation specified above, the nodes of $\mathcal{H}(\mathcal{C})$ can be organized in layers, where all sets in each layer are incomparable with respect to that set containment relation, and for each node W in one layer, whose upper layer is not empty, there must exist one node W' in the upper layer such that $(W', W) \in E_\mathcal{H}$. In other words, edges are defined only among nodes sitting in different layers. Owing to this graphical nature, we call $\mathcal{H}(\mathcal{C})$ a hierarchical structure. For each subset $\tilde{V} \subseteq V_\mathcal{H}$, let $\subseteq^*_{\tilde{V}}$ be the finite transitive extension of the set containment relation over $\tilde{V}$, where for all $W, W' \in V_\mathcal{H}$, $W \subseteq^*_{\tilde{V}} W'$ if there exist a finite sequence of nodes $W_1, W_2, \cdots, W_n \in \tilde{V}$ such that $W \subseteq W_1$, $W_i \subseteq W_{i+1}$ for $i \in \{1, \cdots, n-1\}$, and $W_n \subseteq W'$. Let $R : \tilde{V} \to 2^{\tilde{V}}$ be a map, where for all $W \in \tilde{V}$, $R(W) := \{W' \in \tilde{V} | W \subseteq^*_{\tilde{V}} W'\}$. We now define a binary relation $\simeq_{\tilde{V}} \subseteq \tilde{V} \times \tilde{V}$ such that $W \simeq W'$ iff there exist a sequence $W_1, \cdots, W_n \in \tilde{V}$ such that $R(W) \cap R(W_1) \neq \varnothing$, $R(W_i) \cap R(W_{i+1}) \neq \varnothing$ for $i \in \{i, \cdots, n-1\}$, and $R(W_n) \cap R(W') \neq \varnothing$.

Proposition 2: $\simeq_{\tilde{V}}$ is an equivalence relation. □

For each $W \in \tilde{V}$ let $[W]_{\tilde{V}} := \{W' \in \tilde{V} | W \simeq_{\tilde{V}} W'\}$. Let $\mathcal{L}(\mathcal{H}) \subseteq 2^{V_\mathcal{H}}$ be the collection of all layers of $\mathcal{H}(\mathcal{C})$. For each layer $\mathcal{V} \in \mathcal{L}(\mathcal{H})$, let $\mathcal{P}(\mathcal{V}) := \{[W]_\mathcal{V} | W \in \mathcal{V}\}$ be the partition of the layer $\mathcal{V}$ based on the equivalence relation $\simeq$.

Referring to the example shown in Fig. 5, assume that $\mathcal{S} = \{S_1, S_2, S_3, S_4\}, \Omega(\mathcal{C}) = \{W_1 = \{S_1, S_2, S_3, S_4\}, W_2 = \{S_1, S_2\}, W_3 = \{S_3, S_4\}, W_4 = \{S_1\}, W_5 = \{S_3\}\}$. The hierarchical structure $\mathcal{H}(\mathcal{C})$ is depicted as shown in Fig. 5.

Clearly, we have $(W_2, W_4), (W_3, W_5), (W_1, W_2), (W_1, W_3) \in E_\mathcal{H}$. But $(W_1, W_4) \notin E_\mathcal{H}$ because there exists $W_2 \in V_\mathcal{H}$ such that $W_4 \subseteq W_2 \subseteq W_1$. The layered structure is clearly seen in the picture, where in each layer, each coset of the equivalence relation $\simeq$ is a singleton, i.e., no two nodes in each layer are related to each other.

3) A hierarchical approach for solving ATFRSP: With such a hierarchical structure, we propose the following air flow routing and scheduling strategy. The calculation will proceed in a bottom-up manner. Starting from the bottom layer, for each layer $\mathcal{V} \in \mathcal{L}(\mathcal{H})$, for each $W \in \mathcal{V}$, we heuristically assign a sequential order on nodes in $[W]$. In our approach, the nodes in $[W]$ with smaller index of the layer get higher priority in calculation and the nodes in $[W]$ with the same index of the layer are sequenced by their calculation complexity from higher to lower, where the calculation complexity is determined by the total number of decision variables and constraints. Without loss of generality, assume that $[W] = \{W_1, W_2, \cdots, W_i = W, W_{i+1}, \cdots, W_n\}$ for some $n \in \mathbb{N}$. In addition, assume that $Q(W)$ is the collection of children nodes of W. Computation on node W will be triggered only after all calculations from its children nodes and nodes preceding it in $[W]$ are finished. In other words, the earliest starting time for processing W is determined by the longest calculations time on its children nodes and the sum of the total computation times for those nodes preceding W in $[W]$. The nodes will send their flow management results to W, which are reflected in the updated link capacities in W. More explicitly, suppose link j is an air link in W, the capacity of link j at time interval t in the layer $\mathcal{V}$ is determined as follows,

$$\tilde{C}_j(t) = C_j(t) - \sum_{p \in Q(W) \cup \{W_1, \cdots, W_{i-1}\}} N_p(t). \quad (22)$$

Recalling the proposed ATFRSP formulation, C2-Link capacity constraints and C5-Flight density limit constraints are related to the link capacities, which are required to be updated first before subsequent flow routing and scheduling computation on nodes in W can be carried out.

B. A Hierarchical Heuristic Algorithm for solving the ATFRSP

With this hierarchical structure, we use a heuristic approach to solve the proposed ATFRSP problem and generate the flight plans. Firstly, the sub-problems in lower levels are solved in parallel. Secondly, the air traffic flow management in the lower levels will be fixed and the related link capacities will be updated and sent to the higher level. Then the computations will be taken at higher levels and finally reached the root node. The aforementioned hierarchical flow routing and scheduling approach is summarized in **Algorithm 1** below:

Algorithm 1 A hierarchical heuristic algorithm for ATFRSP

Step 1: Initialization

1) Input: the network $G = (V, E)$ and all O-D pairs $\mathcal{C}$;
2) Construct the hierarchical structure $\mathcal{H}(\mathcal{C})$. Assume that the list of layers is $\{\mathcal{V}_1, \cdots, \mathcal{V}_n\}$, where $\mathcal{V}_1$ is the bottom layer and $\mathcal{V}_n$ is the top one.

Step 2: Bottom-up iteration over layers: $i = 1, \cdots, n$

1) For each cluster $\kappa \in \mathcal{P}(\mathcal{V}_i)$, and for each node $W \in \kappa$, check whether all its children nodes and preceding nodes in κ have done the computation. If not yet, wait. Otherwise, update the link capacities with (22), and solve the ATFRSP problem in (20) for W, and send updating information to other suffixing nodes in κ;
2) If $i = n$, the calculation is terminated; otherwise, each node in $\mathcal{V}_i$ sends its updating information to its parent node in $\mathcal{V}_{i+1}$, set $i := i + 1$. Jump to **Step 2.1**. □

In this bottom-up hierarchical approach, computation among nodes in each layer, which are not related to each other with respect to that equivalence relation $\simeq$, can be carried out in parallel, which can significantly speed up the overall computation. Nevertheless, for some sub-problems the computational burdens are still heavy. This can be alleviated by adopting some distributed computational approaches such as the Lagrangian multiplier method proposed in our previous work [26], which also utilizes metaheuristics or incremental optimization methods to speed up computation in the relevant nodes of each layer in the hierarchy.

C. An Incremental Approach for Flight Plan Generation

In Sections II and III, we propose the problem formulation for ATFM and solve it via a hierarchical heuristic algorithm. Till now, what we have achieved is the flow information. To obtain the flight plan (or the trajectory) for each individual aircraft, we adopt an incremental strategy and dispatch the flight in a sequence based on the flow information. Assume that we have the flow routing and scheduling results obtained from **Algorithm 1**. With the consideration of the travel distance for each aircraft as well as some extra constraints specified by airlines, an optimization problem can be built to describe this procedure.

The flow assignment on each link can be denoted as a matrix shown below:

$$\tilde{\mathbf{F}} = \begin{matrix} & \\ \text{link 1} \\ \text{link 2} \\ \vdots \\ \text{link m} \end{matrix} \begin{matrix} \begin{matrix} 1 & 2 & \cdots & k \end{matrix} \\ \begin{bmatrix} \tilde{f}_{11} & \tilde{f}_{12} & \cdots & \tilde{f}_{1k} \\ \tilde{f}_{21} & \tilde{f}_{22} & \cdots & \tilde{f}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{f}_{m1} & \tilde{f}_{m2} & \cdots & \tilde{f}_{mk} \end{bmatrix} \end{matrix}, \quad (23)$$

where $1, 2, \cdots, k$ are the time slots, and link i is the air route i. $\tilde{f}_{ik}$ is the flow assignment of flights in link i at time slot k, which is the result obtained from Algorithm 1. Suppose each aircraft $\mathbf{F}_i$ corresponds to a space-time matrix to denote its location at associated time, and the total number of flights is denoted as n.

$$\mathbf{F}_i = \begin{matrix} & \\ \text{link 1} \\ \text{link 2} \\ \vdots \\ \text{link m} \end{matrix} \begin{matrix} \begin{matrix} 1 & 2 & \cdots & k \end{matrix} \\ \begin{bmatrix} \delta_{11} & \delta_{12} & \cdots & \delta_{1k} \\ \delta_{21} & \delta_{22} & \cdots & \delta_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \delta_{m1} & \delta_{m2} & \cdots & \delta_{mk} \end{bmatrix} \end{matrix}, \quad (24)$$

where δ_{ij} is a binary variable for each flight. If the flight is at link i during time slot k, then $\delta_{ik} = 1$, otherwise, $\delta_{ik} = 0$. Our goal is to determine these binary numbers, δ_{ik}, for all aircraft $\mathbf{F}_i$ to generate the flight trajectory.

Some conditions are required to be added to this $\mathbf{F}_i$ matrix. Firstly, the summation of each column vector in $\mathbf{F}_i$ should be no more than 1, which means at most one link could be selected at each time interval.

$$(\forall j \in [1, k]) \sum_{i=1}^{k} \delta_{ij} \leq 1 \quad (25)$$

Secondly, all the columns with non-zero summation should be adjacent with each other. Thirdly, each two adjacent column with non-zero summation should satisfy the link connection constraints which are shown in the following part.

The flight trajectories are required to be connected subgraphs regarding the whole air traffic network. The link connection constraints are shown as follows. Based on the Laplacian matrix of the air traffic network, we generate a set of mapping from the air link set E to the binary set $\{0, 1\}$, which is denoted as $\mathcal{M}_i$, $i \in E$.

$$\mathcal{M}_i : \{k | k \in E\} \rightarrow \{0, 1\} \quad (26)$$

If link k is in the downstream link set of link i, $\mathcal{M}_i(k) = 1$, otherwise $\mathcal{M}_i(k) = 0$. With this mapping, the link connection constraints can be described as follows, $\forall j + 1$,

$$[\delta_{ij} = 1] \wedge [\delta_{k(j+1)} = 1] \Rightarrow \mathcal{M}_i(k) = 1, \quad (27)$$

where δ_{ij} means at time interval j the link i is selected for the trajectory and $\delta_{k(j+1)}$ means at time interval $j+1$ the link k is selected. This condition implies that for any two adjacent time intervals, the selected link for the later time interval must be in the downstream set of the link for the former time interval.

In the dispatching process, we must guarantee flow assignment constraints and traveling distance constraints which are related to the fuel consumption constraints in the real-world applications. We denote the upper bound of traveling distance for each flight $\mathbf{F}_i$ as $\tilde{T}_i$. Denote the length for each air link in a vector, i.e, L, then the traveling distances of flights can be described as:

$$\mathbf{f} := \begin{bmatrix} ||f_1'L||_1 \\ ||f_2'L||_1 \\ \vdots \\ ||f_n'L||_1 \end{bmatrix} \leq \begin{bmatrix} \tilde{T}_1 \\ \tilde{T}_2 \\ \vdots \\ \tilde{T}_n \end{bmatrix} := \tilde{T}_L \tag{28}$$

Then our optimization problem can be captured by:

$$\text{minimize} \quad D = \sum_{i=1}^{n} trace(\mathbf{F}_i^T \mathbf{F}_i) \tag{29-1}$$

subject to

(1) Flow constraint:

$$\sum_{i=1}^{n} \mathbf{F}_i \preceq \tilde{\mathbf{F}} \tag{29-2}$$

(2) Link connection constraint:

$$[\delta_{ij} = 1] \wedge [\delta_{kj'} = 1] \Rightarrow \mathcal{M}_i(k) = 1 \tag{29-3}$$

(3) Traveling distance constraint:

$$\mathbf{f} \leq \tilde{T}_L \tag{29-4}$$

In this problem formulation, the objective function (29-1) is designed to minimize the number of occupied links for each aircraft while the flow constraint (29-2), which is referring to the obtained air traffic flow information, is to ensure the generated flight plans could meet the flow information. The link connection constraint (29-3) and the traveling distance constraint (29-4) are to guarantee the physical feasibility of the generated flight plans. To this end, we apply a heuristic flight plan generation strategy, shown below in **Algorithm 2**.

In **Algorithm 2**, we adopt the flow assignment information generated by the hierarchical heuristic algorithm and then generate the flight trajectory information based on an incremental approach. We release the flight one by one based on the time sequence, i.e., First-Come-First-Serve principle, and reduce the corresponding flow value on the air-routes passing by this flight with consideration of the capacity constraints and flow assignments. The feasibility of the solutions obtained by incremental strategies are guaranteed since we could always use ground holding approaches to delay the flights.

D. Comments on the hierarchical heuristic approach for solving ATFRSP

As a summary of this technical development, some comments on the proposed hierarchical heuristic algorithm are listed in this section.

Algorithm 2 Obtaining individual flight plans

1) Input: the network G and a list of aircraft with their O-D pairs, and expected departure-arrival times. For each link and airport, obtain the outgoing flow rate in each time interval from **Algorithm 1**.
2) At each airport, for each O-D pair, create a FIFO queue of aircraft according to their expected departure time.
3) Iteration:
   a) Pick one airport and one O-D pair, whose queue is not empty, release the first aircraft in the queue and determine its route and schedule based on the remaining link capacities and outgoing flow rates associated with that O-D pair by the First-Come-First-Serve principle.
   b) Update the corresponding link capacities and outgoing flow rates associated with the O-D pair by reducing one, if the concerned aircraft goes through those links at relevant time intervals.
   c) Continue Step (3.a) until all queues are empty.
4) Output the resulting aircraft routes and schedules. □

1) *"Zoom In"/"Zoom Out" technology*. When solving the ATFRSP with a "small-scale" system, e.g., an air traffic system which only involves one or two FIRs, it looks like we *zoom in* to the map and solve a small-scale system with a large number of flights, and when solving the ATFRSP with a "large-scale" system, e.g., the intercontinental air traffic network, it looks like we *zoom out* of the map and solve a large-scale system with a small number of flights. This *"Zoom In"/"Zoom Out" technology* provides the flexibility to multiple scale systems with properly managed computational complexity.
2) *The Priority list for solving ATFRSP*. In this architecture, we first solve the ATFRSP with the smallest scale air traffic system, then fix the solution and solve a larger scale system, which means we give the ATFRSP with the smallest scale system the highest priority. In practice, this sequence means that we give the highest priority to the local flights.
3) *User-defined priority*. In real applications, the user may not want to arrange the flights with this priority list. For example, the air traffic controller would like to give the highest priority to the international flights to reduce the impact on other adjacent air traffic systems. In this case, we solve the ATFRSP problem in a bottom-up manner. Actually, the proposed hierarchical heuristic algorithm provides the flexibility of the priority assignment to the users, i.e., the user can decide the priority list for the subsystem and solve the ATFRSP for those subsystems based on the predefined priority. Thus, this methodology could be adopted under different ATC requirements.
4) *The temporal and spatial partition*. In the proposed algorithm, we divide the air traffic system based on spatial decomposition. However, some temporal partitions could further reduce the computational complexity. For

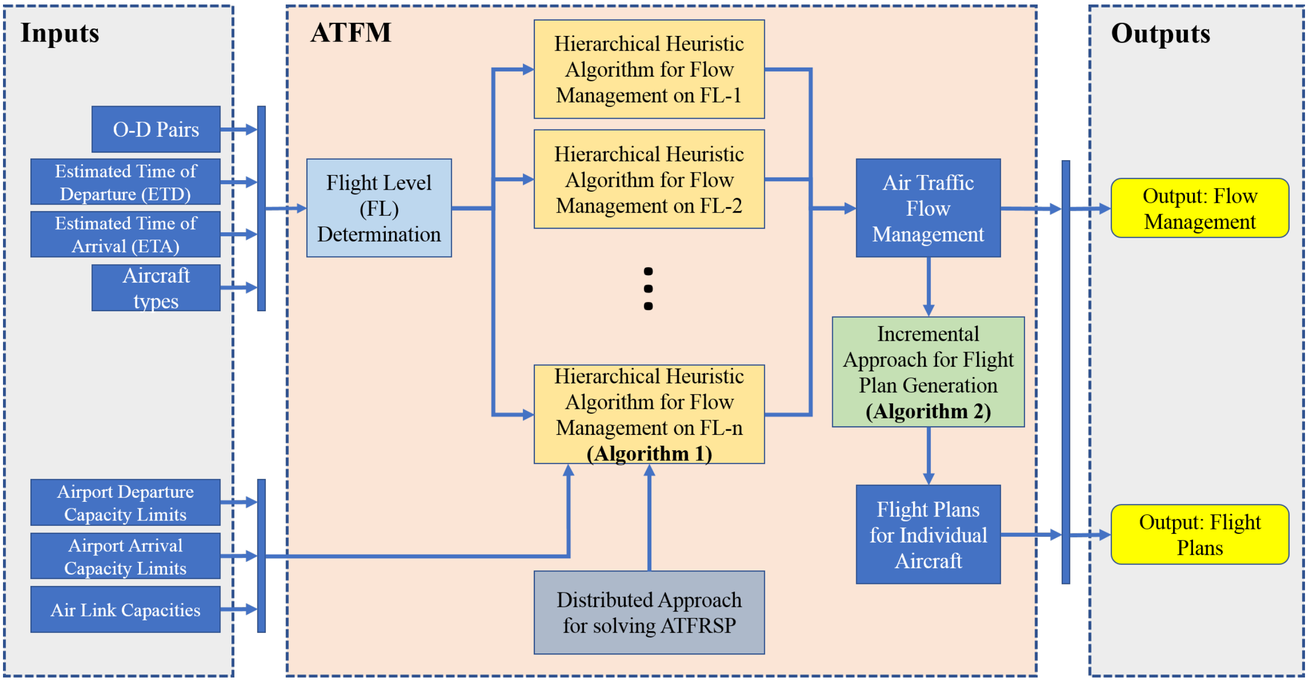

Fig. 6. The whole procedure for the proposed algorithm for solving ATFRSP

example, in a single receding horizon, it may not require to calculate all the flights in this region, because only a subset of the total flights exist in this time horizon.

E. Air Traffic Flow Management System Design

As a summary, the whole methodology for solving this ATFM problem is shown in Fig. 6. The input data for solving this ATFM problem include the flight information which is obtained from the air traffic planner and the air traffic operating information which is obtained from the air traffic operators. The flight information includes the O-D pair information, the estimated time of departure (ETD), the estimated time of arrival (ETA) and the type information, while the air traffic operating information includes the estimated capacities for airport and air links. All these data are collected and sent to the ATFM system to generate the flow assignment as well as the flight trajectory information. The ATFM problem is solved with the proposed hierarchical heuristic algorithm (Algorithm 1) with consideration of the uncertainties in the airports' and air links' capacities. The flight trajectory information will be generated from the ATFM results based on the incremental approach which is shown in Algorithm 2. The final outputs from this system include the ATFM information which could be used to analyze the effectiveness of the system utilization and flight plans or trajectory information for air traffic control service.

IV. Simulation Results

In the first case study, we test the proposed hierarchical heuristic algorithm for solving ATFRSP with an air traffic grid to show the computational efficiency. In the second and third case studies, we utilize the ASEAN air traffic network to generate our air traffic scheduling results and adopt them on a commercial simulation platform to show the computational efficiency and the performance enhancement.

A. Case Study I: Air Traffic Grid

A small-scale air traffic grid is utilized and the structure is shown in Fig. 7. A total of 16 airports and 80 waypoints are included in this system. We define 48 regional O-D pairs and 12 global O-D pairs for this system. The problem is solved by *Gurobi* [31] in *MATLAB (R2015a)* [32] and the results are shown below.

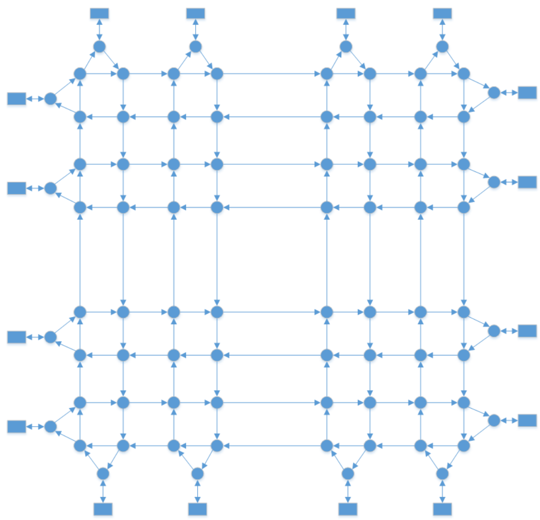

Fig. 7. A small-scale air traffic system

A comparison on the processing time for solving the MIQP problem with or without the proposed hierarchical heuristic approach with different prediction horizons is shown in Table II.

TABLE II
A COMPARISON ON PROCESSING TIME FOR SOLVING MIQP WITH/WITHOUT HIERARCHICAL HEURISTIC APPROACH

Length of Prediction Horizon	Average Processing Time Without Hierarchical Heuristic Approach	Average Processing Time With Hierarical Heuristic Approach
$H_p = 9$	0.627s	0.740s
$H_p = 10$	0.854s	0.816s
$H_p = 11$	5.600s	1.675s
$H_p = 12$	22.811s	2.106s
$H_p = 13$	472.953s	5.259s

TABLE III
A COMPARISON ON NETWORK-WISE FLIGHT DEVIATION FOR SOLVING MIQP WITH/WITHOUT HIERARCHICAL HEURISTIC APPROACH

Length of Prediction Horizon	Network-wise Flight Deviation Without Hierarchical Heuristic Approach	Network-wise Flight Deviation With Hierarchical Heuristic Approach	Differences between two deviations (%)
$H_p = 9$	1121	1208	7.76%
$H_p = 10$	1297	1336	3.01%
$H_p = 11$	1375	1464	6.47%
$H_p = 12$	1531	1625	6.14%
$H_p = 13$	1542	1610	4.41%

We run the simulation for 20 times and calculate the average processing time and the flight deviations for each case, as shown in Table II and III. From Table II, when the length of prediction horizon is short enough, the average processing times for solving the ATFRSP with or without hierarchical heuristic approach are similar, i.e., as the cases with $H_p = 9$ to $H_p = 11$. When the length of prediction horizon increases, the average processing time for the procedure without hierarchical heuristic approach will increase significantly. However, the average processing time for the hierarchical heuristic approach still remains in a reasonable range. In other words, if the partitions are selected properly, the computational complexity will be kept in an acceptable range and the ATFRSP can be solved to suit real-time applications. From Table III, the number of total departure and arrival deviations are listed, which indicate that the performance difference between the outcomes of the approaches with or without using the hierarchical heuristic algorithm are close to each other, and the largest difference is 7.76%. This suggests that the proposed approach has achieved a tremendous gain in reducing computational complexity with an acceptable degree of quality degradation.

B. Case Study II: The ASEAN air traffic System

In this case study, we test the proposed hierarchical heuristic algorithm for solving the ATFRSP problem in an air traffic network within the Association of Southeast Asian Nations (ASEAN) region which includes four FIRs, i.e., Bangkok FIR (BKK), Kuala Lumpur FIR (KL), Kota Kinabalu FIR (KK) and Singapore FIR (SIN), as shown in Fig. 8.

The real-world flight information shows that a great number of flights only travel within a single FIR and only about one

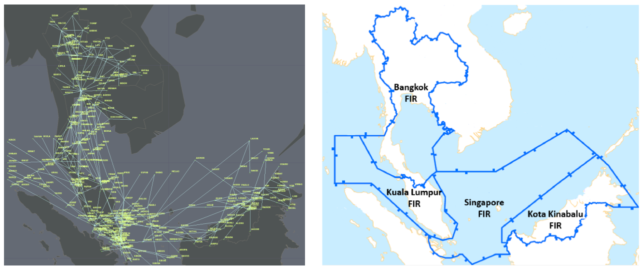

Fig. 8. A Partition of the ASEAN air traffic System with four FIRs

TABLE IV
PROCESSING TIME FOR THE HIERARCHICAL HEURISTIC ALGORITHM FOR SOLVING ATFRSP IN ASEAN REGION

FIRs	Number of Flight Plans	Processing Time
BKK, KL, SIN, KK	1	0.06s
BKK, KL, SIN	14	22.10s
KL, SIN, KK	26	44.96s
BKK, KL	19	92.71s
KL, SIN	7	1.25s
SIN, KK	6	0.32s
BKK	78	385.23s
KL	27	63.62s
KK	28	24.55s

- BKK, Bangkok FIR; KL, Kuala Lumpur FIR; SIN, Singapore FIR; KK, Kota Kinabalu FIR.

third of flights will travel among multiple FIRs. For example, there are a total number of 206 O-D pairs in these four FIRs in the ASEAN region. However, 78 O-D pairs only travel in BKK FIR, 27 O-D pairs only travel in KL FIR and 28 flight plans only travel in KK FIR. Only 73 O-D pairs cover these four FIRs. Among those 73 O-D pairs, we find that 32 O-D pairs cover two FIRs, 40 O-D pairs cover three FIRs and only one flight plan covers four FIRs. The hierarchical structure for this system is shown in Fig. 9. Four levels with nine nodes are involved in this system. We test a total of 1164 flights departing and arriving in this four FIRs in one-day time period. A total of 45 airports, 147 waypoints and 484 air-routes are involved in this air traffic system. The sampling time is set to be 6 minutes and a total of 240 time intervals are involved. The processing time with the flight plans involved for each part is shown in Table IV. The bottlenecks in the calculation are marked and shown in Fig. 9.

From Table IV, we can see that the processing time for solving ATFRSP in four FIRs only takes about 0.06s since only one O-D pair is transferred in this network. The processing time for solving this problem is about 9 minutes which is suitable for real-time applications and we notice that more than half of the processing time is spent on solving the ATFRSP in BKK FIR which includes the 78 O-D pairs, 58 waypoints, 210 air-routes and 3144960 decision variables. A comparison between the proposed model and the segment-based model in [26] on the computational complexity is shown in Table V. Regarding the proposed model, we show the highest computational complexity from all the subsystems Table V shows that the newly proposed model could reduce the computational complexity significantly. Some possible approaches to solve this issue include (1)

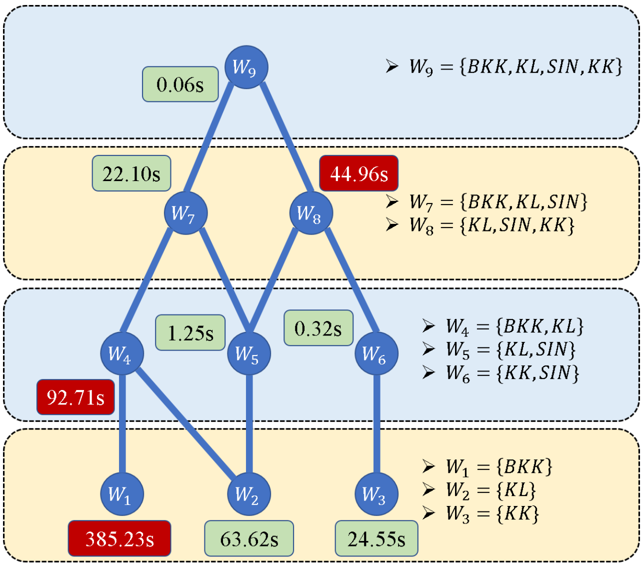

Fig. 9. The hierarchical architecture of this ASEAN air traffic system

TABLE V
A COMPARISON BETWEEN THE PROPOSED AIR TRAFFIC FLOW MODEL AND THE SEGMENTED-BASED MODEL

Item	Proposed Model	Segment-Based Model
Number of Variables	$\sim 3 \times 10^6$	$\sim 2 \times 10^7$
Complexity	$\sim 1 \times 10^{10}$	$\sim 1 \times 10^{13}$

Lagrange multiplier method, i.e., to solve this problem via partitioning the BKK FIR into some sub-FIRs by relaxing some boundary constraints with Lagrangian multipliers, (2) incremental algorithms, (3) some meta-heuristics such as evolutionary algorithms.

C. Case Study III: Simulations for ASEAN region Air Traffic System on AitTOp platform

The air traffic scheduling results obtained by our proposed algorithm are tested on AirTOp (Air Traffic Optimization) [33] platform, which is a commercial simulation platform that can be used in the context of departure, en-Route, approach simulations as well as for airport ground movements studies. We build the Air traffic system in 4 FIRs in ASEAN region,

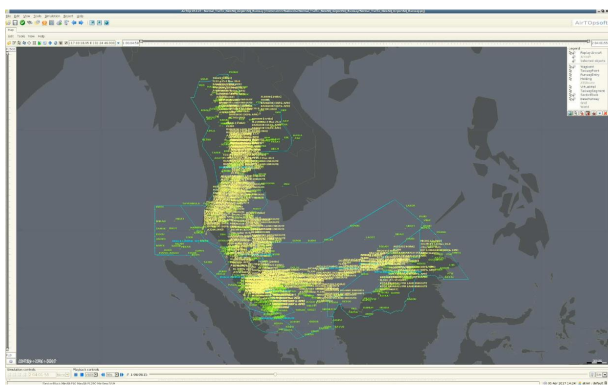

Fig. 10. ASEAN region Air Traffic System on AitTOp platform

which includes waypoints, airports with runways, ATC routes, ATC sectors, etc. The flight plan data is abstracted from real aviation data downloaded from FlightRadar24 [34], which includes callsign, airline, type, departure time, arrival time, origin airport, destination airport, flight level, cruising speed, etc. A total of 1507 flights depart from and arrive at the 46 airports in this region in 1-day period. To make the traffic congestion more significant, we double the traffic volume and a total of 3014 flights are involved in our experiment. The results for the comparisons on the average delay time are shown in Table VI.

TABLE VI
COMPARISONS ON THE AVERAGE DELAY TIME IN AIRTOP SIMULATION

Flight Plans	Average Delay Time	Reduction
Original flight plans	90 mins 40 sec	–
Flight plans generated by Incremental Strategy (ground holding first)	74 mins 45 sec	16.46%
Flight plans generated by Incremental Strategy (rerouting first)	74 mins 27 sec	16.79%
Flight plans generated by Hierarchical Heuristic Algorithm	64 mins 41 sec	28.66%

In Table VI, we compare the network-wise air traffic delay time obtained from the proposed hierarchical heuristic algorithm to the results from three other flight scheduling strategies. The original flight plan data are collected from FlightRadar24 [34] and the delay time is estimated in AirTOp [33]. Based on the original data, we generate the flight plans with incremental strategies and hierarchical heuristic algorithm, respectively. The incremental strategies are carried out based on the shortest path algorithm with consideration of the capacities for each air-route and two different options are taken into consideration, i.e., the ground holding first strategy and the rerouting first strategy. The average delay time for incremental strategies are reduced by about 16% to 17% compared to the results from the original data, while the average delay time for hierarchical heuristic algorithm is reduced by about 29%. These results show the effectiveness of the proposed algorithm.

V. CONCLUSIONS

In this paper, we propose a hierarchical heuristic approach to minimize the airport departure and arrival schedule deviations. The scheduling problem is formulated based on an en-route air traffic system model consisting of air routes, waypoints and airports. A novel link flow dynamic model is provided to describe the system dynamics under safety related constraints such as the capacities of air links and airports, and the speed limits. To lower the computational complexity, a novel hierarchical routing and scheduling approach is presented, where the whole network is partitioned into regions, which contain FIRs as the minimum component units, and regions are organized in a hierarchical architecture induced by the containment relationship among regions. Air flow routing and scheduling is carried out in a bottom-up manner, and by maximizing the parallelism among regions at the same level of the hierarchy, the approach has a great potential to incur low computational complexity. By adopting the Lagrangian multiplier method together with metaheuristics, we may reduce the computational complexity even further. The effectiveness of our proposed hierarchical approach is illustrated in a traffic network consisting of four FIRs of the ASEAN region.

References

[1] M. Ball, C. Barnhart, M. Dresner, M. Hansen, K. Neels, A. Odoni, E. Peterson, L. Sherry, A. A. Trani, and B. Zou, "Total delay impact study: a comprehensive assessment of the costs and impacts of flight delay in the united states," 2010.

[2] B. Sridhar and P. Menon, "Comparison of linear dynamic models for air traffic flow management," in *IFAC World Congress*, 2005.

[3] D. Bertsimas and S. S. Patterson, "The air traffic flow management problem with enroute capacities," *Operations research*, vol. 46, no. 3, pp. 406–422, 1998.

[4] K. D. Bilmoria, S. Banavar, G. B. Chatterji, K. S. Sheth, and S. Grabbe, "Facet: Future atm concepts evaluation tool," 2000.

[5] A. M. Bayen, R. L. Raffard, and C. J. Tomlin, "Eulerian network model of air traffic flow in congested areas," in *American Control Conference, 2004. Proceedings of the 2004*, vol. 6. IEEE, 2004, pp. 5520–5526.

[6] ——, "Adjoint-based control of a new eulerian network model of air traffic flow," *Control Systems Technology, IEEE Transactions on*, vol. 14, no. 5, pp. 804–818, 2006.

[7] B. Sridhar, T. Soni, K. Sheth, and G. Chatterji, "Aggregate flow model for air-traffic management," *Journal of Guidance, Control, and Dynamics*, vol. 29, no. 4, pp. 992–997, 2006.

[8] D. Sun and A. M. Bayen, "Multicommodity eulerian-lagrangian large-capacity cell transmission model for en route traffic," *Journal of guidance, control, and dynamics*, vol. 31, no. 3, pp. 616–628, 2008.

[9] C. F. Daganzo, "The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory," *Transportation Research Part B: Methodological*, vol. 28, no. 4, pp. 269–287, 1994.

[10] ——, "The cell transmission model, part ii: network traffic," *Transportation Research Part B: Methodological*, vol. 29, no. 2, pp. 79–93, 1995.

[11] A. Agustı, A. Alonso-Ayuso, L. F. Escudero, C. Pizarro *et al.*, "On air traffic flow management with rerouting. part ii: Stochastic case," *European Journal of Operational Research*, vol. 219, no. 1, pp. 167–177, 2012.

[12] G. B. Dantzig and P. Wolfe, "Decomposition principle for linear programs," *Operations research*, vol. 8, no. 1, pp. 101–111, 1960.

[13] D. Bertsimas and S. S. Patterson, "The air traffic flow management problem with enroute capacities," *Operations research*, vol. 46, no. 3, pp. 406–422, 1998.

[14] J. Rios and K. Ross, "Massively parallel dantzig-wolfe decomposition applied to traffic flow scheduling," *Journal of Aerospace Computing, Information, and Communication*, vol. 7, no. 1, pp. 32–45, 2010.

[15] P. Wei, Y. Cao, and D. Sun, "Total unimodularity and decomposition method for large-scale air traffic cell transmission model," *Transportation Research Part B: Methodological*, vol. 53, pp. 1–16, 2013.

[16] A. Agustín, A. Alonso-Ayuso, L. F. Escudero, and C. Pizarro, "Mathematical optimizationg models for air traffic flow management: A review." *Stud. Inform. Univ.*, vol. 8, no. 2, pp. 141–184, 2010.

[17] L. Bianco, P. Dell'Olmo, and A. R. Odoni, *Modelling and simulation in air traffic management*. Springer Science & Business Media, 2012.

[18] D. Bertsimas, G. Lulli, and A. Odoni, "The air traffic flow management problem: An integer optimization approach," in *Integer programming and combinatorial optimization*. Springer, 2008, pp. 34–46.

[19] E. P. Gilbo, "Airport capacity: Representation, estimation, optimization," *Control Systems Technology, IEEE Transactions on*, vol. 1, no. 3, pp. 144–154, 1993.

[20] ——, "Optimizing airport capacity utilization in air traffic flow management subject to constraints at arrival and departure fixes," *Control Systems Technology, IEEE Transactions on*, vol. 5, no. 5, pp. 490–503, 1997.

[21] C. G. Panayiotou and C. G. Cassandras, "A sample path approach for solving the ground-holding policy problem in air traffic control," *Control Systems Technology, IEEE Transactions on*, vol. 9, no. 3, pp. 510–523, 2001.

[22] D. Bertsimas and S. S. Patterson, "The traffic flow management rerouting problem in air traffic control: A dynamic network flow approach," *Transportation Science*, vol. 34, no. 3, pp. 239–255, 2000.

[23] Y. Zhang, R. Su, Q. Li, C. Cassandras, and L. Xie, "Distributed flight routing and scheduling in air traffic flow management," in *Decision and Control (CDC), 2016 IEEE 55th Conference on*. IEEE, 2016, pp. 1080–1085.

[24] Y. Zhang, Q. Li, and R. Su, "Sector-based distributed scheduling strategy in air traffic flow management," *IFAC-PapersOnLine*, vol. 49, no. 3, pp. 365–370, 2016.

[25] Q. Li, Y. Zhang, and R. Su, "A flow-based flight scheduler for en-route air traffic management," *IFAC-PapersOnLine*, vol. 49, no. 3, pp. 353–358, 2016.

[26] Y. Zhang, R. Su, Q. Li, C. Cassandras, and L. Xie, "Distributed flight routing and scheduling in air traffic flow management," *Intelligent Transportation Systems, IEEE Transactions on*, vol. 99, pp. 1–12, 2017.

[27] ICAO, "Icao doc. 4444 - procedures for air navigation services - air traffic management. 15th ed." *The International Civil Aviation Organization (ICAO)*, 2007.

[28] A. Bemporad and M. Morari, "Control of systems integrating logic, dynamics, and constraints," *Automatica*, vol. 35, no. 3, pp. 407–427, 1999.

[29] G. G. N. Sandamali, R. Su, Y. Zhang, and Q. Li, "Flight routing and scheduling with departure uncertainties in air traffic flow management," in *Control & Automation (ICCA), 2017 13th IEEE International Conference on*. IEEE, 2017, pp. 301–306.

[30] "Snowflake software," https://snowflakesoftware.com, accessed: 2018-04-11.

[31] Gurobi, "Gurobi optimizer reference manual," *Gurobi Inc.*, 2015.

[32] Mathworks, "Matlab: R2015a," *Mathworks Inc, Natick*, 2015.

[33] "Airtop software," http://http://airtopsoft.com, accessed: 2017-04-17.

[34] "Flightradar24," https://www.flightradar24.com, accessed: 2017-04-17.

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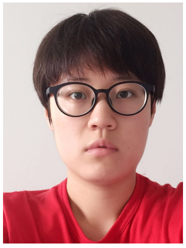

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