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# Space-time dynamics of electricity markets incentivize technology decentralization

Yue Shao, Victor M. Zavala*

*Department of Chemical and Biological Engineering, University of Wisconsin-Madison, 1415 Engineering Dr., Madison, WI 53706, USA*

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ABSTRACT

We study economic incentives provided by space-time dynamics of day-ahead and real-time electricity markets. Specifically, we seek to analyze to what extent such dynamics promote decentralization of technologies for generation, consumption, and storage (which is essential to obtain a more flexible power grid). Incentives for decentralization are also of relevance given recent interest in the deployment of small-scale modular technologies (e.g., modular ammonia and biogas production systems). Our analysis is based on an asset placement problem that seeks to find optimal locations for generators and loads in the network that minimize profit risk. We show that an unconstrained version of this problem can be cast as an eigenvalue problem. Under this representation, optimal network allocations are eigenvectors of the space-time price covariance matrix while the eigenvalues are the associated profit variances. We also construct a more sophisticated placement formulation that captures different risk metrics and constraints on types of technologies to systematically analyze trade-offs in expected profit and risk. Our analysis reveals that space-time market dynamics provide significant incentives for decentralization and strategic asset placement but that full mitigation of risk is only possible through simultaneous investment in generation and loads (which can be achieved using batteries or microgrids).



## 1. Introduction

Decentralization of technologies for power generation (e.g., power plants), consumption (e.g., manufacturing facilities and data centers), and storage (e.g., batteries) is an ongoing industrial trend (Ramshaw, 1999; Stankiewicz and Moulijn, 2000). From the perspective of an independent system operator (ISO) of the power grid, decentralization is desirable as it can provide spatial flexibility to control network flows and to overcome limited transmission infrastructure (Buchholz, 2010; Kim et al., 2017a). In addition, large centralized power generation and consumption facilities can become liabilities during extreme weather or cyber attacks (Lier and Grünewald, 2011). To give an idea of the risk that large centralized facilities pose to the power grid, consider the fact that the load of a conventional ammonia manufacturing plant is around 64 MW (Egenhofer et al., 2017) and that the load of a large data center reaches 50 MW (Avgerinou et al., 2017) (equivalent to the load of tens of thousands of homes). Similarly, the power supply of a large centralized power plant such as the Hammond plant in Georgia is 800 MW Georgia Power. The growing demand from large data centers is of particular concern as it is projected that, within the next decade, the loads from such facilities will represent over 20% of the total grid load (Kim et al., 2017a). Another issue associated with centralized facilities is that they provide limited investment flexibility to mitigate long-term risks in electricity prices and policy. The need to mitigate investment risks is promoting the development and deployment of smaller-scale (modular) technologies (Guo et al., 2009; Kim et al., 2017b; Palys et al., 2018; Wu et al., 2009). On the other hand, it is well-known that large centralized systems benefit from economies of scale and thus a strong trade-off exists between expected profit and risk.

Because electricity prices are a key driving factor in the revenue/cost of facilities, space-time price fluctuations must be considered in investment and operating decisions. For instance, power generation and consumption facilities often sell/purchase electricity in the Day-Ahead Energy Market (DAM) as opposed to the Real-Time Energy Market (RTM) to minimize risk, as the former is far less volatile ISO New England,(Conejo et al., 2005; Dowling et al., 2017). The growing share of renewable power in the supply portfolio is also introducing stronger market volatility and risk (Johnson and Oliver, 2016), as unpredictable weather events can disrupt these renewable technologies and thus cause intermittent and volatile electricity supply. This issue is exacerbated by the lack of sufficient elastic (flexible) demand. The temporal and spatial volatility of electricity prices in RTM is illustrated in

* Corresponding author.
*E-mail address:* victor.zavala@wisc.edu (V.M. Zavala).

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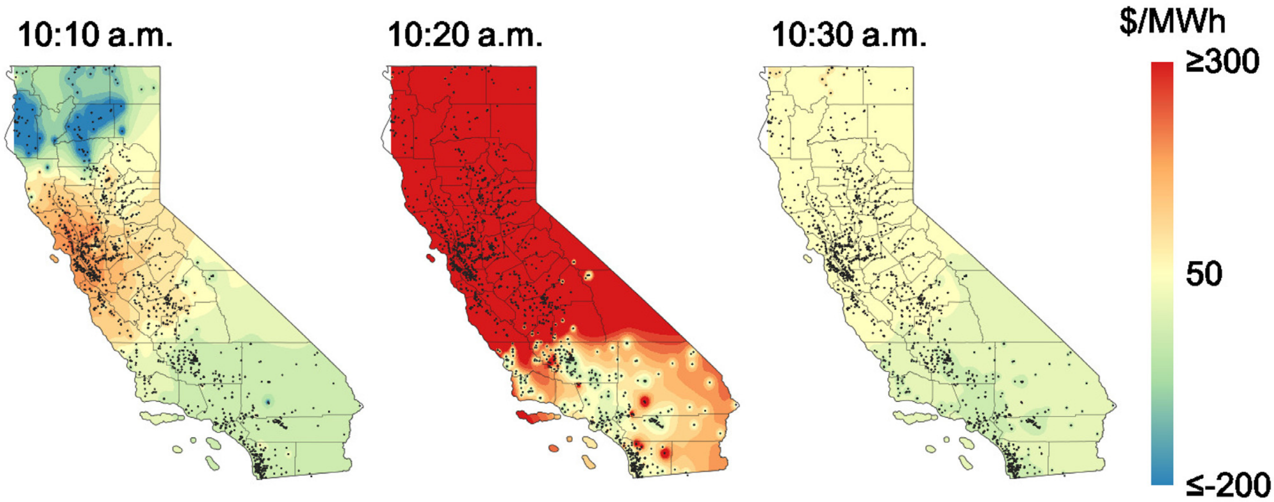


**Fig. 1.** Electricity price fluctuation in RTM on February 5, 2015 in CAISO.

Fig. 1. Here, we show the nodal price change over 20 min for a specific day in California. We see that, under a 20-min period, the average electricity price increases from 48.42 USD/MWh to 592.33 USD/MWh and then drops to 35.15 USD/MWh. Here, we also see that such fluctuations are less abrupt at some network locations. Price volatility is less severe in day-ahead markets; in fact, day-ahead markets are precisely designed to pre-allocate generation and loads in the network in order to help participants mitigate profit risk (Zavala et al., 2017). On the other hand, the average RTM price is typically lower than the average DAM price. Consequently, there exists a premium to participate in the DAM (in order to avoid RTM volatility and associated risk). This suggests that there exists an economic incentive to decentralize (diversify) generation and load assets over multiple network locations in order to exploit spatial correlations in DAM and RTM prices (and with this avoid large premia). Similarly, spatial variations in DAM and RTM prices can be exploited by decentralized facilities to maximize profit. For instance, large cloud computing providers are currently placing data centers strategically in the network in order to avoid large electricity costs (Kim et al., 2017a). One could also envision that small modular manufacturing facilities can be relocated to exploit more favorable prices. A challenge that arises in this context is that DAM and RTM prices exhibit complex spatio-temporal dynamics and correlation patterns (Wang and Hobbs, 2014). As a result, it is *non-trivial* to identify suitable degrees of asset decentralization and optimal locations for such assets.

In this work, we propose a computational framework for analyzing economic incentives created by space-time dynamics of electricity markets. Our framework is based on an asset placement formulation that seeks to find optimal locations for generation and load (consumption) assets in the network that minimize profit risk. We show that an unconstrained version of this problem can be cast as an eigenvalue problem. Under this representation, optimal network allocations are eigenvectors of the space-time price covariance matrix, while the eigenvalues are the profit variances that result from such allocations. Consequently, risk analysis can be performed in a systematic and computationally efficient manner by using principal component analysis (PCA). We construct a constrained placement problem that captures constraints on the types of assets and that trade-offs risk and expected profit. Unfortunately, for the ISO-scale data sets of interest, this problem is a large-scale mixed-integer quadratic programming (MIQP) problem that cannot be solved with current solvers. We use the mean absolute deviation as an alternative risk measure to obtain a more scalable (but still challenging) mixed-integer linear program (MILP). Analysis using the California ISO (CAISO) market data for 2015 reveals that space-time market dynamics provide significant incentives for strategic diversification and asset placement but that complete mitigation of revenue risk is only possible by *simultaneous* investment in decentralized generation and load assets (which can also be achieved by using batteries or hybrid systems such as microgrids or other prosumers). These results are of relevance given the recent interest in the deployment of small-scale modular technologies. We highlight that our work focuses on the use of real (but historical) data to conduct analysis; as such, the study is realistic but has limited predicted power. Unfortunately, existing forecasting techniques for economic time-series data focus on uni-dimensional data (Ledoit and Wolf, 2004), while the market data set considered here is high-dimensional (reaching thousands of locations that are correlated in space and time). As part of future work, we will investigate forecasting strategies for such high-dimensional data sets.

The paper is structured as follows. In the following section, we motivate our discussion by conducting a basic space-time analysis of electricity markets in California. In Section 2 we formulate the technology placement problem, interpret it as an eigenvalue problem, and provide scalable constrained variants. A detailed analysis of the California ISO data set using the placement problem formulations is provided in Section 3.

## 2. Optimal placement problem

In this section, we derive different variants of the optimal placement problem that will allow us to explore incentives provides by space-time dynamics of electricity markets.

### 2.1. Unconstrained formulation

We capture the space-time price data in a matrix $\Pi \in \mathbb{R}^{m\times n}$. Here, the number of columns $n$ is the number of spatial network locations (nodes) and the number of rows $m$ is the number of time points. The matrix entry $\Pi_{i,j}$ is interpreted as the price at time $i$ and at node $j$. We use $p_i := \Pi_{i,:} \in \mathbb{R}^n$, $i = 1, \ldots, m$ to denote all node prices at time $i$. We denote the set of spatial locations as $\mathcal{N} := \{1, \ldots, n\}$ and the set of all time realizations as $\mathcal{M} := \{1, \ldots, m\}$. All prices have units of USD/MWh and we construct separate matrices for DAM and RTM.

Given the space-time price data, we seek to identify optimal locations for loads and generators in the network that minimize the temporal profit variance (variance is used as a standard measure of risk and can also be interpreted as profit volatility). We define a node allocation vector $w \in \mathbb{R}^n$ and the profit function at time $i$ as $\varphi(w, p_i) := \sum_{j \in \mathcal{N}} w_j \Pi_{i,j}$. We interpret a positive node allocation $w_j > 0$ as an injection of power (a generation asset incurring revenue for a positive price) and a negative node allocation $w_j < 0$ as a withdrawal of power (a load asset incurring cost for a positive price). The node allocations $w_j$ have units of MWh. If the prices are negative, a positive allocation incurs cost and a negative allocation incurs a revenue. In other words, installing generators maximizes revenue, but we will see that the simultaneous installation of generators and loads is needed to minimize risk.

The temporal average of the profit is given by:

$$\mu_\varphi(w) = \frac{1}{m} \sum_{i \in \mathcal{M}} \varphi(w, p_i) \tag{2.1}$$

and the temporal variance is

$$\Sigma_\varphi(w) = \frac{1}{m-1} \sum_{i \in \mathcal{M}} (\varphi(w, p_i) - \mu_\varphi(w))^2. \tag{2.2}$$

The optimal placement problem consists of finding the allocation vector $w$ that minimizes the profit risk. This problem is stated as:

$$\min_w \ \Sigma_\varphi(w). \tag{2.3}$$

We assume that the optimal allocation vector (denoted as $w_1^*$) satisfies the constraint $\|w_1^*\|_2 = 1$ (it is a vector of unit length), where $\|\cdot\|_2$ denotes the Euclidean norm. This constraint is interpreted as the distribution of a finite amount of power among the network nodes. We note that the placement problem is scale-invariant. In other words, replacing $w \to \gamma w$ for some $\gamma > 0$ in the optimization problem yields the same optimal allocations. Consequently, imposing a constraint of the form $\|w\|_2 = 1/\gamma$ will yield the same optimal allocation obtained with the constraint $\|w\|_2 = 1$. This formulation seeks to exploit the space-time dynamics of the prices to identify node allocations for generation or load that minimize risk. This is a large-scale and continuous *quadratic program* (QP).

### 2.2. Eigenvalue interpretation

An interesting observation that we make is that, under the special case with no temporal price correlations, the optimal placement problem described above can be interpreted as an *eigenvalue problem*. This connection reveals some interesting properties of the market prices. In the absence of temporal price correlations, the price at the spatial location (network node) $j$ can be modeled as a random variable (denoted as $P_j$) and we use $P = \{P_1, \dots, P_n\}$ to denote a random vector containing all node prices. Consequently, the matrix entry $\Pi_{i,j}$ is interpreted as the $i$th time realization of the price $P_j$ and we assume that the probability of the realization is $1/m$. In this case, $p_i$ denotes the $i$th realization of the spatial price vector $P$. Under this setting, the sample average of the profit approximates the expected value of the profit:

$$\mu_\varphi(w) \approx \mathbb{E}[\varphi(w, P)] \tag{2.4}$$

and the sample variance approximates the variance:

$$\Sigma_\varphi(w) \approx \mathbb{V}[\varphi(w, P)]. \tag{2.5}$$

Here, we recall that $\mathbb{V}[\varphi(w, P)] = \mathbb{E}[\varphi(w, P)^2] - \mathbb{E}[\varphi(w, P)]^2$. The key observation is that the profit variance is related to the price covariance as $\mathbb{V}[\varphi(w)] = w^T \mathbb{E}[(P - \mathbb{E}[P])(P - \mathbb{E}[P])^T] w$. This result can be from the following series of implications:

$$\begin{aligned} \mathbb{V}[\varphi(w)] &= \mathbb{E}[\varphi(w, P)^2] - \mathbb{E}[\varphi(w, P)]^2 \\ &= \sum_{j \in \mathcal{N}} \sum_{k \in \mathcal{N}} w_j w_k \mathbb{E}[P_j P_k] - \sum_{j \in \mathcal{N}} \sum_{k \in \mathcal{N}} w_j w_k \mathbb{E}[P_j] \mathbb{E}[P_k] \\ &= \sum_{j \in \mathcal{N}} \sum_{k \in \mathcal{N}} w_j w_k \mathrm{Cov}(P_j, P_k) \\ &= w^T \mathbb{E}[(P - \mathbb{E}[P])(P - \mathbb{E}[P])^T] w. \end{aligned} \tag{2.6}$$

One can derive a similar relationship between the sample profit and covariance matrix to establish $\Sigma_\varphi(w) = w^T \Sigma w$. Consequently, the optimal placement problem (2.3) can also be written as:

$$\min_w \ w^T \Sigma w \text{ s.t. } \|w\|_2 = 1. \tag{2.7}$$

This reveals that the placement problem is an *eigenvalue problem*. Accordingly, the optimal allocation vector $w_1^*$ is the eigenvector corresponding to the minimum eigenvalue $\lambda_1^*$ of the price covariance matrix $\Sigma$. Moreover, the minimum eigenvalue is the minimum profit variance ($\lambda_1^* = \Sigma_\varphi(w_1^*)$). The eigenvalue problem is also a QP but this can also be solved efficiently using standard techniques (e.g., QR or SVD).

The eigenvalue problem is the basis of principal component analysis (PCA). The first principal component is given by $(w_1^*)^T p_i$, $i \in \mathcal{M}$. In PCA, one extracts the entire eigenvalue spectrum of the price matrix to obtain all the principal components. For instance, to obtain the second smallest eigenvalue and corresponding eigenvector we add the linear orthogonality constraint $w^T w_1^* = 0$ to the eigenvalue problem (2.7). The solution of the new problem yields the eigenvector $w_2^*$ and corresponding eigenvalue $\lambda_2^* = \Sigma_\varphi(w_2^*)$. Since adding the orthogonality constraint restricts the feasible space, we have that $\Sigma_\varphi(w_2^*) \geq \Sigma_\varphi(w_1^*)$. This procedure is repeated to obtain the entire set of eigen-pairs $w_j^*, \lambda_j^*$, $j \in \mathcal{N}$, where $\lambda_n^* = \Sigma_\varphi(w_n^*)$ is the maximum possible cost variance (obtained with the loading allocation $w_n^*$). In our context, this procedure provides useful information because it allows us to obtain a family of allocations $w_j^*$, $j \in \mathcal{N}$ and to rank them according to their profit variance. The eigenvectors can also be used to form a matrix $W$ that can be used to project any price realization $p_i$ into the space of the principal components as $Wp_i$. The projection can be used to identify clusters and/or outliers in the price data by analyzing only a subset of principal components.

### 2.3. Constrained formulation

While mitigating profit variance is an important investment goal, obtaining a maximum expected profit is also important. Moreover, one often has constraints on the nature and capacity of assets that can be installed. We thus extend the placement problem (2.3) to capture these features. We impose an $\ell_1$-norm constraint on the allocation vector $w$ so that the total amount of power allocated adds up to one MWh and we add a condition that only one type of asset is allowed to be built at one location (either generation or load). Consequently, we can decompose the node allocation $w_j$ into a generation $0 \leq w_{j,l} \leq 1$ and a load component $-1 \leq w_{j,g} \leq 0$ (which are mutually exclusive). This gives the following conflict resolution (multi-objective optimization) problem:

$$\max_w \ \{\mu_\varphi(w), -\Sigma_\varphi(w)\} \tag{2.8a}$$

$$\text{s.t.} \sum_{j \in \mathcal{N}} \left(|w_{j,l}| + |w_{j,g}|\right) = 1 \tag{2.8b}$$

$$0 \leq w_{j,g} \leq z_{j,g}, \ j \in \mathcal{N} \tag{2.8c}$$

$$-z_{j,l} \leq w_{j,l} \leq 0, \ j \in \mathcal{N} \tag{2.8d}$$

$$z_{j,l} + z_{j,g} \leq 1, \ j \in \mathcal{N} \tag{2.8e}$$

$$z_{j,l}, z_{j,g} \in \{0, 1\}, \ j \in \mathcal{N}. \tag{2.8f}$$

where $z_{j,l}$ and $z_{j,g}$ are binary variables that indicate if either a load or generation asset is installed at a particular location $j$. The constraint $z_{j,l} + z_{j,g} \leq 1$ indicates that either a load or a generator (but not both) can be installed at one location. Consequently, we have that $\sum_{j\in\mathcal{N}} \left(|w_{j,l}| + |w_{j,g}|\right) = \sum_{j\in\mathcal{N}} |w_j| = \|w\|_1$. This constraint is used to avoid degeneracy of the solution (e.g., adding a load and a generator in a given node has the same net effect as installing one generator or load). The objective function captures the trade-off between expected profit and risk (which are often conflicting). The use of binary variables allows us to enforce a sharp separation between loads and generators (a continuous formulation does not allow for this). This facilitates interpretability of the solution. Specifically, we aim to use the placement formulation to explore how space-time price dynamics provide incentives to install decentralize facilities for loads and generators.

Unfortunately, the constrained placement problem is a large-scale mixed-integer QP. This problem is intractable for the ISO-scale data sets considered in this work. Motivated by this limitation, we consider the mean absolute deviation as a risk measure. This is given by:

$$\mathrm{MD}(w) = \frac{1}{m} \sum_{i\in\mathcal{M}} |\varphi(w, p_i) - \mu_\varphi(w)| \approx \mathbb{E}[|\varphi(w, P) - \mu_\varphi(w)|]. \tag{2.9}$$

This risk measure is used to formulate the placement problem:

$$\max_w \ \{\mu_\varphi(w), -\mathrm{MD}(w)\} \tag{2.10a}$$

$$\text{s.t.} \sum_{j\in\mathcal{N}} \left(|w_{j,l}| + |w_{j,g}|\right) = 1 \tag{2.10b}$$

$$0 \leq w_{j,g} \leq z_{j,g}, \ j \in \mathcal{N} \tag{2.10c}$$

$$-z_{j,l} \leq w_{j,l} \leq 0, \ j \in \mathcal{N} \tag{2.10d}$$

$$z_{j,l} + z_{j,g} \leq 1, \ j \in \mathcal{N} \tag{2.10e}$$

$$z_{j,l}, z_{j,g} \in \{0, 1\}, \ j \in \mathcal{N} \tag{2.10f}$$

which can be cast as a mixed-integer linear program that is still large-scale scale but tractable with existing tools. The constrained placement problem is also scale-invariant. In other words, replacing $w \to \gamma w$ for some $\gamma > 0$ yields the same optimal allocations. This is because $MD(\gamma w) = \gamma MD(w)$, and $\mu_\varphi(\gamma w) = \gamma \mu_\varphi(w)$ (resulting in a linear scaling of the objective function). Consequently, imposing a constraint of the form $\|w\|_1 = 1/\gamma$ will yield the same allocation obtained with the unit-length constraint $\|w\|_1 = 1$. The constraints set capacity of load and generators, and the constraint on the binary variables ensures that only one type of technology is allowed at each location. We can use the above formulation to understand the impacts of installing only certain types of assets or at certain locations. For instance, if we only wish to install generation assets, we set all $z_{j,l}$ to zero.

## 3. Results and discussion

In this section, we conduct a basic statistical analysis for an electricity market data set of CAISO and use the optimal placement formulation to analyze economic incentives created by the DAM and RTM.

### 3.1. Volatility analysis of electricity markets

In the DAM, electricity prices are updated hourly and market participants commit to buy or sell power one day before real-time operation, thus avoiding price volatility. This market produces one financial settlement per day. In the RTM market, prices are updated every 5 min and participants commit to buy or sell electricity over the course of the operating day. This market seeks to balance discrepancies between the day-ahead commitments and the actual real-time generation and loads seen in the power grid (e.g., due to unexpected variations in renewable power supply, equipment failures, and so on). The DAM and RTM work together to produce a multi-settlement system that balances power at different timescales and at thousands of network locations (Dowling et al., 2017). Usually, electricity prices in the DAM are usually less volatile but are on average higher than RTM prices, and thus market participants can participate strategically in either or both of these markets.

We conducted a basic statistical analysis to compute space-time price averages and standard deviations for the CAISO data set for the year of 2015. This dataset is open-access and was collected from CAISO Open Access Same-time Information System (OASIS) (AM and EC, 2014). The dataset includes complete electricity price profiles for the year at 2234 different network locations. The data set contains over 19,569,840 price points for the DAM (one-hour time resolution) and 234,838,080 price points for the RTM (5-min time resolution). We use this data to construct a space-time covariance matrix $\Sigma$ form both the DAM and RTM.

The results are visualized in Figs. 2–4. Fig. 2 illustrates that the time-average price for both markets is in the range of 27–50 USD/MWh. The space-time average RTM price is 32.71 USD/MWh, which is 2.62% lower than the corresponding average DAM price of 33.59 USD/MWh. The differences illustrate that there is a premium in the DAM. Spatial patterns for both markets are quite similar, indicating that prices are dictated by the network topology. Fig. 3 demonstrates temporal price volatility (standard deviation) at all locations. The temporal volatility in the DAM is consistently under 10 USD/MWh in most locations while the volatility in the RTM is in the range of 60–70 USD/MWh and reaches levels of 90 USD/MWh in some locations. The spatial average of the temporal volatilities was found to be 62.41 USD/MWh for the RTM, almost four times larger than in the DAM, which was only 12.93 USD/MWh. These results clearly indicate that RTM possesses greater temporal volatility. Fig. 4 presents spatial volatility through time. We see that the DAM shows low spatial volatility (except in a few instances in the summer months) while the RTM shows more frequent spikes in spatial volatility. Based on this analysis we conclude that the RTM is more volatile than the DAM in both time and space. We also found that the temporal average of the spatial volatility was found to be 8.85 USD/MWh for the RTM and 5.60 USD/MWh for the DAM. We can thus see that, on average, spatial volatility is less significant than temporal volatility (which are 12.93 USD/MWh for DAM and 62.42 USD/MWh for the RTM).

We also computed the spatial correlation matrix based on the covariance matrix. Our results show that, in the DAM, the average correlation is 0.67, that 99% of the total number of locations are positively correlated, and that the minimum correlation is −0.22. In the RTM, the average correlation is 0.82 and the minimum correlation is 0.00083. We conclude that a strong positive correlation exists in both electricity markets (prices at different locations tend to move in the same direction). This indicates that there is tight physical network coupling. As we will see next, strong positive correlation indicates that it is *impossible* to eliminate investment risk by simply investing in either generation or loads (a combination of both is needed). This would not be the case if we had a strong negative correlation in the market.

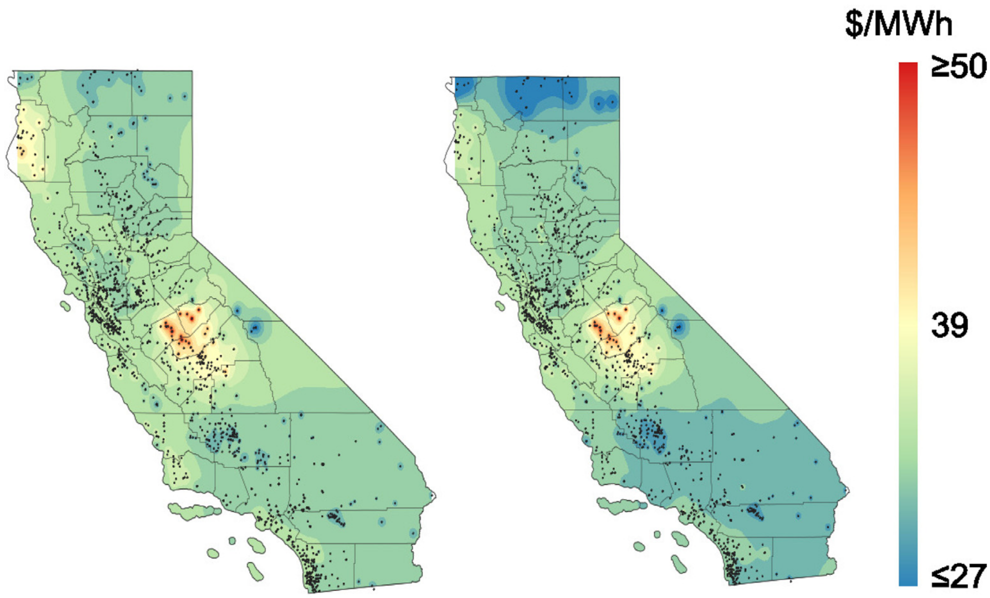


**Fig. 2.** Temporal average price (at different spatial locations) for CAISO in day-ahead (left) and real-time (right) markets.

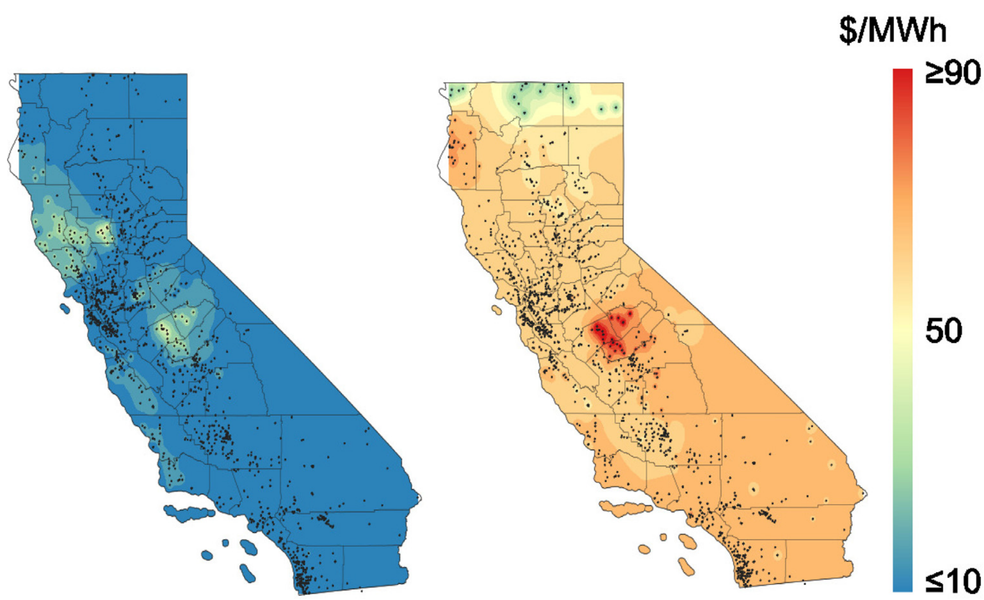


**Fig. 3.** Temporal average standard deviation (at different spatial locations) for the DAM (left) and RTM (right).

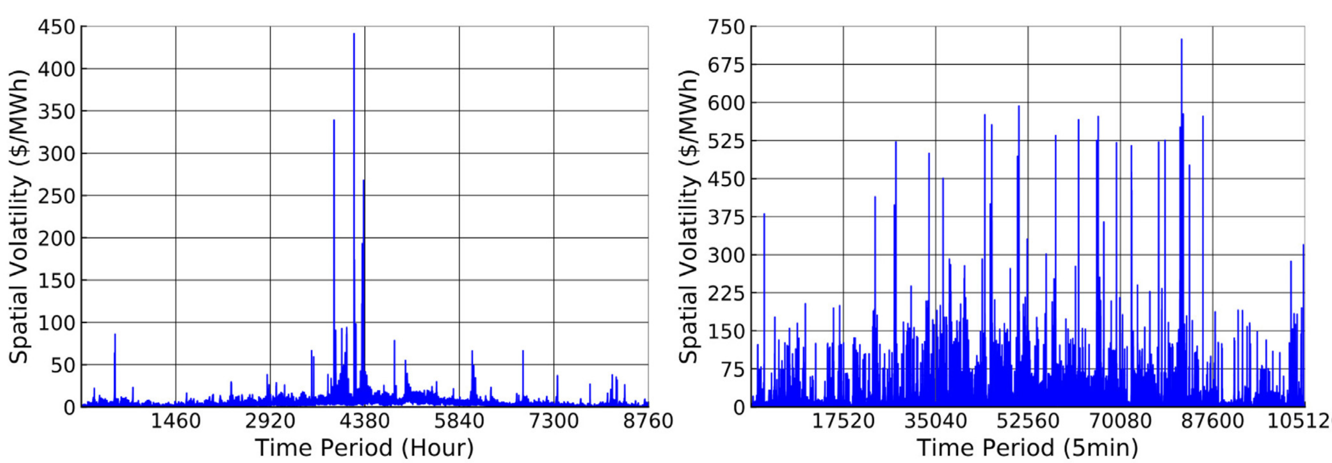


**Fig. 4.** Spatial average standard deviation (at different temporal locations) for the DAM (left) and RTM (right).

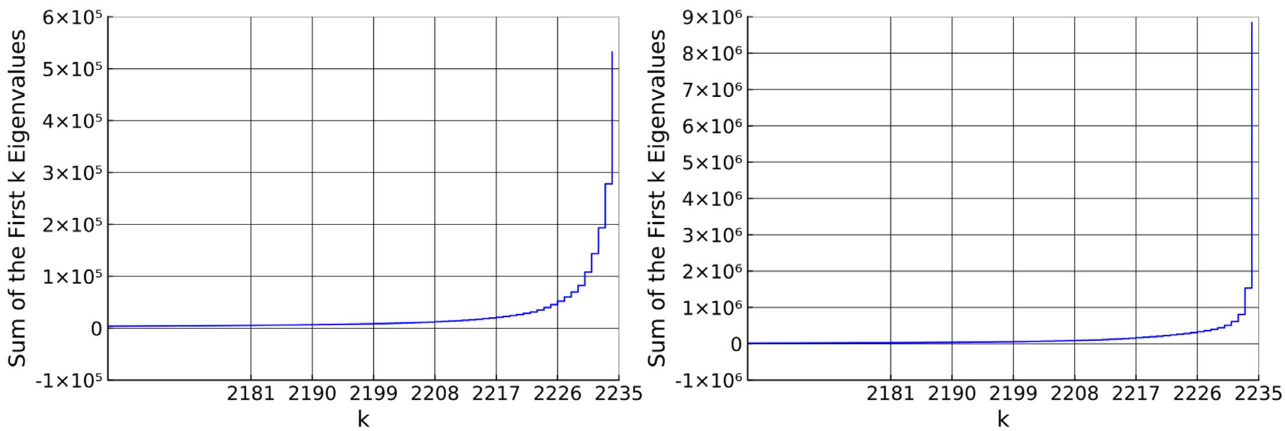


**Fig. 5.** Cumulative eigenvalue spectrum for the DAM (left) and the RTM (right) covariances.

**Table 1**
Eigenvalues for DAM and RTM covariance matrices.

| Eigenvalue | DAM | RTM |
|---|---|---|
| $\lambda_1$ | $-4.25 \times 10^{-12}$ | $-4.59 \times 10^{-11}$ |
| $\lambda_{10}$ | $-2.09 \times 10^{-14}$ | $-1.90 \times 10^{-12}$ |
| $\lambda_{100}$ | $-2.86 \times 10^{-16}$ | $-5.74 \times 10^{-16}$ |
| $\lambda_{500}$ | $2.91 \times 10^{-18}$ | $5.80 \times 10^{-17}$ |
| $\lambda_{1000}$ | $5.78 \times 10^{-4}$ | $4.68 \times 10^{-5}$ |
| $\lambda_{1500}$ | 0.24 | 0.016 |
| $\lambda_{2000}$ | 5.87 | 9.20 |
| $\lambda_{2100}$ | 18.35 | 74.15 |
| $\lambda_{2200}$ | 300.62 | 2806.04 |
| $\lambda_{2234}$ | $2.54 \times 10^5$ | $7.31 \times 10^6$ |

**Table 2**
Risk vs. expected profit trade-off for DAM.

| Risk ($/MWh) | Std. dev. ($/MWh) | Expected Profit ($/MWh) | # of Loads | # of Generators |
|---|---|---|---|---|
| 24.03 | 46.66 | 52.76 | 0 | 2 |
| 18.15 | 33.78 | 49.15 | 0 | 3 |
| 12.19 | 19.52 | 44.22 | 0 | 5 |
| 8.31 | 11.64 | 39.24 | 0 | 12 |
| 5.75 | 7.85 | 34.16 | 0 | 8 |
| 5.06 | 7.00 | 31.85 | 0 | 7 |
| 4.25 | 5.53 | 28.15 | 3 | 9 |
| 3.15 | 4.08 | 22.13 | 13 | 9 |
| 1.33 | 1.75 | 10.63 | 29 | 12 |
| 0.16 | 0.23 | 1.92 | 142 | 112 |
| 0.055 | 0.011 | 0.80 | 287 | 254 |

### *3.2. Eigenvalue analysis of space-Time covariance matrix*

Solving the basic placement formulation is equivalent to solving an eigenvalue problem. In Table 1 and Fig. 5, we summarize the eigenvalue spectrum in ascending order for both the DAM and RTM price covariance matrices (recall that the eigenvalues are the variances of the profit). Recall that both the DAM and RTM matrices have a total of 2234 eigenvalues. The first 1180 eigenvalues of the DAM price covariance are close to zero. For the RTM, the first 1454 eigenvalues are close to zero (below a threshold value of $O(10^{-2})$). This indicates that many eigenvectors (allocations) give zero variance, meaning that many combinations of asset locations (given by the corresponding eigenvectors) can eliminate profit variance. An optimal strategy to eliminate risk is to place combinations of loads and generators at neighboring nodes (those with similar temporal price profiles). This can be visualized in Fig. 6, where we show the optimal placement of assets (the eigenvectors) corresponding to the minimum eigenvalues. As can be seen, allocations of generation and load always appear in pairs next to each other and are of equal magnitude.

The largest eigenvalue (the maximum possible profit variance) is $O(10^5)$ for the DAM and $O(10^6)$ for the RTM, indicating that there is more volatility in the RTM (reinforcing the observations made with basic statistical analysis). In Fig. 7, we present the optimal allocations corresponding to the maximum variance. The strategy here is to place the same asset type (in this case power generation) at all nodes. The maps also reveal areas the are strongly positively correlated (so the strategy to maximize variance is to allocate more generation at such locations). Obviously, this strategy is not optimal from an investment standpoint but highlights some interesting properties of the behavior of electricity prices.

### *3.3. Risk vs. mean profit trade-off for the DAM*

We used the placement formulation to analyze trade-offs between risk and expected profit. In Table 2 and Fig. 8 we present the optimal trade-off solutions (Pareto optimal solutions) for the DAM. The Pareto solutions were identified using an $\epsilon$-constrained approach. From these results, we can make a number of interesting observations. First, it is clear that to maximize expected profit it is optimal to centralize facilities (these facilities are simply installed at locations with large mean price). In this case, obviously, the type of asset to install is generation and the expected profit is 52.76 $/MWh. This strategy, however, results in a large risk (an MD value of 24.03 $/MWh). We can also see that the mean deviation is significant, representing half of the expected profit, which is due to the high temporal volatility of the prices. The trade-off trends also indicate that installation of a larger number of smaller power generators (diversifying generation among multiple locations) can substantially decrease risk. For instance, by increasing the number of generators to five, we see that the risk is decreased by 50% and this only decreases the expected profit by 15%. This illustrates that there is a strong nonlinear trade-off between expected profit and risk. The mean absolute deviation gives linear penalties for large and small deviations (compared to the standard deviation, which attributes quadratic penalties). In Table 2 we present the standard deviation values for each placement problem solved. Note that the trend of standard deviation agrees with that of the mean absolute deviation. Therefore, choosing the mean absolute deviation as the risk measure for the optimal placement problem is consistent. In other words, one can recover elements of the Pareto frontier corresponding to the standard deviation by using the mean absolute

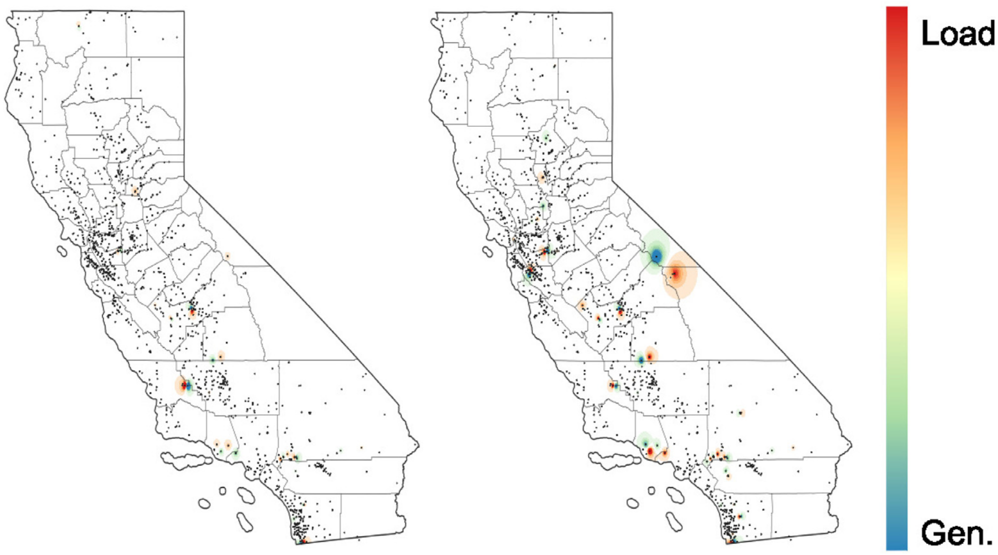


**Fig. 6.** Optimal placement leading to zero risk for the DAM (left) and RTM (right).

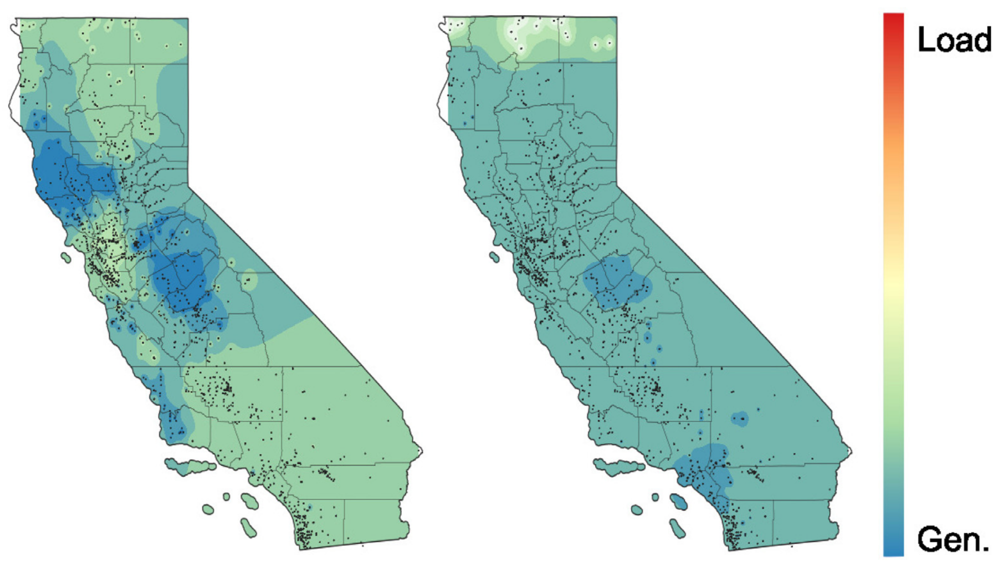


**Fig. 7.** Optimal placement leading to maximum risk the DAM (left) and RTM (right).

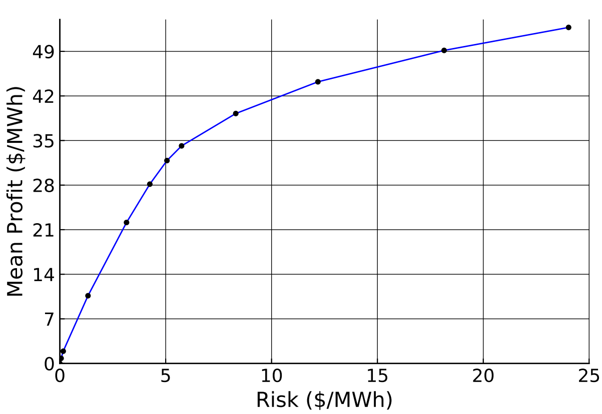


**Fig. 8.** Risk vs. expected profit trade-off for theDAM.

deviation (there is a one-to-one corresponding between the risk measures).

From Table 2 and Fig. 8 we see that further reductions in risk require the installation of both generation and loads. In particular, elimination of risk cannot be achieved through the use of either just generation or just loads (due to the positive correlation of prices). In the hypothetical case in which market prices were negatively correlated, installing the same asset type would be sufficient to fully mitigate risk. Consequently, the limiting value of risk for a single asset type is an indicator of the degree of positive correlation in the market. Fig. 9 shows optimal placement locations for low-risk and high-risk cases. We see that high-risk is achieved by placing only generation assets while low-risk is achieved by diversifying loads and generation.

An interesting trade-off point that we see in Table 2 is that in which we obtain a risk of $MD = 5.06$ USD/MWh and expected profit of $\mu_{\varphi} = 31.85$ USD/MWh (this is the solution for minimum possible risk achieved with only generation assets). In this solution, seven generation locations achieve a mean absolute deviation of 5.06 MWh and an expected profit of 31.85 MW (78.94% of the risk is reduced while 39.63% of the profit is sacrificed). In Table 3 we show the power allocation to each of the seven locations. We see that two locations share 90% of the total generation (these seek to maximize expected profit) while 10% of the generation is split in small generators (these seek to minimize risk). From Table 2, we

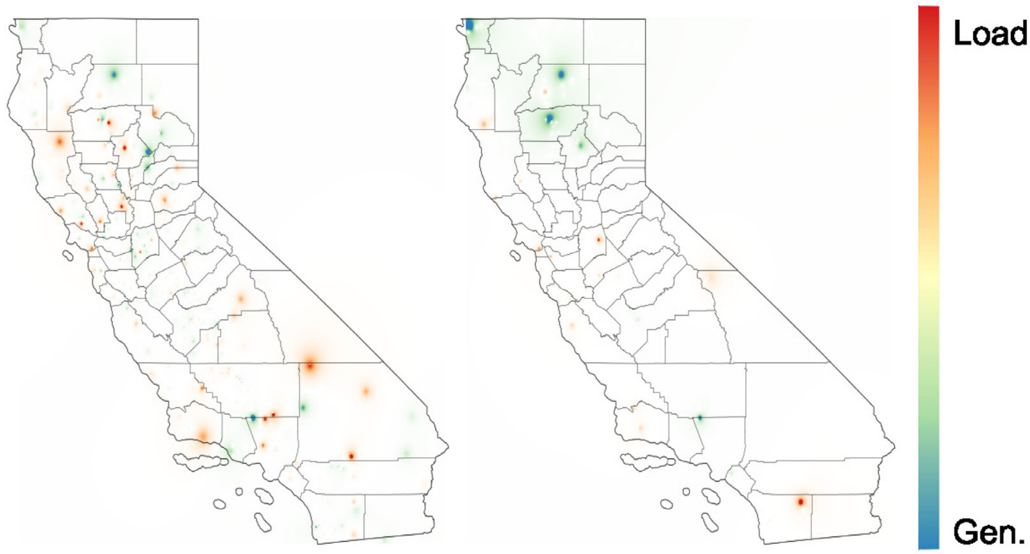


**Fig. 9.** Optimal placement for low risk (left) and high risk (right) in the DAM.

**Table 3**
Optimal allocation for case with $MD = 5.06$ USD/MWh and $-\mu_{\varphi} = 31.85$ USD/MW in the DAM.

| Location | $w_i$ (MWh) |
|---|---|
| NEORBLF_7_B1 | 0.60 |
| JBBLACK1_7_B1 | 0.31 |
| DELNORTE_LNODED50 | 0.037 |
| HMBUNIT2_7_GN010 | 0.023 |
| HMBLTBY_6_N003 | 0.017 |
| TOPAZC1_7_N021 | 0.010 |
| BAFCOG12_7_B1 | 0.00090 |

**Table 4**
Risk vs. expected profit trade-off for the RTM.

| Risk (USD/MWh) | Std. dev. (USD/MWh) | Expected Profit Loads | # of Generators | # of |
|---|---|---|---|---|
| 36.85 | 94.55 | 51.59 | 0 | 1 |
| 29 | 79.96 | 47.03 | 0 | 3 |
| 24 | 68.47 | 43.75 | 0 | 4 |
| 19 | 57.79 | 40.22 | 0 | 4 |
| 16 | 54.11 | 37.70 | 0 | 6 |
| 13 | 44.04 | 33.98 | 0 | 13 |
| 10 | 31.08 | 29.46 | 0 | 22 |
| 8 | 20.86 | 25.43 | 0 | 21 |
| 6 | 14.53 | 19.71 | 6 | 18 |
| 4 | 9.87 | 13.44 | 21 | 22 |
| 2 | 4.98 | 6.88 | 45 | 47 |
| 1 | 2.55 | 3.53 | 71 | 62 |
| 0.6 | 1.56 | 2.17 | 87 | 84 |
| 0.4 | 1.06 | 1.49 | 103 | 99 |
| 0.2 | 0.54 | 0.76 | 130 | 157 |

see that the use of just two generators incurs a large risk. Consequently, investing in smaller generators is key to mitigate risk. From these results, we also conclude that further diversification of generation does not provide significant benefits in risk mitigation.

### 3.4. Risk vs. expected profit trade-off for the RTM

Trade-off analysis for the RTM was performed by using price data with a time resolution of 20 min. The reason is that the placement problem is intractable at higher resolutions. The Pareto analysis results are summarized in Table 4 and Fig. 10. Here, we report standard deviation values in order to highlight how the mean

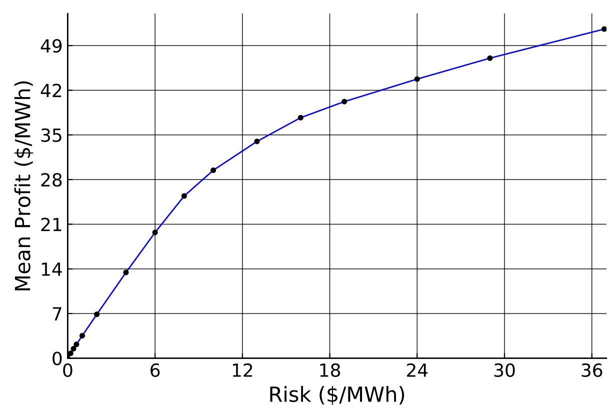


**Fig. 10.** Risk vs. expected profit trade-off for RTM.

and standard deviation follow the same trend. The results for RTM have similar trends to those found in the DAM. In contrast with the DAM results, however, the risk for RTM is higher (which is consistent with the results obtained using the eigenvalue analysis). Compared to the DAM, more diversification of generation is needed to decrease the risk by the same amount (due to the higher volatility in RTM). We also observe that a combination of loads and generators is needed to fully eliminate risk and that the expected profit obtained with the RTM and DAM are similar.

Fig. 11 shows high-risk and low-risk allocations. High-risk allocations with large expected profit favor centralization of assets while low-risk ones favor decentralization of assets. Moreover, this indicates that assets capable of providing simultaneous provision of generation and load (e.g., microgrids or batteries) can be used to mitigate risk. Our analysis also indicates that electricity markets provide significant incentives to modularize power-intensive assets (e.g., manufacturing facilities and data centers). For instance, decentralization of ammonia systems can help mitigate risk associated with the high consumption of electricity in refrigeration systems.

The risk estimated with the 20-min formulation underestimates that of the 5-min counterpart. We can see, however, that the 20-min resolution data already reveals that much higher risk is observed in RTM relative to DAM. This observation is also confirmed using the eigenvalue analysis (which was performed using the

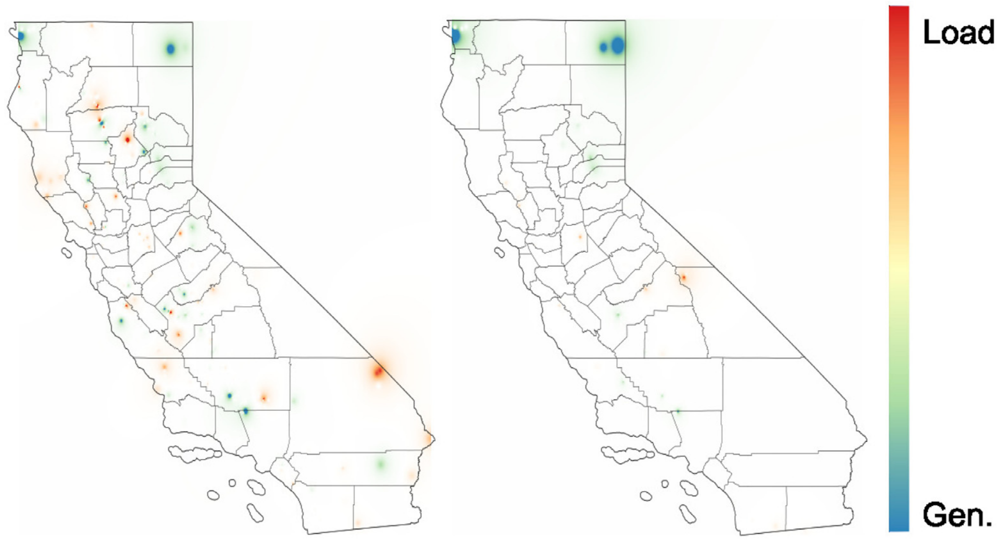


**Fig. 11.** Optimal placement for low risk (left) and high risk (right) in the RTM.

5-min resolution data). Moreover, we expect similar trade-off trends by using higher time resolutions.

### 3.5. Computational considerations

The unconstrained placement problem is an eigenvalue problem that can be readily solved for both the DAM and the RTM data (even at 5 min resolutions). The constrained placement formulation (2.10), on the other hand, is a large-scale mixed-integer linear program. The RTM problem (with 20-min resolution) contains 85,544 constraints and 61,496 variables (4,468 binary) while the DAM problem contains 32,985 constraints, 21,989 continuous variable (4,468 binary). The problems were solved using `Gurobi` with default relative MIP gap of 0.01% and solution times range from 1.5 h to 5 h (on a standard personal computer). The long times are due to significant symmetries in the problem (i.e., many allocation combinations achieve the same optimal objective). This degeneracy was revealed by the eigenvalue analysis (which indicates that the price covariance has a large number of zero eigenvalues). The RTM problem is intractable with time resolutions below 20 min. We are currently investigating strategies to decompose the placement problem in order to be able to scale to higher time resolutions. In particular, this problem has the interesting property that it only has a single coupling constraint. Consequently, one can develop specialized Lagrangian decomposition (Fisher, 2004; Held and Karp, 1971) schemes that achieve high parallel execution efficiency.

## 4. Conclusions and future work

This paper examines economic incentives created by space-time dynamics of day-ahead and real-time electricity markets. We developed an optimal technology placement formulation that seeks to identify optimal strategies to maximize expected profit and minimize risk. We have shown that a pure risk minimization formulation can be cast as an eigenvalue problem. We also developed more sophisticated formulations that capture different technology asset types (e.g., generation or loads) and risk measures using mixed-integer programming techniques. Our analysis for the CAISO market reveals that significantly more temporal (as opposed to spatial) volatility is observed in both DAM and RTM markets (the RTM also has more volatility in general). Our analysis also reveals that both markets exhibit positive spatial correlation in prices, indicating that it is *impossible* to fully eliminate risk by using only either generators or loads. Consequently, decentralizing technologies of the same type has significant but limited impacts on risk mitigation. Full risk mitigation can only be achieved by combinations of generation and load assets (which can be achieved with microgrids, prosumers, or batteries). Our analysis also indicates that electricity markets provide significant incentives to modularize power-intensive technologies (e.g., manufacturing and data centers). This is of particular relevance due to recent interest in the deployment of small-scale modular technologies.

Our analysis is retroactive in nature (uses historical data) and thus lacks predictive capabilities. Enabling predictability requires us to develop advanced forecasting methods that capture simultaneous spatial and temporal correlations. Specifically, we are interested in investigating recently-developed dynamic principal component analysis and dynamic mode decomposition techniques to conduct space-time analysis and forecasting of market data (Dong and Qin, 2018; Vanhatalo et al., 2017). Such techniques exploit space-time correlations to identify dominant modes in the data.

It is also necessary to extend or placement formulations to analyze effects of temporal flexibility. In particular, our current formulation assumes that technologies provide a fixed capacity all the time, while new technologies can provide ramping capacity to account for uncertainty due to demand and renewable forecasting errors. Therefore, we are interested in developing advanced formulations that can capture technologies that can shift load/generation in time. These formulations are intractable with off-the-shelf tools (due to a dramatic increase in the number of decision variables) and we will thus investigate decomposition algorithms for their solution. Specifically, such formulations reach tens to hundreds of millions of variables but note that the placement formulation exhibits sparse coupling (the total installed capacity constraint). As a result, one can envision using Lagrangian dual decomposition techniques to tackle this problem. We are also interested in including nonlinear effects of economies of scales in the placement, which can be done by using piece-wise linear approximations.

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## Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.compchemeng.2019.05.005.

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