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Gravitationally-Small Gravitational Antennas, the Chu Limit, and Exploration of Veselago-Inspired Notions of Gravitational Metamaterials

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Abstract – **Recent results provide an analytic expression for the quality factor, or Q, of gravitationally-small gravitational quadrupoles, drawing upon similarities to the Chu limit for electrically-small electromagnetic antennas. Thus, in some sense gravitationally-coupled systems are similar to metamaterials, having high-Q resonant elements separated by much less than a wavelength, and supporting gravitational wave propagation through the resonant elements. These new results raise the question of whether the notions of gravitational Q and gravitationally-small elements may lead to insights for the development of gravitational metamaterials. Here, we review recent gravitational Q results, and explore potential inspiration from electromagnetic metamaterial frameworks where unit cells may be considered as electrically-small antennas. Finally, a gravitoelectromagnetic framework is used to explore a Veselago-inspired approach to hypothetical gravitational metamaterials, should a gravitational unit cell be found. In an astrophysics context, interstellar distances are also shown to be less than a gravitational wavelength for certain planetary orbits.**

I. Introduction

The successful observation of gravitational waves by LIGO in 2015 has led to something of a resurgence of activity in reconsidering gravitational phenomena [1]. Ever since Heaviside proposed a magnetic analog in gravitational fields, electromagnetics have continued to serve as a useful tool for understanding gravitational phenomena [2, 3, 4] Most recently, we have proposed a gravitational analog of the electromagnetic Chu limit [5]. This context of gravitationally-small gravitational radiators brings to mind metamaterial approaches where unit cells were considered as electrically-small antenna elements [6].

In the following, we motivate our exploration of gravitational metamaterials by reviewing recent results in finding an analytic expression for the Q (quality factor) of gravitationally-small (size $\ll \lambda/\pi$) gravitational quadrupole radiators. This gravitationally-small antenna notion is viewed in light of earlier work on electromagnetic metamaterials using small dipoles in unit cells. The subsequent section draws upon gravitational analogs of Maxwell equations to construct a Veselago-inspired proposition for gravitational metamaterials. Recent work in electric quadrupole polarization and quadrupole coupling are noted, along with considerations of sub-wavelength coupling between planetary orbits at interstellar distances.

II. Gravitationally-Small Antenna Metamaterial as a Unit Cell

Before discussing gravitationally-small quadrupole elements as potential gravitational-metamaterial unit cells, we review earlier metamaterials using dipole unit cells. Tretyakov [6] considered electromagnetic metamaterials, comprised of arrays of loaded electrically-small dipole unit cells. In this earlier work, each unit cell was comprised of an electrically-small dipole with appropriate circuit loading to generate the desired polarizability of the metamaterial unit cell "atom." The loading of the dipoles usually consist of inductance or capacitance that can shift resonant frequencies of the unit cells. In addition, Clausius-Mossotti relations were used to compute desired dipole loading from desired effective parameters of the metamaterial array.

These earlier electromagnetic metamaterials comprised of small dipoles, combined with recent gravitationally-small radiator theory, inspire our consideration of the potential for similar gravitational metamaterials. If the array of small electric dipoles forms an electromagnetic metamaterial, it may invite the analogous conjecture that some sort of array of gravitationally-small antennas could lead toward development of a gravitational metamaterial. This is in part supported by the close parallels between Maxwell's equations and gravitoelectromagnetic equations (GEM) discussed below, and partly by recent results showing gravitationally-small antenna properties of gravitational quadrupoles.

As a candidate gravitational unit cell, we have recently given the analytic expression for the Q (quality factor) of a gravitationally-small quadrupole radiator, similar to the Chu limit for electrically-small antennas [5]. The quadrupole is the lowest order radiating gravitational multipole, since there is no negative mass, whereas dipoles are the lowest order electromagnetic radiators. In this, the Q of a gravitationally-small quadrupole of two circular-orbit masses was shown to be

$$Q = \frac{20m_1^7 G}{m_2 c^2 (m_1+m_2)^5}\left(\frac{(k\,a_s)^{-7}}{2a_{min}} - \frac{(k\,a_s)^{-5}c^2(m_1+m_2)^2}{8m_1^3 G}\right), \tag{1}$$

where the masses are m_1 and m_2 in kg, a_s is orbital radius in meters, gravitational constant $G = 6.7\times10^{-11}$ N·(m/kg)^2, $c = 3\times10^8$, and a_{min} is the final radius of the orbit around the barycenter at coalescence. From prior theoretical results in [5], a_{min} at coalescence can be found from the gravitational-wave frequency at coalescence.

III. Hypothetical Vesalago-Inspired Gravitational Metamaterials

In light of the gravitationally-small behavior of quadrupoles in Weldon [5] and the small electric-dipole metamaterials described by Tretyakov [6], it seems appropriate to consider hypothetical gravitational metamaterials, much along the lines of the seminal paper by Veselago [7]. Fortunately, the gravitoelectromagnetic (GEM) approximation to general relativity provides a gravitational framework for the analysis that closely resembles the Maxwell equations for electromagnetics [4, 8] . To begin, we first note that similarities between electromagnetic and gravitational forces and fields led Heaviside to propose a gravitational magnetic field analog, as early as 1893 [2]:

$$\mathbf{F} = \frac{q_1 q_2}{4\pi\varepsilon r^2}\hat{\mathbf{r}} \qquad \Rightarrow \qquad \mathbf{F} = \frac{-Gm_1m_2}{r^2}\hat{\mathbf{r}} \tag{2}$$

$$\mathbf{E} = \frac{q_1}{4\pi\varepsilon r^2}\hat{\mathbf{r}} \qquad \Rightarrow \qquad \mathbf{g} = \mathbf{E}_g = \frac{-Gm_1}{r^2}\hat{\mathbf{r}}\ , \tag{3}$$

where the left-hand equations for electric field and forces are clearly similar to the right-hand equations for gravitational fields and forces. More recently, GEM field approximations to general relativity have been used in the study of gravitational phenomena [3, 4]. In GEM, gravitational analogs of the electric field and magnetic fields result in [4].

$$\nabla\cdot\mathbf{E} = \frac{\rho_e}{\varepsilon} \qquad \Rightarrow \qquad \nabla\cdot\mathbf{E}_g = 4\pi G\rho_m = \frac{\rho_m}{\varepsilon_g} \tag{4}$$

$$\nabla\cdot\mathbf{B} = 0 \qquad \Rightarrow \qquad \nabla\cdot\mathbf{B}_g/2 = 0 \tag{5}$$

$$\nabla\times\mathbf{B} = \mu\mathbf{J}_e + \varepsilon\mu\frac{\partial\mathbf{E}}{\partial t} \qquad \Rightarrow \qquad \nabla\times\mathbf{B}_g/2 = \frac{4\pi G}{c}\mathbf{J}_g + \frac{1}{c}\frac{\partial\mathbf{E}_g}{\partial t} = \mu_g\mathbf{J}_g + \frac{1}{c}\frac{\partial\mathbf{E}_g}{\partial t}; \quad \mathbf{J}_g = \rho_m v, \tag{6}$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \qquad \Rightarrow \qquad \nabla\times\mathbf{E}_g = -\frac{1}{c}\frac{\partial\mathbf{B}_g/2}{\partial t} \tag{7}$$

$$\nabla^2\mathbf{E} = \mu\varepsilon\frac{\partial^2\mathbf{E}}{\partial t^2} \qquad \Rightarrow \qquad \nabla^2\mathbf{E}_g = \frac{1}{c^2}\frac{\partial^2\mathbf{E}_g}{\partial t^2}\ , \tag{8}$$

where by comparison the left-hand equations are for electromagnetic fields and the right-hand equations are for gravitational fields (where we retain the "$\mathbf{B}_g/2$" terms in the equations above, following the form of equations in [4]). Thus the GEM field equations are quite similar to Maxwell's equations.

Taking inspiration from Veselago [7], if we may hypothesize the possibility of gravitational relative permittivity ε_g and/or gravitational relative permeability μ_g (perhaps through the resonance of quadrupoles), we may suppose

$\mathbf{B}_g = \mu_g \mathbf{H}_g$ and $\mathbf{D}_g = \varepsilon_g \mathbf{E}_g$. Then, along the lines of Veselago [7], we may hypothesize the existence of gravitational equivalents of double-negative metamaterials ($\mu_g < 0$ and $\varepsilon_g < 0$), single-negative metamaterials ($\mu_g < 0$ or $\varepsilon_g < 0$), and the full range of possible gravitational candidates. At present, we are also looking into gravitational quadrupole interaction with gravitational waves along the lines of electric quadrupole polarization in [9], and the coupling of gravitational waves to quadrupole modes of mechanical resonators in [10], where quadrupole modes couple to the spin-2 gravitational waves [10]. Just as electromagnetic resonators in metamaterials can provide positive and negative effective μ and ε above and below resonance, we are investigating the potential for generating positive and negative effective μ_g and ε_g using gravitational and mechanical quadrupoles.

Lastly, we note possible applications of metamaterial notions to astrophysics, where the separation between gravitational quadrupoles of planetary orbits between two different solar systems is gravitationally small. For example, the nearest star system to our solar system is Proxima Centauri at a distance of 1.3 pc, but is well within the distance of one wavelength corresponding to the orbital period of Jupiter with 12 year orbit and $\lambda = 3.7$ pc, or even more within the wavelength distance corresponding to the orbital period of Neptune with 165 year orbit and $\lambda = 50$ pc. Surprisingly, interstellar distances can be much less than a wavelength of the gravitational quadrupoles of planetary orbits, corresponding well with conventional metamaterial constraints of unit cell spacing being much less than a wavelength. The 250 million year orbital period of the sun within the milky way would also suggest that the 140 closest galaxies within 4 Mpc are well within one wavelength distance.

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