

Accurate Appearance Preserving Prefiltering for Rendering Displacement-Mapped Surfaces

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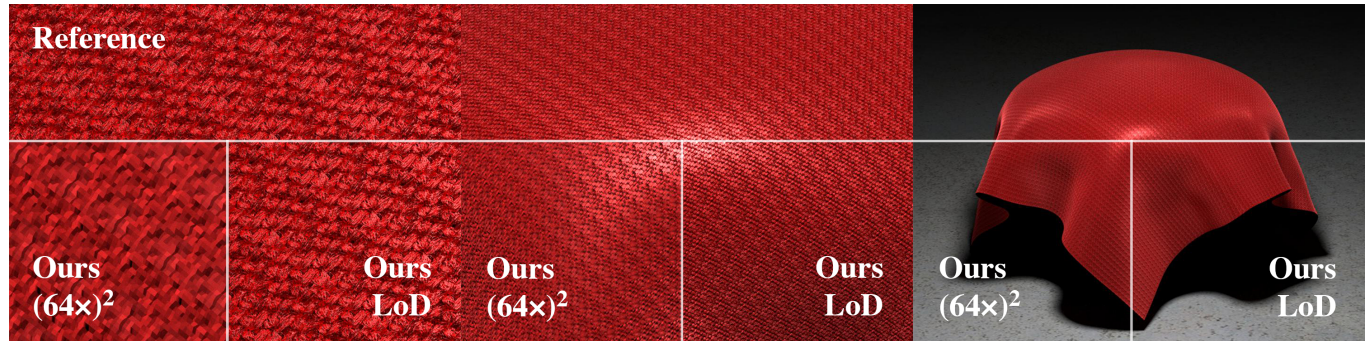


Fig. 1. We present a new approach to prefilter high-resolution displacement maps while preserving the input appearance. High-resolution displacement maps can produce rich geometric details (top-left) but they are difficult to prefilter. Our prefiltered model handles the change of shadowing, masking and interreflections caused by downsampling the displacement map. At a single scale, although the detailed micro-structures are different (see ours $(64\times)^2$ at left), our prefiltered model preserves the original appearance accurately when we view the object from a distance (see the right image). We can also combine our models at multiple downsampling scales to form a mipmap, enabling accurate and anti-aliased LoD rendering.

Prefiltering the reflectance of a displacement-mapped surface while preserving its overall appearance is challenging, as smoothing a displacement map causes complex changes of illumination effects such as shadowing-masking and interreflection. In this paper, we introduce a new method that prefilters displacement maps and BRDFs jointly and constructs SVBRDFs at reduced resolutions. These SVBRDFs preserve the appearance of the input models by capturing both shadowing-masking and interreflection effects. To express our appearance-preserving SVBRDFs efficiently, we leverage a new representation that involves spatially varying NDFs and a novel scaling function that accurately captures micro-scale changes of shadowing, masking, and interreflection effects. Further, we show that the 6D scaling function can be factorized into a 2D function of surface location and a 4D function of direction. By exploiting the smoothness of these functions, we develop a simple and efficient factorization method that does not require computing the full scaling function. The resulting functions can be represented at low resolutions (e.g., 4^2 for the spatial function and 15^4 for the angular function), leading to minimal additional storage. Our method generalizes well to different types of geometries beyond Gaussian surfaces. Models prefiltered using

our approach at different scales can be combined to form mipmaps, allowing accurate and anti-aliased level-of-detail (LoD) rendering.

CCS Concepts: • **Computing methodologies** → **Rendering**.

Additional Key Words and Phrases: multi-resolution, level of detail, prefiltering, global illumination

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1 INTRODUCTION

High-resolution displacement maps are commonly used to describe detailed micro-geometries that can produce richly diverse appearances. Compared to normal mapping, displacement mapping is more physically consistent and can offer more realistic self-shadowing and silhouettes. However, such realism comes at the cost of difficult prefiltering: smoothing a displacement map usually weakens its intrinsic shadowing and results in brightened overall appearance. Therefore, rendering a high-resolution displacement map without introducing severe aliasing generally requires significant super-sampling, which is computationally expensive.

Previous displacement mapping techniques such as LEAN [Olano and Baker 2010] and LEADR [Dupuy et al. 2013] can produce anti-aliased renderings of rough surfaces. However, they assume the normals of the input surfaces to have Beckmann distributions, which is usually violated in practice and fundamentally limits the accuracy

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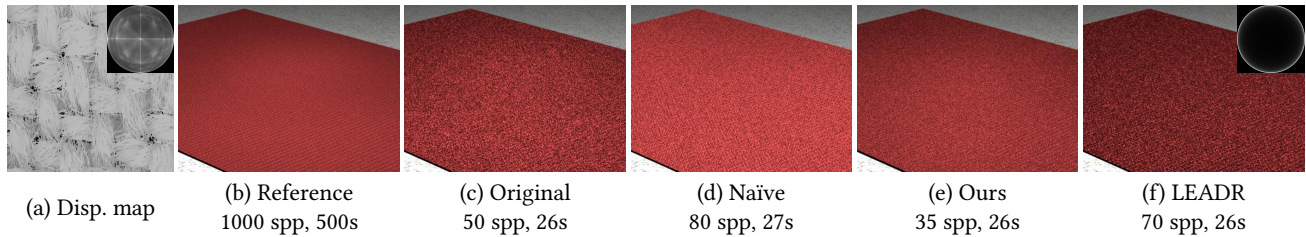


Fig. 2. **Equal-time renderings with only direct illumination.** Given a high-resolution displacement map (a) and a highly glossy base BRDF as input, directly rendering using them results in aliased results (c). Simply downsampling the displacement map to a lower resolution but keeping the original base BRDF leads to inaccurate appearance (d). Our method (e) prefilters the displacement map and the base BRDF while preserving the input appearance (b), which produces accurate and anti-aliased renderings. Using the microfacet BRDF proposed in LEADR [Dupuy et al. 2013] generates inaccurate appearance (f) since the Gaussian surface assumption is violated. We show the ground truth NDF visualization of this displacement map in the top-right inset of (a), and the NDF estimated from LEADR in the top-right inset of (f), which is very different.

of these methods (Figure 2(f)). To handle a wider range of surfaces, some bi-scale appearance models [Wu et al. 2011; Iwasaki et al. 2012] precompute the overall surface reflectance by averaging light reflections at the micro-scale. This yields one spatially macro-scale effective BRDF with no spatial variation, which has difficulties in reproducing micro-scale details at close-up views. Both approaches neglect interreflection and can lead to significant energy loss.

In this paper, we introduce a novel method to prefilter displacement-mapped (opaque) surfaces while accurately preserving their overall appearances under both direct and global illuminations (e.g., Figures 1, 2). Given a high-resolution displacement map and an isotropic base micro-BRDF, we seek displacement maps and accompanying surface reflectance models that have reduced resolutions and closely preserve the macro-scale appearance of the input model. To this end, we leverage a generalized version of the effective BRDF formulation [Wu et al. 2011] that takes both shadowing-masking and interreflection effects into account. Based on this formulation, we directly match the effective BRDFs of the input model and our prefiltered variants.

Our method (Figure 3) starts with downsampling the input displacement map. This is achieved via an optimization that aims to preserve the surface’s meso-scale geometries (e.g., normals) and minimize potential energy loss. The resulting surface, when coupled with the input reflectance model, generally leads to a different appearance (see Figure 2(d) for an example) due to the change of shadowing structures, distribution of normals, and interreflections caused by the downsampling. The main focus of this paper, therefore, is to find proper surface reflectance models (i.e., BRDFs) so that, when coupled with the downsampled geometries, they closely resemble the appearance of the input.

To fully preserve the detailed appearance of the input model using the downsampled geometry, a 6D spatially varying effective BRDF is generally required. Unfortunately, explicitly expressing this SVBRDF requires significant computation and storage, negating the benefits of displacement map downsampling. Instead, we decompose the 6D function into a 4D spatially varying normal distribution function (SVNDF) [Han et al. 2007] and a novel scaling function. The SVNDF, which neglects both shadowing-masking and interreflection, primarily encodes spatial variations of the effective BRDFs and can be represented as 2D spatially varying parametric distributions. The scaling function, in contrast, accounts for

the shadowing-masking and interreflection effects and is vital for matching the input appearance. To efficiently describe this scaling function, which is 6D itself, we factorize it into a 2D spatial scaling function of surface location and a 4D angular scaling function of direction (e.g., Figure 10). Further, we exploit the fact that the scaling functions are usually low-frequency and smooth in practice to introduce a simple and efficient factorization method to compute them from a sparse set of samples. With a resolution up to 4^2 for the spatial function and 15^4 for the angular one, our scaling functions only consume 200–400 KB of storage, making our prefiltered models compact and practically useful. Our contributions include:

- We develop a novel method to prefilter the surface reflectance of a high-resolution displacement map while accurately preserving the overall appearance (Figures 1, 16, 17, 18). This is achieved by first downsampling the displacement map to minimize the meso-scale average surface slopes (§4.1), then separating a 6D SVBRDF into a SVNDF (§4.2) and a scaling function (§4.3).
- To match the target appearance, we utilize two scaling functions. One captures only shadowing-masking (§4.3) and the other effectively handles both shadowing-masking and interreflection (§5.2). To model interreflection within the micro-geometry, which is generally missing in previous work, we introduce a generalized formulation of effective BRDFs (§5.1).
- We show the scaling function can be factorized as the product of a single spatial scaling function and a single angular scaling function. Exploiting their low-frequency property and smoothness, we present a simple and efficient method to compute the spatial and angular scaling functions (§6).
- To enable level-of-detail (LoD) rendering, we present a linear interpolation method that provides smooth transitions between our models prefiltered at varying scales (§7.2).

2 RELATED WORK

We review several main research areas in the following paragraphs. Comparison between our method and the most related techniques is summarized in Table 1.

Surface appearance prefiltering. We refer readers to [Bruneton and Neyret 2012] for a comprehensive review of surface appearance prefiltering techniques. Traditional normal/BRDF map filtering methods [Fournier 1992; Tan et al. 2005; Toksvig 2005; Han et al.

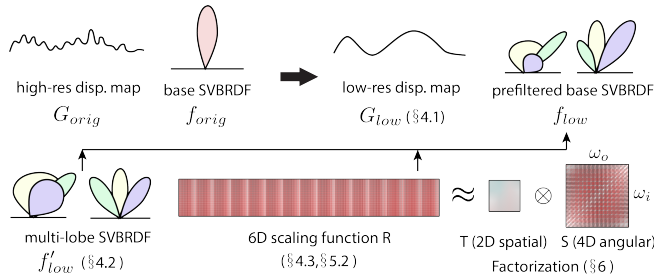


Fig. 3. **Workflow of our method.** We convert a high-resolution displacement map and base BRDF (top-left) to a low-resolution one and prefiltered SVBRDF (top-right). The prefiltered SVBRDF is obtained as a product of the SVBRDF from a multi-lobe NDF (bottom-left) and a 6D scaling function (bottom-right). This 6D function can be further factorized as the product of a single 2D spatial scaling function and a single 4D angular scaling function.

2007; Kaplanyan et al. 2016; Xu et al. 2017] ignore shadowing and masking effects. Tan et al. [2008] approximate shadowing and masking using horizon map distributions. LEADR [Dupuy et al. 2013] extends from LEAN mapping [Olano and Baker 2010] by incorporating a physically based shadowing-masking term derived from microfacet theory. In LEADR, they use one-lobe NDFs for simplicity and efficiency in a real-time rendering context. We use multi-lobe NDFs to achieve better accuracy, especially at glossy highlights (Figure 16). Recently, Loubet and Neyret [2017] proposed a hybrid mesh-volume model to construct LoDs of complex objects. They leverage volumetric models to handle self-occlusions caused by micro-geometries, while our method generates pure surface reflectance models that are simpler and more efficient to use. In addition, none of these methods take interreflections into account, since interreflections combined with shadowing and masking are complicated to analyze and prefilter.

Bidirectional texture functions (BTF) [Cabral et al. 1987; Dana et al. 1999] capture spatially varying and view-dependent surface appearance. Generating 6D BTF data requires expensive precomputation and a large amount of storage. Although BTF filtering methods [Ma et al. 2005; Wu et al. 2009; Jarabo et al. 2014] provide a direct solution to surface appearance prefiltering, the high dimensionality limits their practical applications. Our method separates a full 6D function into a 4D SVNDF and a 6D scaling function. The latter scaling function can be further factorized into a single 2D spatial scaling function and a single 4D angular scaling function, which is cheaper to compute and compact to store. Complex shadowing-masking and interreflections can be captured by the spatial and angular scaling functions accurately.

Bi-scale material design. A series of works model object’s macro-scale appearance by manipulating its micro-scale details [Westin et al. 1992; Heidrich et al. 2000; Wu et al. 2011; Iwasaki et al. 2012]. Westin et al. [1992] explicitly simulate ray tracing on micro-geometry. Heidrich et al. [2000] speed up the simulation on height fields using precomputed visibility. Wu et al. [2011] propose an interactive bi-scale material editing system. They precompute rotated BRDF values and bidirectional visible normal distribution functions (BVNDF) and compress them as low-rank matrices. This work has been extended

Table 1. We compare our method with LEADR [Dupuy et al. 2013], bi-scale appearance models [Wu et al. 2011; Iwasaki et al. 2012] and multi-scattering microfacet models [Heitz et al. 2016; Lee et al. 2018; Xie and Hanrahan 2018]. Our method can handle both shadowing-masking and interreflections but is not limited to Gaussian/GGX surfaces, at the expense of some precomputation.

Method	LEADR	bi-scale	microfacet	ours
Shadowing and masking	✓	✓	✓	✓
Interreflections	✗	✗	✓	✓
General surfaces	✗	✓	✗	✓
No Precomputation	✓	✗	✓	✗

to edit highly glossy materials by using mixtures of spherical Gaussians (SG) or anisotropic spherical Gaussians (ASG) [Iwasaki et al. 2012; Xu et al. 2013]. The shadowing-masking term is controlled by the weights of SG/ASG lobes. Most of these bi-scale appearance modeling techniques focus on the average large-scale appearance and do not consider spatial variation. In addition, the interactive approaches have no interreflection components. Recently, several methods for rendering glinty surfaces have been developed by simulating specular reflection on micro-surfaces [Yan et al. 2014; Jakob et al. 2014; Yan et al. 2016; Chermain et al. 2018]. Zirr and Kaplanyan [2016] propose a bi-scale microfacet model to render micro-details in real-time. Their methods focus on spatially varying NDFs, neglecting shadowing-masking and interreflections.

Microfacet models. Microfacet models describe the aggregate reflectance from a statistical representation of rough surfaces, i.e. the orientations of microfacets, resulting in a number of physically based BRDFs [Cook and Torrance 1982; Oren and Nayar 1994; Walter et al. 2007]. The Smith model [Smith 1967; Heitz 2014] gives an accurate approximation of the microfacet shadowing-masking function with the assumption of independence between heights and normals. In particular, the shadowing-masking function has analytic solutions if Gaussian or GGX surfaces are given. Ashikmin et al. [2000] derive a 4D BRDF from a 2D NDF, in which the shadowing-masking term is numerically computed from the NDF. LEADR [Dupuy et al. 2013] assumes Gaussian surfaces and leverages the Smith model to handle masking and shadowing effects. Recent works [Heitz et al. 2016; Lee et al. 2018; Xie and Hanrahan 2018] extend microfacet theory by modeling multiple scattering inside the micro-surface. Though microfacet models enable efficient computation of shadowing-masking and interreflections, they are limited by Gaussian surfaces, GGX surfaces or V-grooves. Our method can handle general surfaces without assumptions for specific micro-geometries.

Inverse rendering. Inverse rendering optimizes for scene parameters that produce the best match to target appearance. Previous methods [Hašan and Ramamoorthi 2013; Gkioulekas et al. 2013; Khungurn et al. 2015; Zhao et al. 2016; Gkioulekas et al. 2016] require global inverse rendering, which solves for a number of scattering parameters and involves expensive Monte Carlo path tracing during iterative optimizations. We just need to perform standard normal mapping and precompute simpler effective BRDFs once to obtain the prefiltered scattering parameters. Unlike inverse rendering techniques that depend on specific scene configurations, our model can

Table 2. Definitions of commonly used symbols.

Symbol	Meaning	Def.
$\mathcal{P}$	Surface patch	§3
$\mathcal{G}(\mathcal{P})$	Micro-geometry defined by a surface patch (subset) $\mathcal{P}$ of a displacement map	§3
f	Isotropic base BRDF	§3
$f^{\text{eff}}(\mathcal{G}, f)$	Effective BRDF determined by micro-geometry $\mathcal{G}$ and base BRDF f	§3, Eq. (6)
$\mathcal{G}_{\text{orig}}$	Micro-geometry defined by the original high-resolution displacement map	§4
f_{orig}	Original input base BRDF	§4
$\mathcal{G}_{\text{low}}$	Micro-geometry defined by the low-resolution displacement map	§4
f_{low}	Prefiltered spatially varying base BRDF	§4
f'_{low}	Spatially varying multi-lobe BRDF	§4.2, Eq. (11)
$f_{\text{ir}}^{\text{eff}}$	Effective BRDF with interreflections	§5.1, Eq. (17)
R, R_{ir}	6D scaling function without/with interreflections	§4.3, Eq. (12) §5.2, Eq. (20)
T, T_{ir}	2D spatial scaling function without/with interreflections	§4.3, Eq. (14) §6, Eq. (21)
S, S_{ir}	4D angular scaling function without/with interreflections	§4.3, Eq. (14) §6, Eq. (21)

generalize to different lighting and viewing conditions as we match the effective BRDFs rather than rendered images.

Height field rendering. Rendering displacement-mapped surfaces closely relates to height field rendering. Self-shadowing on height fields can be computed in real-time by determining horizon angles for a set of azimuthal directions [Snyder and Nowrouzezahrai 2008; Timonen and Westerholm 2010]. Fast height field rendering with global illumination [Nowrouzezahrai and Snyder 2009] approximates visibility and indirect radiance using low-order SH basis functions, which cannot capture high-frequency reflection and shadowing effects. In these methods, height fields are attached on a planar base surface. Our method allows perturbing displacements on a general surface representation such as triangular meshes.

3 PRELIMINARIES

In this section, we provide preliminaries of displacement mapping, normal mapping, and effective BRDFs. Table 2 summarizes all symbols commonly used in this paper.

Displacement mapping. Displacement maps provide an efficient way to describe detailed micro-geometries. Mathematically, a *displacement map* is a function $h : [0, 1]^2 \rightarrow \mathbb{R}$ that specifies the distance which individual surface points are shifted along the normal directions. To be precise, given a texture-mapped base surface and a displacement map, the resulting geometry is obtained by moving each surface point with texture coordinate (u, v) along its normal direction for a distance of $h(u, v)$. Further, a *surface patch* $\mathcal{P} \subseteq [0, 1]^2$ denotes a small and locally flat region of the base surface. For notational simplicity, the patch area is normalized using some filter kernel $k_{\mathcal{P}}^1$ (i.e., $\int_{\mathcal{P}} k_{\mathcal{P}}(\mathbf{p}) d\mathbf{p} = 1$). For each $\mathbf{p} \in \mathcal{P}$, we

¹We use a box filter in this paper.

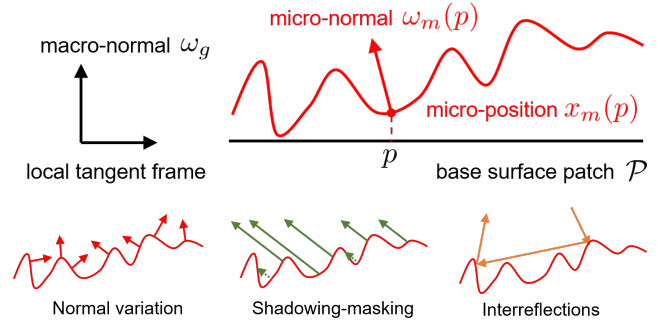


Fig. 4. Illustrations of micro-geometry and related illumination effects. They are shown in 2D for simplicity, but fully developed in 3D.

denote its final position (perturbed by displacement mapping) and micro-normal as $\mathbf{x}_m(\mathbf{p})$ and $\omega_m(\mathbf{p})$, respectively. Lastly, the micro-geometry given by a base patch $\mathcal{P}$ and a displacement map can be expressed by a collection of micro-positions and micro-normals: $\mathcal{G}(\mathcal{P}) = \{(\mathbf{x}_m(\mathbf{p}), \omega_m(\mathbf{p})) \mid \mathbf{p} \in \mathcal{P}\}$.

Meso-geometry. For a surface patch $\mathcal{P}$, we define a local tangent frame under which the macro-normal ω_g of the patch equals to $(0, 0, 1)$. Then, the meso-scale geometry of $\mathcal{G}(\mathcal{P})$ can be described with the *average slope*:

$$\tilde{s} = \int_{\mathcal{P}} s_m(\mathbf{p}) k_{\mathcal{P}}(\mathbf{p}) d\mathbf{p}, \quad (1)$$

where $s_m(\mathbf{p}) = (-x_m/z_m, -y_m/z_m)$ is the micro-slope with x_m, y_m , and z_m given by the micro-normal at $\mathbf{p}$ (i.e., $(x_m, y_m, z_m) = \omega_m(\mathbf{p})$) under the local tangent frame.

Normal mapping. Provided a displacement map, we can extract the *normal distribution function* (NDF) [Dupuy et al. 2013] for any given surface patch $\mathcal{P}$ via

$$D(\omega) = \int_{\mathcal{P}} \frac{\delta(\omega - \omega_m(\mathbf{p}))}{\langle \omega_m(\mathbf{p}), \omega_g \rangle} k_{\mathcal{P}}(\mathbf{p}) d\mathbf{p}, \quad (2)$$

where $\langle \cdot, \cdot \rangle$ represents the dot product clamped to 0, and δ denotes the Dirac delta function. A physically valid NDF must satisfy

$$\int_{\mathcal{H}^2} D(\omega) \langle \omega, \omega_g \rangle d\omega = \int_{\mathcal{P}} k_{\mathcal{P}}(\mathbf{p}) d\mathbf{p} = 1, \quad (3)$$

where $\mathcal{H}^2$ denotes the unit hemispherical domain. It means that the projected area of the micro-geometry onto the macro-normal ω_g should be equal to the area of the base surface patch.

An NDF D can be approximated by a mixture of von Mises-Fisher (vMF) lobes [Han et al. 2007],

$$D(\omega) \approx \sum_{i=1}^m \alpha_i \gamma(\omega; \kappa_i, \mu_i), \quad (4)$$

where α_i represents the lobe's amplitude. Each vMF lobe with bandwidth κ_i and the center direction μ_i is defined as $\gamma(\omega; \kappa_i, \mu_i) = \frac{\kappa_i}{4\pi \sinh \kappa_i} \exp(\kappa_i \langle \omega, \mu_i \rangle)$. This vMF-based parameterization allows efficient description of spatially varying NDFs (SVNDFs).

From an NDF D and an isotropic base BRDF f , the composite multi-lobe BRDF can be formulated as [Han et al. 2007]

$$\rho(\omega_i, \omega_o; D, f) = \frac{1}{\cos \theta_i} \int_{\mathcal{H}^2} f(R\omega(\omega_i), R\omega(\omega_o)) \langle \omega, \omega_i \rangle D(\omega) d\omega. \quad (5)$$

Note that ω_i and ω_o are in the macro-scale tangent frame defined by the macro-normal ω_g . We need a rotation function $R_\omega(\cdot)$ to transform ω_i and ω_o to the local frame defined by the micro-normal ω . The multi-lobe BRDF keeps the orientations of the micro-normals, but neglects both shadowing-masking and interreflections caused by the micro-geometry.

Effective BRDFs. To include shadowing-masking effects, the effective BRDF f^{eff} is used to describe the overall reflectance of a patch $\mathcal{P}$ [Wu et al. 2011]. It depends on the micro-geometry $\mathcal{G}(\mathcal{P})$ and the micro-BRDF f . The effective BRDF can be viewed as the average of the cosine-weighted and shadowed micro-BRDFs weighted by the visible projected area along the viewing direction ω_o :

$$f^{\text{eff}}(\omega_i, \omega_o; \mathcal{G}, f) = \frac{1}{A_{\mathcal{G}}(\omega_o)} \int_{\mathcal{P}} f(\mathbf{x}_m(\mathbf{p}), \omega_i, \omega_o) \langle \omega_m(\mathbf{p}), \omega_i \rangle V(\mathbf{x}_m(\mathbf{p}), \omega_i) A_{\mathcal{G}}(\mathbf{p}, \omega_o) k_{\mathcal{P}}(\mathbf{p}) d\mathbf{p}, \quad (6)$$

where $V(\mathbf{x}, \omega)$ denotes the binary visibility function (indicating whether a ray starting from point $\mathbf{x}$ along direction ω is occluded), and $A_{\mathcal{G}}(\mathbf{p}, \omega_o)$ is the visible projected area along ω_o given by

$$A_{\mathcal{G}}(\mathbf{p}, \omega_o) = \frac{\langle \omega_o, \omega_m(\mathbf{p}) \rangle}{\langle \omega_g, \omega_m(\mathbf{p}) \rangle} V(\mathbf{x}_m(\mathbf{p}), \omega_o). \quad (7)$$

The total visible projected area of $\mathcal{G}(\mathcal{P})$ is the normalization factor of the weighted average [Dupuy et al. 2013; Heitz 2014],

$$A_{\mathcal{G}}(\omega_o) = \int_{\mathcal{P}} \frac{\langle \omega_o, \omega_m(\mathbf{p}) \rangle}{\langle \omega_g, \omega_m(\mathbf{p}) \rangle} V(\mathbf{x}_m(\mathbf{p}), \omega_o) k_{\mathcal{P}}(\mathbf{p}) d\mathbf{p} = \frac{\langle \omega_o, \omega_n \rangle}{\langle \omega_g, \omega_n \rangle}, \quad (8)$$

where ω_n denotes the meso-scale normal direction.

The effective BRDF (6) has the cosine term and the visibilities baked in and captures the shadowing-masking caused by micro-scale self-occlusions. We will generalize the formulation in §5.1 to capture interreflections.

4 PREFILTERING REFLECTANCE PARAMETERS

Our method takes as input a high-resolution displacement map h_{orig} and an isotropic base micro-BRDF f_{orig} (which could be spatially varying, e.g., Figure 14). We denote $\mathcal{G}_{\text{orig}}(\mathcal{P})$ as the micro-geometry defined by the original displacement map on a surface patch $\mathcal{P}$. Then, we prefilter the input model and obtain a lower-resolution displacement map h_{low} (with $\mathcal{G}_{\text{low}}(\mathcal{P})$ representing its micro-geometry on $\mathcal{P}$) associated with a new spatially varying BRDF f_{low} . Our goal is to have the appearance of the prefiltered model closely resemble the input. Notice that, even if the input base BRDF f_{orig} is spatially invariant, the prefiltered reflectance f_{low} may need to have spatial variations to accurately reproduce the detailed appearance of the input model.

To this end, our technique starts with computing the downsampled displacement map h_{low} by minimizing differences of the meso-scale slopes (§4.1). Then, we seek a prefiltered SVBRDF f_{low} that preserves the original appearance for the downsampled displacement map. This, however, is nontrivial as f_{low} generally needs to be spatially varying. We introduce a novel two-step approach to compute f_{low} . First, for each base patch of the downsampled displacement map h_{low} , we compute its corresponding *patch NDF* from the original displacement map h_{orig} . Each patch NDF implies a spatially varying multi-lobe BRDF without considering shadowing-masking and interreflections. This step generates a spatially varying

base BRDF f'_{low} as an initial solution (§4.2). Then, we scale f'_{low} by a 6D scaling function $R(\mathbf{x}, \omega_i, \omega_o)$ to match the effective BRDFs (§4.3). The final prefiltered SVBRDF $f_{\text{low}} = R \cdot f'_{\text{low}}$ is able to reproduce the original appearance. Our method is also illustrated in Figure 3.

Although the scaling function $R(\mathbf{x}, \omega_i, \omega_o)$ is 6D, we will show in §6 that R can be factorized into a 2D spatial function of location $\mathbf{x}$ and a 4D angular function of directions ω_i and ω_o . Further, these functions are usually smooth in practice, and therefore can be computed efficiently as low-resolution tabulated functions, allowing our prefiltered models to be compactly represented.

4.1 Downsampling Displacement Maps

As stated in §3, displacement maps are defined mathematically as continuous 2D scalar functions. In practice, we represent displacement maps as piecewise linear functions using 2D textures. Specifically, the original high-resolution displacement map h_{orig} is defined at each vertex of a dense grid: $\{(u \cdot 2^{-l}, v \cdot 2^{-l})\}$ for $u, v = 0, \dots, 2^l$. The prefiltered low-resolution displacement map h_{low} uses a coarse grid $\{(u \cdot 2^{-l'}, v \cdot 2^{-l'})\}$ for $u, v = 0, \dots, 2^{l'}$ with some $l' < l$.

Obtaining h_{low} requires specifying the displacement values at the vertices of the coarse grid. For every patch $\mathcal{P}_{uv} = [u \cdot 2^{-l'}, (u+1) \cdot 2^{-l'}] \times [v \cdot 2^{-l'}, (v+1) \cdot 2^{-l'}]$ covering a grid cell of h_{low} , our goal is to find out the optimal heights at its four corners (i.e., grid points) such that the average slope $\tilde{s}_{uv}$ of the bilinear patch closely matches the reference slope $\tilde{s}_{uv}^*$ of the original micro-surface $\mathcal{G}_{\text{orig}}(\mathcal{P}_{uv})$. The reference average slope $\tilde{s}_{uv}^*$ can be calculated using Eq. (1). On the other hand, the average slope of the bilinear patch $\mathcal{P}_{uv}$ is

$$\tilde{s}_{uv} = \left(\frac{h_{11} + h_{10} - h_{01} - h_{00}}{2}, \frac{h_{11} + h_{01} - h_{10} - h_{00}}{2} \right), \quad (9)$$

where $h_{00}, h_{10}, h_{01}, h_{11}$ denote the displacement values at the four cell corners. Please refer to Appendix A for the derivation. To minimize the differences between the two average slopes robustly, we further introduce a regularization term for local smoothness, yielding the final objective function:

$$L(h_{\text{low}}) = \sum_u \sum_v \left\| \tilde{s}_{uv} - \tilde{s}_{uv}^* \right\|^2 + w \left\| \Delta h_{\text{low}} \left(\frac{u}{2^{l'}}, \frac{v}{2^{l'}} \right) \right\|^2, \quad (10)$$

where Δ is the Laplacian operator and $w = 0.01$ is the weight of the regularization term. By minimizing this objective function using least-squares, the optimal solution gives the downsampled displacement map h_{low} and its corresponding micro-geometry $\mathcal{G}_{\text{low}}$.

4.2 Spatially Varying Multi-Lobe BRDF

For a cell from the coarse grid of the downsampled displacement map h_{low} and its corresponding patch $\mathcal{P}_{uv}$, the patch NDF $D_{uv}(\omega)$ defined in Eq. (2) provides a compact approximation of the original micro-geometry $\mathcal{G}_{\text{orig}}(\mathcal{P}_{uv})$. Previous works such as LEADR [Dupuy et al. 2013] typically assume the NDFs to follow Beckmann distributions. Our method, in contrast, does not enforce any restriction on the NDFs and is therefore more general. We use the normal mapping technique [Han et al. 2007] to fit an NDF with a mixture of vMF lobes (4), resulting in a spatially varying NDF at a lower resolution. The SVNDF is technically 4D but can be described compactly using spatially varying vMF parameters (stored as 2D textures).

From these NDFs that approximate $\mathcal{G}_{\text{orig}}$ and the original base micro-BRDF f_{orig} , we use Eq. (5) to formulate the initial multi-lobe BRDF for each patch $\mathcal{P}_{uv}$:

$$f'_{\text{low}}(\mathbf{x}, \omega_i, \omega_o) = \rho(\omega_i, \omega_o; D_{uv}, f_{\text{orig}}). \quad (11)$$

Here the micro-position $\mathbf{x}$ is within the patch $\mathcal{P}_{uv}$.

4.3 Scaling Function

Although the multi-lobe SVBRDF f'_{low} is a good start for matching the input appearance, it is incomplete as it neglects the shadowing-masking and interreflection effects. To address this problem, we first consider the single-bounce case (i.e., direct illumination that only involves shadowing and masking). We will discuss the multiple-bounce case that handles interreflections in §5.

To capture shadowing and masking, we multiply the initial multi-lobe SVBRDF f'_{low} with another scaling function R . Suppose R has a spatial resolution of M^2 , we uniformly subdivide the base surface into M^2 patches. Let $\mathcal{P}_{\mathbf{x}}$ denote the patch containing $\mathbf{x}$. We define the scaling function $R(\mathbf{x}, \omega_i, \omega_o)$ as the ratio² between the original effective BRDF (6) and the prefiltered effective BRDF over $\mathcal{P}_{\mathbf{x}}$:

$$R(\mathbf{x}, \omega_i, \omega_o) = \frac{f^{\text{eff}}(\omega_i, \omega_o; \mathcal{G}_{\text{orig}}(\mathcal{P}_{\mathbf{x}}), f_{\text{orig}})}{f^{\text{eff}}(\omega_i, \omega_o; \mathcal{G}_{\text{low}}(\mathcal{P}_{\mathbf{x}}), f'_{\text{low}})}, \quad (12)$$

The final prefiltered base SVBRDF can then be expressed as

$$f_{\text{low}}(\mathbf{x}, \omega_i, \omega_o) = R(\mathbf{x}, \omega_i, \omega_o) f'_{\text{low}}(\mathbf{x}, \omega_i, \omega_o). \quad (13)$$

It is easy to verify that the final effective BRDF $f^{\text{eff}}(\mathcal{G}_{\text{low}}, f_{\text{low}})$ (with ω_i and ω_o omitted for notational simplicity) over $\mathcal{P}_{\mathbf{x}}$ equals the original effective BRDF, since the single-bounce effective BRDF is linear in the scaling factors, i.e., $f^{\text{eff}}(\mathcal{G}_{\text{low}}, R \cdot f'_{\text{low}}) = R \cdot f^{\text{eff}}(\mathcal{G}_{\text{low}}, f'_{\text{low}})$. The identical effective BRDFs indicate a good match of aggregate reflectance between the original and the prefiltered models.

Due to the high dimensionality of the scaling function R , it is challenging to obtain and store. We will show in §6 that R can be accurately approximated as the product of a function $T(\mathbf{x})$ of location and a function $S(\omega_i, \omega_o)$ of directions:

$$R(\mathbf{x}, \omega_i, \omega_o) \approx T(\mathbf{x}) \cdot S(\omega_i, \omega_o). \quad (14)$$

We first generalize our scaling function to interreflections in §5. Then, in §6, we develop the factorization above and show that the factorized scaling functions T and S can be computed directly without fully computing the 6D function R .

5 INTERREFLECTIONS

We now describe how to capture interreflection effects using a scaling function similar to Eq. (12). We first generalize the effective BRDF formulation (6) to measure the average surface reflectance with interreflections (§5.1). Then, we provide an approximate solution to the scaling function with interreflections handled (§5.2).

5.1 Effective BRDF with Interreflections

The effective BRDF formulation depicted in §3 models only direct (i.e., single-bounce) illumination of the micro-geometry. This leads to significant energy loss due to the neglect of interreflections within the micro-geometry. To address this problem, we adapt Veach's path integral formulation [1997] to express effective BRDFs with interreflections.

A ray that bounces within the micro-geometry $\mathcal{G} = \{(\mathbf{x}_m, \omega_m)\}$ can be described by a light transport path $\bar{\mathbf{x}} = (\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_k)$ as well as the viewing direction ω_o and the lighting direction ω_i . Notice that, unlike the traditional formulation where the endpoints of a path respectively lay on the light source and the sensor, all the path vertices $\mathbf{x}_1, \dots, \mathbf{x}_k$ are located on the micro-surface in our case. The path contribution $h(\bar{\mathbf{x}})$ is given by the product of BRDF terms and cosine terms, followed by a visibility term at the last path vertex $\mathbf{x}_k$ along the lighting direction ω_i :

$$h(\bar{\mathbf{x}}) = V(\mathbf{x}_k, \omega_i) \prod_{j=1}^k f(\mathbf{x}_j, \omega_{j \rightarrow j+1}, \omega_{j \rightarrow j-1}) \langle \mathbf{n}(\mathbf{x}_j), \omega_{j \rightarrow j+1} \rangle, \quad (15)$$

where $\omega_{i \rightarrow j} := \frac{\mathbf{x}_j - \mathbf{x}_i}{\|\mathbf{x}_j - \mathbf{x}_i\|}$ indicates the normalized direction from $\mathbf{x}_i$ to $\mathbf{x}_j$ (with $\omega_{1 \rightarrow 0} := \omega_o$ and $\omega_{k \rightarrow k+1} := \omega_i$), and $\mathbf{n}(\mathbf{x}_j)$ denotes the micro-normal at $\mathbf{x}_j$.

We define the path space $\Omega(\mathbf{x}, \omega_i, \omega_o)$ as a collection of light transport paths $\bar{\mathbf{x}} = (\mathbf{x}_1, \dots, \mathbf{x}_k)$ with $\mathbf{x}_1 = \mathbf{x}$, and the viewing and illumination directions given by ω_o and ω_i , respectively. At a given position $\mathbf{x}_m(\mathbf{p}) \in \mathcal{G}$, the reflectance including interreflections aggregates contributions from all the valid paths:

$$r(\mathbf{x}_m(\mathbf{p}), \omega_i, \omega_o) = \int_{\Omega(\mathbf{x}_m(\mathbf{p}), \omega_i, \omega_o)} h(\bar{\mathbf{x}}) d\mu(\bar{\mathbf{x}}), \quad (16)$$

where $d\mu(\bar{\mathbf{x}})$ denotes the path throughput measure, which is a product of solid angle measures on the path directions.

According to Eqs. (6) and (16), we express the multi-bounce effective BRDF as the average path contribution over $\mathcal{G}$ weighted by the visible projected area along ω_o :

$$f_{\text{ir}}^{\text{eff}}(\omega_i, \omega_o; \mathcal{G}, f) = \frac{1}{A_{\mathcal{G}}(\omega_o)} \int_{\mathcal{P}} r(\mathbf{x}_m(\mathbf{p}), \omega_i, \omega_o) A_{\mathcal{G}}(\mathbf{p}, \omega_o) k_{\mathcal{P}}(\mathbf{p}) d\mathbf{p}. \quad (17)$$

Under this formulation, the single-bounce effective BRDF (6) becomes a special case where each path $\bar{\mathbf{x}}$ is restricted to contain only one vertex.

5.2 Computing the Scaling Function

Following a similar idea as §4.3, we scale the initial multi-lobe SVBRDF f'_{low} by a scaling function R_{ir} :

$$f_{\text{low}}(\mathbf{x}, \omega_i, \omega_o) = R_{\text{ir}}(\mathbf{x}, \omega_i, \omega_o) f'_{\text{low}}(\mathbf{x}, \omega_i, \omega_o), \quad (18)$$

so that the resulting multi-bounce effective BRDF (17) of the prefiltered model matches that of the input. Namely, $f_{\text{ir}}^{\text{eff}}(\mathcal{G}_{\text{low}}, f_{\text{low}}) = f_{\text{ir}}^{\text{eff}}(\mathcal{G}_{\text{orig}}, f_{\text{orig}})$. Since $f_{\text{ir}}^{\text{eff}}$ is nonlinear in R_{ir} due to multiple scattering (i.e., $f_{\text{ir}}^{\text{eff}}(\mathcal{G}_{\text{low}}, R_{\text{ir}} \cdot f'_{\text{low}}) \neq R_{\text{ir}} \cdot f_{\text{ir}}^{\text{eff}}(\mathcal{G}_{\text{low}}, f'_{\text{low}})$ in general), R_{ir} cannot be obtained using Eq. (12). Although one can optimize R_{ir} iteratively using inverse rendering methods (e.g., [Khungurn et al. 2015; Zhao et al. 2016]), the optimization would be very expensive, if not impractical, due to the large number of unknowns involved.

²In practice, we need to add a small ϵ to avoid division by zero.

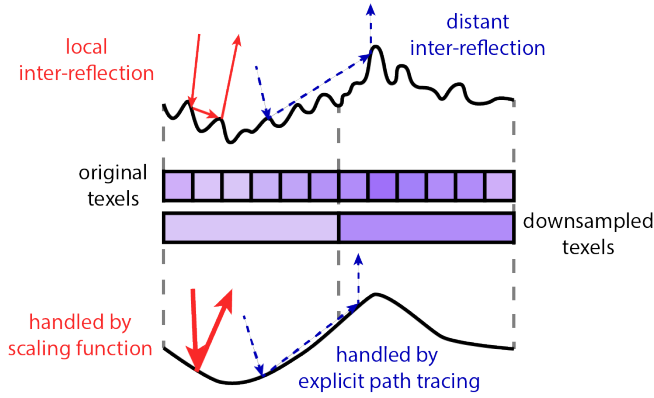


Fig. 5. **Local and global interreflections.** Our method tries to match local interreflections using a scaling function, while the path tracer handles distant (global) interreflections.

Instead, we seek R_{ir} without performing global optimizations. To this end, we make a simplifying assumption: the high-order components in $f_{ir}^{eff}(\mathcal{G}_{low}, f_{low})$ are negligible. In other words,

$$f_{ir}^{eff}(\mathcal{G}_{low}, f_{low}) \approx f^{eff}(\mathcal{G}_{low}, f_{low}). \quad (19)$$

This is because the high-order component mainly consists of *distant* interreflections (i.e., those across multiple surface patches at the reduced resolution) that are supposed to be handled with explicit path tracing (see the blue arrows in Figure 5). The *local* interreflection, on the other hand, is usually weakened due to the downsampling of an object’s micro-geometry, which we compensate using the scaling function R_{ir} (see the red arrows in Figure 5).

Under this formulation, our goal becomes to match $f_{ir}^{eff}(\mathcal{G}_{orig}, f_{orig})$ and $R_{ir} \cdot f^{eff}(\mathcal{G}_{low}, f'_{low})$, yielding the scaling function:

$$R_{ir}(\mathbf{x}, \omega_i, \omega_o) = \frac{f_{ir}^{eff}(\omega_i, \omega_o; \mathcal{G}_{orig}(\mathcal{P}_{\mathbf{x}}), f_{orig})}{f^{eff}(\omega_i, \omega_o; \mathcal{G}_{low}(\mathcal{P}_{\mathbf{x}}), f'_{low})}. \quad (20)$$

In §8, we show that the simplification usually has minimal affect on accuracy (Figures 17, 18).

6 PROPERTIES OF THE SCALING FUNCTION

Upon establishing the scaling function R_{ir} (20), our problem of prefiltering displacement-mapped surfaces boils down to efficiently representing and computing this function,³ which, unfortunately, is nontrivial as R_{ir} is 6D and generally too expensive to compute and store in brute-force ways.

We tackle this challenge by factorizing the 6D scaling function $R_{ir}(\mathbf{x}, \omega_i, \omega_o)$ into a single spatial scaling function $T_{ir}(\mathbf{x})$ and a single angular scaling function $S_{ir}(\omega_i, \omega_o)$. In practice, both T_{ir} and S_{ir} are smooth (i.e., low-frequency). We exploit their smoothness to achieve efficient factorization using only a sparse sampling of R_{ir} (without explicitly precomputing the full 6D function). In the rest of this section, we provide more details on our factorization method.

³We use the notation R_{ir} in the multiple-bounce case since it is more general. The properties also hold for R in the single-bounce case.

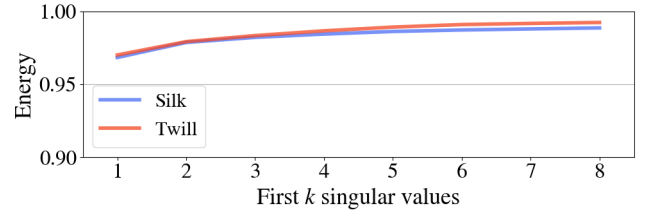


Fig. 6. Graph showing the fractions of energy carried by the top eight singular values of the scaling matrix.

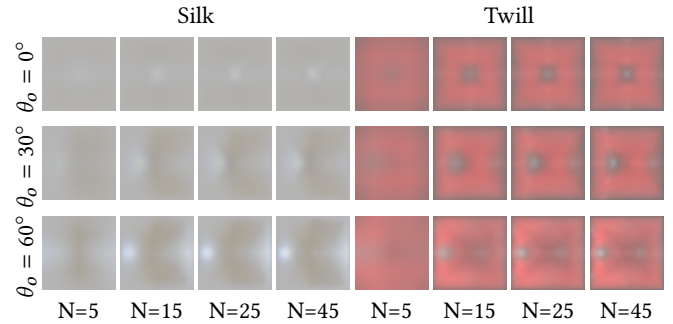


Fig. 7. Scaling function slices (with only ω_i changing and the other parameters fixed) for different angular resolutions. Using $N = 15$ provides adequate accuracy for describing the scaling functions.

Rank-1 factorization. Our experiments indicate that the scaling functions are typically low-rank (Figure 6). To demonstrate this property, we compute the full 6D scaling functions for two example models: “silk” with only direct illumination; and “twill” with global illumination (details for these models are described in §8.2). In both cases, the scaling matrices R_{ir} are tabulated with a resolution $16^2 \times 15^2 \times 15^2$ (16^2 for $\mathbf{x}$, 15^2 for ω_i , 15^2 for ω_o).⁴ After excluding the directions near grazing angles ($\theta > 75^\circ$),⁵ we reshape R_{ir} into a 256×11^4 matrix, which is then decomposed using singular value decomposition (SVD). For both examples, the greatest singular value carries over 95% of the total energy (see Figure 6), suggesting that the scaling function R_{ir} can indeed be accurately approximated by a rank-1 factorization:

$$R_{ir}(\mathbf{x}, \omega_i, \omega_o) \approx T_{ir}(\mathbf{x}) \cdot S_{ir}(\omega_i, \omega_o). \quad (21)$$

Angular smoothness. We now seek the optimal resolution for the scaling function $R_{ir}(\mathbf{x}, \omega_i, \omega_o)$ that provides a good balance between accuracy and data size, starting with the angular resolution N . Specifically, we use the concentric mapping [Shirley and Chiu 1997] to warp a unit hemisphere onto a unit square. Then, both ω_i are ω_o are sampled from a regular 2D grid with resolution N^2 .

Because an effective BRDF captures the aggregate reflectance over a micro-surface, it generally has low angular frequency. Thus, the scaling function, as the ratio of two effective BRDFs, is also expected to be low-frequency angularly. Figure 7 shows 2D slices of the scaling functions (with ω_i changing and the other parameters fixed) for different angular resolutions $N = 5, 15, 25$ and 45 (as references). The results indicate that $N = 15$ provides good accuracy,

⁴This resolution is adequate due to the smoothness of R_{ir} , which will be discussed later in this section.

⁵Numerical evaluation of the scaling functions is unstable near grazing angles.

which is further supported by renderings in Figure 8. Therefore, we choose the resolution of $N = 15$ for all main results in this paper.

Spatial smoothness. To determine the spatial resolution M , we perform a similar experiment by comparing renderings using resolutions of $M = 1, 4, 16$. As shown in Figure 9, the results indicate that increasing the spatial resolution has diminishing benefits. This is because the scaling function varies in a very smooth fashion spatially. We thus use $M = 4$ in most of our results (unless we state explicitly). In certain cases, even using $M = 1$ (that causes the scaling function T_{ir} to be constant-valued) can offer sufficient accuracy.

Efficient factorization. Although computing the full 6D scaling function R_{ir} and decomposing it using SVD provides optimal factorization results, doing so is very costly since both steps require massive computation. We introduce a simple and efficient factorization method that constructs $T_{ir}(\mathbf{x})$ and $S_{ir}(\omega_i, \omega_o)$ from sparse samples of R_{ir} .

According to Eq. (21), $T_{ir}(\mathbf{x})$ can be computed by averaging the 6D scaling function R_{ir} over the angular domain:

$$T_{ir}(\mathbf{x}) = \frac{1}{C} \int_{\mathcal{H}^2} \int_{\mathcal{H}^2} R_{ir}(\mathbf{x}, \omega_i, \omega_o) d\omega_i d\omega_o, \quad (22)$$

where

$$C = \int_{\mathcal{P}_{all}} \int_{\mathcal{H}^2} \int_{\mathcal{H}^2} R_{ir}(\mathbf{x}_m(\mathbf{p}), \omega_i, \omega_o) k_{\mathcal{P}_{all}}(\mathbf{p}) d\omega_i d\omega_o d\mathbf{p}, \quad (23)$$

is a normalization factor, and $\mathcal{P}_{all} = [0, 1]^2$ covers the whole displacement map. To obtain T_{ir} as a tabulated function, we estimate its value at each bin using Monte Carlo integration based on a sparse (joint) sampling of $\mathbf{x}$ (among all locations within the bin), ω_i , and ω_o . We provide more implementation details in §7.

On the other hand, the angular scaling function $S_{ir}(\omega_i, \omega_o)$ can be obtained by averaging R_{ir} spatially:

$$S_{ir}(\omega_i, \omega_o) = \int_{\mathcal{P}_{all}} R_{ir}(\mathbf{x}_m(\mathbf{p}), \omega_i, \omega_o) k_{\mathcal{P}_{all}}(\mathbf{p}) d\mathbf{p}. \quad (24)$$

As a tabulated function, S_{ir} can be computed in a similar way as T_{ir} using Monte Carlo integration.

It is easy to verify that our definitions of T_{ir} and S_{ir} in Eqs. (22, 24) are consistent with Eq. (21). Please see Appendix B for more details.

Figure 10 compares factorization results obtained using our method and SVD (that uses full R_{ir}). Our method offers similar reconstruction errors as SVD does, which is demonstrated in Figure 10(b, c). Further, the corresponding renderings shown in Figure 10(d–f) indicate that the loss in visual quality caused by our method is negligible.

7 IMPLEMENTATION

We now provide details of our implementation. First, we provide details on how our technique works at individual scales (§7.1). Then, we describe how mipmaps of our prefiltered models can be created, enabling anti-aliased level-of-detail (LoD) rendering of displacement-mapped models (§7.2).

7.1 Prefiltering at a Single Scale

Provided a high-resolution displacement map h_{orig} and a downsampling scale, we first optimize for the downsampled displacement

map using least-squares (§4.1) and compute the initial spatially varying multi-lobe BRDF f'_{low} (§4.2). As in prior work [Han et al. 2007], we use six vMF lobes per patch NDF. Then, we seek the scaling function R_{ir} so that the final SVBRDF $f_{low} = R_{ir} \cdot f'_{low}$ preserves the appearance of the input model (§4.3 and §5.2). This is achieved using our efficient factorization of R_{ir} (§6).

To estimate the value of T_{ir} at each tabulation bin, we use 5000 pairs of (ω_i, ω_o) with both directions independently sampled from a cosine-weighted distribution. For evaluating the scaling function R_{ir} via Eq. (20), we trace 2500 paths to estimate the effective BRDFs (17). Lastly, we normalize T_{ir} such that it averages to one. To estimate the scaling function S_{ir} of directions, we use 16 stratified samples for $\mathbf{p}$ and 2500 light transport paths for R_{ir} . We find these sampling rates are sufficient to reproduce accurate appearances.

We implement our method based on the Mitsuba physically based renderer [Jakob 2010]. Running on a workstation equipped with a six-core Intel i7-5930K CPU, computing the factorized functions T_{ir} and S_{ir} takes 20–30 minutes for each input model.

Shell mapping. We use shell mapping [Porumbescu et al. 2005] to deform a flat displacement map to a general shape (Figure 11), instead of explicitly displacing a base mesh. In fact, objects in our scenes are perturbed by extreme high-resolution displacement maps (resolution up to $(200K)^2$, tiled by $(1K)^2$ base displacement maps) to describe very small geometric details. Creating an explicit displacement mesh is not practical.

Importance sampling. Our final prefiltered SVBRDF f_{low} equals to the product of two functions R_{ir} and f'_{low} . Due to the smoothness of the scaling function R_{ir} , we simply follow f'_{low} when importance sampling f_{low} . That said, product importance sampling [Clarberg et al. 2005; Herholz et al. 2016] could be used to further improve the sampling quality.

7.2 Level of Detail

To enable anti-aliased rendering at multiple scales, we prefilter the input model at several downsampling scales. The resulting models effectively form a mipmap. At each mipmap level, we compute and store the lobe parameters as well as the scaling functions T_{ir} and S_{ir} , in addition to the downsampled displacement map. To balance computational and storage overhead and rendering performance, our mipmap uses a sparser set of levels compared to conventional texture mipmaps. In other words, the scales of two contiguous levels of our mipmaps usually differ by more than $2\times$. For example, the mipmap of the “silk” model includes $(16\times)^2$ - and $(64\times)^2$ -downsampled versions (besides the original model), and the mipmap of “twill” has $(8\times)^2$ and $(64\times)^2$ downsampled versions.

At render time, we use ray differentials [Igehy 1999] to compute each pixel’s footprint on the original displacement map and determine the desired mipmap levels. To interpolate between two levels, we trace paths on two different models and linearly interpolate the path contributions. Please refer to Figures 1, 12 and the accompanying video for results rendered with this method.

8 RESULTS

We now show results generated using our method: §8.1 contains experimental evaluations and justifications; §8.2 demonstrates the

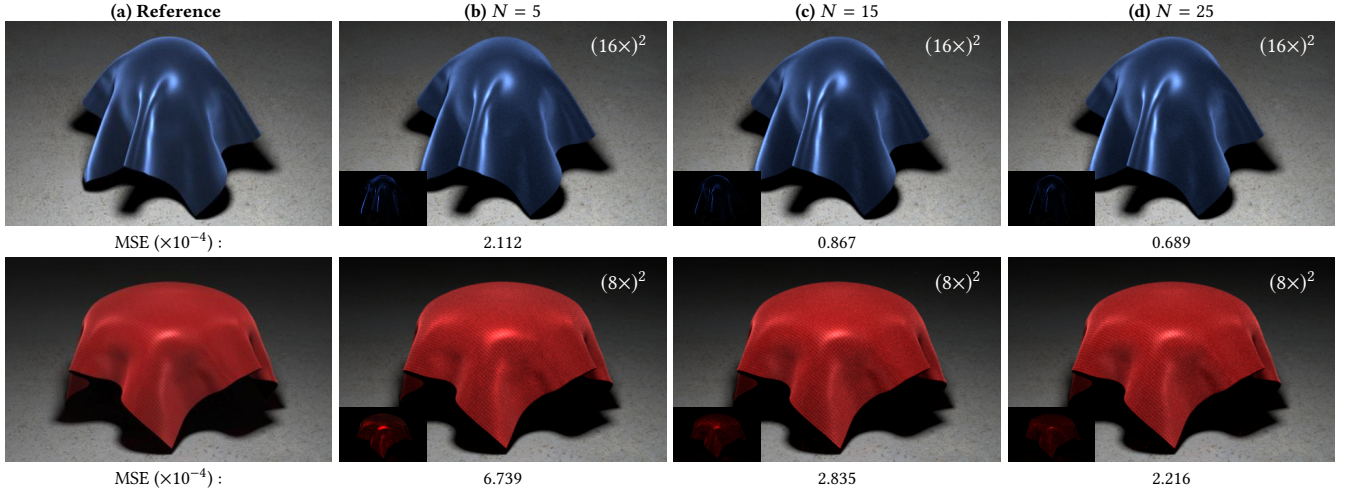


Fig. 8. **Using varying angular resolutions.** Increasing angular resolution improves rendering accuracy, but at the expense of larger computational and storage overhead (with a fixed spatial resolution of $M = 1$). We use a resolution of $N = 15$, which shows the best tradeoff between accuracy and storage/performance. The insets (and those in the next figures 9, 10(e, f)) show $10\times$ squared errors compared to the reference images.

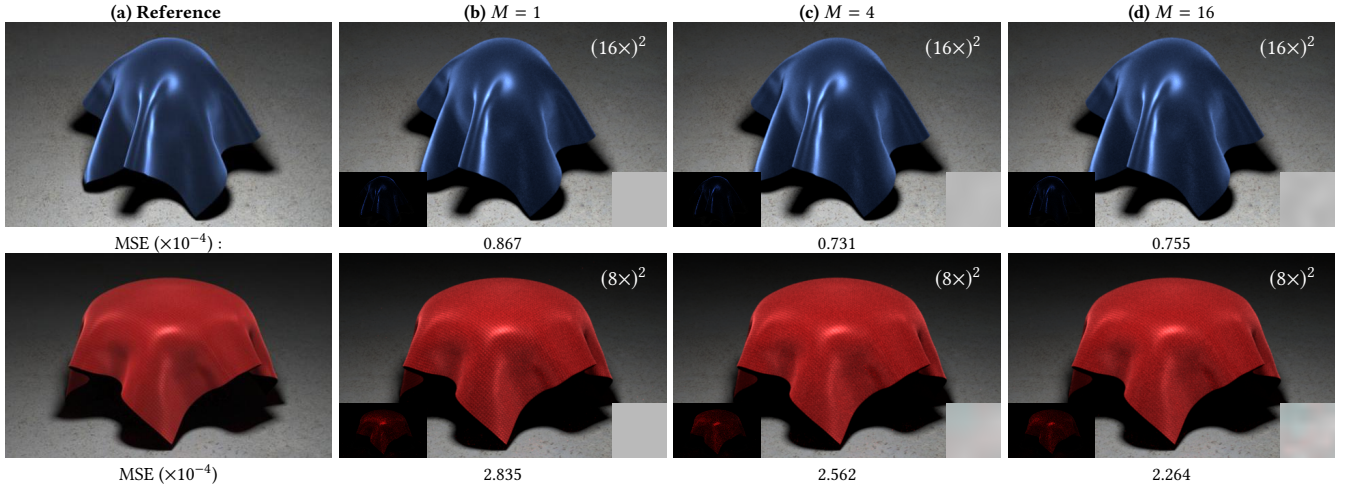


Fig. 9. **Using varying spatial resolutions.** Increasing spatial resolutions results in more accurate rendering results, but the improvement is diminishing after $M = 4$ (errors may become slightly larger because of Monte Carlo noise). We find that $M = 4$ is adequate in most cases, and even using $M = 1$ yields high-quality results. Bottom-right insets visualize the 2D spatial scaling functions for different resolutions.

effectiveness of our technique via a few examples. Finally, we discuss limitations of our methods and future research directions (§8.3).

8.1 Evaluations and Justifications

Energy Conservation. We demonstrate that our prefiltered models conserve energy via white furnace tests (Figure 13). Given a non-absorptive base BRDF and a constant environment light, our prefiltered models without interreflections (Figure 13(b)) lose a significant amount of energy, while those capturing interreflections (Figure 13(c)) preserve most of the energy. The effective albedos of our prefiltered models in Figure 13(c) are 0.988 (twill) and 0.999 (noise).

The ground truth effective BRDFs are energy conserving as they are determined by explicitly simulating light transport within the micro-surfaces. Theoretically, our prefiltered models as approximations of the ground truth effective BRDFs are not guaranteed to conserve energy. Limited energy loss is mainly due to neglecting distant interreflections. In practice, however, we do not observe any problem with energy conservation.

Reciprocity. Our effective BRDFs (17) are reciprocal when weighted by visible projected area (similar to the microflake reciprocity constraints proposed by Heitz et al. [2015]):

$$f_{\text{ir}}^{\text{eff}}(\omega_i, \omega_o; \mathcal{G}, f) A_{\mathcal{G}}(\omega_o) = f_{\text{ir}}^{\text{eff}}(\omega_o, \omega_i; \mathcal{G}, f) A_{\mathcal{G}}(\omega_i). \quad (25)$$

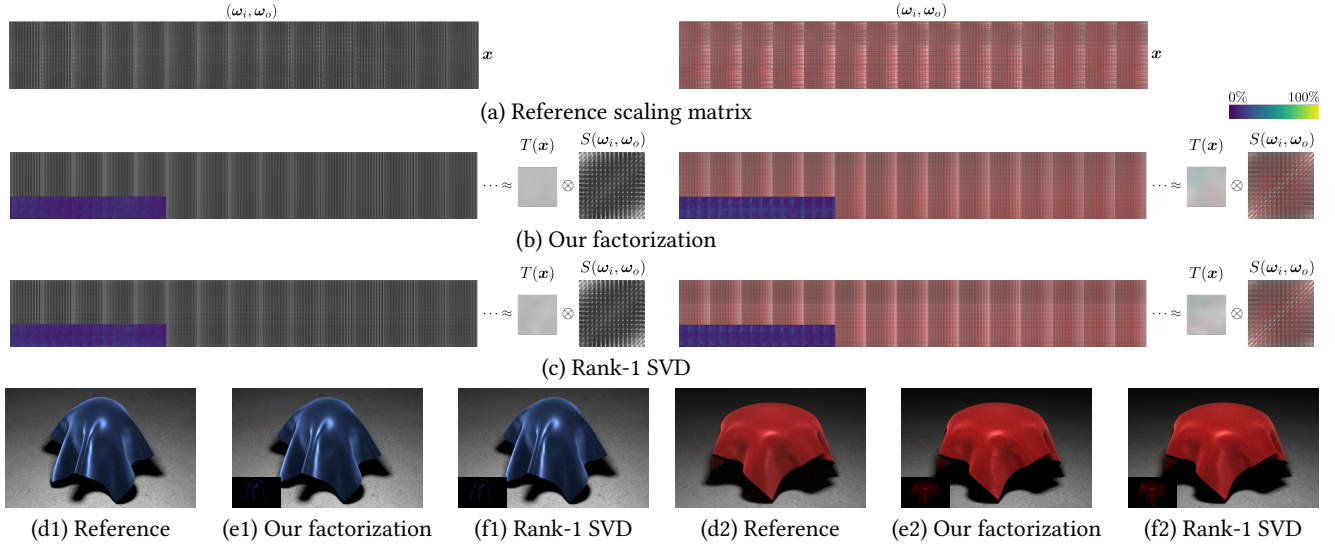


Fig. 10. **Factorization comparison.** We compare our factorization method to rank-1 SVD in terms of scaling function reconstruction (a–c) and rendered images (d–f). We show 6D scaling functions (they are reorganized as scaling matrices) in (a). Even without explicitly computing the full 6D functions, the reconstruction error of our factorization (b) is similar to the error given by rank-1 SVD (c). Both errors are small because of the low-rank nature of the scaling matrices. The insets in (b, c) show the relative error maps of the reconstructed matrices compared to the reference matrices. The colored error bar is shown on the top-right. In terms of visual quality, using the factorized spatial scaling function T and angular scaling function S computed by our method reproduces the reference appearance accurately.

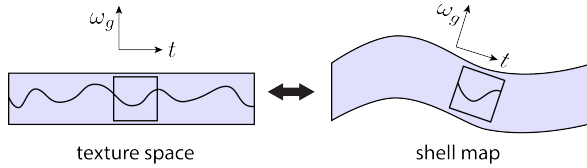


Fig. 11. **Shell map illustration.** We deform a flat displacement map to a general shape using shell maps.

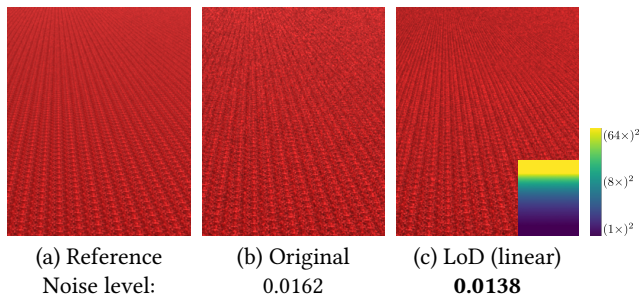


Fig. 12. **LoD rendering.** The reference (a) is rendered using a high-resolution displacement map and 5K spp. The baseline result (b) is rendered using the same displacement map but (10 $\times$)-lower spp. Our equal-time low-spp LoD rendering (c) closely matches the reference (a) at multiple scales, and suffers from less noise (measured using Liu et al. [2012], lower is better) compared to the baseline (b). The mipmap levels are shown in inset.

Since we aim to match the effective BRDFs given by the input models, our prefiltered models are approximately reciprocal.

Correlated Surfaces. Our prefiltering method can handle surfaces with correlations. Figure 14 shows a synthetic two-color sawtooth

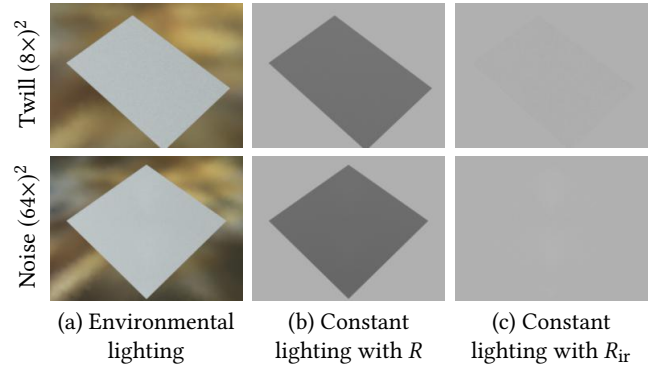


Fig. 13. **White furnace test.** We prefilter displacement maps with a non-absorptive BRDF. Using the scaling function R handling only direct illumination (i.e., single bounce) leads to significant energy loss. Taking inter-reflections into account, the object rendered with the multi-bounce scaling function R_{ir} becomes almost invisible, showing negligible energy loss.

example where the surface normals and colors are strongly correlated. The baseline multi-lobe BRDFs f'_{low} disregard this correlation and produce wrong appearance. Our result, on the other hand, captures this correlation properly and preserves the reference appearance accurately.

8.2 Main Results

We use two sets of scenes to demonstrate the effectiveness of our method. The scene objects and their detailed parameters (before and after prefiltering) are summarized in Table 3. The first set contains two fabric examples (Figures 2, 16, 17). The “twill” and “silk” models

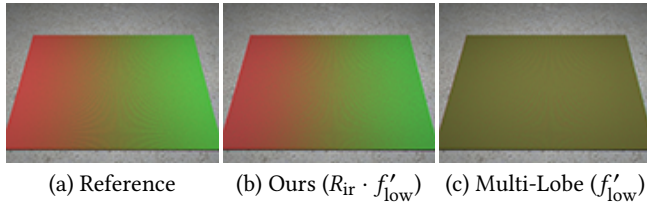


Fig. 14. **Correlated surface.** We prefilter a two-color sawtooth (i.e., V-groove) surface. Thanks to the scaling function R_{ir} , our prefiltered model can effectively capture the normal-color correlation and accurately reproduce the appearance.

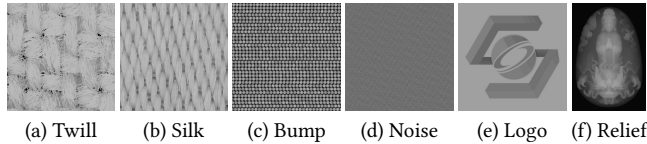


Fig. 15. Displacement maps used in our results.

(Figure 15(a, b)) are extracted from detailed volume data [Zhao et al. 2011]. The resulting surface micro-geometries are highly complex and can cause severe aliasing. The “plane” scene in Figure 2 contains a flat twill fabric. The “twill” scene in Figures 16, 17 (top row) uses the same fabric but shell-mapped. The “silk” scene in Figures 16, 17 (bottom row) has another shell-mapped silk fabric. Although we use isotropic glossy BRDFs in these scenes, the rendered images still show anisotropic glossy highlights because of the complex structured micro-geometries.

Another set of scenes contains objects with more diverse materials. In Figure 18 (top row), we show the “ball” scene that has a bumpy ball on a rough floor. The ball is represented by the “bump” model (Figure 15(c)), while the floor is represented by the “noise” model (Figure 15(d)). Both models use diffuse base BRDFs. In Figure 18 (middle row), we show the “logo” scene that is represented by a bi-scale displacement map (Figure 15(e)). The large-scale displacement map describes the overall shape, while the small-scale displacement map (which is a tiled version of the large-scale one) accounts for the detailed micro-geometry. In Figure 18 (bottom row), we show the “relief” scene that is represented by a composite displacement map (Figure 15(f), which is resampled to keep the image aspect ratio). It is constructed by adding procedural Perlin noise on a height field converted from a gray scale image. In this example, we use a spatial scaling function with resolution $M = 1$ since it leads to sufficient accuracy.

Direct Illumination. Figure 16 contains rendered images generated through the intermediate steps of our method with the effective BRDFs capturing only direct illumination (i.e., neglecting micro-scale interreflections). Directly using original BRDFs f_{orig} with downsampled micro-geometries (the third column) leads to significantly different appearances. Using the spatially varying multi-lobe BRDF f'_{low} without our scaling function $R = ST$ (the fourth column) performs better, but the results are still notably brighter than the references as changes of shadowing and masking are neglected. By applying a further correction using $R = ST$, our models well preserve the input appearances. Additionally, we compare our prefiltered models with those generated using LEADR [Dupuy et al.

2013], which are derived from single-lobe Beckmann NDFs and Smith’s shadowing function. These models fail to reproduce the appearance of the input since the assumption of Gaussian surfaces does not hold (see the top insets in Figure 2(a, f)). Further, the single lobe representation used by LEADR also limits the result accuracy, especially for glossy base BRDFs.

Global Illumination. The results in Figures 17 and 18 are rendered with full global illumination. In this case, directly using f_{orig} (the third column) still yields poor accuracy. Using multi-lobe BRDFs f'_{low} improves the glossy highlights quality but cannot fully capture the change of interreflections. Our full model with f'_{low} and the scaling functions $R_{ir} = S_{ir} T_{ir}$ resembles the input appearance accurately. To demonstrate the effectiveness and generality of our technique, we also show rendering results under different lighting and viewing configurations in Figure 17 and the accompanying video.

Performance and Storage. Table 3 summarizes the storage size and precomputation time of our prefiltered models and Table 4 shows equal-time mean squared error (MSE) comparisons. Our prefiltered models can offer significant storage reduction at a single downsampling scale. Breaking down individual components of our model, the multi-lobe SVNDFs are most storage-consuming (except “noise” and “logo” as they are greatly downsampled) while our scaling functions T and S only introduce minimal overhead.

Our model also benefits rendering performance by reducing ray tracing cost. We demonstrate this by rendering a few high-resolution models and their prefiltered counterparts (generated with our method) in equal-time and compare MSEs of the results (see Table 4). Using our prefiltered models leads to lower errors in all the scenes and anti-aliased renderings with better visual qualities (see Figure 2(c, e)). In the accompanying video, we demonstrate that our prefiltered models preserve input appearance accurately and reduce flickering significantly.

8.3 Limitations and Future Work

Figure 19 shows a failure example where our method does not accurately reproduce the original appearance caused by interreflections. This “statue” scene contains a $(4K)^2$ displacement map and a moderately glossy base BRDF. We downsample it by $(16\times)^2$ and use a spatial scaling function of resolution $M = 1$. Compared to the “relief” scene, the vertical displacements in this scene are much greater. In this case, distant interreflections can only be partially handled by explicit path tracing. Since our method neglects these interreflections (for enabling efficient computation of S_{ir} and T_{ir}), the results have visible accuracy loss shown as darkening at the steep edges (see (b1) and (b2)). On the other hand, although our method produces inaccurate edge appearance, the overall surface reflectance (i.e. average image intensity) still matches the reference because the two effective BRDFs are matched by our scaling functions. For direct illumination results ((a1) and (a2)), the accuracy is not affected.

Since our method considers micro-geometries and base BRDFs jointly, which is necessary for accurately handling complex light transport effects such as shadowing-masking and interreflections, changing the micro-geometry or base BRDF requires recomputing our prefiltered models. In the future, our technique may be

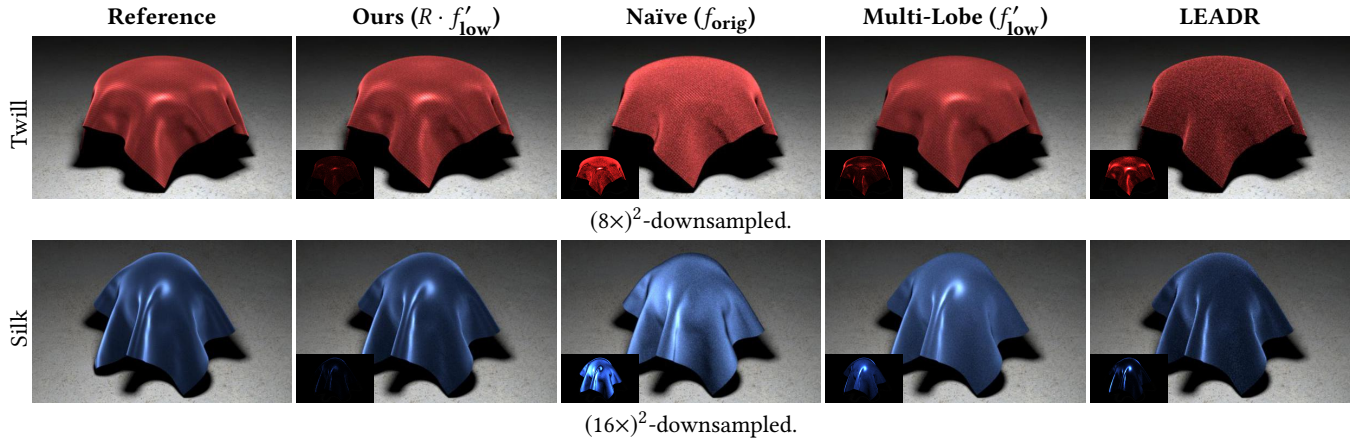


Fig. 16. **Direct illumination results.** Our method accurately matches the reference even with significant downsampling, and is much more accurate than naive downsampling, multi-lobe BRDFs without taking shadowing-masking or interreflections into account, and LEADR. The insets (and those in the next figures 17, 18) show 10× squared errors compared to the reference images.

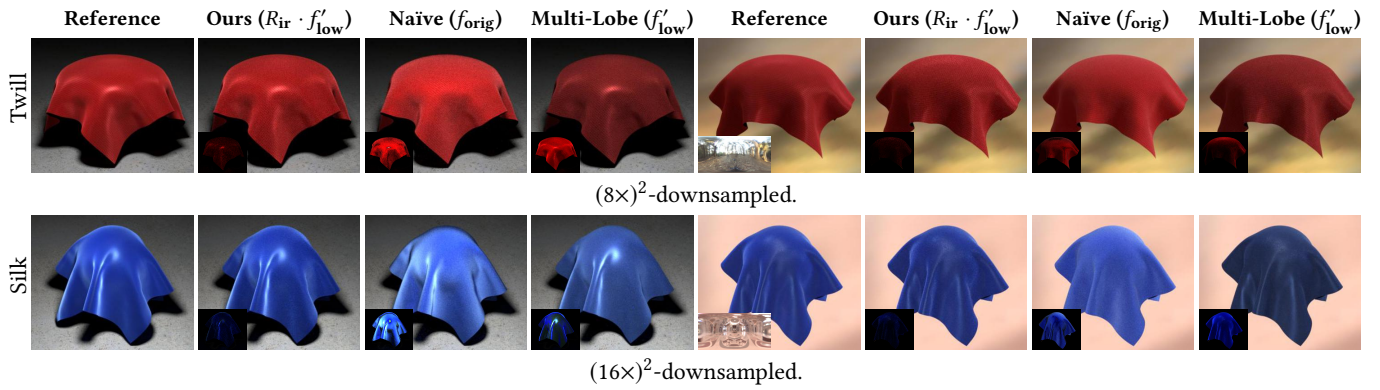


Fig. 17. **Global illumination results.** The four columns on the left are rendered under local area lights, while the four columns on the right are rendered under environmental lightings (environment maps are visualized as insets of the reference images) from a different view. Our method is able to capture interreflections well, closely matching the reference. Naive downsampling and the multi-lobe BRDFs do not model interreflections and have significant errors, while LEADR does not perform well even for direct illumination and is not designed for interreflections (and is therefore not shown here).

Table 3. **Model Information.** We list the detailed parameters used for prefiltering the input model at a single downsampling scale. We also provide the data size of our prefiltered models and their corresponding precomputation time.

Name	Original model				Our prefiltered model						Precomputation time			
	Resolution	Tiling	Total size (KB)	Base BRDF	Downsampling scale	Total size (KB)	h_{low} (KB)	f'_{low} (KB)	T/T_{ir} (KB)	S/S_{ir} (KB)	Total time (min)	h_{low} and f'_{low} (min)	T/T_{ir} (min)	S/S_{ir} (min)
Twill	$(1K)^2$	100^2	3502	glossy	$(8\times)^2$	2377	65	1915	1	396	22	1	7	14
Silk	$(1K)^2$	200^2	2724	glossy	$(16\times)^2$	1085	17	478	1	589	22	1	7	14
Bump	$(1K)^2$	4^2	4015	diffuse	$(16\times)^2$	704	17	484	1	202	24	1	3	20
Noise	$(1K)^2$	100^2	3954	diffuse	$(64\times)^2$	239	2	34	1	202	22	1	4	17
Logo ⁶	$(1K)^2$	$(1K)^2$	723	glossy	$(1K\times)^2$	589	~0	2	1	586	36	1	12	23
Relief	$(4K)^2$	1^2	33114	glossy	$(16\times)^2$	8367	238	7619	0 ($M=1$)	510	26	1	0	25

combined with material editing techniques such as Hašan and Ramamoorthi [2013] to allow changing the base BRDF with a single precomputation.

⁶We provide only parameters of the small-scale displacement map, since the large-scale one remains unchanged when downsampling.

Interreflection is an effect that is well known to be challenging to analyze due to its high nonlinearity. A theoretical or data-driven analysis can be an interesting topic to explore. To balance accuracy and efficiency, we use relatively low resolutions for the spatial and

Table 4. **Equal-time performance.** Our prefiltered models not only reduce the storage size, but also bring benefits in rendering. Given equal time (around 60s for 50-100 spp), rendering using our models results in anti-aliased images, which have lower MSE than those rendered using the original models.

Scene	Twill	Silk	Ball	Logo	Relief
Original model's MSE ($\times 10^{-3}$)	1.095	1.269	2.055	2.072	2.946
Our model's MSE ($\times 10^{-3}$)	0.571	0.622	1.850	0.498	1.692

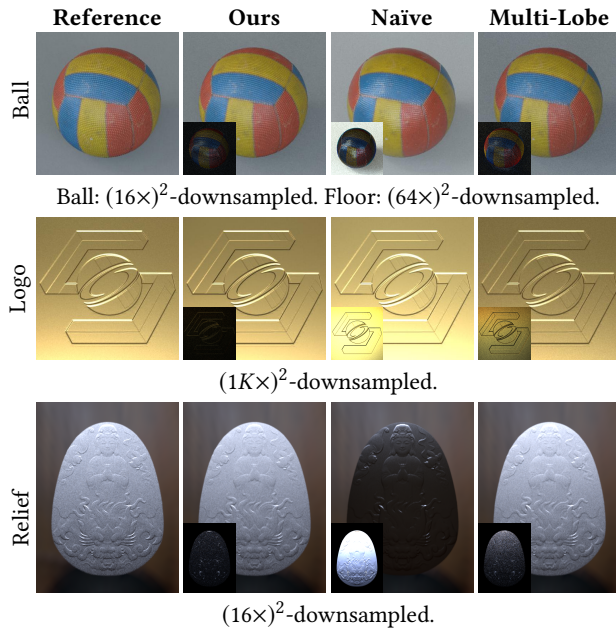


Fig. 18. **Global illumination results.** We show results of more materials. Images rendered using our prefiltered models best match the references.

angular scaling functions empirically. We leave how to determine the optimal resolutions as future work.

We assume the base BRDFs to be isotropic because the anisotropic micro-geometries are capable of reproducing anisotropic appearance. Using anisotropic base BRDFs would increase the model expressiveness but at the price of more expensive precomputation. We leave this extension for future exploration.

Additionally, as our technique only introduces minimal storage overhead, it may benefit real-time and interactive rendering applications.

9 CONCLUSION

We introduce a novel method to prefilter the surface reflectance generated by high-resolution displacement maps while accurately preserving the input appearance. Our prefiltered models leverage SVBRDFs expressed by a SVNDF coupled with a 6D scaling function that captures the change of shadowing-masking and interreflection effects caused by the downsampling of micro-geometries. We further introduce an efficient approach to factorize this 6D function into a 2D function of location and a 4D function of direction. Our models

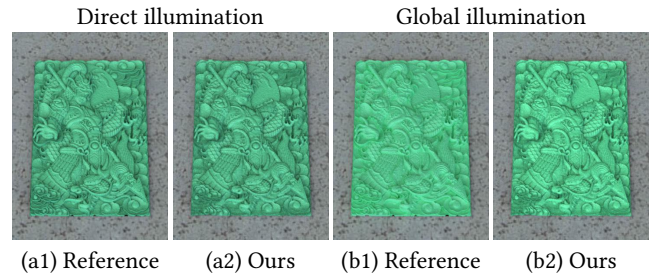


Fig. 19. **Failure case.** Our method accurately handles direct illumination, but is unable to match the appearance with global illumination, since distant interreflections are not handled with sufficient accuracy.

are capable of producing accurate appearance that matches the original one. In practice, our prefiltered models offer significant storage reduction and efficient anti-aliased rendering. Additionally, multiple models generated using our technique with varying downsampling scales can be combined to form a mipmap, allowing level-of-detail rendering of detailed surfaces in an efficient and consistent manner.

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A AVERAGE SLOPE OF A BILINEAR PATCH

Let $h_{00}, h_{10}, h_{01}, h_{11}$ be the height values at the four corners of a unit patch $[0, 1]^2$. Then, the height at any point (u, v) within the patch can be obtained via a bilinear interpolation:

$$h(u, v) = (1 - u)(1 - v)h_{00} + u(1 - v)h_{10} + v(1 - u)h_{01} + uvh_{11}.$$

The slope at each point can be computed as

$$x_s(u, v) = \frac{h(u + \Delta u, v) - h(u, v)}{\Delta u} = h_{10} - h_{00} + v(h_{00} + h_{11} - h_{10} - h_{01}),$$

$$y_s(u, v) = \frac{h(u, v + \Delta v) - h(u, v)}{\Delta v} = h_{01} - h_{00} + u(h_{00} + h_{11} - h_{10} - h_{01}).$$

So the average slope of the bilinear patch is

$$x_{\bar{s}} = \int_0^1 \int_0^1 x_s(u, v) du dv = \frac{1}{2}(h_{11} + h_{10} - h_{01} - h_{00}),$$

$$y_{\bar{s}} = \int_0^1 \int_0^1 y_s(u, v) du dv = \frac{1}{2}(h_{11} + h_{01} - h_{10} - h_{00}).$$

B FACTORIZATION OF R_{ir}

Expanding $T_{\text{ir}}(\mathbf{x})$ as Eqs. (22, 23) and $S_{\text{ir}}(\omega_i, \omega_o)$ as Eq. (24) gives

$$T_{\text{ir}}(\mathbf{x})S_{\text{ir}}(\omega_i, \omega_o) = \frac{\int_{\mathcal{H}^2} \int_{\mathcal{H}^2} R_{\text{ir}}(\mathbf{x}, \omega_i, \omega_o) d\omega_i d\omega_o}{\int_{\mathcal{P}_{\text{all}}} \int_{\mathcal{H}^2} \int_{\mathcal{H}^2} R_{\text{ir}}(\mathbf{x}_m(\mathbf{p}), \omega_i, \omega_o) k_{\mathcal{P}_{\text{all}}}(\mathbf{p}) d\omega_i d\omega_o d\mathbf{p}} \\ \times \int_{\mathcal{P}_{\text{all}}} R_{\text{ir}}(\mathbf{x}_m(\mathbf{p}), \omega_i, \omega_o) k_{\mathcal{P}_{\text{all}}}(\mathbf{p}) d\mathbf{p}$$

As $R_{\text{ir}}(\mathbf{x}, \omega_i, \omega_o) \approx T_{\text{ir}}(\mathbf{x}) \cdot S_{\text{ir}}(\omega_i, \omega_o)$, we can simplify this formula by factoring out R_{ir} and regrouping the terms:

$$\frac{T_{\text{ir}}(\mathbf{x}) \int_{\mathcal{H}^2} \int_{\mathcal{H}^2} S_{\text{ir}}(\omega_i, \omega_o) d\omega_i d\omega_o}{\int_{\mathcal{P}_{\text{all}}} T_{\text{ir}}(\mathbf{x}_m(\mathbf{p})) k_{\mathcal{P}_{\text{all}}}(\mathbf{p}) \left[\int_{\mathcal{H}^2} \int_{\mathcal{H}^2} S_{\text{ir}}(\omega_i, \omega_o) d\omega_i d\omega_o \right] d\mathbf{p}} \\ \times S_{\text{ir}}(\omega_i, \omega_o) \int_{\mathcal{P}_{\text{all}}} T_{\text{ir}}(\mathbf{x}_m(\mathbf{p})) k_{\mathcal{P}_{\text{all}}}(\mathbf{p}) d\mathbf{p} \\ = \frac{T_{\text{ir}}(\mathbf{x}) \int_{\mathcal{H}^2} \int_{\mathcal{H}^2} S_{\text{ir}}(\omega_i, \omega_o) d\omega_i d\omega_o}{\left[\int_{\mathcal{P}_{\text{all}}} T_{\text{ir}}(\mathbf{x}_m(\mathbf{p})) k_{\mathcal{P}_{\text{all}}}(\mathbf{p}) d\mathbf{p} \right] \left[\int_{\mathcal{H}^2} \int_{\mathcal{H}^2} S_{\text{ir}}(\omega_i, \omega_o) d\omega_i d\omega_o \right]} \\ \times S_{\text{ir}}(\omega_i, \omega_o) \int_{\mathcal{P}_{\text{all}}} T_{\text{ir}}(\mathbf{x}_m(\mathbf{p})) k_{\mathcal{P}_{\text{all}}}(\mathbf{p}) d\mathbf{p} \\ = \frac{T_{\text{ir}}(\mathbf{x})}{\int_{\mathcal{P}_{\text{all}}} T_{\text{ir}}(\mathbf{x}_m(\mathbf{p})) k_{\mathcal{P}_{\text{all}}}(\mathbf{p}) d\mathbf{p}} S_{\text{ir}}(\omega_i, \omega_o) \int_{\mathcal{P}_{\text{all}}} T_{\text{ir}}(\mathbf{x}_m(\mathbf{p})) k_{\mathcal{P}_{\text{all}}}(\mathbf{p}) d\mathbf{p} \\ = T_{\text{ir}}(\mathbf{x}) S_{\text{ir}}(\omega_i, \omega_o).$$

Therefore, the rank-1 factorization is consistent with our definition of T_{ir} and S_{ir} .