Mechanism and Machine Theory 142 (2019) 103584

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmachtheory

Research paper

Developable mechanisms on regular cylindrical surfaces

Jacob R. Greenwood, Spencer P. Magleby, Larry L. Howell*

Brigham Young University, Department of Mechanical Engineering, 350 EB, Provo, UT 84602, United States

ARTICLE INFO

Article history:
Received 21 May 2019
Revised 24 July 2019
Accepted 1 August 2019
Available online xxx

Keywords:
Developable mechanism
Developable surface
Cylindrical developable mechanism
Morphing mechanism

ABSTRACT

Developable mechanisms can provide high functionality and compact stowability. This paper presents engineering models to aid in the design of cylindrical developable mechanisms. These models take into account the added spatial restrictions imposed by the developable surface. Equations are provided for the kinematic analysis of cylindrical developable mechanisms. A new classification for developable mechanisms is also presented (intramobile, extramobile, and transmobile) and two graphical methods are provided for determining this classification for single-DOF planar cylindrical developable mechanisms. Characteristics specific to four-bar cylindrical developable mechanisms are also discussed.

1. Introduction

Many factors are pushing for products to be lighter, less expensive, and more compact, while still maintaining a high standard of performance. This is particularly apparent in the space, transportation, and medical industries. For example, surgical instruments need to be highly sophisticated while still being as small as possible for minimally invasive procedures [1]. However, a shrinking design space and increased functionality are often competing objectives.

Several techniques have been proposed in the literature to address these issues. One technique to achieve compactness and functionality is to design mechanisms that have links shaped specific to the application. This been used in the literature to design rolling robots [2,3], mechanical characters [4], and a variety of morphing mechanisms such as airfoils [5–7], extrusion dies [5,7–9], and antennas [7]. Other techniques include the use of origami-inspired mechanisms [10–12], scissor-based linkages [13], Bricard linkages [14–16], and Myard linkages [17,18].

Developable mechanisms are a class of mechanisms that can address some of these emerging needs [19]. A developable mechanism is a mechanism which, at some point in its motion, is constrained to a specific shape and orientation on a predefined developable surface (eg. cylinder, cone, or tangent-developed surface). Developable mechanisms use shaped links to achieve high functionality while allowing for compact storage space. Developable mechanisms are specifically designed to be placed on (or embedded in) developable surfaces, enabling them to be ultra-compact. This opens the door to new functionality because they actuate off of the exterior/shell of a device instead of being contained within the device's shell (similar to how lamina-emergent mechanisms emerge out of a planar sheet [20,21]) as shown in Fig. 1. For example, a mechanism might be embedded in the fuselage of a rocket during liftoff, but later could actuate and open the side of the rocket once it is in space. Developable mechanisms can provide new functionality while allowing predictable motion in one configuration and being compactly stored in another.

* Corresponding author.
E-mail address: lhowell@byu.edu (L.L. Howell).

https://doi.org/10.1016/j.mechmachtheory.2019.103584
0094-114X/

Fig. 1. Example of a developable mechanism actuating off of a cylindrical surface. In this case, the mechanism emerges from the surface. Developable mechanisms can also be placed on the outer (or inner) developable surface of a rigid body. In both cases the links are shaped to conform to the developable surface.

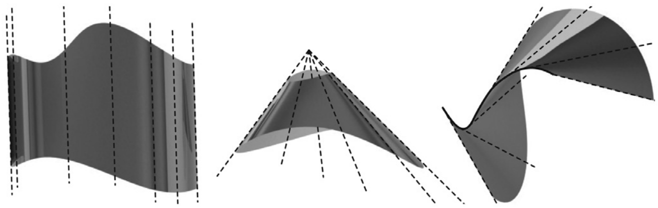


Fig. 2. Generalized cylindrical (left), conical (center), and tangent developed (right) surfaces with some of their ruling lines shown.

Engineering models that predict the motion or enable the design of mechanisms that conform to cylindrical surfaces are needed. Because existing formulas do not account for the added spatial restrictions imposed by a developable surface, new models are needed to help engineers model, analyze, and design developable mechanisms on regular cylinders.

This paper presents an approach to designing developable mechanisms on regular cylindrical surfaces. First, the background of developable mechanisms will be discussed and specifically applied to regular cylindrical surfaces. Second, three behaviors of developable mechanisms are proposed and evaluated. These behaviors and corresponding mechanism types can help engineers design mechanisms that meet common motion requirements on developable surfaces. Next, terminology for cylindrical developable mechanisms is introduced. This includes defining new engineering models to account for the added spatial restriction imposed by the developable surface (in this case, a regular cylinder).

2. Background

2.1. Developable surfaces

A developable surface is any surface that can be formed by bending a planar sheet (such as a piece of paper) without stretching, compressing, or tearing [22]. There are four types of developable surfaces (the plane, cylinder, cone, and tangent developed), each of which is classified by the orientation of the ruling lines on its surface, as shown in Fig. 2. The trivial case of the plane has ruling lines in every direction. The ruling lines for a generalized cylinder are all parallel to each other while the ruling lines for a generalized cone converge to a single point. Tangent-developed surfaces have ruling lines that are tangent to a spatial curve.

Developable surfaces have been used in mechanism design to design rolling-contact joints [12,23], gear profiles [24,25], and origami-based mechanisms [26,27].

2.2. Instant centers

Instant centers (also called instantaneous centers of rotation or instantaneous velocity centers) are useful in graphical mechanism synthesis methods for predicting the relative motion of links in rigid-body mechanisms. Instant centers have been used in a variety of different mechanism design applications, such as in mechanism synthesis [28], in generating line-envelopes [29], in compliant mechanism design [30,31], and for predicting singularities in parallel mechanisms [32].

The annotation for instant centers, IC_{ab}, denotes the relative motion of link b with respect to link a (see Fig. 3). It is important to note that $IC_{ab} = IC_{ba}$ and instant centers only predict the instantaneous motion of the links because some instant centers move as the mechanism is actuated.

2.3. Developable mechanisms

A mechanism is called a *developable mechanism* if, at some point in its motion, all the links are simultaneously constrained to a specific shape and orientation on a pre-defined developable surface. Developable mechanisms are possible on all four types of developable surfaces by aligning the joint axes with a developable surface's ruling lines [19,33]. Because they are designed using developable surfaces, developable mechanisms can simplify manufacturing by being fabricated flat then bent with the surface into their final shape [34].

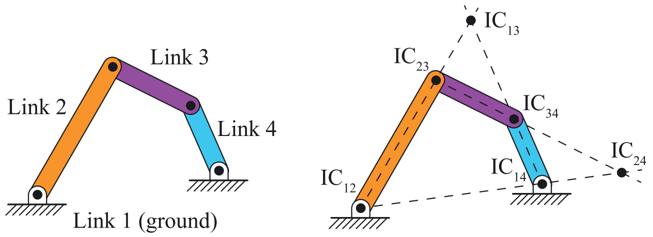


Fig. 3. Instant centers of a typical four-bar mechanism.

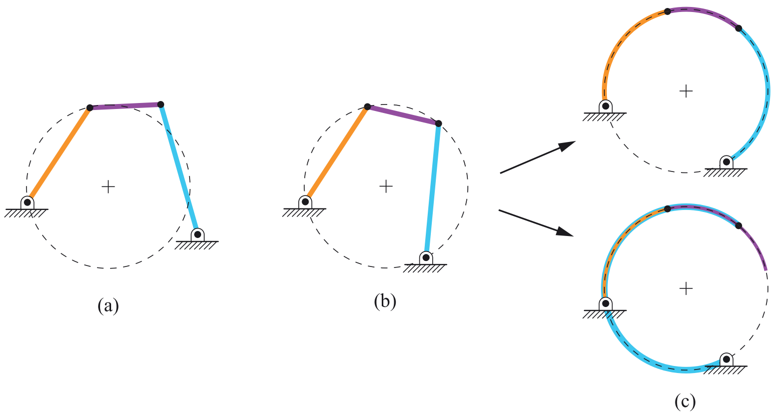


Fig. 4. A mechanism with the two developable mechanism conditions applied. (a) Basic mechanism. (b) Applying the hinge-axis ruling condition. (c) Applying the link shape condition. This yields many different possibilities, two of which are shown. Note that the mechanisms with straight links (b) and shaped links (c) are kinematically equivalent because their joint axes are at the same location and the links are rigid.

Developable mechanisms are "contained within or conform to developable surfaces when both are modeled with zero-thickness" [19]. For example, a mechanism could be placed on the wing of an airplane or embedded in the shaft of a surgical instrument. The *conformed position* of a developable mechanism is the position where all the links are simultaneously conformed to and aligned with the reference surface, as shown in the far left image of Fig. 1. The *reference surface* is the developable surface to which the mechanism's joint axes must align. Hence, the mechanism type depends on the reference surface.

Engineering models are needed that predict the motion or enable the design of mechanisms that conform to a specific surface. Existing formulas do not account for the added restrictions imposed by a developable surface. This paper presents models to help engineers describe, design, and analyze developable mechanisms on regular cylindrical surfaces.

3. Developable mechanism conditions

Developable mechanisms are constrained to have their joint axes coincident with ruling lines on a developable surface in their conformed position [33]. Nelson et al. referred to this constraint as the *hinge-axis ruling condition* [19].

Traditional engineering formulas for kinematic analysis assume that all links in a mechanism are rigid and only the distance between joints, not the shape of the links, matters [35]. However, for developable mechanisms, the links conform to the developable surface (in the conformed position), so their shape does matter. This will be referred to as the *link-shape condition*.

These two conditions apply to developable mechanisms and are formalized below.

Condition 1 Hinge-axis ruling condition: The joint axes must be coincident with ruling lines on the developable surface.
Condition 2 Link-shape condition: The links must conform to the developable surface.

An example of these conditions for regular cylindrical developable mechanisms is shown in Fig. 4.

Cylindrical developable mechanisms can be modeled as planar mechanisms because cylindrical developable surfaces have parallel ruling lines. However, the synthesis of cylindrical developable mechanisms is more complicated than for planar mechanisms because of the two conditions stated above. For developable mechanisms on regular cylinders, the reference surface is a regular cylinder (often represented as a circle). The joint axes must align with this surface and the shape of the links (in the conformed position) are circular arcs, as shown in Fig. 4. Links can be shaped to curve in either direction around the circle and can also extend beyond the joint axis locations. This is a useful tool because the points of interest on a mechanism are often not located at the joint locations (for example, a coupler point). This paper presents models that take into account these added conditions.

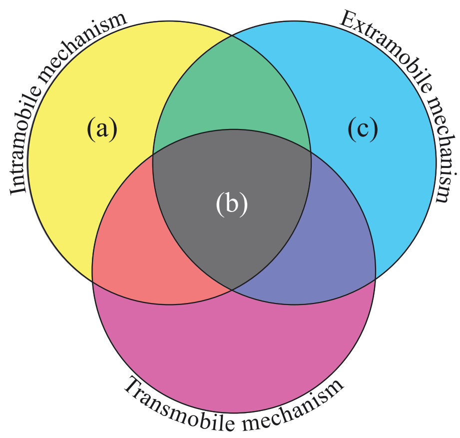


Fig. 5. Venn diagram showing how intramobile, extramobile, and transmobile mechanisms overlap. There are no known examples of (a) exclusively intramobile and (c) exclusively extramobile mechanisms. The only known examples that are (b) all three types have more than one DOF.

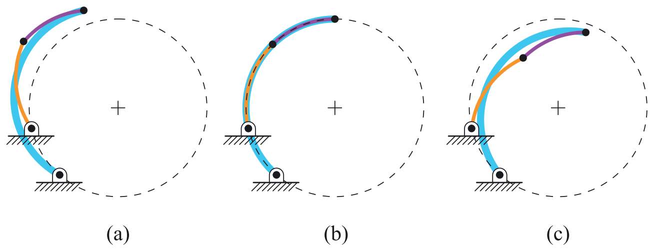


Fig. 6. An example 1-DOF developable mechanism that is an extramobile and intramobile mechanism (top middle of the Venn diagram in Fig. 5). When the mechanism is actuated (a) counterclockwise, all links can enter the exterior of the reference surface. When the mechanism is actuated (c) clockwise, all links can enter the interior of the reference surface. The mechanism is shown in its conformed position in (b).

4. Intramobile, extramobile, and transmobile mechanisms

The location of links and joints throughout a morphing mechanism's motion may need to be taken into consideration during the design process. For example, a morphing airplane wing mechanism should be completely contained inside the wing [5,36]. Because developable mechanisms conform to a developable surface, it is important to predict the behavior of the mechanism with relation to that surface—specifically the behavior of mechanisms as they deploy from the conformed position. This section identifies three behaviors that developable mechanisms can exhibit with respect to a developable surface and three corresponding mechanism types. It also provides two methods for predicting these behaviors for regular cylindrical developable mechanisms.

Intramobility: The ability of a mechanism to completely enter (or remain on the edge of) the interior of the reference surface when moved from the conformed position.
Extramobility: The ability of a mechanism to completely enter (or remain on the edge of) the exterior of the reference surface when moved from the conformed position.
Transmobility: The ability of a mechanism to simultaneously enter both the interior and exterior of the reference surface when moved from the conformed position.

If a mechanism exhibits intramobility, it is an *intramobile mechanism*. Similarly, *extramobile mechanisms* and *transmobile mechanisms* exhibit extramobility and transmobility, respectively. These three mechanism types (intramobile, extramobile, transmobile) are not mutually exclusive, as shown in Figs. 5 and 6. The types and behaviors are also independent of the amount of motion possible. For example, an extramobile mechanism may remain exterior to the reference surface for a large range of motion while another may do so for a much smaller range of motion.

The terms presented in this section (intramobility, intramobile, etc.) describe possible behaviors of mechanisms on closed developable surfaces with respect to the developable surface. Mechanisms on non-closed developable surfaces may use these terms simply by defining the “exterior” and “interior” of the surface. These terms are applicable to mechanisms with single or multiple DOF. An example of these concepts (non-closed surface, multiple DOF) is shown in Fig. 7.

Intramobility and extramobility are especially useful if the developable surface lies on the interior or exterior of a rigid body. For example, a regular cylindrical mechanism inside of a pipe would not be able to actuate unless it exhibited intramobility. Similarly, a developable mechanism on the outside of a pipe would require extramobility to actuate.

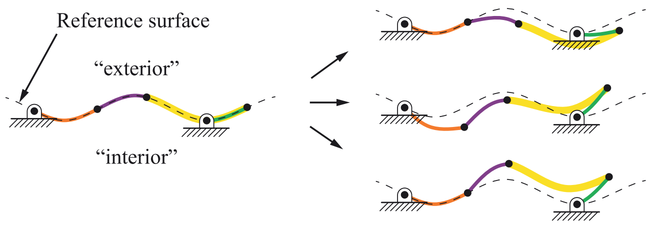


Fig. 7. An example 2-DOF developable mechanism on a generalized cylindrical reference surface. Because this is a non-closed surface, the "interior" and "exterior" of the reference surface are chosen. This mechanism is an intramobile, extramobile, and transmobile mechanism (center of the Venn diagram in Fig. 5) because all links can be move from its (left) conformed position to the (right top) interior, to (right middle) both sides, and to the (right bottom) exterior of the reference surface.

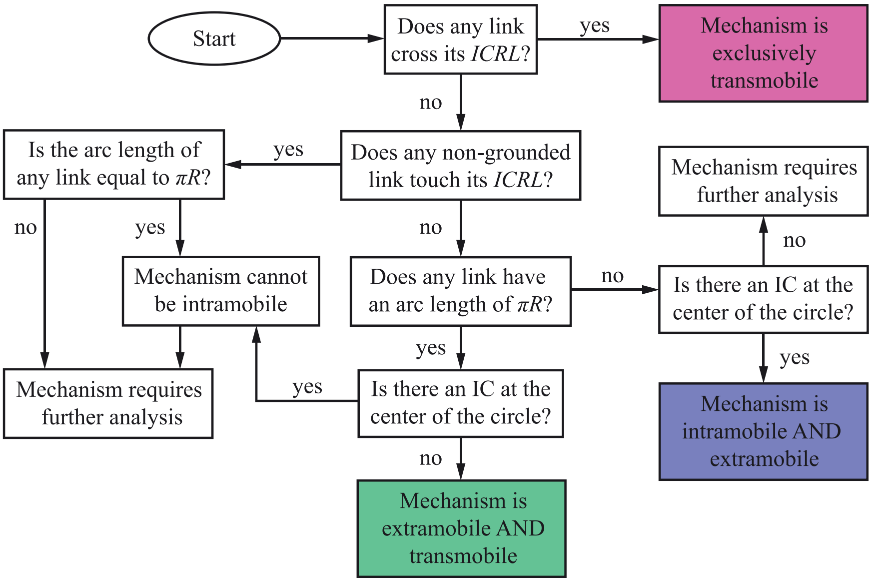


Fig. 8. Flow diagram for determining if a mechanism is intramobile, extramobile, and/or transmobile using the ICRL method. The colors match those used in the Venn diagram (Fig. 5).

4.1. Intramobility, extramobility, and transmobility for regular cylindrical developable mechanisms

Regular cylindrical surfaces (circular tubes or arc segments of a circular tube) are especially common in engineering applications, such as rocket fuselages, pressure tanks, and utility poles. This commonality of regular cylinders makes it advantageous to have rapid methods to analyze the behaviors of these mechanisms. Two graphical methods for determining the intramobility, extramobility, and transmobility for single-DOF regular cylindrical developable mechanisms are presented below.

4.2. Instant center reference line method

The instant center reference line (ICRL) method determines the classification (intramobile, extramobile, transmobile) of a single-DOF regular cylindrical developable mechanism. This method uses the fact that mechanisms are comprised of individual links and instant centers describe instantaneous link motion with respect to the other links.

The ICRL method is shown as a flow diagram in Fig. 8. The *instant center reference line* for link i ($ICRL_i$) is the line that passes through the center of the circular developable surface and instant center IC_{1i}, as shown in Fig. 9. For most cases this method determines if the mechanism is intramobile, extramobile, and/or transmobile (i.e. the mechanism's location in the Venn diagram shown in Fig. 5). In some cases further analysis may be necessary as described below. An example of this method being applied to a four-bar regular cylindrical developable mechanism is shown in Fig. 10.

4.2.1. Derivation

The ICRL method was derived by considering the motion of link i relative to IC_{1i} and considering the interaction between connected links. The principles described below were used to create the flow diagram (Fig. 8).

Principle 1: Links that cross their ICRL cause the mechanism to be transmobile. Instantaneously, link i rotates around IC_{1i}. $ICRL_i$ indicates the portions of link i that will move one way with respect to the circular reference surface (e.g. inside)

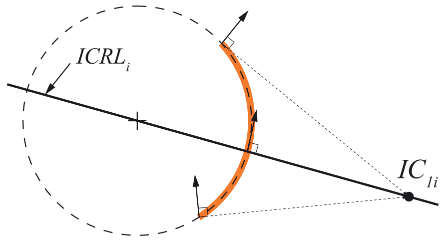


Fig. 9. Link i rotates around IC_{1i}. The instant center reference line ($ICRL_i$) is also shown.

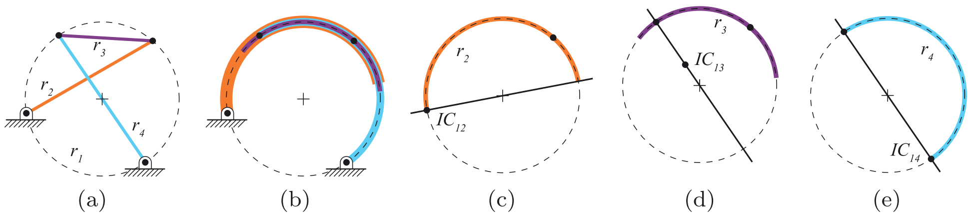


Fig. 10. The application of the ICRL method for an example four-bar developable mechanism. (a) Skeleton diagram of mechanism. (b) Shaped links (note that links 2 and 3 extend past their joint axes). (c–e) The three moving links and their corresponding ICRLs. Because link 3 is on both sides of $ICRL_3$, the mechanism is exclusively transmobile.

from the portions of link i that will move the opposite way with respect to the surface (e.g. outside). If any link can simultaneously move inside and outside the reference surface the mechanism must be exclusively transmobile (in which case the other links will not change this). Therefore, if any link crosses its ICRL, the mechanism is exclusively transmobile.

Principle 2: If the mechanism has at least one link with an arc length of πR (where R is the radius of the reference surface), the mechanism cannot be intramobile. The mechanism cannot be intramobile because the link cannot enter the circle without also exiting the circle.

Principle 3: If a non-grounded link touches its ICRL, further analysis is necessary. Portions of a link touching the ICRL can instantaneously move tangent to the circle. Because the instant centers of non-grounded links can move as the mechanism moves, the ICRL may move, indicating a change in behavior of the mechanism. Hence, further analysis is required to determine if the mechanism can be intramobile, extramobile, and transmobile (such as putting the mechanism in CAD and watching it move).

Principle 4: A mechanism cannot be transmobile if no link touches or crosses its ICRL (excluding fixed joint locations). If a link does not touch or cross its ICRL, the direction of its instantaneous motion is either inside or outside the reference surface. Grounded links touch their ICRL at their fixed pivot (and instant center) location. However, this point on the link, by definition, does not move from the reference surface.

Principle 5: If there is an instant center at the center of the circle, further analysis is necessary. If an instant center is at the center of the circle, no ICRL can be drawn for that link. The direction the instant center will move (and where ICRL will be) is not easily determined with graphical methods alone. Therefore, determining the behavior of the mechanism requires further analysis.

4.3. Shadow method

The shadow method is a particularly useful method during the initial design of a 1-DOF regular cylindrical developable mechanism, provided it has a four-bar loop. This method is much quicker than the ICRL method because it does not require finding the instant centers and it can analyze the four-bar loop as a whole (instead of analyzing each link individually). For example, the shadow method can be done through rough hand sketches while the ICRL method may require the user to use CAD or other software. However, it does not take into account the shape of the links and is therefore less conclusive. This method only determines if a mechanism is exclusively transmobile or if the mechanism cannot be intramobile. As such, it may be used as a quick "first check" before using the ICRL method.

For any four-bar loop with one fixed link, shade in the area within the circle that is bounded by the vectors of the two side links (i.e. $\vec{r_2}$ and $\vec{r_4}$ for traditional four-bar mechanisms) and contains the ground and coupler links, as shown in Fig. 11. Next, determine the location of the center point of the circle. Three cases are possible: If the center point of the circle is inside the shaded area, the mechanism is exclusively transmobile. If it is outside the shaded area, the mechanism may be intramobile, extramobile and/or transmobile. If it is on the edge of the shaded area, the mechanism may be extramobile and/or transmobile (but cannot be intramobile).

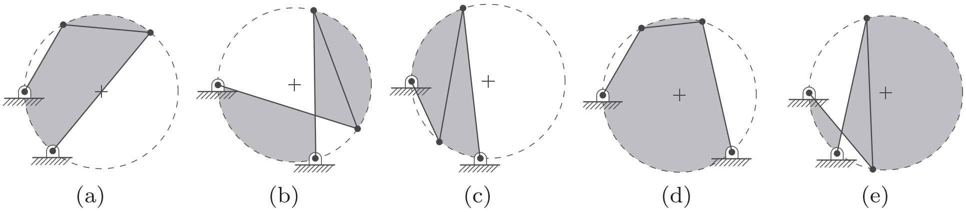


Fig. 11. Skeleton diagrams of six developable mechanisms, each of which is analyzed by the shadow method. Mechanism (a) cannot be an intramobile mechanism. Mechanisms (b and c) may be intramobile, extramobile and/or transmobile mechanisms. Mechanisms (d-e) are exclusively transmobile mechanisms.

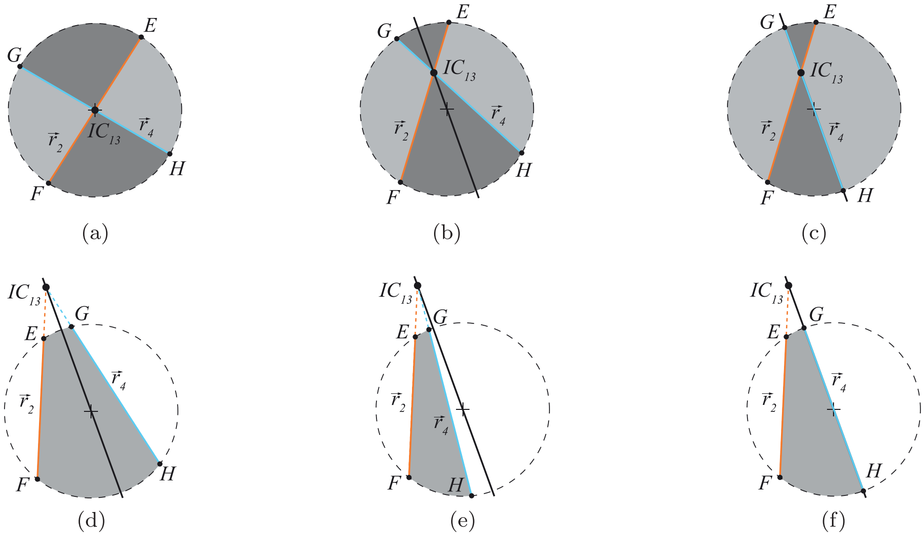


Fig. 12. The six possible scenarios of $\vec{r_2}$ and $\vec{r_4}$ with respect to $ICRL_3$ for a four-bar mechanism. Note that in (a-c), if $\vec{r_3} = \overrightarrow{EH}$ or $\overrightarrow{FG}$ the shadow would be the dark-grey shaded area while if $\vec{r_3} = \overrightarrow{EG}$ or $\overrightarrow{FH}$ the shadow would be the light-grey shaded area. Swapping $\vec{r_2}$ and $\vec{r_4}$ does not change the intramobility, extramobility, or transmobility of the mechanism. Each scenario is analyzed by the ICRL method.

4.3.1. Derivation

The shadow method was derived by applying the ICRL method to a four-bar loop with one fixed link. This method is concerned only about the skeleton diagram of the four-bar mechanism, so only the loop vectors ($\vec{r_1}, \vec{r_2}, \vec{r_3}, \vec{r_4}$) and their endpoints (the joint locations) matter. For a four-bar loop, IC_{13} is coincident with link vectors $\vec{r_2}$ and $\vec{r_4}$. This is true regardless of their directions, ($\overrightarrow{EF}$ or $\overrightarrow{FA}$, and $\overrightarrow{GH}$ or $\overrightarrow{HG}$ in Fig. 12).

Six separate scenarios of $\vec{r_2}$ and $\vec{r_4}$ must be considered. Each scenario has four possible configurations for $\vec{r_3}$: $\overrightarrow{EG}$, $\overrightarrow{EH}$, $\overrightarrow{FG}$, or $\overrightarrow{FH}$. Examples of these scenarios are shown in Fig. 12.

(a) Both vectors cross at the circle's center.
(b) Both vectors cross $ICRL_3$ at a point other than the center.
(c) One vector crosses $ICRL_3$ and the other is collinear with it.
(d) The vectors are on separate sides of $ICRL_3$.
(e) The vectors are on one side of $ICRL_3$.
(f) One vector is collinear with $ICRL_3$ and the other is on one side.

For scenarios (c) and (f) it is assumed that $\vec{r_2}$ does not pass through the center point of the circle (and $\vec{r_4}$ does). If this is not the case, then $\vec{r_2}$ and $\vec{r_4}$ are swapped for this derivation.

The ICRL method is used for each scenario to determine if a link causes the mechanism to be exclusively transmobile or if a link prohibits the mechanism from being intramobile (Principles 1 and 2 from the derivation of the ICRL method). The mechanism is exclusively transmobile if the two endpoints of $\vec{r_3}$ are on opposite sides of $ICRL_3$. Since only the link vectors are used (instead of the shaped links), $\vec{r_2}$ and $\vec{r_4}$ cannot cross $ICRL_2$ and $ICRL_4$, respectively. The mechanism cannot be intramobile if a link must have an arc length of at least πR. A summary of the results for all three moving links ($\vec{r_2}, \vec{r_3}, \vec{r_4}$) is shown in Table 1.

Table 1
Summary of shadow method derivation results. For scenarios (a-c), there are two different possible locations of the shadow area, depending on where $\vec{r_3}$ is located. These are noted as light-grey and dark-grey, as shown in Fig. 12.

Scenario	Does the link cause the mechanism to be exclusively transmobile?			Does the link prevent intramobility?			Location of center point with respect to the shadow area
	$\vec{r_2}$	$\vec{r_3}$	$\vec{r_4}$	$\vec{r_2}$	$\vec{r_3}$	$\vec{r_4}$	
a (light-grey)	no	no	no	yes	no	yes	On the edge
a (dark-grey)	no	no	no	yes	no	yes	On the edge
b (light-grey)	no	no	no	no	no	no	Outside
b (dark-grey)	no	yes	no	no	no	no	Inside
c (light-grey)	no	no	no	no	no	yes	On the edge
c (dark-grey)	no	no	no	no	no	yes	On the edge
d	no	yes	no	no	no	no	Inside
e	no	no	no	no	no	no	Outside
f	no	no	no	no	no	yes	On the edge

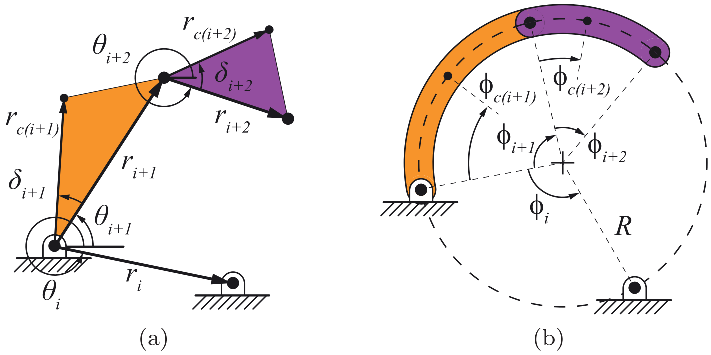


Fig. 13. (a) Traditional method for describing the position and orientation of links and coupler points in a mechanism. (b) Proposed method for describing the position and orientation of links and coupler points in a regular cylindrical developable mechanism.

4.4. General rules for intramobility and extramobility on regular cylindrical surfaces

These two methods yield two general rules for designing intramobile and extramobile mechanisms on regular cylinders.

1. No link should have an arc length greater than πR.
2. Links with one fixed-pivot joint should only extend in one direction past their grounded joint.

Beyond being used in the ICRL method, the ICRL can help designers predict how far the arc length of a link can be extended past its joint axis without changing the intramobility or extramobility of the mechanism. If the arc length of link i is extended without touching $ICRL_i$, this does not change the classification (intramobile, extramobile, transmobile) of the mechanism. (Although it may change the amount of motion possible inside or outside the reference surface.) If link i is extended until it touches $ICRL_i$, further analysis (such as using CAD) should be used.

5. Regular cylindrical developable mechanisms

An approach for regular cylindrical developable mechanism synthesis is presented below. This approach takes into account the two conditions discussed in Section 3 and uses the foundation offered by traditional kinematics.

Links, which traditionally are defined by a link length (r_i) and initial orientation angle (θ_i), will now be defined by the radius of the zero-thickness reference cylinder (R) and a link angle (ϕ_i), as shown in Fig. 13. The link length, link angle, and cylinder radius are related by

$$r_i = 2R\sin\frac{\phi_i}{2} \tag{1}$$

For an n-bar loop, the sum of all link angles must be an integer multiple of 360°, as

$$\sin(2(\phi_1 + \phi_2 + \ldots + \phi_n)) = 0 \tag{2}$$

5.1. Four-bar cylindrical developable mechanisms

Four-bar mechanisms are often the fundamental unit of multi-loop mechanisms. A solid understanding of four-bar cylindrical developable mechanisms will enable the creation of more complex developable mechanisms.

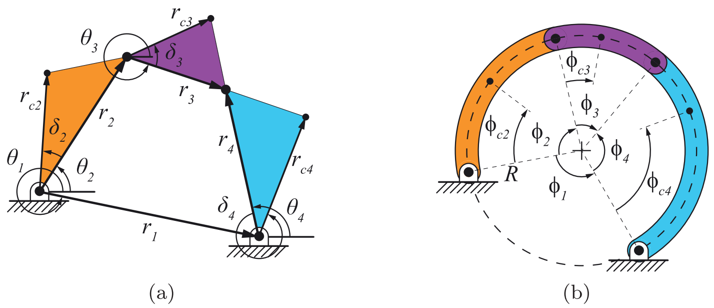


Fig. 14. (a) Traditional method for describing the position and orientation of links and coupler points in a four-bar mechanism. (b) Proposed method for describing the position and orientation of links and coupler points in a regular cylindrical four-bar developable mechanism.

Six variables are needed to describe a traditional four-bar mechanism: r_1, r_2, r_3, r_4, the circuit (μ), and ($\theta_2 - \theta_1$). Only four variables are needed to describe the conformed position of a four-bar mechanism on a regular cylinder (see Fig. 14), since the four link angles need to add up to an integer multiple of 360°,

$$\sin(2(\phi_1 + \phi_2 + \phi_3 + \phi_4)) = 0 \tag{3}$$

The four specified variables (eg. ϕ_1, ϕ_2, ϕ_3, R) can be used to find the fifth (eg. ϕ_4).

The angles at the conformed position and the circuit can be found using the following equations

$$\theta_2 - \theta_1 = -180^\circ - \frac{\phi_1}{2} - \frac{\phi_2}{2} \tag{4}$$

$$\theta_3 - \theta_1 = -180^\circ - \frac{\phi_1}{2} - \phi_2 - \frac{\phi_3}{2} \tag{5}$$

$$\theta_4 - \theta_1 = -90^\circ - \mu 90^\circ + \frac{\phi_4}{2} + \frac{\phi_1}{2} \tag{6}$$

$$\mu = \begin{cases} 1 \text{ (open)} & \text{if } \sin\frac{\phi_4}{2} < 0 \\ -1 \text{ (crossed)} & \text{if } \sin\frac{\phi_4}{2} > 0 \end{cases} \tag{7}$$

Eqs. (4)–(7) apply only in the conformed position of the mechanism.

5.1.1. Intramobility and extramobility of cylindrical developable four-bar mechanisms

There are six different configurations for which cylindrical developable four-bar mechanisms will be intramobile and/or extramobile mechanisms. Table 2 contains these configurations and the link angles that define them.

This table is a useful starting point for designing intramobile and extramobile mechanisms. For example, if the design requires that the mechanism be a double rocker intramobile mechanism, the design must fall into classes 1A, 1B, 2A, or 2B. Note that classes 1A and 1B are isomorphisms, as are classes 2A and 2B, and 3A and 3B. To not repeat mechanisms or create infeasible mechanisms, the link angles are restricted as follows: ($0^\circ < \phi_1 \leq 180^\circ$), ($0^\circ < \phi_2 < 360^\circ$), and ($0^\circ < \phi_3 < 360^\circ$).

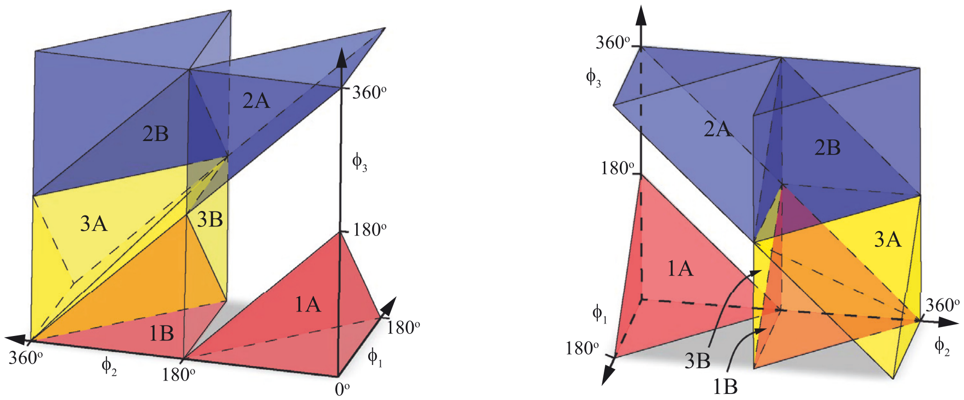
The table was derived using the shadow method (discussed in Section 4.3). Hence, configurations provide only the skeleton diagram of the mechanism and do not take into account the shape of the links or links that extend beyond their joint axes. The ICRL method should be used for a more complete analysis of the entire final mechanism.

The link angles for intramobility and extramobility are nearly identical. For example, in class 1A a mechanism that obeys $\phi_1 + \phi_2 + \phi_3 < 180^\circ$ can be an intramobile mechanism and a mechanism that obeys $\phi_1 + \phi_2 + \phi_3 \leq 180^\circ$ can be an extramobile mechanism. The overlap shows where the mechanism is intramobile AND extramobile (an example is shown in Fig. 6). When $\phi_1 + \phi_2 + \phi_3 = 180^\circ$, the arc length of link 4 must be at least πR and therefore cannot be part of an intramobile mechanism, as shown in Fig. 11a. If $\phi_1 + \phi_2 + \phi_3 > 180^\circ$, the mechanism is exclusively transmobile because the shadow would include the center point of the circle, as shown in Fig. 11d.

For four-bar cylindrical developable mechanisms, the mechanism type (intramobile, extramobile, transmobile) is independent of the radius of the reference cylinder (R). This non-dimensional design space has only three variables, ϕ_1, ϕ_2, and ϕ_3, which means the design space can be plotted in 3D. Fig. 15 shows the design space with the classes, represented as polyhedra, labeled.

5.2. Six-bar cylindrical developable mechanisms

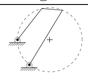
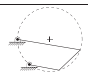
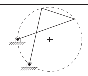
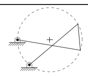
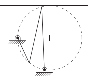
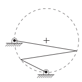
Single-DOF six-bar mechanisms are possible on cylindrical developable surfaces by aligning all the joint axes with the ruling lines on the cylindrical reference surface and by shaping the links so they conform to the surface.



(a) From the back (b) From the front

Fig. 15. Two different views of the four-bar cylindrical developable mechanism design space, with the polyhedra that represent the intramobile/extramobile classes labeled. The polyhedra are defined by the link angles in Table 2. Classes 1A and 1B are represented by red polyhedra, classes 2A and 2B are represented by blue polyhedra, and classes 3A and 3B are represented by yellow polyhedra. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 2
Intramobile and extramobile link angles for all four-bar developable mechanisms on regular cylinders. *For extramobility, the inequality $<$ should be $\leq$, and $>$ should be $\geq$. **Using Barker's classification of planar four-bar linkages [37].

Class	Skeleton Diagram	Link angles for intramobility	Possible Barker Configurations**
1A		$\phi_1+\phi_2+\phi_3<180°*$	GCCC, GCRR, GRCR, RRR2
1B		$\phi_2>180°*$ $\phi_1+\phi_2+\phi_3<360°$	GCCC, GRCR, GRRC, RRR4
2A		$\phi_2<180°*$ $\phi_1+\phi_2+\phi_3<540°*$ $\phi_2+\phi_3\geq 360°$	GCCC, GRCR, CP3X, CP2X
2B		$\phi_2>180°*$ $\phi_1+\phi_2+\phi_3>540°*$ $\phi_1+\phi_2\leq 360°$	GCCC, GRCR, CP3X, CP2X
3A		$\phi_1+\phi_2\geq 360°$ $\phi_2+\phi_3\geq 360°$ $\phi_1+\phi_2+\phi_3<540°*$	GCRR, CP3X, CP2X
3B		$\phi_2>180°*$ $\phi_1+\phi_2\leq 360°$ $\phi_2+\phi_3\leq 360°$ $\phi_1+\phi_2+\phi_3>360°$	GRRC, CP3X, CP2X

5.2.1. Intramobility and extramobility of six-bar cylindrical developable mechanisms

The techniques presented in Section 4.1 can determine the mechanism type (intramobile, extramobile, transmobile) for six-bar mechanisms. If the mechanism has a four-bar loop (Watt 1 and 2, Stephenson 1 and 3), it is especially useful to use the shadow method (to check if intramobility and/or extramobility is possible) before finding the instant centers and using the ICRL method. Two design examples, a Stephenson 2 linkage and a Watt 1 linkage, are discussed below and shown in Figs. 16 and 17, respectively.

Stephenson 2 extramobile mechanism: First, a skeleton diagram of the linkage is drawn and all the joint axes are placed along the reference surface (Fig. 16a). The links are shaped to conform to the reference surface (Fig. 16b). Since this linkage does not have a grounded four-bar loop, the shadow method cannot be used. The instant centers with respect to ground are located (Fig. 16c). The ICRL method is used (Fig. 16d–h). None of the links cross their ICRLs, no non-grounded link touches its ICRL, no link has an arc length of πR, and there is not an instant center at the center of the circle. Hence, the mechanism is intramobile and extramobile.

Watt 1 extramobile mechanism: First, a skeleton diagram of the linkage is drawn and all the joint axes are placed along the reference surface (Fig. 17a). The links are shaped to conform to the reference surface (Fig. 17b). Since this link-

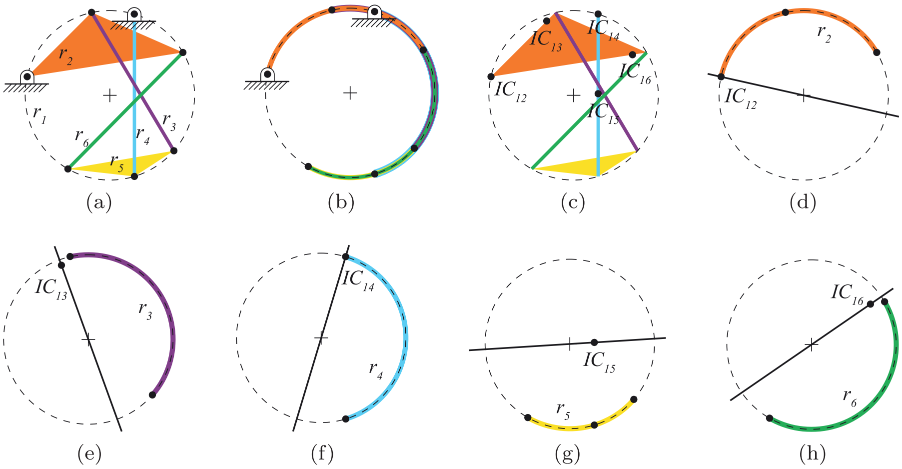


Fig. 16. Checking an example six-bar (Stephenson 2) developable mechanism for intramobility, extramobility, and transmobility. For more discussion, see Section 5.2.1. (a) Skeleton diagram. (b) Shaped links. (d–h) Using the ICRL method: (c) link 2, (d) link 3, (e) link 4, (f) link 5, (g) link 6.

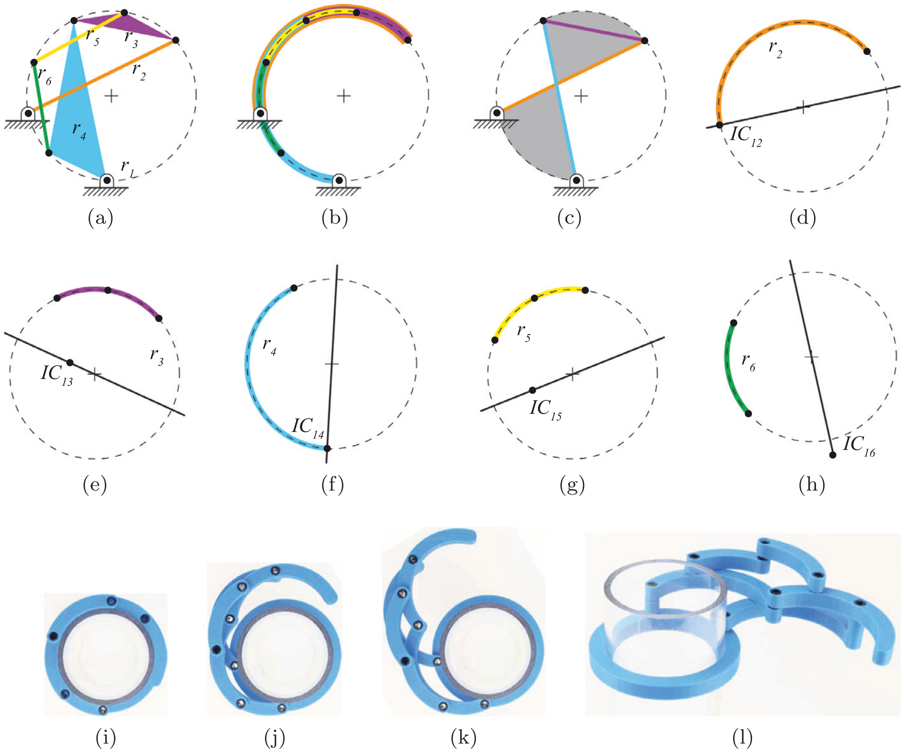


Fig. 17. Checking an example six-bar (Watt 1) developable mechanism for intramobility, extramobility, and transmobility. For more discussion, see Section 5.2.1. (a) Skeleton diagram. (b) Shaped links. (c) Using the shadow method on the mechanism's four-bar loop. (d-h) Using the ICRL method: (d) link 2, (e) link 3, (f) link 4, (g) link 5, (h) link 6. (i–k) Mechanism deploying off the outside of a clear tube. (l) Isometric view of deployed mechanism.

age has a grounded four-bar loop it is checked with the shadow method (Fig. 17c). This mechanism may be intramobile and/or extramobile because the shaded area does not include the center point of the circle. Because intramobility and/or extramobility is still a possibility, the ICRL method is used (Fig. 17d–h). None of the links cross their ICRLs, no non-grounded link touches its ICRL, no link has an arc length of πR, and there is not an instant center at the center of the circle. Hence, the mechanism is intramobile and extramobile. The mechanism is modeled in CAD and link 3 is extended until $ICRL_3$, as discussed in Section 4.4. For further analysis, the kinematic model is moved and by visual analysis it is determined that this extension of link 3 does not change the extramobility of the mechanism. (Figs. 17i–l) show the mechanism deploying off of a clear tube.

Six-bar mechanisms can be intramobile and extramobile regular cylindrical developable mechanisms by carefully arranging the geometry. Example six-bar mechanisms that are intramobile and extramobile have been identified for all five types of six-bar single-DOF mechanisms (Watt 1 and 2, and Stephenson 1–3).

6. Conclusion

Developable mechanisms can provide performance and functionality that is not possible with traditional mechanisms. They can provide a high level of functionality while being able to be stored in a compact position when not in use. These mechanisms can be placed on or embedded within developable surfaces. This can open up previously unused real estate and will create new opportunities for mechanism design in the ever-shrinking design space.

This paper introduces a new classification for developable mechanisms—intramobile, extramobile, and transmobile. This classification applies well to developable mechanisms and mechanisms that conform to the exterior or interior of a rigid body. It can also be used to describe other mechanisms when it is important to understand the mechanism's movement with respect to a surface. Two methods were presented for graphically determining this classification for regular cylindrical developable mechanisms.

Developable mechanisms on regular cylindrical developable surfaces are especially useful because regular cylindrical developable surfaces are common in engineering applications. The models presented take into account the added spacial restrictions imposed by the cylindrical reference surface. The concepts and material in this paper aid in the design of cylindrical developable mechanisms by bridging the gap between existing kinematic models and the additional constraints necessary for developable mechanisms.

Acknowledgments

This material is based on work supported by the National Science Foundation under NSF Grant no. 1663345. Thanks to Jared Butler for technical review and assistance in preparing the manuscript for publication.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.mechmachtheory.2019.103584.

References

[1] J.J. Lim, A.G. Erdman, A review of mechanism used in laparoscopic surgical instruments, Mech. Mach. Theory 38 (11) (2003) 1133–1147.
[2] Y. Wang, C. Wu, L. Yu, Y. Mei, Dynamics of a rolling robot of closedfive-arc-shaped-bar linkage, Mech. Mach. Theory 121 (2018) 75–91.
[3] S.-C. Chen, K.-J. Huang, W.-H. Chen, S.-Y. Shen, C.-H. Li, P.-C. Lin, Quattroped: a leg–wheel transformable robot, IEEE/ASME Trans. Mechatron. 19 (2) (2014) 730–742.
[4] B. Thomaszewski, S. Coros, D. Gauge, V. Megaro, E. Grinspun, M. Gross, Computational design of linkage-based characters, ACM Trans. Graph. 33 (4) (2014) 64.
[5] L.W. Funke, J.P. Schmiedeler, K. Zhao, Design of planar multi-degree-of-freedom morphingmechanisms, J. Mech. Robot. 7 (1) (2015) 011007.
[6] K. Zhao, J.P. Schmiedeler, A.P. Murray, Design of planar, shape-changing rigid-bodymechanisms for morphing aircraft wings, Journal of Mechanisms and Robotics 4 (4) (2012) 041007.
[7] S.A. Shamsudin, A.P. Murray, D.H. Myszka, J.P. Schmiedeler, Kinematic synthesis of planar, shape-changing, rigidbody mechanisms for design profiles with significant differences in arclength, Mech. Mach. Theory 70 (2013) 425–440.
[8] K.S. Giaier, D.H. Myszka, W.P. Kramer, A.P. Murray, Variable geometry dies for polymer extrusion, ASME 2014 International Mechanical Engineering Congress and Exposition, American Societyof Mechanical Engineers, 2014. V02AT02A023–V02AT02A023
[9] B. Li, A.P. Murray, D.H. Myszka, Designing variable-geometry extrusion dies that utilize planar shape-changing rigid-body mechanisms, in: ASME 2015 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, American Society of Mechanical Engineers, 2015. V05BT08A070–V05BT08A070
[10] M. Johnson, Y. Chen, S. Hovet, S. Xu, B. Wood, H. Ren, J. Tokuda, Z.T.H. Tse, Fabricating biomedical origami: a state-of-the-art review, Int. J. Comput. Assist. Radiol. Surg. 12 (11) (2017) 2023–2032.
[11] P. Liyanage, H. Mallikarachchi, Origami based folding patterns for compact deployables tructures, in: Proceedings of 4th International Conferenceon Structural Engineering and Construction Mangement, 2013.
[12] T.G. Nelson, R.J. Lang, S.P. Magleby, L.L. Howell, Curved-folding-inspired deployable compliant rolling-contact element (D-CORE), Mech. Mach. Theory 96 (2016) 225–238.
[13] Y. Yang, Y. Peng, H. Pu, H. Chen, X. Ding, G.S. Chirikjian, S. Lyu, Deployable parallel lower-mobility manipulators with scissor-like elements, Mech. Mach. Theory 135 (2019) 226–250.
[14] J. Cui, H. Huang, B. Li, Z. Deng, A novel surface deployable antenna structure based onspecial form of bricard linkages, in: Advances in Reconfigurable Mechanisms andRobots I, Springer, 2012, pp. 783–792.
[15] J. Wang, X. Kong, Deployable mechanisms constructed by connectingorthogonal bricard linkages, 8R or 10r single-loop linkages using sjoints, Mech. Mach. Theory 120 (2018) 178–191.
[16] H. Huang, Z. Deng, B. Li, Mobile assemblies of large deployable mechanisms, J. Space Eng. 5 (1) (2012) 1–14.
[17] C.-H. Liu, Y.-A. Yao, R.-M. Li, Y.-B. Tian, N. Zhang, Y.-Y. Ji, F.-Z. Kong, Rolling 4R linkages, Mech. Mach. Theory 48 (2012) 1–14.
[18] X. Qi, Z. Deng, B. Li, R. Liu, H. Guo, Design and optimization of large deployable mechanismconstructed by Myard linkages, CEAS Space J. 5 (3–4) (2013) 147–155.
[19] T.G. Nelson, T.K. Zimmerman, R.J. Lang, S.P. Magleby, L.L. Howell, Developable mechanisms on developable surfaces, Sci. Robot. 4 (27) (2019) 5171.
[20] S.E. Wilding, L.L. Howell, S.P. Magleby, Spherical lamina emergent mechanisms, Mech. Mach. Theory 49 (2012) 187–197.
[21] J.O. Jacobsen, B.G. Winder, L.L. Howell, S.P. Magleby, Lamina emergent mechanisms and their basic elements, J. Mech. Robot. 2 (1) (2010) 011003.
[22] D.J. Struik, Lectures on Classical Differential Geometry, Courier Corporation, 1961.
[23] T.G. Nelson, J.L. Herder, Developable compliant-aided rolling-contact mechanisms, Mech. Mach. Theory 126 (2018) 225–242.

[24] V. Zelenỳ, I. Linkeová, J. Sỳkora, P. Skalník, Mathematical approach to evaluate involute gearprofile and helix deviations without using special gear software, Mech. Mach. Theory 135 (2019) 150–164.
[25] D.B. Dooner, On the third law of gearing: a study on hypoid geartooth contact, Mech. Mach. Theory 134 (2019) 224–248.
[26] T. Tachi, G. Epps, Designing one-dof mechanisms for architecture by rationalizing curved folding, International Symposium on Algorithmic Design for Architecture and Urban Design (ALGODE-AIJ). Tokyo, 2011.
[27] R.J. Lang, Origami Design Secrets: Mathematical Methods for an Ancient Art, AKPeters/CRC Press, 2011.
[28] R. Sancibrian, E.G. Sarabia, A. Sedano, J.M. Blanco, A general method for the optimal synthesis of mechanisms using prescribed instant center positions, Appl. Math. Model. 40 (3) (2016) 2206–2222.
[29] H.-A. Hsieh, K.-L. Hsu, Line-envelope generations using the concept of instant center, Mech. Mach. Theory 116 (2017) 360–370.
[30] C.J. Kim, S. Kota, Y.-M. Moon, An instant center approach toward the conceptual design of compliant mechanisms, J. Mech. Des. 128 (3) (2006) 542–550.
[31] A. Humer, Design and optimization of large-deformation compliant mechanisms, PAMM 18 (1) (2018) e201800469.
[32] M. Slavutin, O. Shai, A. Sheffer, Y. Reich, A novel criterion for singularity analysis of parallel mechanisms, Mech. Mach. Theory 137 (2019) 459–475.
[33] T.G. Nelson, R.J. Lang, N.A. Pehrson, S.P. Magleby, L.L. Howell, Facilitating deployable mechanisms and structures via developable lamina emergent arrays, J. Mech. Robot. 8 (3) (2016) 031006.
[34] T. Zimmerman, J. Butler, D. Frandsen, D. Burrow, D. Fullwood, S. Magleby, L. Howell, Modified material properties in curved panels through lamina emergent torsional joints, in: 2018 International Conference on Reconfigurable Mechanisms and Robots (ReMAR), IEEE, 2018, pp. 1–9.
[35] L.L. Howell, Compliant Mechanisms, John Wiley & Sons, 2001.
[36] L.W. Funke, J.P. Schmiedeler, Simultaneous topological and dimensional synthesis of planar morphing mechanisms, J. Mech. Robot. 9 (2) (2017) 021009.
[37] C.R. Barker, A complete classification of planar four-bar linkages, Mech. Mach. Theory 20 (6) (1985) 535–554.