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CONCEPTUALIZING STABLE STATES IN ORIGAMI-BASED DEVICES USING AN ENERGY VISUALIZATION APPROACH

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ABSTRACT

In many origami-based applications a device needs to be maintained in one or more fold states. The origami stability integration method (OSIM) presented in this paper provides an approach for graphically combining various techniques to achieve stability. Techniques are divided into four groups based on whether they are intrinsic or extrinsic to the origami pattern and whether they exhibit differentiable or non-differentiable energy storage behaviors. These categorizations can help designers select appropriate techniques for their application. The paper also contains design considerations and resources for several intrinsic techniques. Finally, two case studies are presented which use the OSIM and the technique guidelines to conceptualize stability in origami-based devices.

1 INTRODUCTION

Many advantages of origami capitalize on its shape-changing ability—how origami moves as it folds. Devices based on origami can be simple to manufacture, starting in a planar state then folding into a desired shape [1–3]. Origami can provide complex motions with simple actuation and low degrees of freedom (DOF) [4–6]. This allows a device to quickly transform from a compact shape to a deployed shape.

However, the ability of origami-based devices to fold can also make them mechanically unstable and lead to undesired motion or behaviors. The balance between retaining foldability and providing adequate stability is a fundamental challenge for origami-based design. For example, what can be done to allow an origami-inspired stool to both fold flat and support a person's weight? While many techniques exist for creating stability in origami-based devices, a major difficulty for designers lies in determining which techniques to use and how they will interact with the chosen origami pattern, the loading conditions, and other techniques.

This paper addresses this difficulty by presenting the *origami stability integration method* (OSIM), an approach for combining stability techniques and conceptualizing their energy behavior in origami-based devices. The paper consists of six sections: (1) review of background material, (2) presentation of foundational concepts, (3) description of the OSIM, (4) discussion on stability technique categorization, (5) detailing attributes of select stability techniques, (6) presentation of two case studies using the OSIM.

2 Background

This work builds on the concepts of viewing origami as a linkage and visualizing energy storage behavior graphically. It also references a variety of existing origami stability techniques.

2.1 Origami as a Kinematic Linkage

Researchers have shown that the motion of origami can be modeled as a kinematic linkage [7–9]; the facets (or panels) are modeled as links, the creases as hinge joints, and origami vertices as spherical mechanisms [10], as shown in Fig. 1. Thinking of origami as a kinematic linkage allows designers to apply engineering concepts (such as mechanical advantage [11], motion prediction [12], and stability [13]) to origami design.

2.2 Visualizing Stability

A *stable equilibrium* is a state of a system in which the sum of the net forces is zero and the energy stored in the system is at a local minima. This is often visualized using a "ball on a hill" analogy, where all energy in the system is shown as potential energy [14]. The stable equilibria are the local minima, as illustrated in Fig. 2. This method has been used in origami research to depict multi-stable vertices [15], bistable leaf-out origami [16], and a bistable waterbomb pattern [17]. Origami devices with multiple DOF can be represented on a single axis by imposing symmetry or other constraints [16, 17].

The OSIM also uses the "ball on a hill" analogy for conceptualizing stable equilibria. This paper is concerned with combining stability techniques rather than calculating specific energy components. For this reason the examples in this paper use approximated energy storage behaviors. While the OSIM is intended to be used early in the design process, it could be used later on in conjunction with higher fidelity models that include components such as damping, dynamics, and force-deflection interactions.

2.3 Origami Stability Techniques

A myriad of stability techniques are seen in origami-based products and origami literature [2–4, 10, 15, 18–25], as well as in a number of reviews [25, 26].

Many techniques are incorporated as part of the creases or facets. Some examples include strain energy stored in the joints during folding [15–17, 27–30] and introducing bias at the joints using other stimuli, such as heat, magnetic fields, or surface tension [2,3,23–25]. Some of these stimuli-actuated techniques even have reversible folding capabilities [23, 31].

Other techniques not incorporated as part of the creases or facets include: clasps, magnets, actuators, and other constraints [32, 33]. For example, an origami kayak uses supporting bulkheads, straps, and retaining channels to retain a fold state. (Installation of the retaining channels is shown in Fig. 3.)

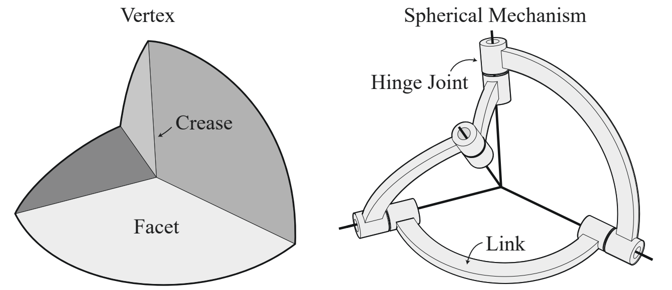

FIGURE 1: The motion of an origami vertex can be modeled as that of a spherical mechanism.

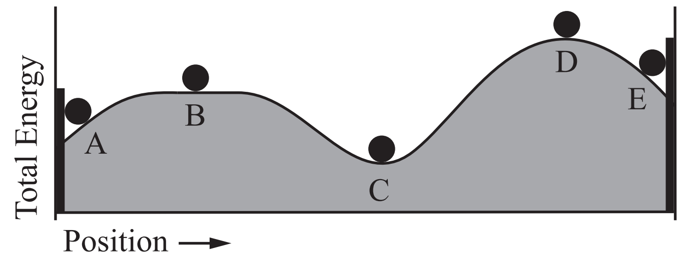

FIGURE 2: A "ball on a hill" diagram for visualizing stability through potential energy. Positions A, C, and E are stable equilibrium positions. Position D is an unstable equilibrium position. Position B is a neutrally stable position.

3 Foundational Concepts

To facilitate the discussion of origami in the OSIM two foundational concepts, *origami linkage* and *fold-state continuum*, are introduced. The term *origami linkage* provides a way to discuss the facets and creases of a fold state without referring to the fold angles. The *fold-state continuum* provides a flexible way to represent a range of fold states.

3.1 Origami Linkage

Most origami-based devices utilize the movement of multiple vertices. However, the common terms used to discuss origami as a kinematic linkage (such as *origami vertex*, *origami figure*, *configuration*, *fold state*, and *crease pattern*) either do not refer

FIGURE 3: An origami kayak. The retaining channel being installed is one of several techniques that create stability. Others not shown include bulkheads and straps. (Image used with permission from Oru Kayaks^{TM}, downloaded from www.orukayak.com)

to a generalized set of facets and creases or they imply a static position. To address these limitations, a new term, *origami linkage*, is presented.

An *origami linkage* is a set of planar facets and their interconnecting creases. This term builds on the traditional use of the term *linkage* in engineering to emphasize that the members exist independent of their position and to strengthen the concept that origami can be an engineering tool. Because we want to refer to sets of facets and creases generally (instead of specific fold states with fixed fold angles), the term origami linkage is used in the OSIM.

An origami linkage can be as simple as a single vertex or multiple vertices, shown in Fig. 4. Typically an origami linkage has an equivalent spherical mechanism. A *fold state* is an origami linkage in a defined position.

In the presentation of the OSIM it is assumed that an origami linkage has been defined. A variety of resources are available that provide specific guidance for designing or selecting an origami linkage such as those provided by Morgan et al., Hernandez et al., Lang, and Avila et al. [34–37].

3.2 Fold-State Continuum of Origami Linkages

As an origami linkage folds, it progresses through a continuous set of fold states that can form a continuum. A fold-state continuum is a useful tool in analyzing and understanding the range of motion of an origami linkage and the properties of the linkage throughout the folding process.

For origami linkages with N DOF, the fold-state continuum has N dimensions. Planning for stability in these cases can be complex. One method for representing the motion of origami

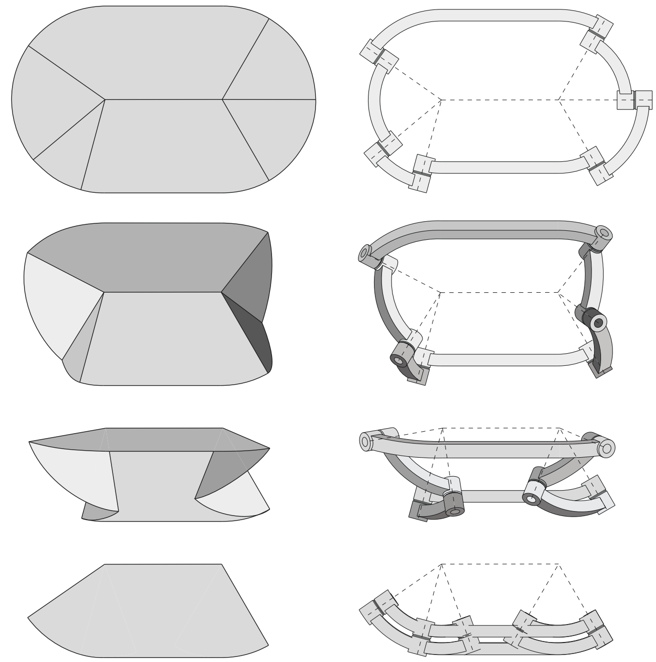

FIGURE 4: A two-vertex origami linkage represented as an origami linkage (left) and a set of two, kinematically equivalent spherical mechanisms (right), shown in four corresponding fold states.

with multiple DOF systems is to remove degrees of freedom using symmetry or other constraints (such as constraints applied by a user during manipulation) [16, 17, 38]. While this is a simplification, in many cases it may be acceptable, especially in the initial stages of the design process. The fold-state continuum for a single DOF origami linkage can be plotted along a single line, as seen in Fig. 5. Care should be taken to ensure that origami linkages deploy correctly from any change-point position (such as the fully unfolded state). This 1D continuum is used as the x-axis in the OSIM to represent the motion of the origami linkage.

Ideally, origami linkages are stable in a single, predetermined fold state (i.e. one position along the 1D continuum). However, some margin of error is often acceptable. In the OSIM, designers specify "desired stability zones" along the fold-state continuum that correspond with this acceptable stability tolerance.

4 Origami Stability Integration Method (OSIM)

This section describes the steps of the OSIM, along with an example. The *origami stability integration method (OSIM)* is a design tool for visualizing and planning stable equilibria in an origami linkage. In the method desired fold states are indicated along a 1D fold-state continuum. The x-axis is the 1D fold-state continuum (see section 3.2), and the y-axis is the amount of po-

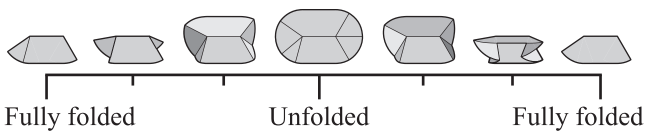

FIGURE 5: A 1D fold-state continuum of a single DOF origami linkage in Fig. 4. Seven fold states along the continuum are shown and marked on the continuum. Three fold states are labeled.

tential energy stored in the system (see section 2.2). The OSIM is intended as a conceptual design tool.

The steps of the method are as follows:

1. Select an origami linkage. (See section 3.1)
2. Assemble a 1D fold-state continuum for the linkage. (See section 3.2.)
3. Designate zones critical to the function of the origami-based device, as shown in Fig. 6.
   (a) Desired stability zones: where stable fold states are desired in the final device.
   (b) Unacceptable motion zones: fold states that the origami linkage should not reach (such as zones that contain bifurcation points). This zone should be "blocked", meaning that it should have a high energy storage behavior.
4. Determine (approximate or calculate) the energy components inherent in the system (such as gravity, strain in the hinges, and other loading conditions). Graph the energy stored by each component along the fold-state continuum. (See section 2.2.)
5. Sum the energy components from step 4.
6. Superimpose a basic energy curve so that stable equilibria and blocked fold states fall in the prescribed zones from step 3.
7. Select stability techniques to obtain this energy curve and determine (approximate or calculate) their energy behavior along the continuum. (Resources for selecting techniques are given in sections 5 and 6.)
8. Sum each energy component from steps 4 and 7. Compare the result of the sum to desired energy curve. Prototype and revisit steps as necessary.

Different combinations of techniques and linkages can produce favorable results. As such, the steps of the OSIM are meant to be an iterative process. Throughout the process, the origami linkage could be modified or replaced. A well-designed origami linkage and loading conditions can reduce the number of techniques necessary for achieving the desired energy conditions. If the device has multiple DOF, each presumed stable equilibrium should be evaluated to ensure that it is not a saddle point.

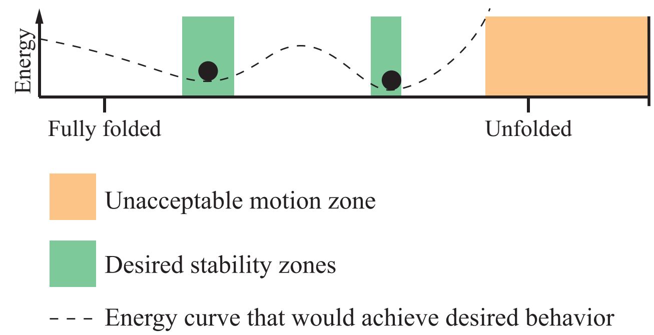

FIGURE 6: An example of how to indicate "desired stability zones" and "unacceptable motion zones", with an energy curve that achieves the desired behavior.

4.1 Example of the OSIM

Suppose a designer is designing a device to have three stable fold states, one as a container, channel, and compact shape. The following OSIM steps correspond with illustrations in Fig. 7.

Step 1: A degree-5 origami vertex is chosen as the origami linkage because it can create the desired shapes. Fig. 7 step 1 shows three fold states that could perform the desired functions.

Step 2: The fold states are placed along a 1D fold-state continuum.

Step 3: Three "desired stability zones" are designated along the continuum, one corresponding with each function of the device. An "unacceptable motion zone" is also added to prevent the contents of the container being crushed.

Step 4: There are two inherent energy components: (1) facet interference and (2) energy storage in the creases from their folding. Their approximated energy behaviors are shown in Fig. 7.

Step 5: The sum of the inherent energy components would result in a stable equilibrium at the unfolded state and a set of blocked fold states. The blocked fold states correspond with the "unacceptable motion zone", but there are no stable equilibrium in either "desired stability zones".

Step 6: The designer superimposes an energy path that satisfies the requirements.

Step 7: The designer selects two techniques and estimates their energy behavior.

Step 8: The sum of the techniques and inherent energy components satisfy the desired conditions because the stable fold states are within the "desired stability zones" and the energy conditions prevent the device from entering the "unacceptable motion zone."

Two additional examples of this process are given in section 7 as case studies.

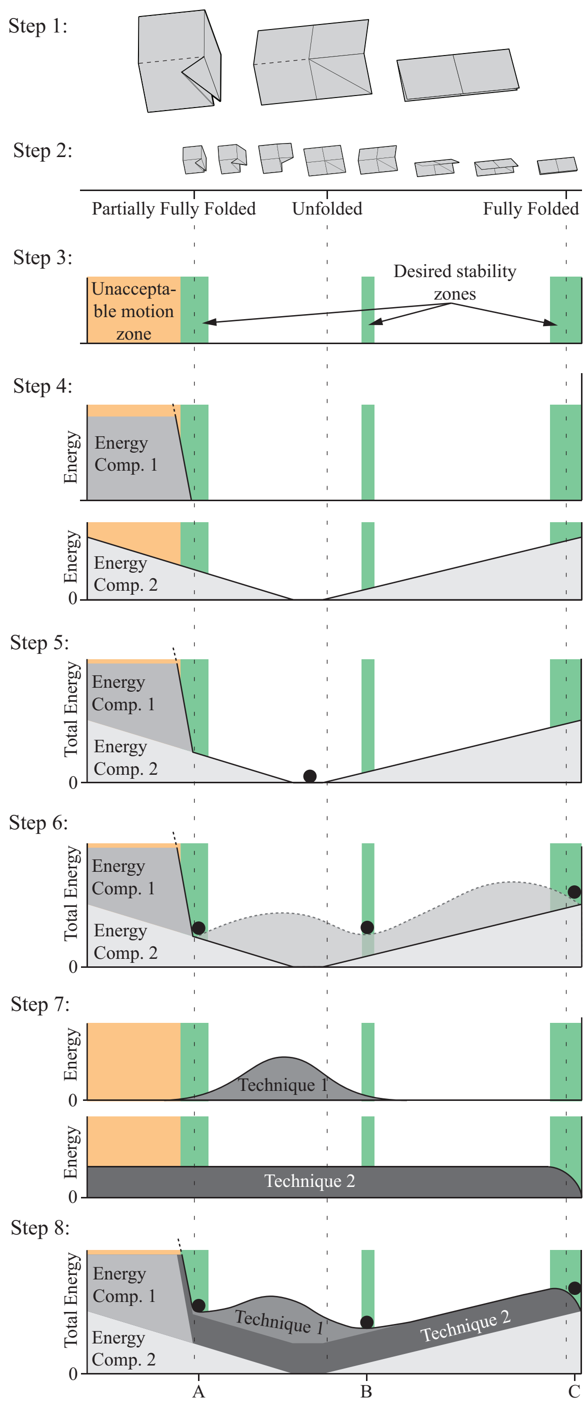

FIGURE 7: Steps of the origami stability integration method (OSIM) demonstrated with a fictitious example in subsection 4.1. Note that the final stable states (marked as A, B, and C in Step 8) are within the desired stability zones.

5 Technique Classification

Selecting stability techniques is a critical part of the OSIM (step 7). Therefore, this section presents criteria to help designers select appropriate techniques.

5.1 Technique Classification Criteria

Two criteria are introduced: intrinsic vs. extrinsic, and differentiable vs non-differentiable. This results in four groups of techniques: Intrinsic Differentiable, Intrinsic Non-differentiable, Extrinsic Differentiable, Extrinsic Non-differentiable. Figure 8 shows how frequently each group has been used in a sample of 69 origami-based devices [37]. (Details about each group and its techniques are given in section 6.)

5.1.1 Intrinsic vs. Extrinsic This criteria classifies how the technique is applied to the origami linkage. An *intrinsic stability technique* is a technique that assists in realizing stable equilibria, using only the creases and facets of an origami linkage. These are techniques such as hinge interference, compliant joints, and non-rigidly foldable linkages [4]. In this study, stimuli-actuated techniques are also considered intrinsic despite the need for outside influence.

An *extrinsic stability technique* is any technique that does not use just the facets and creases of an origami linkage to help realize stable equilibria. There are a wide variety of extrinsic techniques, such as clasps, magnets, straps, and actuators.

5.1.2 Differentiable vs. Non-differentiable This criteria is meant to classify the energy storage behavior of the technique.

A *differentiable stability technique* creates a change in the energy of the linkage that is differentiable at all fold states in the continuum (i.e. a gradual change in energy). Examples include torsional and linear springs, magnets, and non-rigidly foldable linkages. While the influence of a differentiable techniques affects the energy storage of all the fold states in the continuum, its may be negligible over a range of fold states (such as a spring that goes slack).

Non-differentiable stability techniques have a non-differentiable change in energy. In other words, if there is a sharp change in energy (i.e. piecewise behavior), it is classified as a non-differentiable technique. For example, an idealized spring is differentiable, but hard stops would be non-differentiable. Other examples of non-differentiable stability techniques include clasps, telescoping poles, and panel interference.

5.2 Benefits and Drawbacks

This section provides some benefits and drawbacks that pertain to the entire criteria. For example, the benefits of intrin-

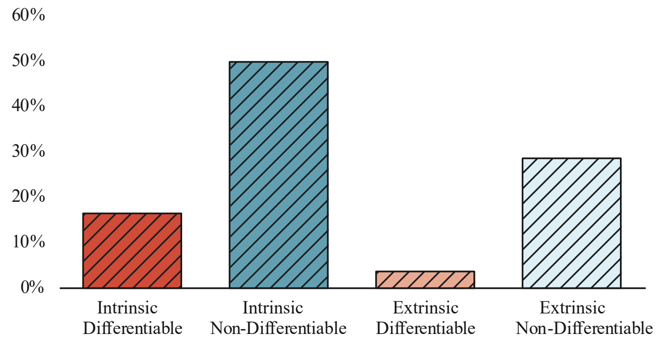

FIGURE 8: The four groups of stability techniques, along with the number of implementations of each type in a sample of 69 origami-based devices [37]. Devices have multiple implementations; the implementations are counted once for each stable equilibrium to which they contribute.

sic techniques apply to both intrinsic differentiable and intrinsic non-differentiable techniques.

5.2.1 Intrinsic Techniques

Benefits The primary advantage of intrinsic techniques is that they only involve the members of the origami linkage. For applications that are concerned with appearance these techniques can highlight the simple, elegant nature of origami.

Drawbacks These techniques are limited by the material of the origami linkage. For example, the stiff aramid fabric of an origami ballistic barrier [39] does not make a good torsional spring for large deflection. However, advances in materials and methods for increasing material compliance are making intrinsic techniques more accessible.

Another challenge is that these techniques usually must be considered when the origami linkage is selected because the two affect each other. For example, if the facet interference technique is selected to help maintain a non-planar fold state, the origami linkage must either be non-flat foldable or have more than one degree of freedom [40, 41].

5.2.2 Extrinsic Techniques

Benefits Extrinsic techniques are useful for applications where origami linkage dimensions are constrained. This may be the case for application with required kinematic properties.

Extrinsic techniques are not limited to the materials used for the origami linkage. For example, a designer could build the origami linkage from a rigid material and add a more compliant material to act as springs.

Drawbacks These techniques usually require added parts and some, such as a clips or bolts, need additional user input to create stability.

5.2.3 Differentiable Techniques

Differentiable techniques have received notable attention in academia [15, 22]. In products, however, differentiable techniques are used much less often than non-differentiable techniques, as shown in Fig. 8.

Benefits Differentiable techniques are useful for assisting actuation because their influence typically extended over a range of folds states. For example, the torque-adaptive wheel stores energy in the creases and facets, resulting in a bias towards the unfolded state [42].

Drawbacks Some of the common methods of creating a differentiable technique, compliant joints and non-rigidly foldable origami, require deformation of the origami linkage. In some cases this may be undesirable. For example, the origami telescope by Early et al. [43] needed to be rigidly foldable or it would plasticly deform.

5.2.4 Non-Differentiable Techniques

The majority of non-differentiable techniques are hard stops: offsets, strings, clasps, retaining channels, detents, and facet interference. A *hard stop* inhibits a device's motion due to interference [18]. Hard stops can be compression hard stops or tension hard stops. Figure 9 illustrates examples of both types as intrinsic and extrinsic techniques in an origami linkage.

Benefits When an application needs a static equilibrium position to fall on a specific fold state, or in a narrow range of fold states, non-differentiable techniques are valuable because they can create distinct, stable equilibria.

Non-differentiable techniques usually have high rates of energy storage. This is useful for creating blocked conditions.

Non-differentiable behaviors can be a result of a user toggling the technique on and off (eg. clips). This is valuable for affecting only energy storage when the linkage is in a specific fold state.

Drawbacks Non-differentiable techniques generally do not assist with actuation.

6 Stability Techniques Design Resource

This section is a resource for those who wish to learn more about specific stability techniques. It contains design considerations for each group of stability techniques and a more detailed discussion of a number of intrinsic techniques. The discussions consists of (1) a brief description, (2) design considerations, (3) resources, and (4) examples.

Common example techniques for each type are also given in Table 1. A summary of benefits and drawbacks (including those from section 5) is given in Table 2.

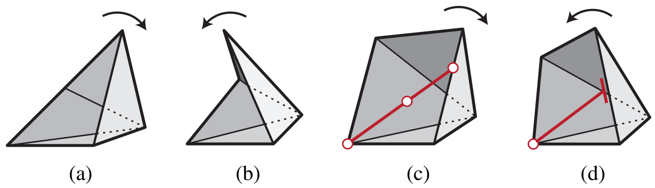

FIGURE 9: Origami linkages (in grey) with intrinsic (a,b) and extrinsic (c,d) hard stops in tension (a,c) and compression (b,d). The red links represent members that are not part of the original origami linkage.

6.1 Intrinsic Differentiable Techniques

Origami linkages have several differentiable (i.e. gradual effect) ways of storing energy in their facets and creases: non-rigid foldability, compliant joints, and stimuli-actuated joints.

6.1.1 Non-Rigid Origami

Description Non-rigidly foldable origami linkages require deformation of the facets or creases in order to fold. In addition to allowing motion, the deformed members also store energy.

A non-rigidly foldable origami linkage behaves similar to a traditional over-constrained, compliant mechanism [44, 45].

Design Considerations This technique is generally useful for creating monostable energy storage behaviors. However, some linkages, such as the flasher, triangulated tube, and square twist, can be bistable [5, 46].

One variation on this technique is to make the origami non-rigidly foldable by offsetting hinges in thick origami. This was demonstrated by creating bistability in an origami antenna [28].

Resources A useful method for calculating energy storage in non-rigidly foldable origami is given by Saito et al. [21].

See the following resources for creating bistablity in traditional compliant mechanisms [13, 44, 47, 48].

Examples The foam origami tub in Fig. 10 is based on a non-rigidly foldable origami linkage. The largest facet bends as it folds, storing energy which biases the tub towards the unfolded state, shown in part (a).

TABLE 1: Examples of stability techniques within each of the four technique groups.

	Differentiable	Non-Differentiable
Intrinsic	Non-Rigid Origami Compliant Joints Stimuli-Actuated Joints	Hinge Interference Limited Facet Extension Global Facet Interference Adjacent Facet Interference
Extrinsic	Magnets Torsional Springs Linear Springs Gravity	Offsets Snaps, Clasps, Buckles Telescoping Poles Hook-and-loop Retaining Channels

6.1.2 Compliant Joints

Description The creases themselves are compliant joints that store strain energy like a torsional spring.

Design Considerations Material is a major consideration in this technique. The materials must be compliant enough to handle maximum deflections, while remaining stiff enough to provide the desired energy storage.

Like the non-rigidly foldable technique, one of the lowest energy states occurs at the manufactured fold state, unless the creases have been modified through plastic deformation or stimuli. (See section 6.1.3.)

Resources Research has shown that the stiffness and low-energy state of a crease in a degree-4 vertex can be tailored to create up to six stable states [15].

TABLE 2: Typical benefits (+) and drawbacks (-) of the four types of stability techniques. Explanations are given in their respective subsections.

	Differentiable	Non-Differentiable
Intrinsic	+ No added parts + Helps actuation - Hinders actuation - Limited by material - Complex to design - Low energy threshold	+ No added parts + Easy to design + Distinct stable points + High energy threshold + Versatile energy storage
Extrinsic	+ Typically passive + Helps actuation - Hinders actuation - Added parts - Low energy threshold	+ High energy threshold + Easiest to implement + Directional effect - Added parts - User input usually required

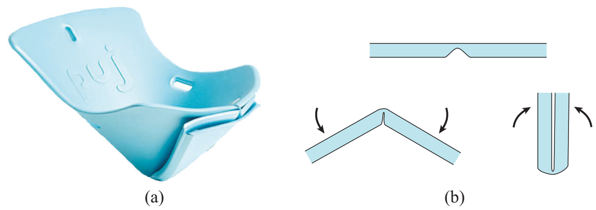

FIGURE 10: (a) The baby bathtub uses several stability techniques discussed in section 7. (b) The linkage is blocked from folding the wrong way because of hinge interference. The fully folded state (shown in (a)) is maintained using magnets and adjacent facet interference, as discussed in section 7.2. (Used with permission from PUJ ®, downloaded from www.puj.com.)

Francis et al. [49] provide a resource for evaluation of various creased materials .

The geometry of the material can be altered to facilitate folding, for example, by using a LET array [20].

Examples Stable behavior outside of the unfolded state can be seen even in paper origami, such as the waterbomb base and kaleidocycle [10, 17, 50].

A number of the origami-based products (colander, bathtub, Kayak, bowl, and glasses case) use living hinges to fold and store energy [33, 51–54].

Because of its limited current use but high potential for application, this is a technique that would benefit from further research into potential applications and design methods.

6.1.3 Stimuli-Actuated Joints

Description Stimuli-actuated joints are different from other intrinsic differentiable techniques because often the energy storage is not due to elastic deformation. The energy storage is more akin to plastic deformation, where the lowest energy state of the hinges is redefined completely.

Design Considerations On the microscopic level, these techniques have a distinct advantage because the materials are often stimulated using a field rather than tactile manipulation [55].

Resources See the resources discussed in section 2, including [2, 3, 23–25, 31].

Examples The origami robot is one of the few macroscopic examples of stimuli-actuated joints. It uses heat to actuate the joints [2].

6.2 Intrinsic Non-Differentiable Techniques

Intrinsic non-differentiable techniques are primarily caused by self-interference—between adjacent facets, from sequential folding, between creases with thickness, between non-adjacent facets, and from facets in tension.

6.2.1 Adjacent Facet Interference

Description The adjacent facet interference technique is intuitive—when adjacent facets interfere with each other they will inhibit folding.

Design Considerations For applications that need a large exposed surface area this technique is not ideal because facets double back on one another.

Adjacent facet interference only blocks one fold direction. Thus, in applications that require fully constrained states, additional techniques will be needed. One benefit is that it is relatively easy to constrain adjacent facets using bolts, magnets, and hook-and-loop fasteners.

This technique is often used in load bearing applications, such as the origami-based chairs and tablet cases [56–59].

This is the most prevalent technique in the sample of origami-based devices, where 51 of the 79 intrinsic non-differentiable techniques are adjacent facet interference, and 29 of those are when the device is either stored or transported.

Resources A resource for designing origami linkages to interfere in non-planar fold states is given by Foschi et al. [40].

Examples Technically the bathtub in Fig. 10 and section 7.2 is fully folded when it functions as a tub, but because it is not rigidly foldable it has a non-planar shape. The adjacent facets are a good technique for this application because the facets are sandwiched between the baby and the sink. The adjacent facets are also constrained by magnets.

6.2.2 Sequential Flap Interference

Description This technique occurs when two creases are made collinear to form a flap in a folds state that can bifurcate. The flap is then folded, blocking other bifurcation modes.

Design Considerations This technique can be difficult to implement if an application requires thick materials because creases that form the flap must be nearly collinear. This means that, without modifications, hinge interference will occur in either fold direction. (See section 6.2.3.)

Resources See the following resources for methods that allow sequential folding in thick materials [60, 61].

Examples In the sample of devices, sequential flap interference only occurs in the battery, ice bucket, and sofa [33, 62]. Flaps are formed in a number of other devices such as the boat, bowl, and camping pot, but they are a slight variation on the technique where the flaps do not create the need for sequential folding [32, 53, 63].

This is one of the few intrinsic techniques that can be toggled on and off by a user (only the ice bucket uses the flap in this way).

6.2.3 Hinge Interference

Description Hinge interference occurs when facets with non-zero thickness obstruct folding of the crease.

Design Considerations This technique is a low-profile intrinsic technique that is beneficial because it only affects the kinematics of the linkage once it is blocked. Facet chamfer angles determine when the facet interferes.

Applications that use thin material are not conducive to this technique because the technique requires out-of-plane thickness.

Resources Huffman and Lang et al. [5, 64] provide fold angle equations. Tachi shows an implementation of this technique in the tapered panel technique (a thickness accommodation technique) [4]. Ku gives a novel variation for creating creases with interference [65]. Directional hinge techniques that bridge the gap between extrinsic and intrinsic techniques are provided by Shemenski et al. [18].

Examples The linkage shown in Fig. 11 is blocked from reaching the unfolded state by hinge interference.

Figure 10 (b) shows the crease interference technique used to block the tub from folding in the wrong direction. This example is also used in case study 7.2.

6.2.4 Global Facet Interference

Description Global facet interference is when two or more non-adjacent facets interfere.

Design Considerations Calculating global interference is more involved than calculating local interference because the position of facets in multiple vertices must be evaluated.

Resources One method for simplifying implementation of this technique is the use of symmetry. An example of how symmetry is used to achieve global interference is shown in Fig. 12.

Examples The majority of the devices that use this technique use symmetry—the kayak (figure 3), canoe, forceps, and several of the chairs [67–71].

6.2.5 Limited Facet Extension

Description Limited facet extension occurs when the crease in between the actuated facet and a grounded facet are unfolded, as seen in Fig. 9. This point is known as a change point, toggle point, or singularity.

Design Considerations The limited facet extension technique is useful for creating containers or devices that are kept in tension. The technique usually only works if the origami linkage is non-developable.

Examples The origami colander (shown in Fig. 13) has a stable state that uses intrinsic hard stops in tension [72]. Since the origami linkage is non-developable the facets that make up the sides reach full extension when the other facets are non-coplanar.

6.3 Extrinsic Differentiable Techniques

Extrinsic differentiable techniques are typically useful when the origami linkage cannot be modified, if more than one stable position is needed, or if the material cannot be changed and

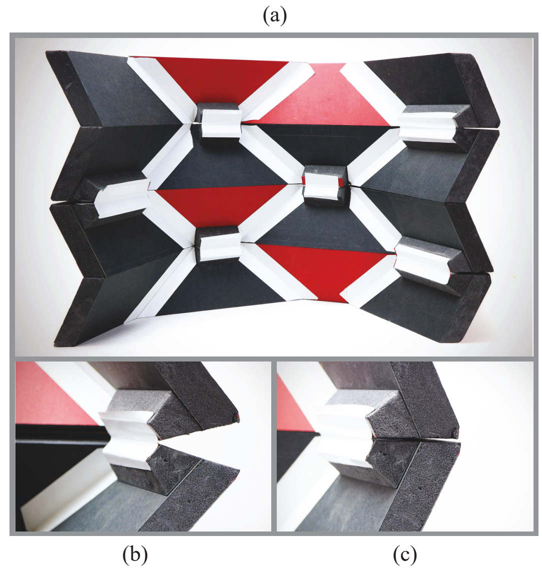

FIGURE 11: The facets of the thick origami linkage (a) are negatively chamfered to create a stable partially folded state [66]. Figures (b) and (c) show details of the hinge interference.

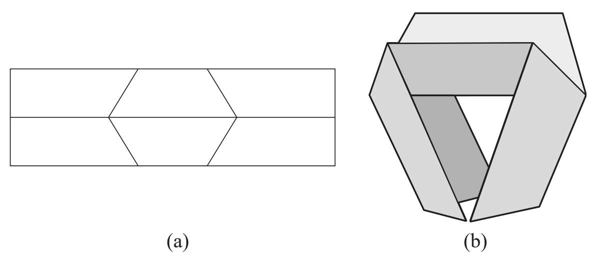

FIGURE 12: A symmetric crease pattern (a) and a fold state with global facet interference (b).

doesn't provide the needed behavior. Both case studies (see section 7) use a extrinsic differentiable technique.

6.4 Extrinsic Non-differentiable Techniques

Extrinsic non-differentiable techniques come in a wide variety and are common in origami-based devices. Snaps, clasps, fasteners, and restraints are easy to add to a device at any fold state.

These techniques generally require user input, such as setting the clips in the origami kayak (Fig. 3). This also gives these techniques versatility; they can have a directional effect on energy storage, where the energy threshold for entering a fold state is low, but exiting is high. This can also be accomplished using a detent or clasp.

FIGURE 13: An origami-based colander that uses the limited facet extension technique. It is shown in the unfolded and UP fold states (Image used with permission from B&R Plastics, Inc™, downloaded from www.brplastics.com)

7 Case studies

Two case studies are included: an origami ballistic barrier and an origami baby tub. These case studies use the OSIM to conceptualize the energy behavior of the device and determine which techniques may help achieve stability.

7.1 Ballistic Barrier Case Study

The origami ballistic barrier is a deployable, bullet-resistant shield that stores fully folded and opens to be a self-standing protection [39,66]. The steps of the OSIM for the ballistic barrier are described below and shown in Fig. 14.

Step 1: The origami linkage is selected based on its crescent shape in the deployed state and its compact stowed state.

Step 2: The origami linkage has a single degree of freedom and is flat foldable which means that the range of the fold-state continuum can be represented as going from one fully folded state to the other.

Step 3: The barrier needs to be stable in the fully folded state and in a partially folded state. The barrier should not reach the unfolded state because it can bifurcate into undesired modes.

Step 4: There are two main energy components: gravity and strain in the hinges. The energy behavior is approximated and graphed along the fold-state continuum.

Step 5: The the two energy components are added together.

Step 6: The desired energy curve is superimposed. Note that we want the deployed stable condition to have lower potential energy than the fully folded stable configuration to aid in actuation.

Step 7: Gas springs are selected for several reasons. (1) Their orientation has a low mechanical advantage in the stored state, allowing for bistable behavior. (2) They are attached to non-adjacent facets to counteract parasitic motion with less hardware. (3) They double as handles. (4) Once reaching full extension, they act as a hard stop, blocking the barrier from entering the unfolded state. Compliant hinges (springback) was considered but not selected because it would required more implimentations and/or it would fatigue the aramid fabric.

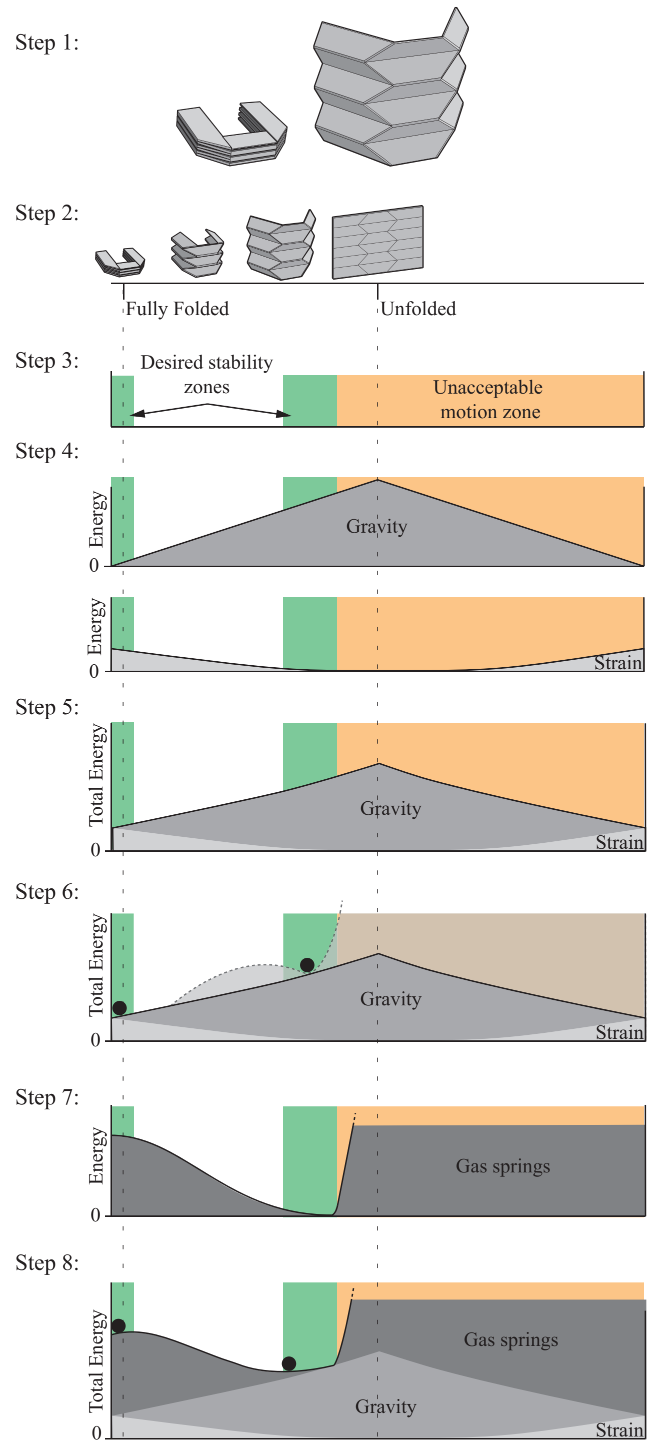

FIGURE 14: The steps of the OSIM for the origami ballistic barrier, as described in section 7.1.

Step 8: The combined energy components results in bistability, with a stable equilibrium in both the fully folded and partially folded state.

7.2 Baby Tub Case Study

While the OSIM was not used to design stability in the origami baby tub, the Puj Tub [52], it is used here as a case study of what could have been done to choose the stability techniques used. The steps of the OSIM for the baby tub are described below and shown in Fig. 15.

Step 1: A 2 DOF origami linkage is selected. This origami linkage cannot fold if the panels remain rigid so a flexible foam is used.

Step 2: Applying symmetry constraints, the 2 DOF motion is simplified to be represented along a 1D fold-state continuum.

Step 3: The tub needs to be stable when unfolded (for storage) and when fully folded (tub shape). The design should be blocked from folding the wrong direction.

Step 4: Strain of the largest panel is the only energy component considered and its energy behavior is graphed along the fold-state continuum.

Step 5: No summing necessary.

Step 6: A desired energy curve is superimposed. The smooth behavior between the stable positions suggests that a differentiable technique might be useful. A non-differentiable technique might also be useful near the unacceptable motion zone.

Step 7: Two techniques are used to provide stability. Magnets, an extrinsic differentiable technique, is chosen because it is passive (i.e. doesn't require additional user input) and it pairs well with the adjacent facet interference technique. Hinge interference, an intrinsic non-differentiable technique (shown in Fig. 10), is chosen because it requires no added parts and it has a high energy threshold.

Step 8: The combined energy components results in the desired energy behavior.

8 Conclusion

The methods, vocabulary, techniques, and examples presented in this paper are tools for planning stable states in origami. The OSIM is a design tool for conceptualizing how different stability techniques may be used to achieve stability in an origami device, given the inherent energy behavior of the origami linkage. The categorization of stability techniques facilitates their comparison and selection.

Future work in this area could include investigating how folding along certain branches of an origami linkage can create different energy storage behaviors. For example, an origami linkage may bifurcate or have multiple stable states along one branch and not along another. Future work could also include how to incorporate dynamic loading in the OSIM.

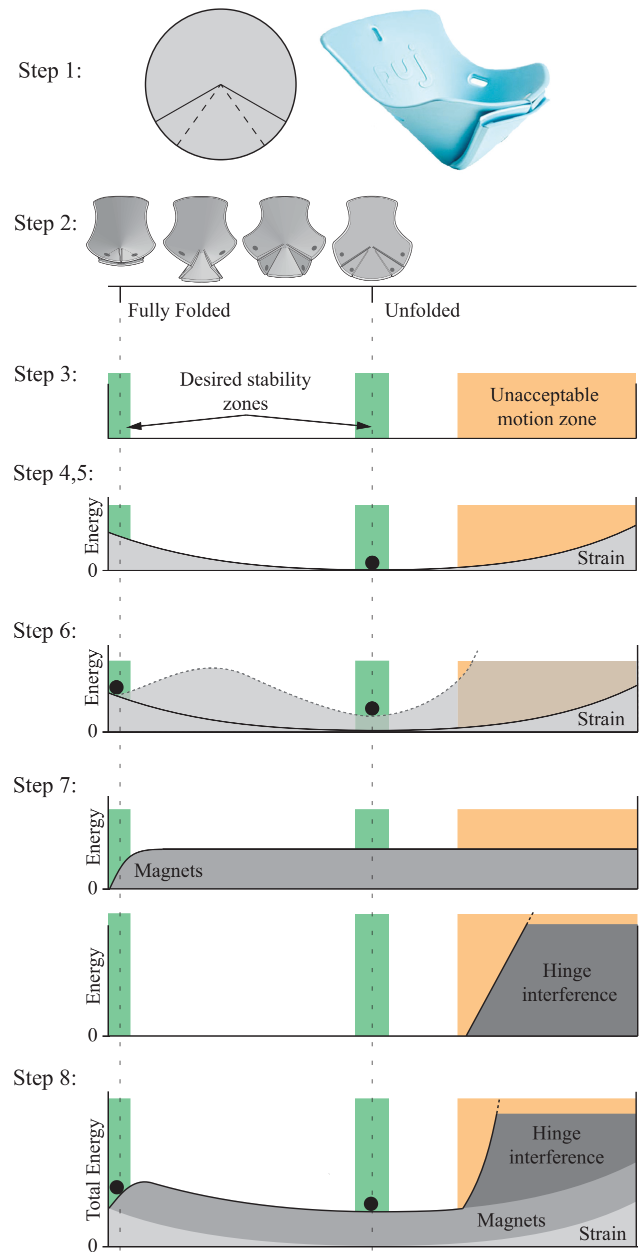

FIGURE 15: The steps of the OSIM for the origami baby tub, as described in section 7.2

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