Soliton Frequency Combs in Dual Microresonators

Zhen Qi and Curtis R. Menyuk
University of Maryland, Baltimore County, 1000 Hilltop Circle, Baltimore, MD 21250, USA
zhenqi1@umbc.edu

Abstract: We study soliton frequency combs generated in dual microresonators with different group velocity dispersion. We obtain stable bright and dark solitons at different pump amplitudes.

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Bright and dark soliton frequency combs have been numerically demonstrated and experimentally observed in Kerr nonlinear microresonators with anomalous [1–3] and normal group velocity dispersion (GVD) [4, 5]. Dual microresonators have been used to generate dark solitons [4, 6]. In this work, we investigate dual microresonators in which the two resonators have opposite dispersion. We show that it is possible to generate both bright and dark solitons.

We consider a silicon nitride (Si_3N_4) dual microresonator system with ring radius $R = 100$ μm, effective area $A_{\mathrm{eff}} = 1$ $\mu\mathrm{m}^2$, quality factor $Q = 5\times10^5$, dispersion parameter $|D_2| = 2\pi\times 33.9$ MHz, and coupling parameter $\bar{\kappa} = 2\pi\times 426$ MHz [4]. It was observed in [4] that dark solitons emerge when $P = 80$ mW. We will solve the coupled Lugiato-Lefever equation (LLE) with anomalous dispersion in one ring and normal dispersion in another ring, and we will show that with a detuning equal to 580 MHz in both rings, a dark soliton is generated when $P = 87.4$ mW and a bright soliton is generated when $P = 361$ mW.

As shown in Fig. 1, one of the rings, denoted the main ring, is pumped by an external continuous wave (CW) laser, while the other ring is denoted the auxiliary ring. Soliton frequency comb generation in this system is governed by the coupled LLE, which after normalization becomes [6, 7]

$$\begin{aligned}\frac{\partial\psi_1}{\partial\tau} &= -(1+i\alpha_1)\psi_1 - i\frac{\partial^2\psi_1}{\partial x^2} + i|\psi_1|^2\psi_1 + i\kappa\psi_2 + F,\\ \frac{\partial\psi_2}{\partial\tau} &= -(\gamma+i\alpha_2)\psi_2 + i\beta\frac{\partial^2\psi_2}{\partial x^2} + i|\psi_2|^2\psi_2 + i\kappa\psi_1,\end{aligned} \tag{1}$$

where subscripts 1 and 2 indicate the main and auxiliary rings respectively. The variables ψ_1 and ψ_2 denote the optical fields, τ is normalized time, x is normalized length along the folded cavity modes [8], with $-L/2 < x < L/2$, where L is the normalized mode circumference, which is the same for the two rings. The parameters α_1 and α_2 denote the detuning in each of the rings, β is the ratio between the dispersion in ring 2 and 1, κ is the linear coupling between the two rings, $\gamma \approx 1$ is the attenuation ratios between rings 2 and 1, and F is the pump amplitude in the main ring.

We consider a case in which the dispersion is normal in the main ring and is anomalous in the auxiliary ring with equal and opposite dispersion, so that $\beta = 1$. We set $\alpha_1 = \alpha_2 = 6$ corresponding to 580 MHz. The parameters $\kappa = 2.2$ and $L = 15$ correspond to the experiments in [4]. Figure 1(a2) shows $||\psi_1||^2 = \int_{-L/2}^{L/2}|\psi_1|^2 dx/L$ as a function of pump amplitude F. The solid (dotted) black, blue, red, and green curves represent stable (unstable) continuous waves, dark solitons, bright solitons, and periodicity-4 cnoidal waves, respectively. A snaking curve that is typical for dark solitons appears when the pump amplitude F is around 3, corresponding to 87.4 mW, as shown in Fig. 1(a3). As in a single ring system, an unstable stationary solution originates from a branch of CW solution, and moves along the snaking curve as the pump amplitude F increases, then the solution undergoes a saddle-node bifurcation, and the stationary solution becomes stable until another saddle-node bifurcation occurs, and so on. No stable stationary solutions originate directly from the CW solutions, indicating that dark solitons must be excited subcritically. Figures 1(c1-c3) show the intensity profiles and (d1-d3) show the corresponding frequency comb spectra for dark solitons, corresponding to the points a, b, and c in panel (a3). The dark soliton solutions move along the snaking curve from top to bottom, gradually become wider and deeper, and more notches appear in the valley of the solutions, corresponding to increasingly higher-order dark soliton molecules [9]. The intensities of dark solitons in the anomalously dispersive auxiliary ring (blue solid) are smaller than those in the normally dispersive main ring (red dashed). The widths of the frequency spectra of dark solitons do not vary apparently.

Bright solitons appear at a larger pump amplitude F around 6, corresponding to 361 mW. Figure 1(a4) shows an expanded view of panel (a2) around the region where bright solitons exist. As in the case of dark solitons, no stable bright solitons originate directly from the CW solutions indicating that bright solitons can only be excited subcritically. Figures 1(c4) shows the intensity profiles and (d4) shows the corresponding frequency comb spectra for bright solitons at point d in panel (a4). Bright solitons in the anomalously dispersive auxiliary ring (blue solid) have a smaller pedestal and a larger peak intensity than those in the normally dispersive main ring (red dashed). Correspondingly, in the spectra of the auxiliary ring solution, the pump line is less apparent, and a larger fraction of the power is distributed to the comb modes.

In conclusion, we have studied frequency combs in the dual-ring microresonator system with opposite GVD and found that both stable dark and bright solitons can be generated at different pump amplitude. Intensities of dark solitons are larger in the normal dispersion ring, while bright solitons are larger in the ring with anomalous dispersion.

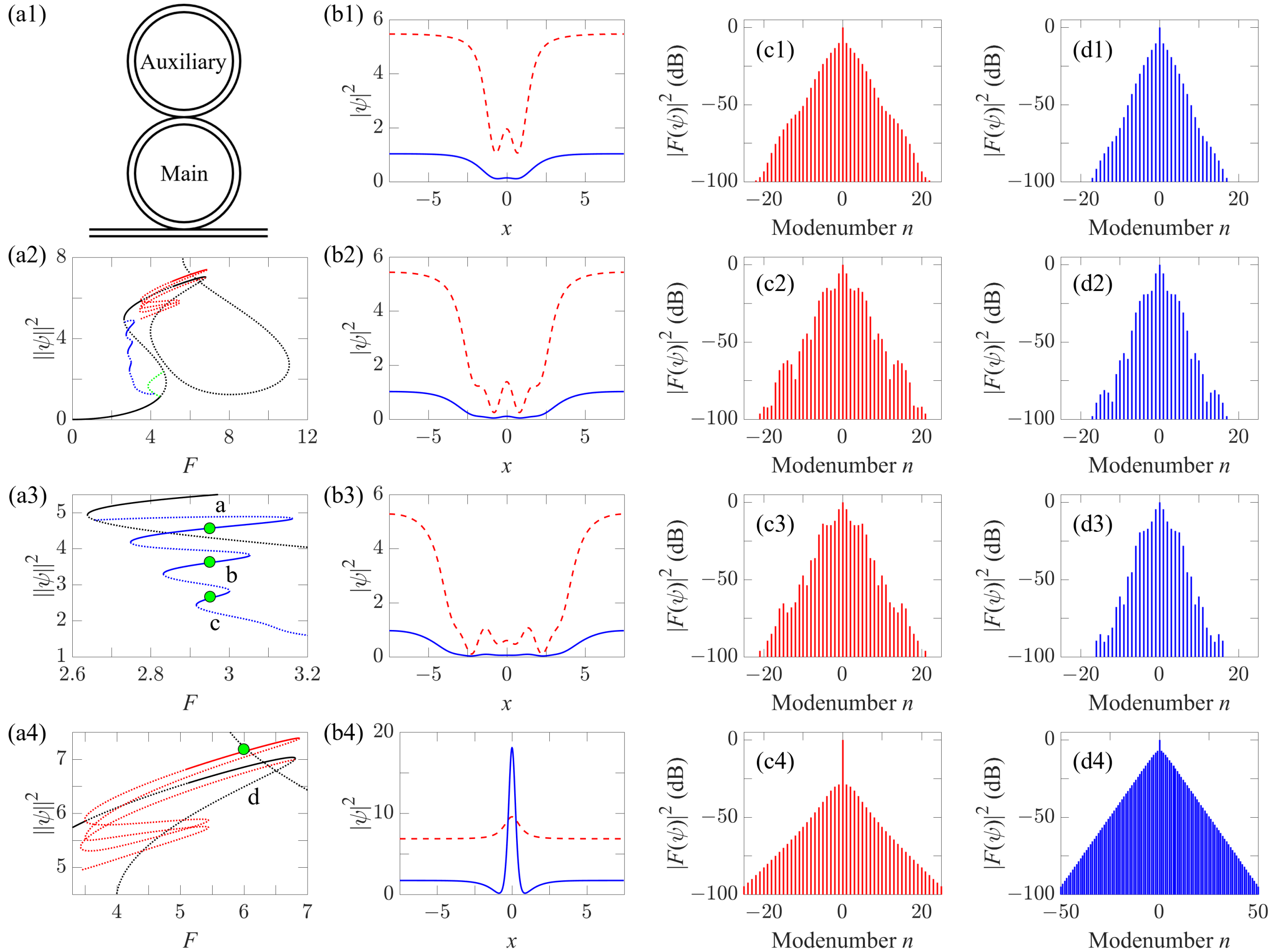

Fig. 1. (a1) Schematic illustration of dual microresonators. (a2) Bifurcation diagram of the normally dispersive main ring. (a3) Expanded view of panel (a2) around the region where dark solitons exist. (a4) Expanded view of panel (a2) around the region where bright solitons exist. The solid (dotted) black, blue, red, green curves represent stable (unstable) CWs, dark solitons, bright solitons, and periodicity-4 cnoidal waves. (b) Profiles of the intensity of the stable solutions on the snaking curve at points (b1) a, (b2) b, and (b3) c for dark solitons in panel (a3), and (b4) point d for bright solitons in panel (a4). Red dashed and blue solid curves represent solitons in the main and auxiliary rings, respectively. Corresponding frequency comb spectra for the main and auxiliary rings in (c) and (d), respectively.

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